D⁰ - D₀ Mixing at BaBar

• Some formalism
• D⁰ → K⁺π⁻
• D⁰ → K⁺K⁻ or π⁺π⁻
• D⁰ → K⁺π⁻π⁰
• D⁰ → K⁺π⁻π⁺π⁻

• Summary
• Produce charmed mesons as flavour eigenstates \( D^0 \) and \( \bar{D}^0 \).

• Decay according to

\[
\frac{i}{\partial t} \left( \frac{D^0(t)}{\bar{D}^0(t)} \right) = \left( M - \frac{i}{2} \Gamma \right) \left( \frac{D^0(t)}{\bar{D}^0(t)} \right)
\]

• making mass and lifetime eigenstates

\[
\begin{align*}
|D_1\rangle &= p|D^0\rangle + q|\bar{D}^0\rangle \\
|D_2\rangle &= p|D^0\rangle - q|\bar{D}^0\rangle
\end{align*}
\]

• which decay thus:

\[
\begin{align*}
|D_1(t)\rangle &= e^{-i(m_1-i\Gamma_1/2)t}|D_1\rangle \\
|D_2(t)\rangle &= e^{-i(m_2-i\Gamma_2/2)t}|D_2\rangle
\end{align*}
\]

\[
\begin{align*}
\Delta M &= m_1 - m_2 \\
\Delta \Gamma &= \Gamma_1 - \Gamma_2 \\
\Gamma &= (\Gamma_1 + \Gamma_2)/2 \\
x &= \Delta M/\Gamma \\
y &= \Delta \Gamma/2\Gamma \\
R_M &= (x^2 + y^2)/2
\end{align*}
\]
Motivation

• Knowledge of mixing needed to measure CP violation.
• Possible source of New Physics.
• Fill in gap between K and B_d, B_s mixing – charm mixing involves down-type quarks.
• Standard Model allows x, y up to ~1%.
• CP violation or |x| >> |y| would establish New Physics.

\[ R_M (%) = 0.021 \pm 0.011\% \]
**Experimental notation**

- Almost always use D⁰ from D* decays for tagging and cleanliness.
- D* decays to slow pion and neutral D; charge of pion tags **production flavour** of D.
- Classify backgrounds:
  - Correct D⁰, random slow pion: ‘mistag’ if pion has wrong charge.
  - D⁰ has correct tracks, but wrong particle assignments, or is missing a track: ‘bad D⁰’ or ‘mis-reconstructed charm’.
  - Ordinary combinatorics.
- For historical reasons, refer to Cabibbo-favoured decays as ‘right-sign’ (RS), doubly-Cabibbo-suppressed as ‘wrong-sign’ (WS).
\[ D^0 \rightarrow K^+\pi^- \]

- Decay-time distribution:

\[
\frac{T_{WS}(t)}{e^{-\Gamma t}} \propto R_D + y' \sqrt{R_D} (\Gamma t) + \frac{1}{4} \left( x'^2 + y'^2 \right) (\Gamma t)^2
\]

\[ |DCS|^2 \]

\[ |Mix|^2 \]

\[
x' = x \cos \delta_{K\pi} + y \sin \delta_{K\pi}
\]

\[
y' = y \cos \delta_{K\pi} - x \sin \delta_{K\pi}
\]

- Allow for CP violation by fitting \( D^0 \) and \( \bar{D}^0 \) samples separately.

Yields from 384 fb\(^{-1}\):

<table>
<thead>
<tr>
<th></th>
<th>Candidates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>1141500 ± 1200</td>
<td></td>
</tr>
<tr>
<td>WS</td>
<td>4030 ± 90</td>
<td></td>
</tr>
</tbody>
</table>

\[ \Delta m = m_{D^0} - m_{D^0} \]
• Fit in three stages:
  • Fit RS and WS to $m_{K\pi}$ and $\Delta m$.
  • Fit RS for D0 lifetime and resolution function.
  • Fit WS decay-time distribution.

• Three different models:
  • Impose no mixing and CP conservation.
  • Allow mixing, but keep CP conservation.
  • Allow both mixing and CP violation.

• Signal time resolution modelled as sum of three Gaussians.

• Combinatorial background sum of two Gaussians, with one power-law tail.

• Mis-reconstructed D$^0$ and mistag have signal time distribution.

Points show difference between data and no-mixing fit; curve shows difference between fit with and without mixing.
### D$^0 \rightarrow K^+\pi^- :$ Results

#### Table:

<table>
<thead>
<tr>
<th>Fit</th>
<th>Parameter</th>
<th>Result [$\times 10^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No CPV, no mix</td>
<td>$R_D$</td>
<td>$3.53 \pm 0.08 \pm 0.04$</td>
</tr>
<tr>
<td></td>
<td>$x'^2$</td>
<td>$3.03 \pm 0.16 \pm 0.10$</td>
</tr>
<tr>
<td></td>
<td>$y'$</td>
<td>$-0.22 \pm 0.30 \pm 0.21$</td>
</tr>
<tr>
<td></td>
<td>$y'^-$</td>
<td>$9.7 \pm 4.4 \pm 3.1$</td>
</tr>
<tr>
<td>No CPV, mix</td>
<td>$R_D$</td>
<td>$3.03 \pm 0.16 \pm 0.10$</td>
</tr>
<tr>
<td></td>
<td>$x'^2$</td>
<td>$-0.24 \pm 0.43 \pm 0.30$</td>
</tr>
<tr>
<td></td>
<td>$y'^+$</td>
<td>$9.8 \pm 6.4 \pm 4.5$</td>
</tr>
<tr>
<td></td>
<td>$y'^-$</td>
<td>$-0.20 \pm 0.41 \pm 0.29$</td>
</tr>
<tr>
<td>CPV and mix</td>
<td>$R_D$</td>
<td>$3.03 \pm 0.16 \pm 0.10$</td>
</tr>
<tr>
<td></td>
<td>$x'^2$</td>
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<td>$-0.20 \pm 0.41 \pm 0.29$</td>
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</tbody>
</table>

#### Diagrams:

- **Left Diagram:**
  - Best fit
  - No mixing

- **Right Diagram:**
  - BaBar $1\sigma$
  - BaBar $2\sigma$
  - BaBar $3\sigma$
  - Belle $2\sigma$ statistical

- **Bottom Diagram:**
  -HFAG-charm
  - FCP 2007

Rolf Andreassen
D mixing changes the decay width of final states with definite CP.

Lifetime for

\[ \tau_{CP}^{+} = \tau_{K\pi} \left[ 1 + \frac{q}{p} \left( y \cos \phi_f - x \sin \phi_f \right) \right]^{-1} \]

\[ \tau_{CP}^{-} = \tau_{K\pi} \left[ 1 + \frac{p}{q} \left( y \cos \phi_f + x \sin \phi_f \right) \right]^{-1} \]

Lifetime for \( \bar{D}^{0} \rightarrow \text{CP state} \)

Lifetime for \( D^{0} \rightarrow \text{K\pi} \)

Extract two quantities:

\[ y_{CP} = \frac{2\tau_{K\pi}}{\tau_{CP}^{+} + \tau_{CP}^{-}} \]

\[ \Delta Y = \frac{2\tau_{K\pi} \tau_{CP}^{+} - \tau_{CP}^{-}}{\tau_{CP}^{+} + \tau_{CP}^{-} \tau_{CP}^{+} + \tau_{CP}^{-}} \]
D⁰ → h⁺h⁻ (ctd)

- Reconstruct three modes:
  - D*⁺ → D⁰ π⁺, D⁰ → Kπ
  - D*⁺ → D⁰ π⁺, D⁰ → KK
  - D*⁺ → D⁰ π⁺, D⁰ → ππ

- High purity allows small systematic uncertainty.

- Fit for lifetime of all modes simultaneously.

- Three contributions to fit function:
  - Signal : Exponential convolved with sum of three Gaussians.
  - Combinatorial background : Two Gaussians, power-law tail.
  - Mis-reconstructed charm: Exponential convolved with single Gaussian. Lifetime extracted from fit to Monte Carlo events.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Size</th>
<th>Purity [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>K⁻π⁺</td>
<td>730880</td>
<td>99.9</td>
</tr>
<tr>
<td>K⁺K⁻</td>
<td>69696</td>
<td>99.6</td>
</tr>
<tr>
<td>π⁺π⁻</td>
<td>30679</td>
<td>98.0</td>
</tr>
</tbody>
</table>
Combining KK and pp results gives 3-sigma evidence of mixing:

\[ y_{CP} = (1.24 \pm 0.39 \pm 0.08)\% \]

- CP violation consistent with zero.

- Combining with earlier measurement using untagged sample:

\[ y_{CP} = (1.03 \pm 0.33 \pm 0.19)\% \]

<table>
<thead>
<tr>
<th></th>
<th>( y_{CP} ) [%]</th>
<th>( \Delta Y ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^+K^- )</td>
<td>1.60 ± 0.46 ± 0.17</td>
<td>−0.40 ± 0.44 ± 0.12</td>
</tr>
<tr>
<td>( \pi^+\pi^- )</td>
<td>0.46 ± 0.65 ± 0.25</td>
<td>0.05 ± 0.64 ± 0.32</td>
</tr>
</tbody>
</table>

\[ \tau_{K\pi}, \tau_{KK}, \tau_{\pi\pi} \]
\( D^0 \rightarrow K^+\pi^-\pi^0 \)

- Wrong-sign decay rate varies across Dalitz plot:

\[
A(m_{K^+\pi^-}, m_{K^-\pi^0}, t) = e^{-\Gamma t} \left[ |\bar{A}_D|^2 + |A_D|^2 (y'' \cos \delta_D - x'' \sin \delta_D) \Gamma t \right] / A_D^2 (x''^2 + y''^2 (\Gamma t)^2)
\]

- Subscript D indicates dependence on position in Dalitz plot.

- Determine \( A_D \) from time-independent fit to right-sign Dalitz plot.

- Fit for \( A_D \) simultaneously with \( x, y \).
D^0 \rightarrow K^+\pi^-\pi^0 (ctd)

- Extract background time and Dalitz-plot distributions from D^0 mass sidebands.
- Yields from 384 fb^{-1}:

<table>
<thead>
<tr>
<th>Category</th>
<th>Right-sign</th>
<th>Wrong-sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>639802 \pm 1538</td>
<td>1483 \pm 56</td>
</tr>
<tr>
<td>Bad D^0</td>
<td>3317 \pm 93</td>
<td>227 \pm 75</td>
</tr>
<tr>
<td>Mistag</td>
<td>2384 \pm 57</td>
<td>765 \pm 29</td>
</tr>
<tr>
<td>Combinatorics</td>
<td>1537 \pm 57</td>
<td>499 \pm 57</td>
</tr>
</tbody>
</table>

Spikes due to limited background statistics
$$D^0 \rightarrow K^+\pi^-\pi^0 : \text{Results}$$

No mixing is excluded at the 99% confidence level.

|  \(x''\) [\%] | 2.39 ± 0.61 ± 0.32 |
|  \(y''\) [\%] | −0.14 ± 0.60 ± 0.40 |
|  \(R_M\) [\%] | 0.029 ± 0.016 |

Stat+syst

68.3%
95.0%
99.0%
99.9%

BABAR
preliminary
**D^0 \rightarrow K^+\pi^-\pi^+\pi^-**

- **Decay time difference integrated over phase space (indicated by tilde):**

\[
\frac{\Gamma_{WS}(t)}{\Gamma_{RS}(t)} = \tilde{R}_D + \alpha\tilde{y}'\sqrt{\tilde{R}_D} (\Gamma t) + \frac{1}{4}(\tilde{x}'^2 + \tilde{y}'^2)(\Gamma t)^2
\]

DCS branching ratio

\[
\begin{align*}
\tilde{x}' &= x \cos \tilde{\delta} + y \sin \tilde{\delta} \\
\tilde{y}' &= y \cos \tilde{\delta} - x \sin \tilde{\delta}
\end{align*}
\]

- **With CP violation:**

\[
\left(\tilde{x}'^2 + \tilde{y}'^2\right) \rightarrow |p/q|^{\pm 2} \left(\tilde{x}'^2 + \tilde{y}'^2\right)
\]

\[
\alpha \tilde{y}' \rightarrow |p/q|^{\pm 1} \left(\alpha \tilde{y}' \cos \tilde{\phi} \pm \beta \tilde{x}' \sin \tilde{\phi}\right)
\]

**Yields from 230 fb^{-1}**

<table>
<thead>
<tr>
<th></th>
<th>D^0 candidates</th>
<th>D^0 candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>351100 ± 600</td>
<td>349200 ± 600</td>
</tr>
<tr>
<td>WS</td>
<td>1162 ± 53</td>
<td>1040 ± 51</td>
</tr>
</tbody>
</table>

**Figure (a)**: BABAR preliminary

**Figure (b)**: BABAR preliminary
D^0 \rightarrow K^+\pi^-\pi^+\pi^- : Results

<table>
<thead>
<tr>
<th>CP conserved</th>
<th>CP violation allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_M ) [%]</td>
<td>( R_M ) [%]</td>
</tr>
<tr>
<td>0.019^{+0.016}_{-0.015} \pm 0.002</td>
<td>0.017^{+0.017}_{-0.016} \pm 0.003</td>
</tr>
<tr>
<td>( \alpha y' ) cos \phi</td>
<td>( \alpha y' ) cos \phi</td>
</tr>
<tr>
<td>( \beta x' ) sin \phi</td>
<td>( \beta x' ) sin \phi</td>
</tr>
<tr>
<td>(</td>
<td>p/q</td>
</tr>
<tr>
<td>( -0.006^{+0.005}_{-0.005} \pm 0.001 )</td>
<td>( -0.006^{+0.008}_{-0.006} \pm 0.006 )</td>
</tr>
<tr>
<td>( 0.002^{+0.005}_{-0.003} \pm 0.006 )</td>
<td>( 1.1^{+1.0}_{-0.6} \pm 0.1 )</td>
</tr>
</tbody>
</table>

- Fit in two stages:
  - Extract D^0 mass and \( \Delta m \) parameters.
  - Fix shape parameters, fit to \( (m_{D^0}, \Delta m, t) \).
- Signal shape is exponential convolved with double Gaussian.
- Combinatorial background modelled by Gaussian with power-law tail.
- Fit RS and WS distributions simultaneously.
- Time-resolution parameters and D^0 lifetime for WS fixed to RS values.
- Data consistent with no-mixing at 4.3% confidence level.
Summary

• BaBar has observed mixing in the neutral D system:
  
  • $D^0 \rightarrow K^+\pi^-$
    • 384 fb$^{-1}$, 3.9 $\sigma$ away from no mixing, $(x', y') = (-0.22\pm0.30\pm0.21, 9.7\pm4.4\pm3.1) \times 10^{-3}$
  
  • $D^0 \rightarrow h^+h^-$
    • 384 fb$^{-1}$, 3.0 $\sigma$ away from no mixing, $y_{CP} = 1.24\pm0.39\pm0.13$
  
  • $D^0 \rightarrow K^+\pi^-\pi^0$
    • 384 fb$^{-1}$, no mixing excluded at 99% CL, $(x'', y'') = (2.39\pm0.61\pm0.32, -0.14\pm0.60\pm0.40)$%
  
  • $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$
    • 230 fb$^{-1}$, compatible with no mixing at 4.3% CL, $R_M = 0.019\pm0.016\pm0.020$

• With the final BaBar dataset of roughly 750 fb$^{-1}$, still more accurate determination of mixing will be possible with these and other modes.