

Boost-invariant Hamiltonian approach to heavy quarkonia

Stanisław D. Głazek

Institute of Theoretical Physics, University of Warsaw

Method: Weak-coupling expansion for QCD in the Fock space

Parameters: $\alpha_Z, m_Z \longrightarrow \alpha_\lambda, m_\lambda \longrightarrow$ masses of $c\bar{c}$ or $b\bar{b}$

Result: Relativistic spectrum and states close to data
in a renormalized canonical Hamiltonian calculus

New approach to QCD: calculations do not involve scattering states for quarks or gluons, Feynman diagrams, path integrals, euclidicity postulate, lattice, or vacuum expectation values

Key:

Similarity Renormalization Group Procedure for Hamiltonians

Developed since S. D. Głazek and K. G. Wilson, PRD48, 5863 (1993); PRD49, 4214 (1994).

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu}^a$$

$$x^\pm = x^0 \pm x^3, \quad t \rightarrow x^+, \quad z \rightarrow x^-, \quad x^\perp$$

$$A^+ = 0$$

$$H_{can} = H_{\psi^2} + H_{A^2} + H_{A^3} + H_{A^4} + H_{\psi A \psi}$$

$$+ H_{\psi A A \psi} + H_{[\partial A A]^2} + H_{[\partial A A](\psi \psi)} + H_{(\psi \psi)^2}$$

$$\mathcal{H}_{\psi^2} = \frac{1}{2} \bar{\psi} \gamma^+ \frac{-\partial^{\perp 2} + m^2}{i\partial^+} \psi,$$

$$\mathcal{H}_{A^2} = -\frac{1}{2} A^\perp (\partial^\perp)^2 A^\perp,$$

$$\mathcal{H}_{\psi A \psi} = g \bar{\psi} \not{A} \psi,$$

$$\mathcal{H}_{(\psi\psi)^2} = \frac{1}{2} g^2 \bar{\psi} \gamma^+ t^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ t^a \psi.$$

At $x^+ = 0$:

$$\psi = \sum_{\sigma c} \int [k] \left[\chi_c u_{k\sigma} b_{k\sigma c} e^{-ikx} + \chi_c v_{k\sigma} d_{k\sigma c}^\dagger e^{ikx} \right],$$

$$A^\mu = \sum_{\sigma c} \int [k] \left[t^c \varepsilon_{k\sigma}^\mu a_{k\sigma c} e^{-ikx} + t^c \varepsilon_{k\sigma}^{\mu*} a_{k\sigma c}^\dagger e^{ikx} \right].$$

Regularization (boost invariant, 7 kinematical symmetries of the LF)

Counterterms

$$H = [H_{can} + H_{CT}]_{reg}$$

RGPEP, no small energy denominators in PT !

$$\begin{aligned} q_\lambda &= U_\lambda q_{can} U_\lambda^\dagger & q_\infty &= q_{can} \\ \frac{d}{d\lambda} H_\lambda &= [\mathcal{T}_\lambda, H_\lambda] & \mathcal{T}_\lambda &= U'_\lambda U_\lambda^\dagger \\ H_\infty &= [H_{can} + H_{CT}]_{reg} & \rightarrow & H_\lambda = H_\infty + \int_\infty^\lambda ds [\mathcal{T}_s, H_s] \end{aligned}$$

$$H_\lambda = f_\lambda G_\lambda$$

$$E.g. : \quad G_\lambda = \int [123] G_\lambda(1, 2, 3) a_{\lambda 1}^\dagger b_{\lambda 2}^\dagger b_{\lambda 3}$$

$$f_\lambda G_\lambda = \int [123] f_\lambda(123) G_\lambda(1, 2, 3) a_{\lambda 1}^\dagger b_{\lambda 2}^\dagger a_{\lambda 3}$$

$$f_\lambda(123) = \exp \left[-(\mathcal{M}_{12}^2 - \mathcal{M}_3^2)^2 / \lambda^4 \right]$$

$$H|P\rangle = M^2|P\rangle \quad \rightarrow \quad H_\lambda|P\rangle = M^2|P\rangle$$

$$|P\rangle = |Q_\lambda \bar{Q}_\lambda\rangle + |Q_\lambda \bar{Q}_\lambda g_\lambda\rangle + \dots$$

$$[H_\lambda] = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & T_3 + V_3 & Y \\ \cdot & Y^\dagger & T_2 + V_2 \end{bmatrix} \rightarrow \begin{bmatrix} T_3 + \mu^2 & Y \\ Y^\dagger & T_2 + V_2 \end{bmatrix}$$

$$V_3 = \mu^2 + \left(V_3 - \frac{\alpha}{\alpha_s} \mu^2 \right) \quad \rightarrow \quad \mu^2 \quad m \gg \Lambda_{QCD}$$

$$[H_\lambda] = \begin{bmatrix} T_3 + \mu^2 & Y \\ Y^\dagger & T_2 + V_2 \end{bmatrix}$$

$$H_{Q\bar{Q}\lambda} = T_{2\lambda} + V_{2\lambda} + Y_\lambda^\dagger \frac{1}{T_3 + \mu^2} Y_\lambda$$

$$H_\lambda |P\rangle = M^2 |P\rangle \rightarrow H_{Q\bar{Q}\lambda} |P\rangle = M^2 |P\rangle$$

$f_\lambda f_\lambda \frac{4m^2}{q_z^2} \frac{\mu^2}{q^2 + \mu^2} \longrightarrow \text{harmonic force in } H_{Q\bar{Q}\lambda}.$

$$|M, P^+, P^\perp\rangle = \int [ij] \tilde{\delta} P^+ \frac{\bar{u}_i \Psi_{ij} v_j}{-4m^2} |ij\rangle, \quad |ij\rangle = b_{\lambda_i}^\dagger d_{\lambda_j}^\dagger |0\rangle$$

$$\frac{\bar{u}_i \Psi_{ij} v_j}{-4m^2} \rightarrow \chi_i^\dagger \phi(\vec{k}_{ij}) \chi_j$$

$$[\vec{p}^2 - k_p \Delta_p] \phi(\vec{p}) - 2 \int \frac{d^3 k}{(2\pi)^3} \mathcal{V} \phi(\vec{k}) = x \phi(\vec{p})$$

$$k_p = \frac{9}{128 \sqrt{2\pi}} \left(\frac{\lambda^2}{\alpha m^2} \right)^3, \quad \mathcal{V} = f \frac{4\pi}{(\vec{p} - \vec{k})^2} (1 + BF)$$

$$f = \exp \left\{ - \left[\frac{\mathcal{M}^2(p) - \mathcal{M}^2(k)}{\lambda^2} \right]^2 \right\}, \quad M = 2m \sqrt{1 + x \left(\frac{2}{3} \alpha \right)^2}$$

Example of J/ψ or Υ : $\phi(\vec{k}) = [a(\vec{k}) + \vec{b}(\vec{k})\vec{\sigma}]$

$$a(\vec{k}) = 0, \quad b^m(\vec{k}) = \left[\delta^{mn} \frac{S(k)}{k} + \frac{1}{\sqrt{2}} \left(\delta^{mn} - 3 \frac{k^m k^n}{k^2} \right) \frac{D(k)}{k} \right] s^n$$

$$\begin{bmatrix} h_{osc} & 0 \\ 0, & h_{osc} + k_p \frac{6}{p^2} \end{bmatrix} \begin{bmatrix} S(p) \\ D(p) \end{bmatrix} = \frac{2}{\pi} \int_0^\infty dk f pk \begin{bmatrix} \mathcal{W}_{ss}, & \mathcal{W}_{sd} \\ \mathcal{W}_{ds}, & \mathcal{W}_{dd} \end{bmatrix} \begin{bmatrix} S(k) \\ D(k) \end{bmatrix}$$

$$h_{osc} = p^2 - k_p \partial_p^2 - x \qquad \mathcal{W}_{ss} = J_0 + \frac{\alpha^2}{3} [(p^2 + k^2) J_0 - 16/9]$$

$$\mathcal{W}_{sd} = \frac{\alpha^2}{3} [p^2 (J_2 - J_0) + 4/3] \frac{\sqrt{2}}{3}, \quad \mathcal{W}_{ds} = \frac{\alpha^2}{3} [k^2 (J_2 - J_0) + 4/3] \frac{\sqrt{2}}{3}$$

$$\mathcal{W}_{dd} = J_2 + (J_2 - J_0)/2 + \frac{\alpha^2}{3} \{ (p^2 + k^2) [J_0 - (J_2 - J_0)/6] - 20/9 \}$$

δ -functions with f_λ

J_0, J_2 are known functions

Hamiltonian RG 2nd-order solution for H_λ (not S -matrix calculus):

$$\alpha_{\lambda_0} = \frac{\alpha_{M_Z}}{1 + [\alpha_{M_Z}/(6\pi)] (11N_C - 2n_f) \ln(\lambda_0/M_Z)}.$$

$$\alpha_{M_Z} \sim 0.12 \quad \rightarrow \quad \alpha_0 \sim 0.33 \quad \text{at} \quad \lambda_0 \sim 3.7 \text{ GeV}$$

TABLE I: $\alpha = 0.32595$, $m = 4856.92$ MeV ($\lambda = 3697.67$ MeV, $\omega = 182.16$ MeV, $k_p = 0.157722$)

meson	theory	experiment	difference
$\Upsilon 10865$	10725	10865	-140
$\Upsilon 10580$	10464	10580	-116
$\Upsilon 3S$	10382	10355	27
$\chi_{22}P$	10276	10269	7
$\chi_{12}P$	10256	10256	0
$\chi_{02}P$	10226	10232	-6
$\Upsilon 2S$	10012	10023	-11
$\chi_{21}P$	9912	9912	-1
$\chi_{11}P$	9893	9893	0
$\chi_{01}P$	9865	9859	5
$\Upsilon 1S$	9551	9460	91
$\eta_b 1S$	9510	9300	210

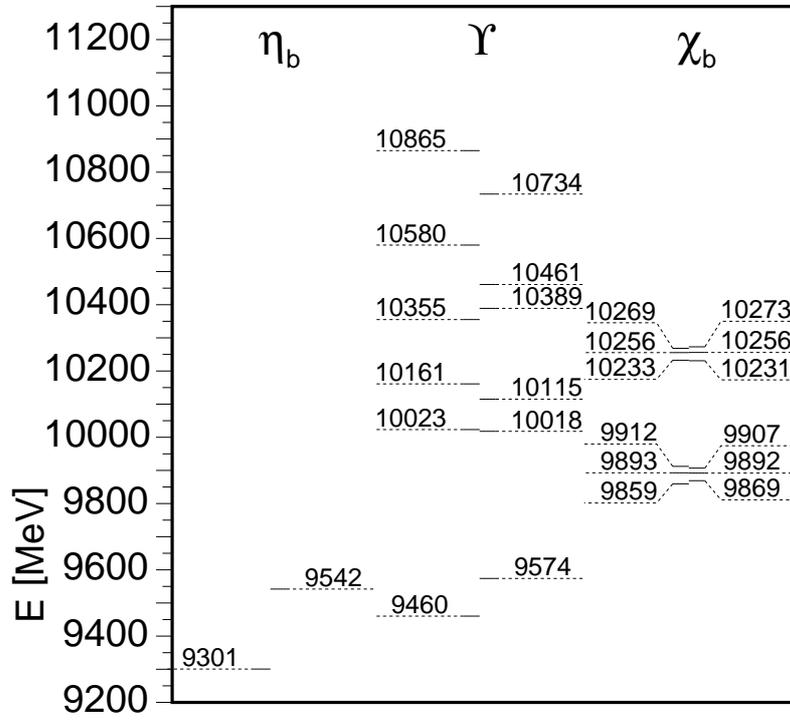


FIG. 1: 7 $b\bar{b}$ middle states: $\alpha = 0.28839$, $m = 4835.9$ MeV ($\lambda = 3779.8$ MeV, $\omega = 184.62$ MeV).

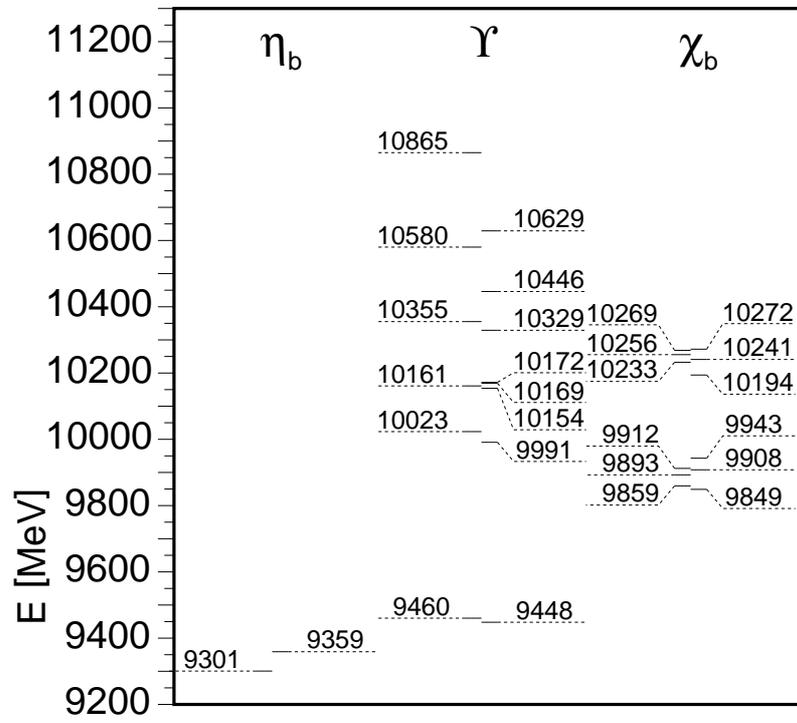


FIG. 2: 12 $b\bar{b}$ states: $\alpha = 0.50738$, $m = 4979.7$ MeV ($\lambda = 3252.3$ MeV, $\omega = 147.11$ MeV).

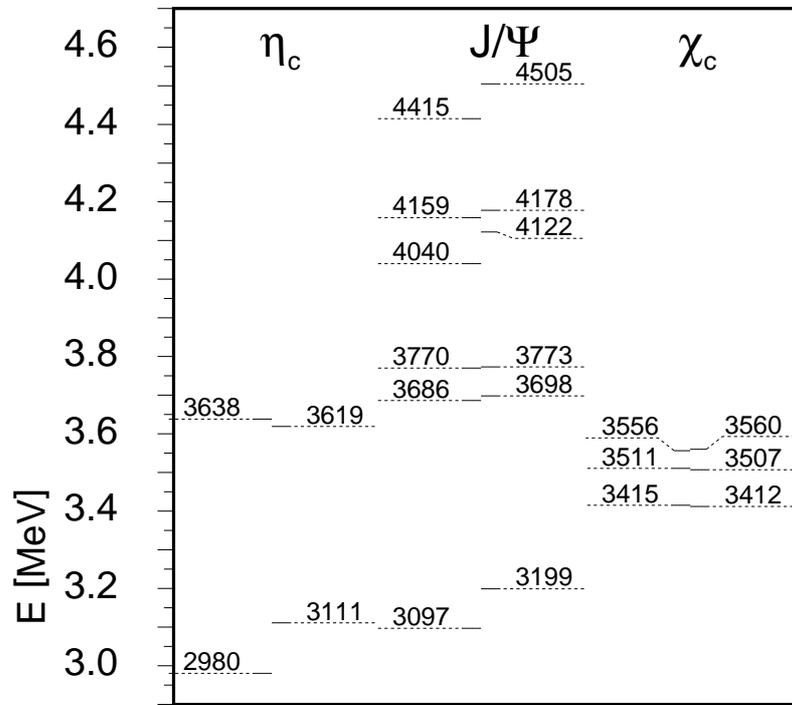


FIG. 3: 3 middle $c\bar{c}$ states: $\alpha = 0.34335$, $m = 1553.3$ MeV ($\lambda = 1990.0$ MeV, $\omega = 284.93$ MeV).

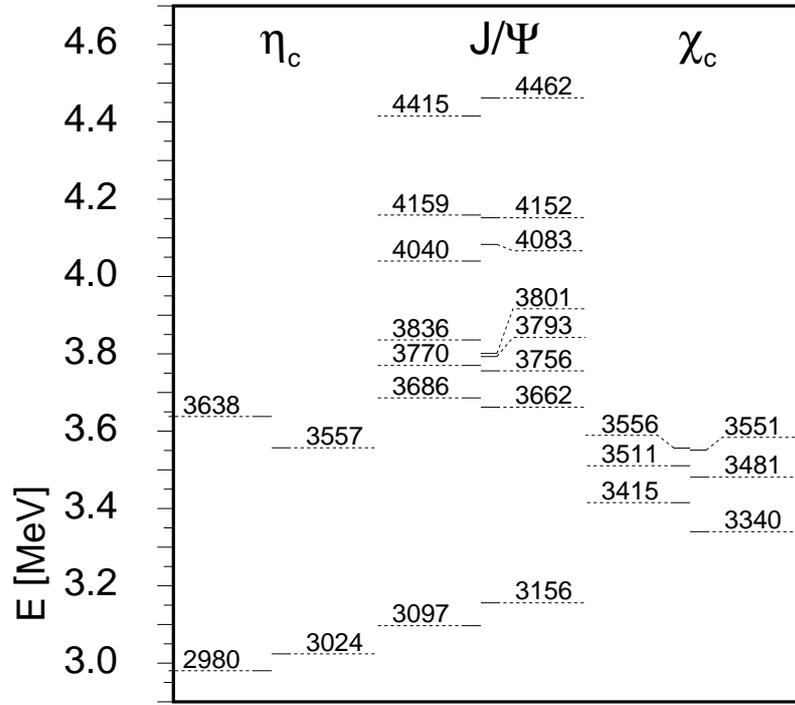
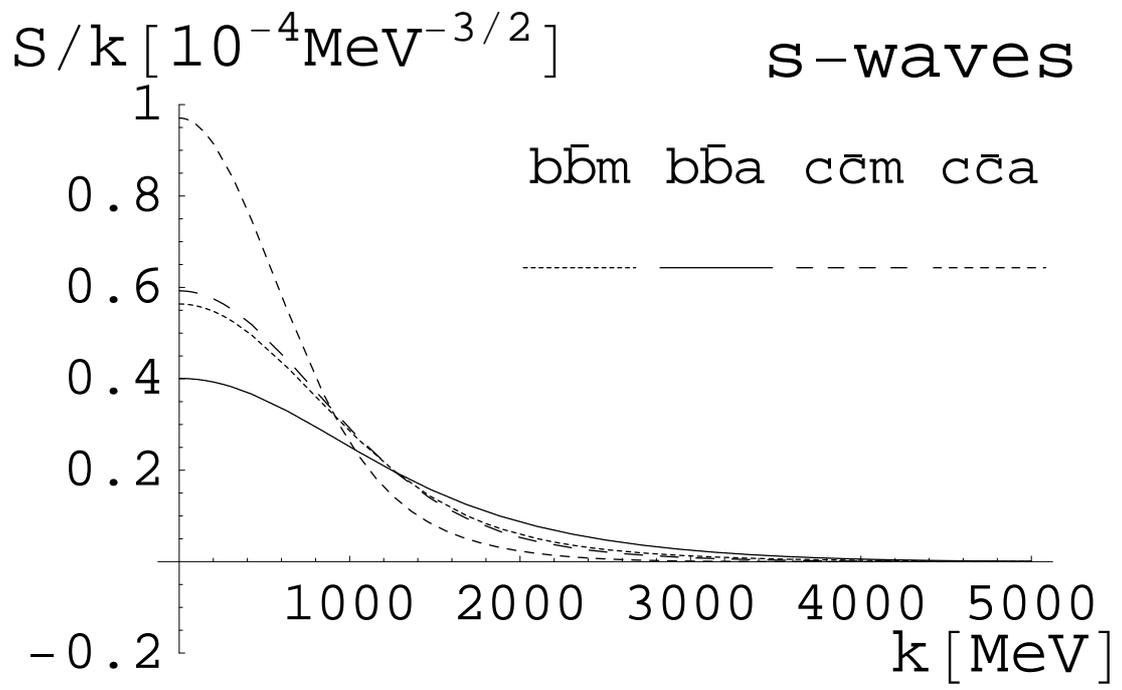
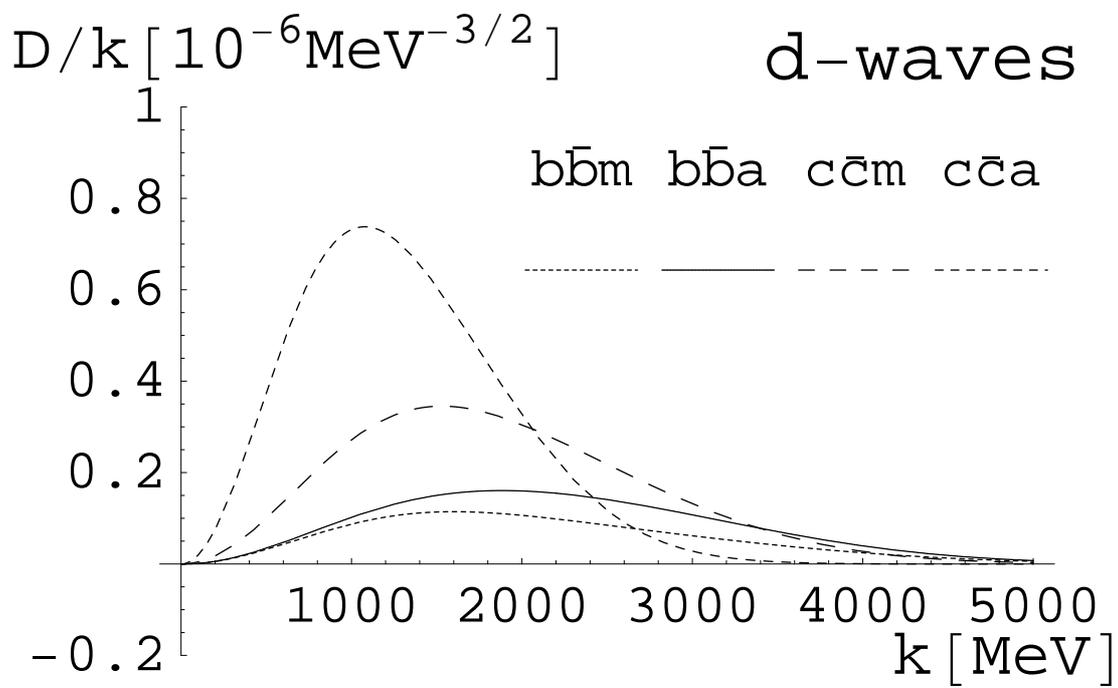


FIG. 4: 11 $c\bar{c}$ states: $\alpha = 0.41443$, $m = 1577.4$ MeV ($\lambda = 1934.2$ MeV, $\omega = 278.72$ MeV).





Summary:

- There exists an approximate constituent picture for heavy quarkonia in 1 flavor QCD: relativistic, simple, usable for fast mesons
- Fits meson masses reasonably well for reasonable α and m
(evolved in one and the same Hamiltonian approach from M_Z)
- Provides specific boost-invariant wave functions in the LF Fock space
→ unequal masses, decays, production, scattering, exclusive processes
- Systematically improvable picture in RGPEP ($\alpha \sim 1/3$)
- Relativistic theory of binding and mass

Concluding quotes:

P. A. M. Dirac (1977):

”Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out.”

K. G. Wilson (2004):

”...the time is ripe for a few accomplished theorists to switch into light-front theory and help build a growing research effort in this area.”

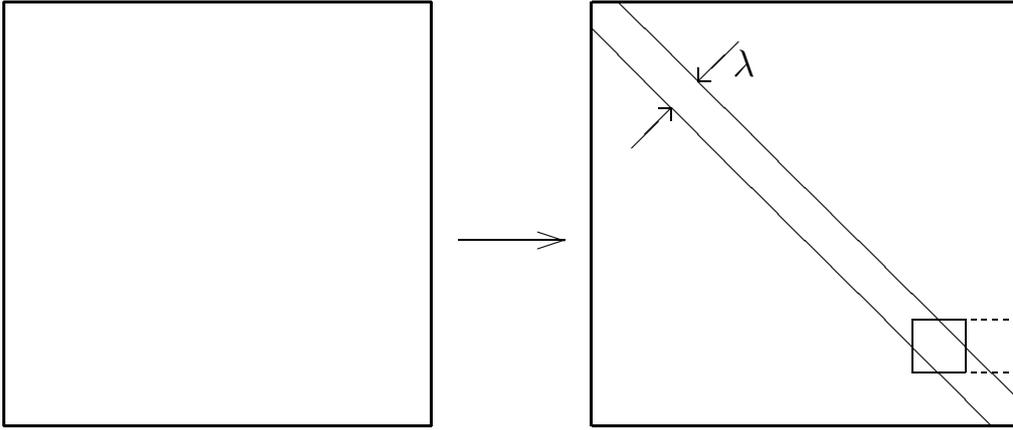
$$a_{\lambda p}^\dagger = a_{can p}^\dagger + g_\lambda a_{1p}^\dagger + g_\lambda^2 a_{2p}^\dagger + \dots$$

$$a_{1p}^\dagger = \begin{array}{c} 1 \\ \diagdown \\ \text{---} \\ \diagup \\ 2 \end{array} \text{---} p + \begin{array}{c} 1 \\ \diagup \\ \text{---} \\ \diagdown \\ 2 \end{array} \text{---} p$$

$$a_{2p}^\dagger = \begin{array}{c} \text{---} \\ \diagdown \\ \text{---} \\ \diagup \\ \text{---} \end{array} p + \begin{array}{c} 1 \\ \diagdown \\ \text{---} \\ \diagup \\ 3 \end{array} \text{---} p + \begin{array}{c} 1 \\ \diagdown \\ \text{---} \\ \diagup \\ 3 \end{array} \begin{array}{c} 1 \\ \diagdown \\ \text{---} \\ \diagup \\ 2 \end{array} p + \begin{array}{c} 1 \\ \diagup \\ \text{---} \\ \diagdown \\ 3 \end{array} \begin{array}{c} 1 \\ \diagdown \\ \text{---} \\ \diagup \\ 2 \end{array} p$$

$$+ \begin{array}{c} 1 \text{---} 3 \\ \diagdown \\ \text{---} \\ \diagup \\ 2 \text{---} p \end{array} + \begin{array}{c} 1 \text{---} p \\ \diagdown \\ \text{---} \\ \diagup \\ 2 \text{---} 3 \end{array} + \begin{array}{c} 1 \\ \diagdown \\ \text{---} \\ \diagup \\ 2 \end{array} \text{---} \text{---} \begin{array}{c} 1 \\ \diagdown \\ \text{---} \\ \diagup \\ 3 \end{array} \text{---} p$$

Poincaré algebra: SDG, T. Masłowski, PRD65, 065011 (2002)



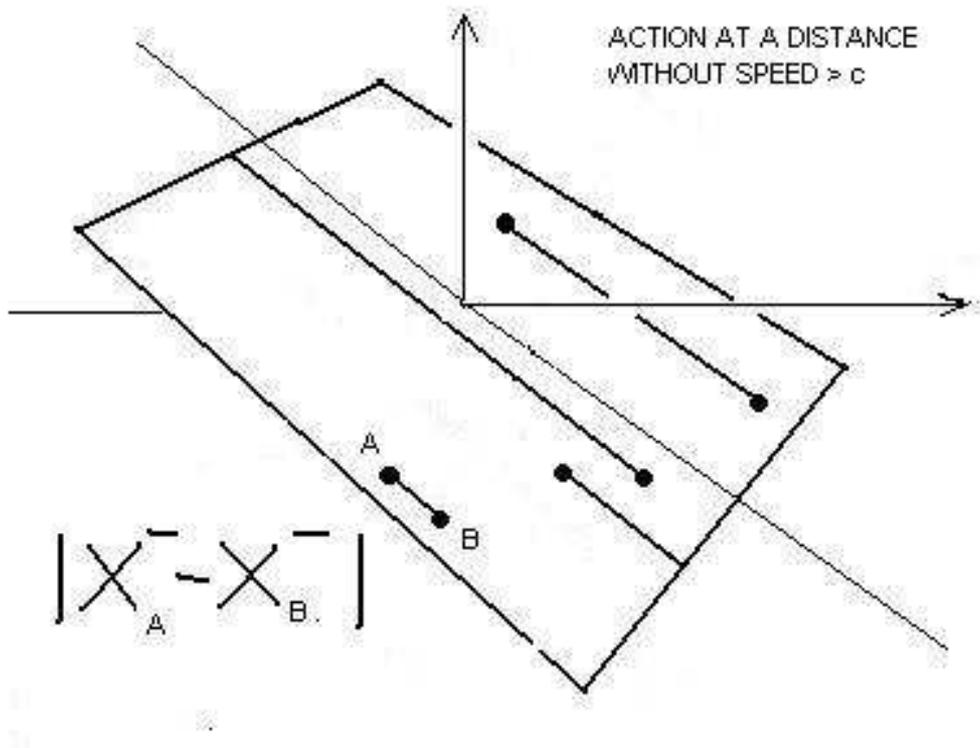
RGPEP has roots in “similarity” RG procedure:

SDG and K. G. Wilson, PRD, '93, '94, '98.

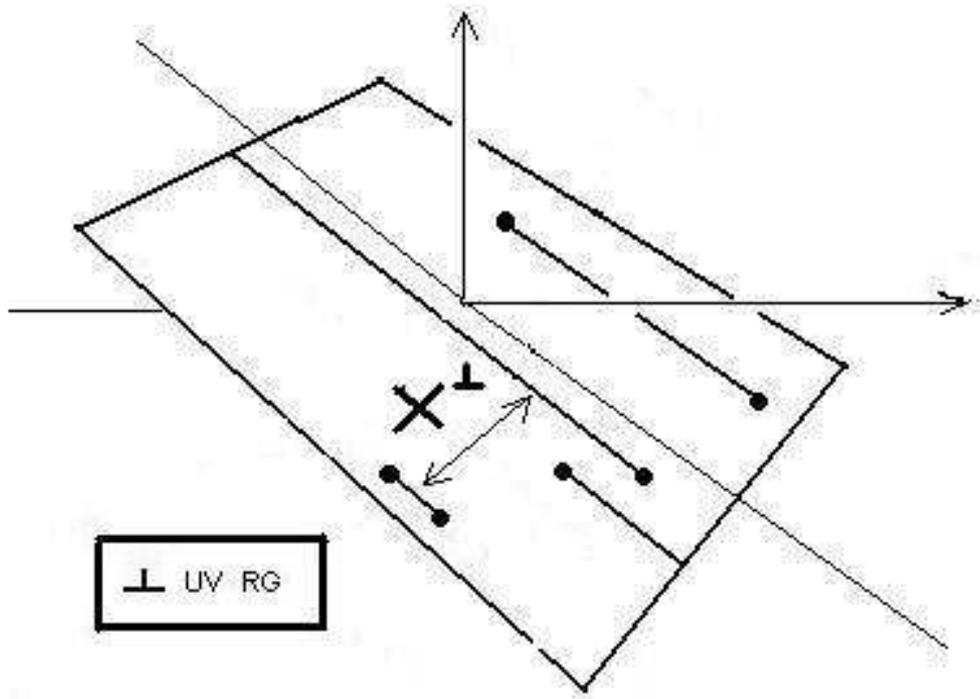
No small denominators in perturbation theory for H_λ !

RGPEP: width λ in Hamiltonian matrices \rightarrow form factors f_λ .

Dirac's 7 kinematical symmetries + relativistic action at a distance



RGPEP: SDG, Acta Phys. Polon. B29, 1979 (1998)



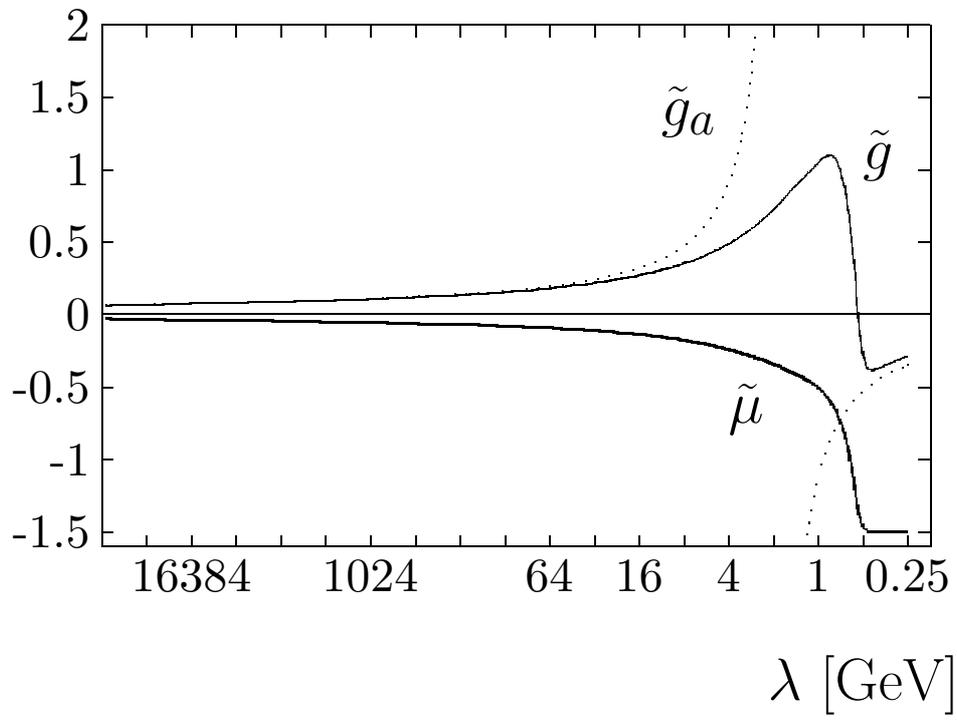
$$H|P\rangle = M^2|P\rangle \quad \rightarrow \quad H_\lambda|P\rangle = M^2|P\rangle,$$

$$|P\rangle = |Q_\lambda \bar{Q}_\lambda\rangle + |Q_\lambda \bar{Q}_\lambda g_\lambda\rangle + \dots$$

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & T_4 + V_4 & Y_1 & Y_2 \\ \cdot & Y_1^\dagger & T_3 + V_3 & Y \\ \cdot & Y_2^\dagger & Y^\dagger & T_2 + V_2 \end{bmatrix} \longrightarrow \begin{bmatrix} T_4 + \mu^2 & Y_1 & Y_2 \\ Y_1^\dagger & T_3 + V_3 & Y \\ Y_2^\dagger & Y^\dagger & T_2 + V_2 \end{bmatrix}.$$

TABLE II: $\alpha = 0.32595$, $m = 4856.92$ MeV, for $\lambda = 3697.67$ MeV ($\omega = 182.16$ MeV, $k_p = 0.157722$)

meson	theory	experiment	difference	precise
$\Upsilon 10865$	10725	10865	-140	10729.7
$\Upsilon 10580$	10464	10580	-116	10466.9
$\Upsilon 3S$	10382	10355	27	10385.2
$\chi_{22}P$	10276	10269	7	10278.5
$\chi_{12}P$	10256	10256	0	10258.0
$\chi_{02}P$	10226	10232	-6	10228.1
$\Upsilon 2S$	10012	10023	-11	10013.8
$\chi_{21}P$	9912	9912	-1	9913.3
$\chi_{11}P$	9893	9893	0	9894.2
$\chi_{01}P$	9865	9859	5	9865.5
$\Upsilon 1S$	9551	9460	91	9551.8
$\eta_b 1S$	9510	9300	210	9510.8



$\tilde{g}_a(\lambda)$ - PT, $\tilde{g}(\lambda)$ - exact, $\tilde{\mu}(\lambda)$ - bound state matrix element.