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# *$\eta$ – $\eta'$ Mixing-From Electromagnetic Transitions to Weak Decays of Charm and Beauty Hadrons*

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# Outline

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- Review of present scenario on  $\eta$ – $\eta'$  mixing
- $\eta$ – $\eta'$  in semileptonic decays of  $D^+$ ,  $D_s^+$  and  $B^+$
- $\eta$ – $\eta'$  in non leptonic B transitions and their CP asymmetries  $\rightarrow$  New physics beyond the Standard Model is advocated to explain the unexpectedly large  $B \rightarrow \eta K$  branching fraction: the role of the meson's wave functions

## $\eta$ - $\eta'$ Mixing

The question of  $\eta$ - $\eta'$  mixing angle has been the subject of discussion almost from the time that SU(3)-flavor symmetry was proposed.

Mixing in SU(3):

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_P & -\sin \theta_P \\ \sin \theta_P & \cos \theta_P \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

Only octet and singlet contribution: quadratic GMO-mass formula  $\theta_P \approx -10^\circ$ ; linear GMO-mass formula  $\theta_P \approx -23^\circ$

Mixing also in quark-flavor basis:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \varphi_P & -\sin \varphi_P \\ \sin \varphi_P & \cos \varphi_P \end{pmatrix} \begin{pmatrix} \eta_{NS} \\ \eta_S \end{pmatrix}$$

with: 
$$\begin{cases} \eta_{NS} = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \\ \eta_S = s\bar{s} \end{cases}$$

$$\theta_P = \phi_P - \arctan \sqrt{2}$$

# Two Mixing Angles Scenario

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- 2000: Leutwyler and Kraiser as well as Kroll, Stech and Feldmann have shown the mixing cannot be adequately described by a single angle; the fact that the decay constants follow the pattern of state mixing is an a-priori assumption

- Due to SU(3) breaking, mixing of decay constants does not follow the same pattern of state mixing

$$\begin{pmatrix} f_{\eta}^8 & f_{\eta}^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8 \cos \theta_8 & -f_0 \sin \theta_8 \\ f_8 \sin \theta_8 & f_0 \cos \theta_8 \end{pmatrix}$$

- In the quark-flavor basis the difference between the two mixing angles is determined by an OZI-rule violating contribution:  $\phi_q$  and  $\phi_s$  nearly coincide

$$\begin{pmatrix} f_{\eta}^q & f_{\eta}^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos \phi_q & -f_s \sin \phi_q \\ f_q \sin \phi_q & f_s \cos \phi_q \end{pmatrix}$$

# Mix with Gluonium

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- The  $\eta'$  meson is a good candidate to have a sizeable gluonic content, (while the  $\eta$  meson is well understood as an SU(3)-flavor octet with a small singlet admixture); we allow a mixing with gluonium

$$\begin{cases} \eta' = X_{\eta'} \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle + Y_{\eta'} |s\bar{s}\rangle + Z_{\eta'} |glue\rangle \\ \eta = X_{\eta} \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle + Y_{\eta} |s\bar{s}\rangle \end{cases}$$
$$\begin{cases} X_{\eta'} = \cos \phi_G \sin \phi_P; Y_{\eta'} = \cos \phi_G \cos \phi_P; Z_{\eta'} = \sin \phi_G \\ X_{\eta} = \cos \phi_P; Y_{\eta} = -\sin \phi_P \end{cases}$$

- Several phenomenological analysis evaluated possible gluonic component, the different determinations are generally consistent but show more model and mode dependence.

# *Radiative $\phi$ and $\psi$ Decays*

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At the beginning of 2007 several papers on the gluonium content of the  $\eta'$  from the analysis of radiative  $\phi$  and  $\psi$  decays has been published:

- 1.KLOE Coll., F. Ambrosino *et al.*, Phys. Lett. B648, 267 (2007)
- 2.G. Li, Q. Zhao and C.-H. Chang, (2007) hep-ph/0701020
- 3.R. Escribano and J. Nadal, JHEP 05, 006 (2007), hep-ph/0703187
- 4.C. E. Thomas, (2007) hep-ph/0705.1500

Different methods and mode decays used

# Radiative $\phi$ Decays

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no gluonium hypothesis

allowing for gluonium

KLOE:  $\phi_\rho = (41.3 \pm 0.3_{stat} \pm 0.7_{sys} \pm 0.6_{th})^\circ$

$$\begin{cases} \phi_\rho = (39.7 \pm 0.7)^\circ \\ Z_{\eta'}^2 = 0.14 \pm 0.04 \end{cases}$$

Escribano:  $\phi_\rho = (41.5 \pm 1.2)^\circ$

$$\begin{cases} \phi_\rho = (41.4 \pm 1.3)^\circ \\ Z_{\eta'}^2 = 0.04 \pm 0.09 \end{cases}$$

Thomas:  
no form factor

$$\phi_\rho = (41.7 \pm 0.5)^\circ$$

$$\begin{cases} \phi_\rho = (41.3 \pm 0.7)^\circ \\ Z_{\eta'}^2 = 0.04 \pm 0.04 \end{cases}$$

with form factor

$$\phi_\rho = (42.8 \pm 0.8)^\circ$$

$$\begin{cases} \phi_\rho = (41.9 \pm 0.7)^\circ \\ Z_{\eta'}^2 = 0.10 \pm 0.08 \end{cases}$$

# Radiative $\phi$ Decays

## KLOE versus Escribano:

OZI-rule reduces considerably possible transitions and their respective VP wave-functions overlaps

KLOE: VP-overlap parameters without gluonium, from Bramon, Escribano and Scadron PLB503 (2001) 271

Escribano: VP-overlap parameters determined allowing for gluonium  $\Rightarrow$  new values

Differently from the Escribano's analysis, Kloe use also constraints from  $\eta' \rightarrow \gamma\gamma$ , where no VP-overlap parameters enter in the analysis: this guarantee “independence” from the parameters in a reasonable range.

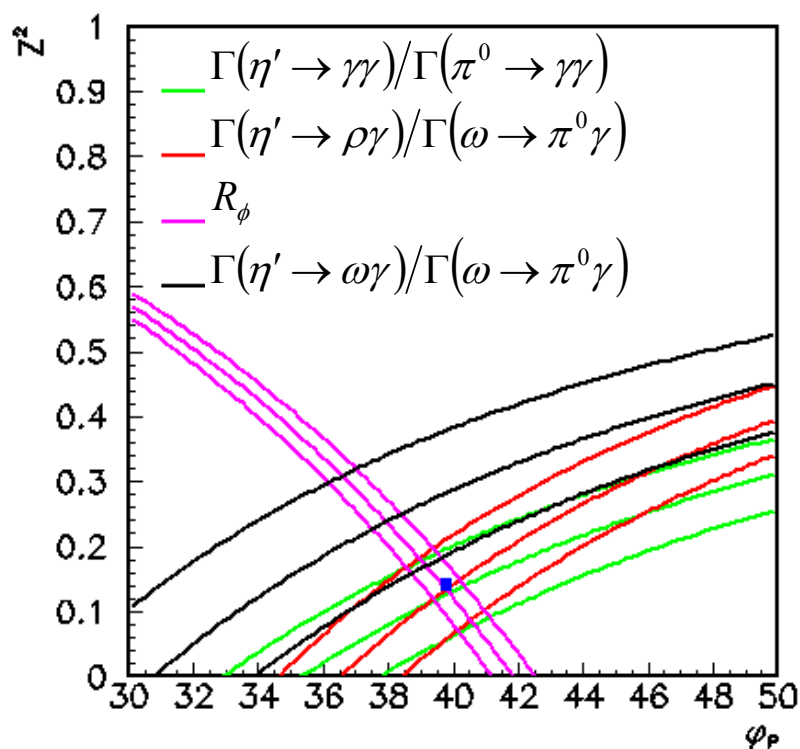
$$\text{KLOE} \left| \begin{array}{l} \Gamma(\phi \rightarrow \eta' \gamma) / \Gamma(\phi \rightarrow \eta \gamma) \\ \Gamma(\eta' \rightarrow \rho \gamma) / \Gamma(\omega \rightarrow \pi^0 \gamma) \\ \Gamma(\eta' \rightarrow \gamma \gamma) / \Gamma(\pi^0 \rightarrow \gamma \gamma) \\ \Gamma(\eta' \rightarrow \omega \gamma) / \Gamma(\omega \rightarrow \pi^0 \gamma) \end{array} \right. \xrightarrow{\text{no VP overlap}}$$

$$\text{Escribano} \left| \begin{array}{l} \Gamma(\phi \rightarrow \eta \gamma) \\ \Gamma(\phi \rightarrow \eta' \gamma) \\ \Gamma(\eta' \rightarrow \rho \gamma) \\ \Gamma(\eta' \rightarrow \omega \gamma) \end{array} \right.$$



# KLOE

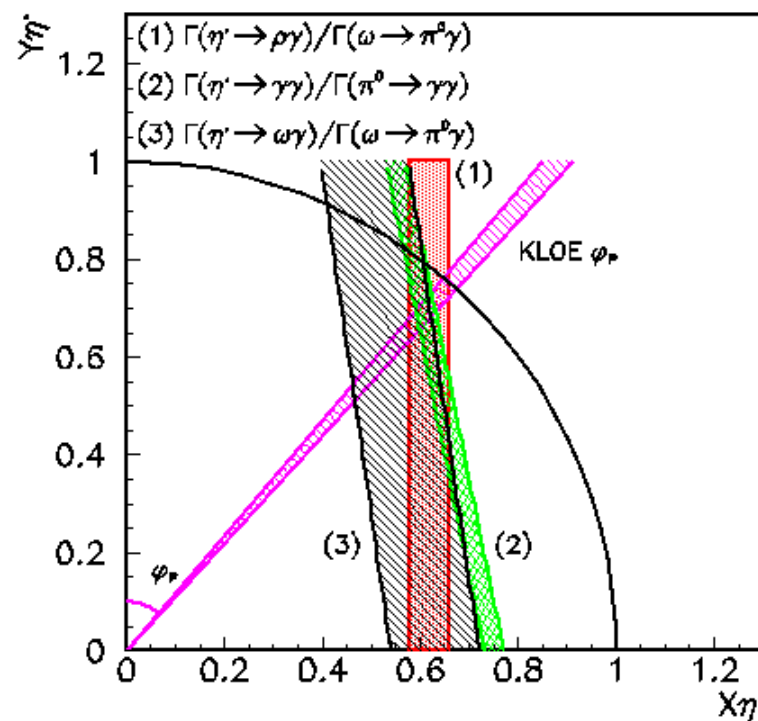
49%  $\chi^2$  Probability



$$\varphi_P = (39.7 \pm 0.7)^\circ$$

$$Z_{\eta'}^2 = 0.14 \pm 0.04$$

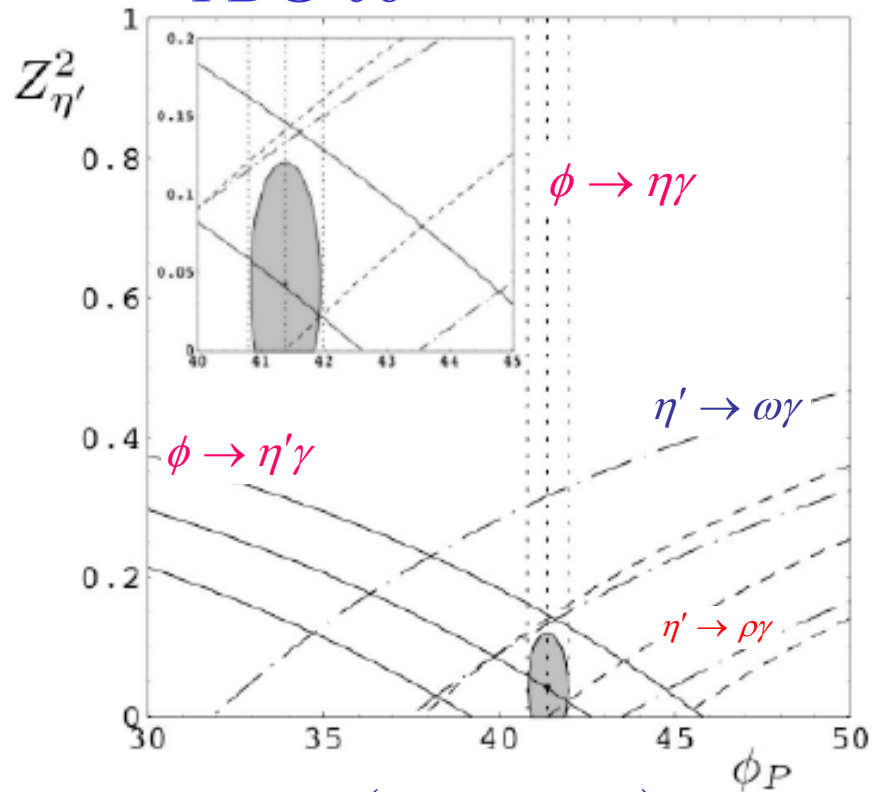
$Z_{\eta'}=0 \Rightarrow 1\% \chi^2$  Probability



KLOE in the hypothesis  
of no gluonium content

# Escribano

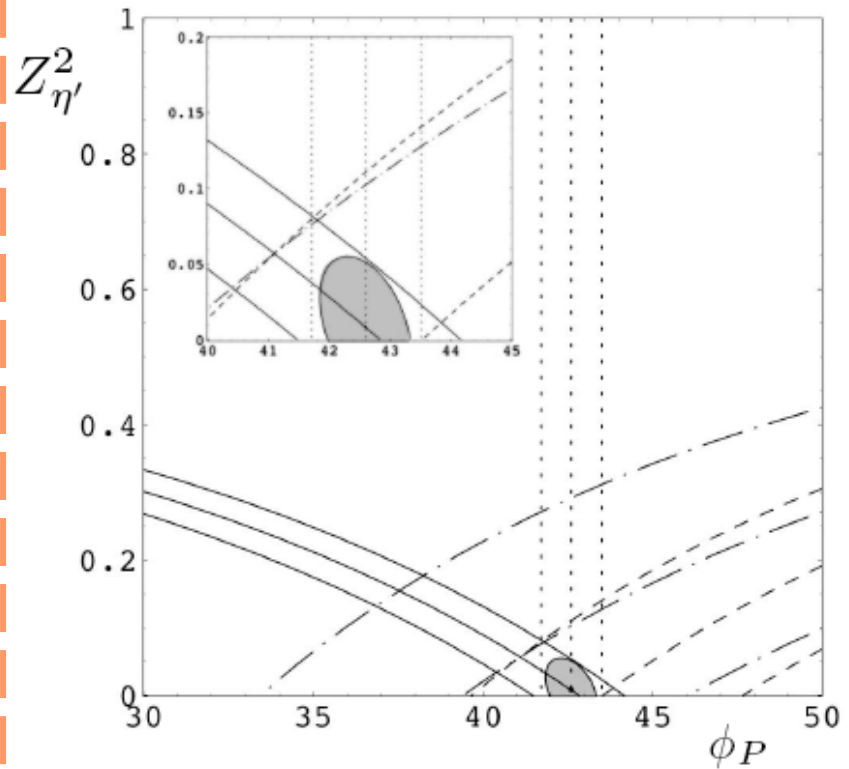
PDG'06



$$\varphi_\rho = (41.4 \pm 1.3)^\circ$$

$$Z_{\eta'}^2 = 0.04 \pm 0.09$$

Latest data



$$\varphi_\rho = (42.6 \pm 1.1)^\circ$$

$$Z_{\eta'}^2 = 0.01 \pm 0.06$$

# Thomas Analysis

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- Difference between KLOE and Escribano: mode used and VP-overlapping parameters treatment.
- VP-overlapping parameter role: the Thomas' results are consistent with no gluonic content and the VP-overlaps play a significant role.
- Thomas perform fit to data with form factors, including an additional momentum dependent factor to the rate  $\Gamma$ :
$$\exp\left(-|\boldsymbol{p}|^2/(8\beta^2)\right)$$
- Including form factors the fits slightly favour gluonium contribution

# Thomas Analysis

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- The analysis has been performed also looking at the decays of light pseudoscalar mesons in to two photons

$$\pi^0 / \eta / \eta' \rightarrow \gamma\gamma$$

- He imposes the value of the mixing angle as obtained from radiative decays and obtains hints at a small gluonium in  $\eta'$

$$\Rightarrow Z_{\eta'}^2 = 0.19 \pm 0.11$$

- The mixing angle is consistent between different determinations and favours  $\phi_P$  close to  $42^\circ$ . There is a hint of a small gluonium in the  $\eta'$  meson.

# *Radiative (and not only) $\psi$ Decays*

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Li, Zhao and Chang: *..in the hadronic decays of charmonia such as  $J/\psi$ ..., the annihilation of the heavy  $c\bar{c}$  pair into intermediate gluons, which then hadronize into final state light mesons and baryons, could favor the production of unconventional hadrons such as glueball....*

## **A look at the analysis:**

•**Phys. Rev. D32, 2883 (1985):** Mark III analysed  $2.7 \cdot 10^6$   $J/\psi$  decays. Branching ratios for  $J/\psi \rightarrow VP$  as  $\rho\pi$ ,  $KK^*(892)$ ,  $\phi\eta$ ,  $\phi\eta'$ ,  $\omega\eta$ ,  $\omega\eta'$ ,  $\omega\pi^0$ ,  $\rho^0\pi$  and  $\rho^0\eta'$ ,  $\phi\pi^0$ , simultaneously fit to a simple model of  $J/\psi$  decays which includes strong and electromagnetic amplitudes and allow for violation of SU(3) invariance: they calculate the strange/nonstrange-quark content of  $\eta$  and  $\eta'$ , and conclude that **(35 $\pm$ 18)% of the  $\eta'$  wave function can be attribute to gluonium** or radial excitation. The analysis is based on the assumption that decays proceed via singly disconnected diagram (SOZI), omitting the doubly disconnected (DOZI) diagram.

# *Radiative (and not only) $\psi$ Decays*

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- **Phys. Rev. D38, 824 (1988):** Seiden et al. reanalyze data combining with two-photon widths of the mesons and stress the importance of the inclusion of DOZI diagrams, neglected in the MarkIII analysis.
  - **Phys. Rev. D38, 2695 (1988):** Mark III analysed  $5.8 \cdot 10^6$   $J/\psi$  decays. Upgrade of branching ratios measurements of  $J/\psi \rightarrow VP$  decays. From the analysis of  $J/\psi \rightarrow \gamma \omega \phi$  decays they shows that **DOZI-suppressed processes contribute to  $J/\psi$  decay rates**, and re-analyze the strange/nonstrange-quark content of  $\eta$  and  $\eta'$ , and any additional component as gluonium or radial excitation.
- In the model **including the DOZI-suppressed contribution**, the  $\eta$  and  $\eta'$  are found to be composed of only light and strange quarks, and **any gluonium contribution to the  $\eta'$  wave function is ruled out**.

# *Radiative $\psi(\prime)$ Decays*

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**Thomas (2007):** From radiative  $J/\psi$  and  $\psi'$  decays the charmonium components of the  $\eta$  and  $\eta'$  are less than 0.1%. No way to extract gluonic component from the ratio  $\Gamma(J/\psi \rightarrow \eta' \gamma) / \Gamma(J/\psi \rightarrow \eta \gamma)$ , the strong decays into VP can be investigated. Following approach by Seiden et al., and then including SOZI and DOZI diagrams, Thomas found the **DOZI diagrams are significant** and form factor does not. In particular any gluonium component comes from production amplitude to be of the same order as the DOZI.

**The fit to the data favours a small gluonic component in the  $\eta'$ .**

Thomas pointed out a significant question: a large DOZI amplitude could be equivalent to a gluonium in the  $\eta'$ . He test this and found the data appears to favour a non-zero DOZI amplitude with small gluonic component of the  $\eta'$ .

The **hadronic  $J/\psi$  and  $\psi'$  decays** furnish a consistent description in terms of one mixing angle with a **suggestion of some gluonic component of the  $\eta'$ .**

# *Remarks on Radiative $\phi$ and $\psi$ Decays*

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- The different determinations of the pseudoscalar mixing angle are generally consistent but show more model and mode dependence as the evaluation of gluonium in the  $\eta'$  meson pointed out.
- Several analysis favours an angle close to  $41.6^\circ$ .
- Data are generally consistent with no additional constituent but there is a hint of a small gluonium component in the  $\eta'$  meson.

A precise determination of mixing angle and gluonium content is necessary in particular for the role of the pseudoscalar mesons in other topics.

We need to put together all the works developed up to now to find a unambiguous determination of the parameters



# *Weak Decays of Charm and Beauty Hadrons*

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- We need to clarify the  $\eta/\eta'$  wave functions to describe semileptonic  $D^+$ ,  $D_S^+$  and  $B^+$  decays
- Knowledge about the structure of  $\eta/\eta'$ , as sizeable  $\eta'$  gluonium content, can help to treat even nonleptonic  $B$  transitions involving  $\eta/\eta'$  and their CP asymmetries, new physics beyond SM is regularly advocated to explain them.

## *Light Flavour Spectroscopy in Semileptonic Decays*

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- Nonstrange D mesons are generally expected not to decay into  $\eta/\eta'$  or  $\phi$  mesons, while the  $D_s^+$  is likely to have both  $s$  and  $\bar{s}$  quarks present after the  $c \rightarrow s W^+$  transition. The observed rates provided novel information on the relative weight of the strange and non-strange  $q\bar{q}$  components of the  $\eta/\eta'$ .
- The weak annihilation provides no more than a nonleading contribution to inclusive rates, it could affect exclusive modes very considerably:  $c$  and  $\bar{s}$  or  $\bar{d}$ , before annihilate into a virtual  $W$ , emit two gluons generating the  $\eta/\eta'$  via a  $gg$  component in the latter's wave functions. [S.Bianco, F.L.Fabbri, D.Benson and I.Bigi, **A Cicerone for the physics of charm** Rivista del Nuovo Cimento Vol.26 N.7-8]
- The progress in the D and B sector by CLEO, FOCUS, Babar and Belle collaborations furnish new observed rates from which we can derive information about the underlying quark level transitions.

## *Light Flavour Spectroscopy in Semileptonic Decays*

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From the ratio of decay widths we can extract information on mixing angle: Feldmann and Kroll [Phys. Rev. D 58, 114006 (1998)] look at decays  $D_S \rightarrow P e \nu$

$$\Gamma(D_S \rightarrow \eta' e^+ \nu) / \Gamma(D_S \rightarrow \eta e^+ \nu) \propto \cot \phi$$

using CLEO data [Phys. Rev. Lett. 75, 3804 (1995)] the pseudoscalar mixing angle has been found  $\phi_P = (41.3 \pm 5.3)^\circ$

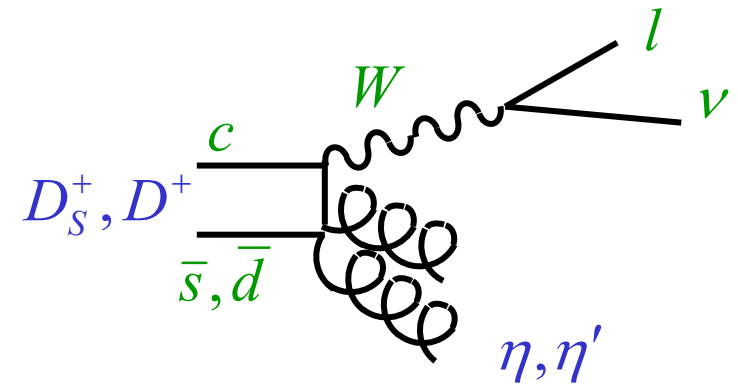
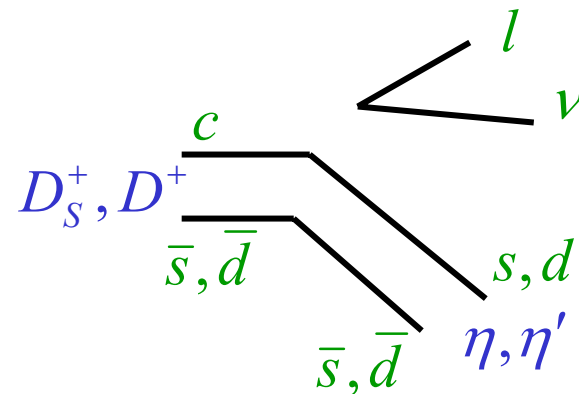
- CLEO: hep-ex/0703042 find evidence for  $B^+ \rightarrow \eta' l^+ \nu$ , upper limit  $BR \leq 1.01 \cdot 10^{-4}$  (90% C.L.), consistent the BABAR upper limit  $\rightarrow$  new data are available

# Light Flavour Spectroscopy in Semileptonic Decays

The wave functions of  $\eta$  and  $\eta'$  in terms of quarks and even gluons enter in the theoretical predictions for exclusive semileptonic modes

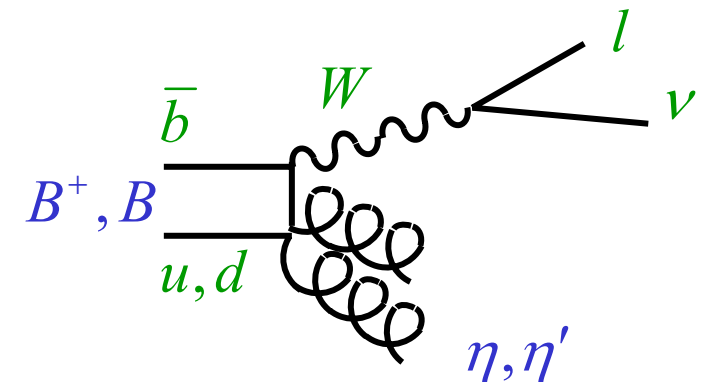
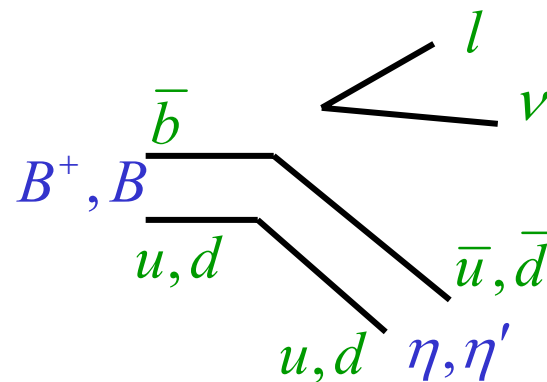
$$D_S^+ \rightarrow \eta(\eta') l^+ \nu$$

$$D^+ \rightarrow \eta(\eta') l^+ \nu$$



$$B^+ \rightarrow \eta(\eta') l^+ \nu$$

$$B \rightarrow \eta(\eta') l^+ \nu$$



# *Charmless Hadronic $B$ Decays*

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- Charmless hadronic  $B$  decays provide valuable tests for the pattern of CP violation in the CKM framework
- Y. Grossman (2003) indicate several observables sensitive to new physics, among them the CP-asymmetry for  $B \rightarrow \eta' K_S$ , whose experimental value is one sigma below the one indicated by SM
- To advocate new physics beyond SM to explain the observed value, we should have fully under control the structure of  $\eta'$  [J.-M.Gerard and E.Kou, PLB616 (2005)]

$$B \rightarrow \eta(\prime)K$$

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CLEO, BELLE and BABAR has been observed an unexpectedly large branching ratio for  $B \rightarrow \eta(\prime)K$  decays.

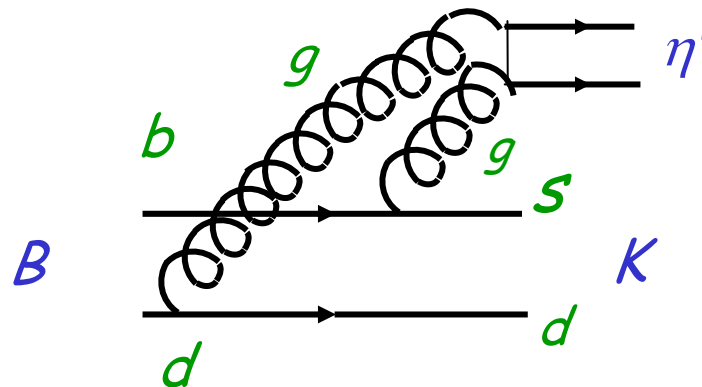
Kou and Sanda [PLB 525 (2002) 240-248]: BR( $B \rightarrow \eta(\prime)K$ ) evaluation in perturbative QCD approach. Using SU(3) relations and allowing for gluonium contribution, a relatively large value of  $\theta_p$  as  $-10^\circ$ , produce a BR( $B \rightarrow \eta(\prime)K$ ) that may be not so small as we expect.

An high value of BR( $B \rightarrow \eta(\prime)K$ ) may imply that we need modify our understanding of  $\eta(\prime)$  meson.

$$B \rightarrow \eta(\prime) K$$


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One of the mechanisms suggested to explain the large BR use the SU(3) singlet contribution: the decays is produced by fusion of gluons, one gluon from  $b \rightarrow sg$  process and another one from spectator. A sizeable gluonium contribution to the  $\eta'$  meson could play an important role: the contribution of the diagram in which two gluons are directly attached to gluonium in  $\eta'$  may be important for  $B \rightarrow \eta' K$  decay.



# Conclusions and Outlooks

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- The different determinations of  $\eta$ – $\eta'$  mixing are generally consistent, but  $\eta'$  gluonium content must be clarified
- Semileptonic  $D^+$ ,  $D_s^+$  and  $B^+$  must be investigated also to check  $\eta'$  gluonium role
- Sizeable gluonium content could help to understand the unexpected high value of  $B \rightarrow \eta K$  decay
- The issue of  $\eta$ – $\eta'$  wave functions is very important for a correct interpretation of CP asymmetry in  $B \rightarrow \eta(^{\prime})K$ . These transitions are Penguin driven in the SM and thus presumably more sensitive to new physics; at the same time we cannot count on a large impact from new physic.

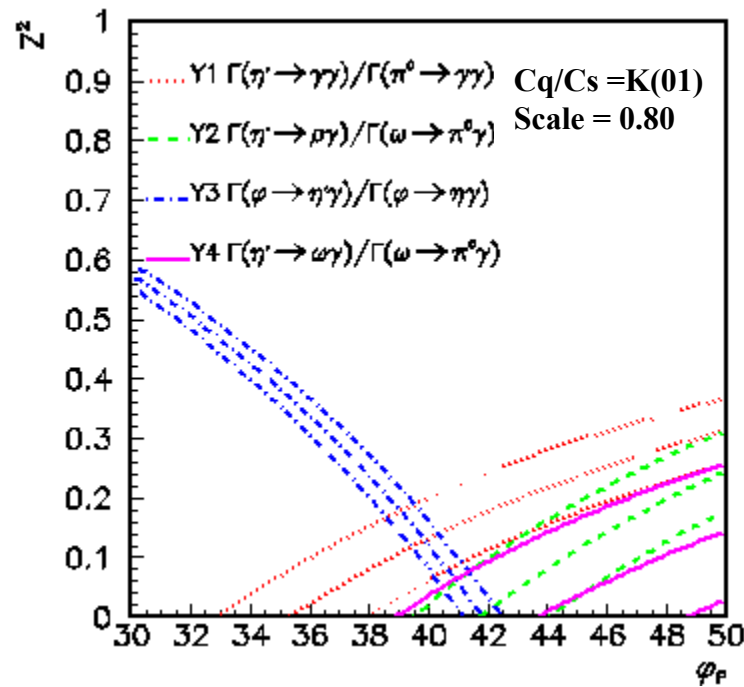
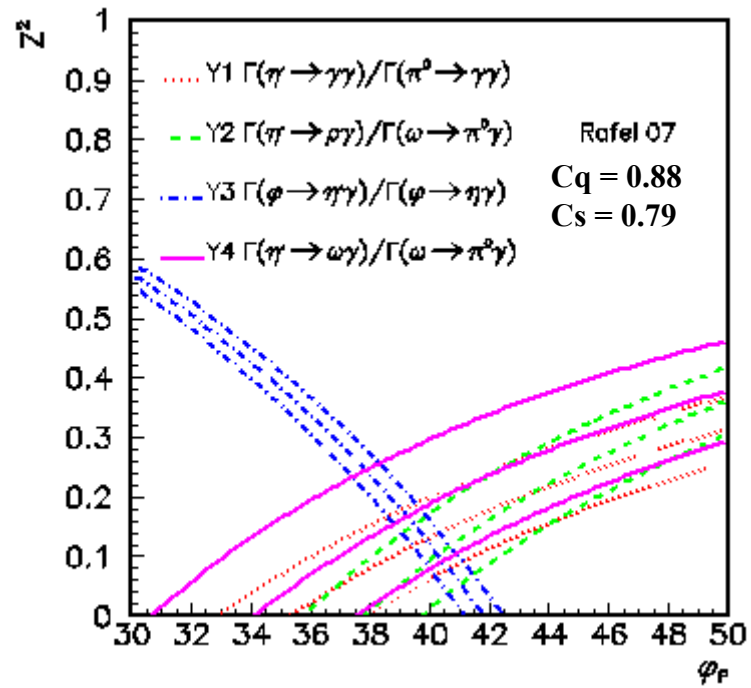
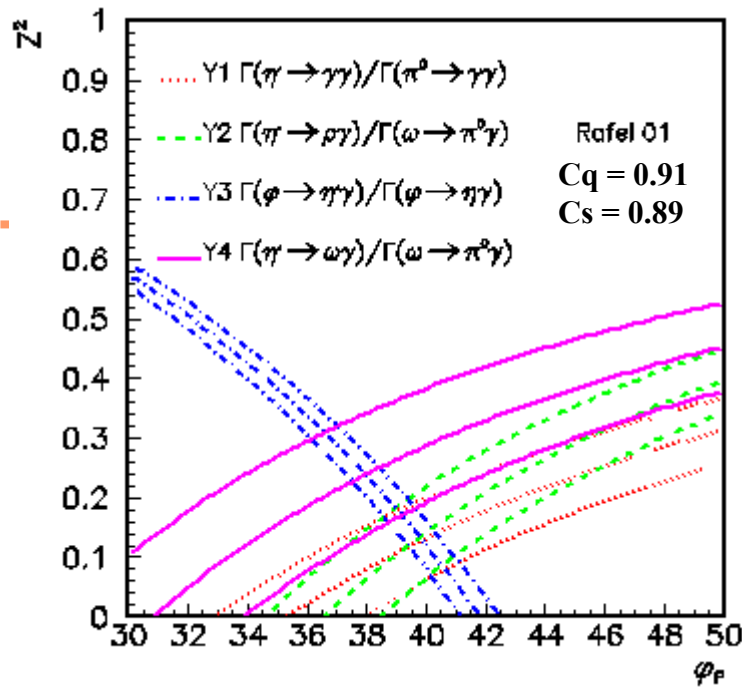




*Spare*

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- QCD brought more dynamical degrees of freedom, gluons, which can form bound states that can mix with neutral mesons
  - Determining  $\eta$ – $\eta'$  mixing is an important task in establishing theoretical control over QCD's nonperturbative dynamics.
  - Gluons mediated strong interactions; presence as independent degrees of freedom demonstrated in hard collisions
  - A purely gluonic component in the  $\eta$  and/or  $\eta'$  wave functions would establish for the first time that gluons play an independent role also in hadronic spectroscopy

# KLOE: check vs VP-overlapping parameters



## *$\eta$ – $\eta'$ Mixing-From Electromagnetic Transitions to Weak Decays of Charm and Beauty Hadrons*

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- $\eta$ – $\eta'$  Wave function composition of SU(3) singlet and octet  $q\bar{q}$  components.
- Gluons can form bound states, gluonium, which can mix with neutral mesons. A purely gluonic component in the mesons wave functions would establish for the first time that gluons play an important role in hadronic spectroscopy
- Semileptonic decays of  $D^+$ ,  $D_s^+$  and  $B^+$  to disentangle the weight of different decay subprocesses
- Weak decay processes involving  $\eta$  and  $\eta'$  mesons as final or intermediate states  $\rightarrow$  New physics beyond the Standard Model is advocated to explain the unexpectedly large  $B \rightarrow \eta K$  branching fraction.

## *Light Flavour Spectroscopy in Semileptonic Decays*

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CLEO: hep-ex/0703042 find evidence for  $B^+ \rightarrow \eta' l^+ \nu$

$$BR(B^+ \rightarrow \eta' l^+ \nu) / BR(B^+ \rightarrow \eta l^+ \nu) > 2.5 (90\% C.L.)$$

Relative rates indicate a significant form factor contribution from the singlet component of the  $\eta'$ , which can bring predictions for  $B \rightarrow \eta' K$  branching fraction into better agreement with experimental measurements

## *$\eta$ – $\eta'$ Mixing and CP asymmetries*

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How observable CP asymmetries are affected by details of the  $\eta$  and  $\eta'$  wave functions?

- Results concerning non-leptonic D and B decays involving  $\eta$  and  $\eta'$  mesons in the final state: a large time dependent CP asymmetry has been established in the channel  $B_d \rightarrow \eta' K_S$ , which is also searched for in  $B_d \rightarrow \eta K_S$  as a potential signature for the intervention of New Physics.
- The measured branching fractions are larger than the SM expectations: SU(3)-singlet couplings unique to the  $\eta'$  meson or new physics contribute to the amplitude.