On assignment of the f_0 - and f_2-mesons from analysis of processes with pseudoscalar mesons

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- Model-independent analysis of isoscalar-scalar sector
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- Discussion and conclusions

Motivation

We present results of the coupled-channel analysis of data on processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ in the the channels with $I^G J^{PC} = 0^+0^{++}$ and 0^+2^{++} . The scalar sector is problematic up to now especially as to an assignment of the discovered mesonic states to quark-model configurations in spite of a big amount of work devoted these problems (see, e.g., *V.V.Anisovich, IJMP A **21**, 3615 (2006)* **and references there). Furthermore, an exceptional interest to this sector is supported by the fact that there, possibly indeed, we deal with a glueball $f_0(1500)$ (see, e.g.,** *C.Amsler, F.E.Close, PR D **53**, 295 (1996); S.Eidelman* et al. *(PDG), PL B **592**, 1 (2004)*).

In the tensor sector, from thirteen discussed resonances, the nine ones ($f_2(1430)$, $f_2(1565)$, $f_2(1640)$, $f_2(1810)$, $f_2(1910)$, $f_2(2000)$, $f_2(2020)$, $f_2(2150)$, $f_2(2220)$) must be confirmed in various experiments and analyses according to the PDG opinion.
Recently in the combined analysis of $p\overline{p} \to \pi\pi, \eta\eta, \eta\eta'$, five resonances – $f_2(1920)$, $f_2(2000)$, $f_2(2020)$, $f_2(2240)$ and $f_2(2300)$ – have been obtained, one of which ($f_2(2000)$) is a candidate for the glueball (*V.V.Anisovich et al., IJMP A **20**, 6327 (2005)*).**

Here we have applied a model-independent method based on the first principles (analyticity and unitarity) directly applied to analysis of experimental data
*D.Krupa, V.A.Meshcheryakov, Yu.S.Surovtsev, NC A **109**, 281 (1996) – KMS, 96*.

Three-coupled-channel formalism in the uniformizing variable method

First we consider the S-waves of processes

$$\pi\pi \longrightarrow \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$$

and should use the 4-channel approach. However, we will apply the uniformizing variable method which is applicable only in the 2- and 3-channel cases.
Variant I: A combined analysis of $\pi\pi \longrightarrow \pi\pi, K\overline{K}, \eta\eta$.
Variant II: Analysis of $\pi\pi \longrightarrow \pi\pi, K\overline{K}, \eta\eta'$.
Influence of the $\eta\eta'$-channel in the I case and of $\eta\eta$ in the II one are taken into account in the background.

The 3-channel S-matrix is determined on the 8-sheeted Riemann surface. The elements $S_{\alpha\beta}$, where $\alpha, \beta = 1(\pi\pi), 2(K\overline{K}), 3(\eta\eta$ or $\eta\eta')$, have the right-hand cuts starting with $4m_\pi^2$, $4m_K^2$, and $4m_\eta^2$ (or $(m_\eta + m_{\eta'})^2$ for variant II), and the left-hand cuts.

The sheets of the Riemann surface are numbered according to the signs of analytic continuations of the channel momenta $k_1 = \sqrt{s/4 - m_\pi^2}$, $k_2 = \sqrt{s/4 - m_K^2}$, $k_3 = \sqrt{s/4 - m_\eta^2}$ (or $k_3 = 1/2\sqrt{s - (m_\eta + m_{\eta'})^2}$) as follows:

	I	II	III	IV	V	VI	VII	VIII
Imk_1	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
Imk_2	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
Imk_3	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$

The resonance representations on the Riemann surface are obtained with the help of formulas from (*KMS, 96)*), expressing analytic continuations of the matrix elements to unphysical sheets in terms of those on sheet I that have only zeros corresponding to resonances.
<u>7 types of resonances</u> corresponding to 7 possible situations when there are resonance zeros on sheet I only in S_{11} – (a); S_{22} – (b); S_{33} – (c); S_{11} and S_{22} – (d); S_{22} and S_{33} – (e); S_{11} and S_{33} – (f); S_{11}, S_{22}, and S_{33} – (g).
A resonance of every type is represented by a pair of complex-conjugate clusters (of poles and zeros on the Riemann surface). The cluster kind is related to the nature of state. The resonance coupled relatively more strongly to the $\pi\pi$ channel than to the $K\overline{K}$ and $\eta\eta$ ones is described by the cluster of type (a); the resonance with dominant $s\bar{s}$ component, by the (e) cluster;; the glueball, by the (g) cluster.

Let us note that whereas cases (a), (b) and (c) can be simply related to the representation of resonances by Breit-Wigner forms, cases (d), (e), (f) and (g) practically are lost at that description.

We can distinguish, in a model-independent way, a bound state of colourless particles (*e.g.***, $K\overline{K}$ molecule) and a $q\bar{q}$ bound state (***D. Morgan, M.R. Pennington, Phys.Rev. D* ***48****, 1185 (1993); KMS, 96***).**

We use the Le Couteur-Newton relations (*K.J.LeCouteur, Proc.Roy.Soc. A* ***256****, 115 (1960); R.G.Newton, J.Math.Phys.* ***2****, 188 (1961); M.Kato, Ann.Phys.* ***31****, 130 (1965)***). They express the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1,\cdots,k_n)$ that is a real analytic function with the only square-root branch-points at $k_i=0$.**

The branch points are taken into account in a proper uniformizing variable:

$$w = \frac{k_2 + k_3}{\sqrt{m_\eta^2 - m_K^2}} \qquad \text{for variant I,}$$

and

$$w' = \frac{k_2 + k_3}{\sqrt{\frac{1}{4}(m_\eta + m_{\eta'})^2 - m_K^2}} \qquad \text{for variant II.}$$

On the w-plane, the Le Couteur-Newton relations are

$$S_{11} = \frac{d^*(-w^*)}{d(w)}, \qquad S_{22} = \frac{d(-w^{-1})}{d(w)}, \qquad S_{33} = \frac{d(w^{-1})}{d(w)},$$

$$S_{11}S_{22} - S_{12}^2 = \frac{d^*(w^{*-1})}{d(w)}, \qquad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-w^{*-1})}{d(w)}.$$

$$d = d_B d_{res}, \qquad d_{res}(w) = w^{-\frac{M}{2}} \prod_{r=1}^{M} (w + w_r^*)$$

M is the number of resonance zeros.

$$d_B = \exp[-i \sum_{n=1}^{3} \frac{k_n}{m_n}(\alpha_n + i\beta_n)],$$

$$\alpha_n = a_{n1} + a_{n\sigma}\frac{s - s_\sigma}{s_\sigma}\theta(s - s_\sigma) + a_{nv}\frac{s - s_v}{s_v}\theta(s - s_v),$$

$$\beta_n = b_{n1} + b_{n\sigma}\frac{s - s_\sigma}{s_\sigma}\theta(s - s_\sigma) + b_{nv}\frac{s - s_v}{s_v}\theta(s - s_v).$$

s_σ – the $\sigma\sigma$ threshold; s_v – the threshold of many opening channels in the range of $\sim$ 1.5 GeV ($\eta\eta',\ \rho\rho,\ \omega\omega$).
In variant II (the uniformizing variable w'),

$$a_{n\eta}\frac{s - 4m_\eta^2}{4m_\eta^2}\theta(s - 4m_\eta^2) \quad \text{and} \quad b_{n\eta}\frac{s - 4m_\eta^2}{4m_\eta^2}\theta(s - 4m_\eta^2)$$

should be added to α'_n and β'_n. In the $\eta\eta'$ background we have only two constants a_{31} and b_{31}.

Model-independent analysis of isoscalar-scalar sector

For the $\pi\pi$ scattering, the data from the threshold to 1.89 GeV are taken from (*B.Hyams* et al., *NP B **64**, 134 (1973); ibid. **100**, 205 (1975); A.Zylbersztejn* et al., *PL B **38**, 457 (1972); P.Sonderegger, P.Bonamy, in Proc. 5th Intern. Conf. on Elem. Part., Lund, 1969, paper 372; J.R.Bensinger* et al., *PL B **36**, 134 (1971); J.P.Baton* et al., *PL B **33**, 525, 528 (1970); P.Baillon* et al., *PL B **38**, 555 (1972); L.Rosselet* et al., *PR D **15**, 574 (1977); A.A.Kartamyshev* et al., *Pis'ma v ZhETF **25**, 68 (1977); A.A. Bel'kov* et al., *Pis'ma v ZhETF **29**, 652 (1979)***).**

For $\pi\pi \to K\overline{K}$, practically all the accessible data are used (*W.Wetzel* et al., *NP B **115**, 208 (1976); V.A.Polychronakos* et al., *PR D **19**, 1317 (1979); P.Estabrooks, PR D **19**, 2678 (1979); D.Cohen* et al., *PR D **22**, 2595 (1980); G.Costa* et al., *NP B **175**, 402 (1980); A.Etkin* et al., *PR D **25**, 1786 (1982)***).**

For $\pi\pi \to \eta\eta$, we exploited data for the quantity $|S_{13}|^2$ from the threshold to 1.72 GeV (*F.Binon* et al., *NC A* ***78****, 313 (1983)***).**

For $\pi\pi \to \eta\eta'$, the data for $|S_{13}|^2$ from the threshold to 1.813 GeV are taken from (*F. Binon* et al., *NC A* ***80****, 363 (1984)***).**

We considered the case with all five resonances discussed below 1.9 GeV.
In <u>variant I</u> the $f_0(600)$ is described by the cluster of type (a); $f_0(1370)$, type (c); $f_0(1500)$, type (g); $f_0(1710)$, type (b); the $f_0(980)$ is represented only by the pole on sheet II and shifted pole on sheet III.

Satisfactory description: for the $\pi\pi$-scattering from about 0.4 GeV to 1.89 GeV
($\chi^2/\mathrm{NDF} = 186.809/(165-33) \approx 1.415$);

for $\pi\pi \to K\overline{K}$, from the threshold to about
1.6 GeV (χ^2/NDF = 155.683/(120 − 33) ≈ 1.789);
for the $|S_{13}|^2$ data of $\pi\pi \to \eta\eta$, from the threshold to
1.72 GeV (χ^2/N.exp.points ≈ 0.86).
The total χ^2/NDF for all three processes is
356.249/(301 − 40) ≈ 1.365.
The background parameters are:
$a_{11} = 0.2006$, $a_{1\sigma} = 0.0141$, $a_{1v} = 0$, $b_{11} = 0$,
$b_{1\sigma} = -0.01025$, $b_{1v} = 0.04898$, $a_{21} = -0.7039$,
$a_{2\sigma} = -1.4213$, $a_{2v} = -5.951$, $b_{21} = 0.0447$, $b_{2\sigma} = 0$,
$b_{2v} = 6.787$, $b_{31} = 0.6456$, $b_{3\sigma} = 0.3348$, $b_{2v} = 0$;
$s_\sigma = 1.638\ \mathrm{GeV}^2$, $s_v = 2.084\ \mathrm{GeV}^2$.

The combined description of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta'$ (<u>variant II</u>) is even better due to the more detailed representation of the background:
the $f_0(600)$ is described by the cluster of type (a′);
$f_0(1370)$, type (b′); $f_0(1500)$, type (d′); $f_0(1710)$, type (c′).
For the
$\pi\pi$-scattering $\chi^2/\mathrm{NDF} = 141.739/(165-29) \approx 1.042$!
for $\pi\pi \to K\overline{K}$ $\chi^2/\mathrm{NDF} = 155.396/(120-32) \approx 1.766$; for $\pi\pi \to \eta\eta'$ $\chi^2/\mathrm{N.exp.points} \approx 0.421$.
The total χ^2/NDF for these three processes is $300.508/(293-38) \approx 1.178$!
The background parameters:
$a_{11} = 0.02411$, $a_{1\eta} = -0.0638$, $a_{1\sigma} = 0$, $a_{1v} = 0.0916$,
$b_{11} = b_{1\eta} = b_{1\sigma} = 0$, $b_{1v} = 0.0388$, $a_{21} = -3.4384$,
$a_{2\eta} = -0.5377$, $a_{2\sigma} = 1.695$, $a_{2v} = -4.953$, $b_{21} = 0.1193$,
$b_{2\eta} = -0.7953$, $b_{2\sigma} = 2.5315$, $b_{2v} = 2.925$, $b_{31} = 0.6731$,
$s_\sigma = 1.638\ \mathrm{GeV}^2$, $s_v = 2.126\ \mathrm{GeV}^2$.

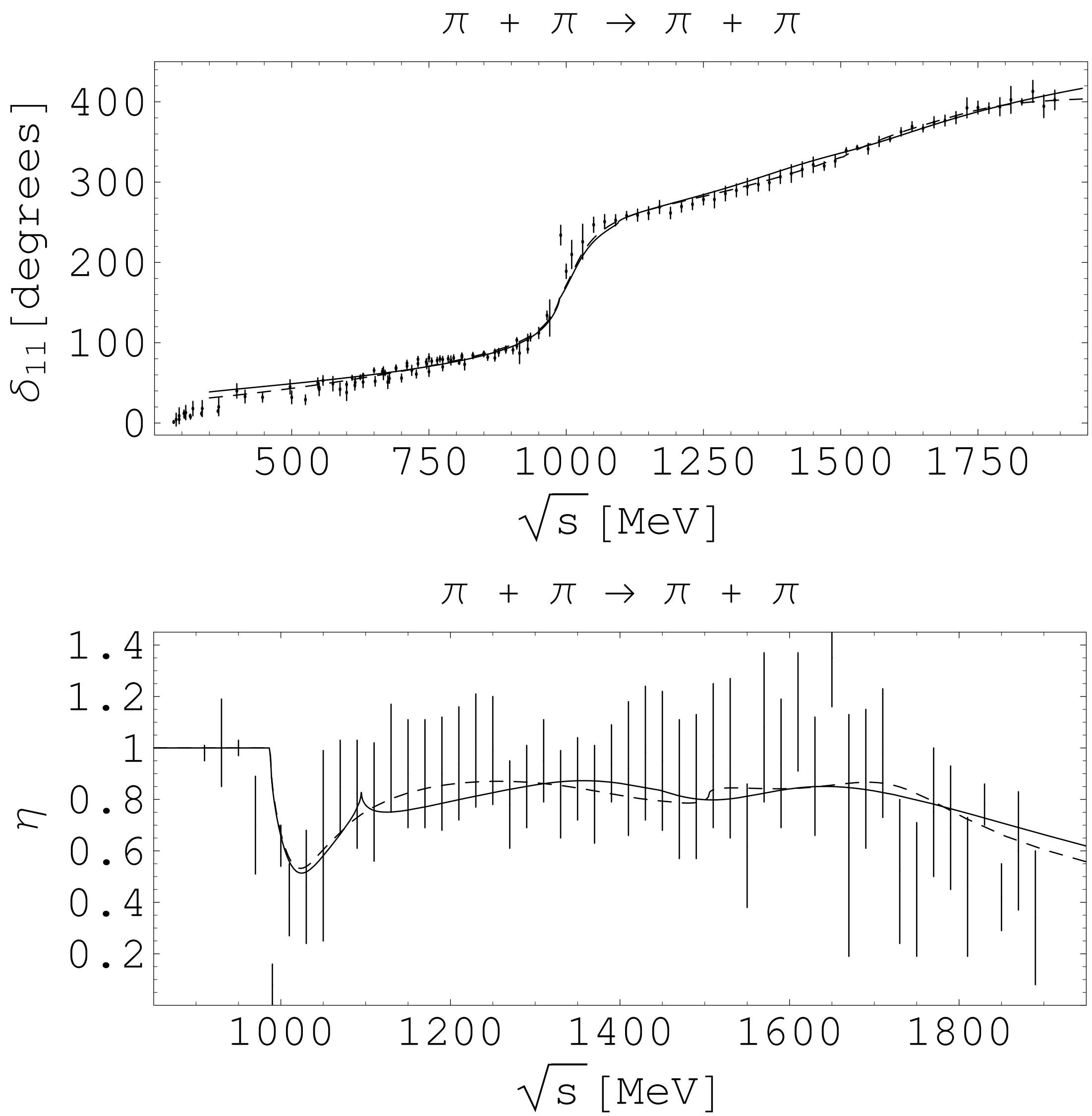


Figure 1: The phase shift and module of the $\pi\pi$-scattering S-wave matrix element. The solid curve – variant I; the dashed curve – variant II.

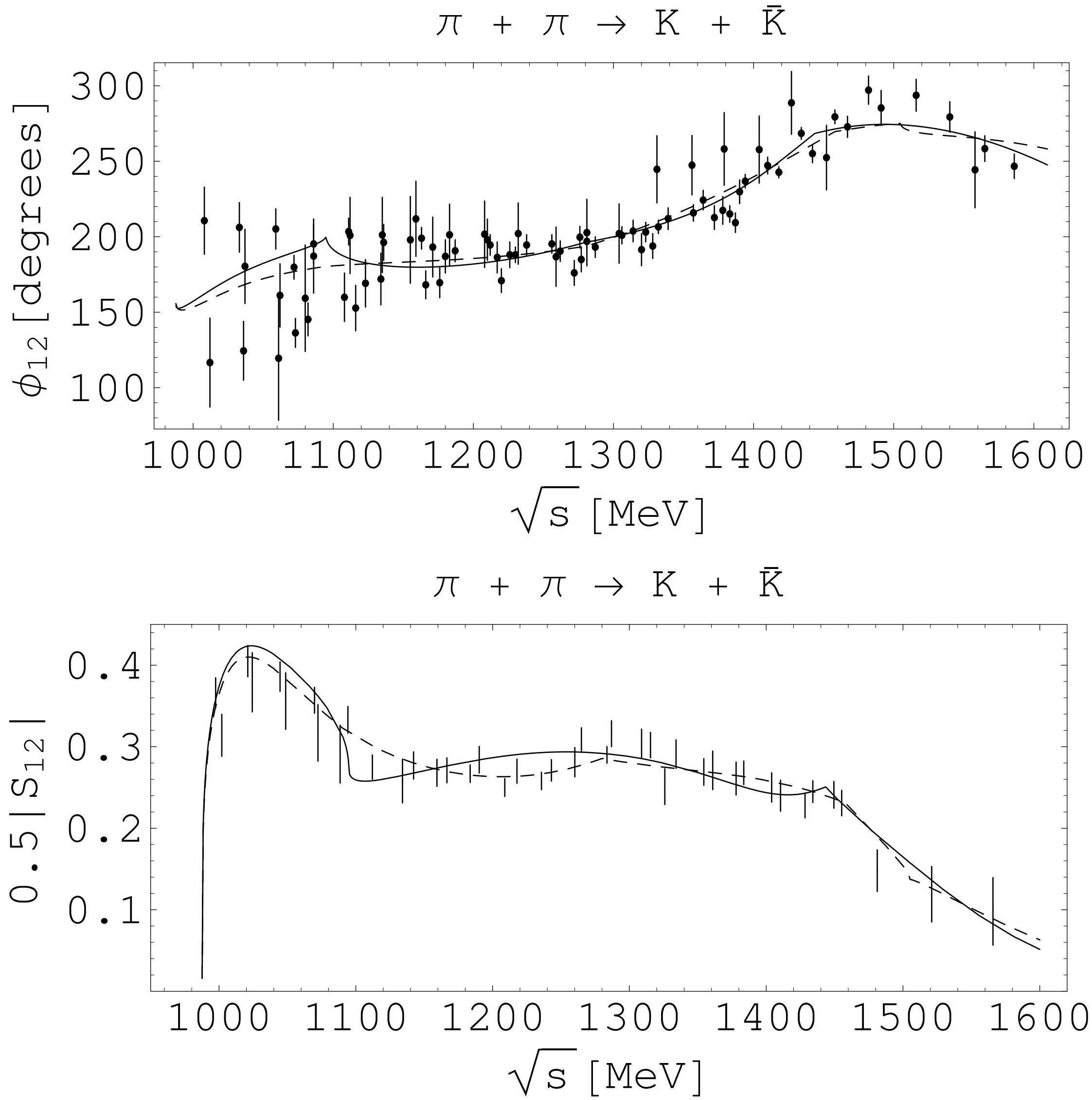


Figure 2: The phase shift and module of the $\pi\pi \longrightarrow K\overline{K}$ S-wave matrix element. The solid curve – variant I; the dashed curve – variant II.

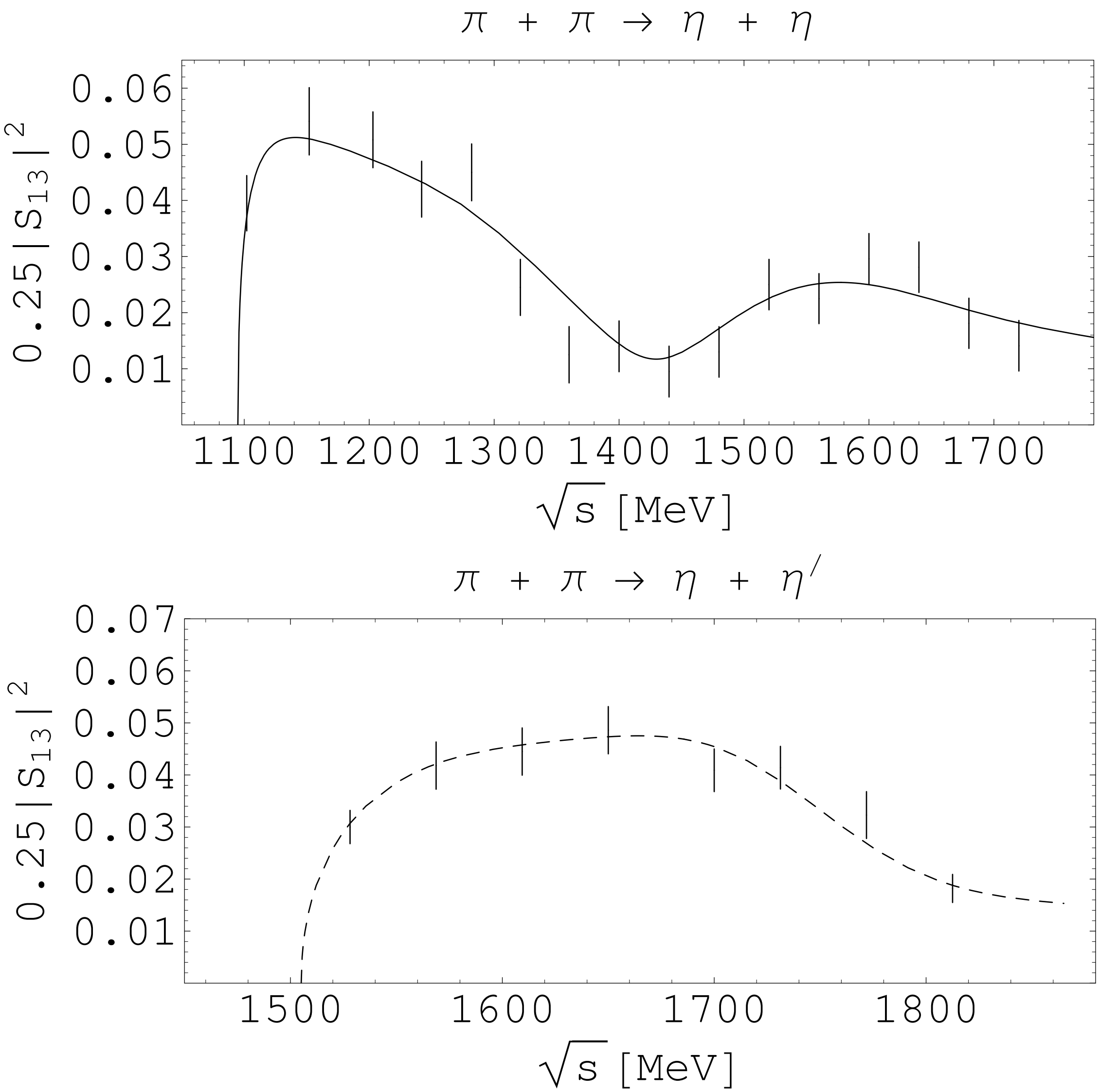


Figure 3: The squared modules of the $\pi\pi \rightarrow \eta\eta$ (upper figure) and $\pi\pi \rightarrow \eta\eta'$ (lower figure) S-wave matrix elements.

Let us indicate the obtained pole clusters for resonances on the complex energy plane $\sqrt{s}$. In variant I, poles on sheets IV, VI, VIII and V, corresponding to the $f_0(1500)$, are of the 2nd and 3rd order, respectively (this is an approximation). In variant II, poles on sheets IV and V, corresponding to the $f_0(1500)$, are of the 2nd order.

Table 1: Pole clusters for the f_0-resonances in variant I.

Sheet		II	III	IV	V	VI	VII	VIII
$f_0(600)$	E_r	683.5±13	673.3±14			593.5±16	603.7±15	
	Γ_r	589±18	589±18			589±18	589±18	
$f_0(980)$	E_r	1013.4±4	984.1±9					
	Γ_r	32.8±6	57.5±10					
$f_0(1370)$	E_r				1398.3±16	1398.3±18	1398.3±18	1398.3±13
	Γ_r				287.5±17	270.5±15	154.9±9	171.9±7
$f_0(1500)$	E_r	1502.6±11	1479.1±13	1502.6±12	1497±12.5	1497.5±16	1496.7±12	1502.6±10
	Γ_r	357.1±15	140.2±12	238.7±13	139.8±14	191.8±17	87.35±11	356.5±14
$f_0(1710)$	E_r		1708.3±12	1708.3±10	1708.3±13	1708.3±15		
	Γ_r		142.3±9	160.3±8	323.1±14	305.1±13		

Table 2: Pole clusters for the f_0-resonances in variant II.

Sheet		II	III	IV	V	VI	VII	VIII
$f_0(600)$	E_r	655.94±10	651.9±13			594.46±16	598.5±14.5	
	Γ_r	606±11	606±12			606±14	606±13	
$f_0(980)$	E_r	1012.8±3	986.3±6					
	Γ_r	31.82±4	57.7±5.5					
$f_0(1370)$	E_r		1391.2±15	1391.2±11	1411.2±17	1411.2±20		
	Γ_r		246.3±12.6	263.7±12	263.7±18	246.3±19		
$f_0(1500)$	E_r	1504.1±12	1499±13	1504.1±13	1504.1±14	1493.8±17	1504.1±15	
	Γ_r	198.7±16	239±15	193.3±13	199±15	193.8±17	193.3±15	
$f_0(1710)$	E_r				1721.2±12	1721.2±13	1721.2±12	1721.2±10
	Γ_r				142.3±9	109.3±8	82.3±8	115.3±7

Note a surprising result obtained for the $f_0(980)$ state. It turns out that this state lies slightly above the $K\overline{K}$ threshold and is described by a pole on sheet II and by a shifted pole on sheet III under the $\eta\eta$ threshold without an accompaniment of the corresponding poles on sheets VI and VII, as it was expected for standard clusters. This corresponds to the description of the $\eta\eta$ bound state.

For subsequent conclusions, let us mention the results for coupling constants from our previous 2-channel analysis (*Yu.S.Surovtsev, D.Krupa, M.Nagy, EPJ A **15, 409 (2002)*): g_1 is the coupling constant with $\pi\pi$; g_2, with $K\overline{K}$.**

	$f_0(665)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$
g_1, GeV	0.652 ± 0.065	0.167 ± 0.05	0.116 ± 0.03	0.657 ± 0.113
g_2, GeV	0.724 ± 0.1	0.445 ± 0.031	0.99 ± 0.05	0.666 ± 0.15

The $f_0(980)$ and the $f_0(1370)$ are coupled essentially more strongly to the $K\overline{K}$ system than to the $\pi\pi$ one, *i.e.*, they have a dominant $s\bar{s}$ component. The $f_0(1500)$ has the approximately equal coupling constants with the $\pi\pi$ and $K\overline{K}$, which apparently could point to its dominant glueball component. In the 2-channel case, $f_0(1710)$ is represented by the cluster corresponding to a state with the dominant $s\bar{s}$ component.

Our 3-channel conclusions on the basis of resonance cluster types generally confirm the ones drawn in the 2-channel analysis (besides the above surprising conclusion about the $f_0(980)$ nature).

Masses and widths of states should be calculated from the pole positions.

$$T^{res} = \sqrt{s}\Gamma_{el}/(m_{res}^2 - s - i\sqrt{s}\Gamma_{tot})$$

Masses and total widths (in the MeV units):

for	$f_0(600)$,	869	and	1178;
for	$f_0(980)$,	1013.4	and	65.6;
for	$f_0(1370)$,	1408.8	and	344;
for	$f_0(1500)$,	1544	and	713;
for	$f_0(1710)$,	1714.2	and	285.

Analysis of isoscalar-tensor sector

We analyze also the isoscalar D-waves of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ considering explicitly also the channel $(2\pi)(2\pi)$ $(i = 4)$. Here it is impossible to use the uniformizing-variable method. Therefore, using the Le Couteur-Newton relations, we generate the resonance poles by some 4-channel Breit-Wigner forms with taking into account a Blatt-Wiesskopf barrier factor (*J.Blatt, V.Weisskopf, Theoretical nuclear physics, Wiley, N.Y., 1952***) conditioned by the resonance spins.**
$d(k_1, k_2, k_3, k_4)$ is taken as

$$d = d_B d_{res}.$$

The 4-channel Breit-Wigner form for the resonance part is taken in the form ($\rho_{rj} = 2k_i/\sqrt{M_r^2 - 4m_j^2}$):

$$d_{res}(s) = \prod_r \left[M_r^2 - s - i \sum_{j=1}^{4} \rho_{rj}^5 R_{rj} f_{rj}^2 \right]$$

f_{rj}^2/M_r is the partial width.

$$R_{rj} = \frac{9 + \frac{3}{4}(\sqrt{M_r^2 - 4m_j^2}\ r_{rj})^2 + \frac{1}{16}(\sqrt{M_r^2 - 4m_j^2}\ r_{rj})^4}{9 + \frac{3}{4}(\sqrt{s - 4m_j^2}\ r_{rj})^2 + \frac{1}{16}(\sqrt{s - 4m_j^2}\ r_{rj})^4}$$

with radii of 0.943 Fermi for all resonances in all channels, except for $f_2(1270)$ and $f_2(1960)$ for which they are: for $f_2(1270)$, 1.498, 0.708 and 0.606 Fermi in channels $\pi\pi$, $K\overline{K}$ and $\eta\eta$, for $f_2(1960)$, 0.296 Fermi in channel $K\overline{K}$.

The background:

$$d_B = \exp\left[-i\sum_{n=1}^{3}\left(\frac{2k_n}{\sqrt{s}}\right)^5 (a_n + ib_n)\right].$$

$$a_1 = \alpha_{11} + \frac{s-4m_K^2}{s}\,\alpha_{12}\,\theta(s-4m_K^2) + \frac{s-s_v}{s}\,\alpha_{10}\,\theta(s-s_v)),$$

$$b_n = \beta_n + \frac{s-s_v}{s}\,\gamma_n\,\theta(s-s_v).$$

$s_v \approx 2.274$ GeV^2 – the combined threshold of channels $\eta\eta',\ \rho\rho,\ \omega\omega$.

The data for the $\pi\pi$ scattering are taken from an energy-independent analysis by B.Hyams et al. (*NP B* ***64****, 134 (1973); ibid.* ***100****,205 (1975)***).**

The data for $\pi\pi \longrightarrow K\overline{K}, \eta\eta$ are taken from works (*S.J.Lindenbaum, R.S.Longacre, PL B* ***274****, 492 (1992); R.S.Longacre* et al.*, PL B* ***177****, 223 (1986)***).**

We obtain a satisfactory description (the total $\chi^2/\mathrm{NDF} = 161.147/(168-65) \approx 1.564$) already with ten resonance $f_2(1270)$, $f_2(1430)$, $f_2'(1525)$, $f_2(1580)$, $f_2(1730)$, $f_2(1810)$, $f_2(1960)$, $f_2(2000)$, $f_2(2240)$ and $f_2(2410)$.

However, the combined analysis of processes $p\overline{p} \longrightarrow \pi\pi, \eta\eta, \eta\eta'$ (*V.V.Anisovich et al., IJMP A* ***20****, 6327 (2005)***) requires one more resonance ($f_2(2020)$) in this region, therefore, we have carried out the additional analysis with the consideration of this state. Description is practically the same one as in the case of ten resonances: the total $\chi^2/\mathrm{NDF} = 156.617/(168-69) \approx 1.582$. Parameters of resonances, obtained in both cases, are shown in the following two tables.**

Table 3: The resonance parameters for ten states (in MeV).

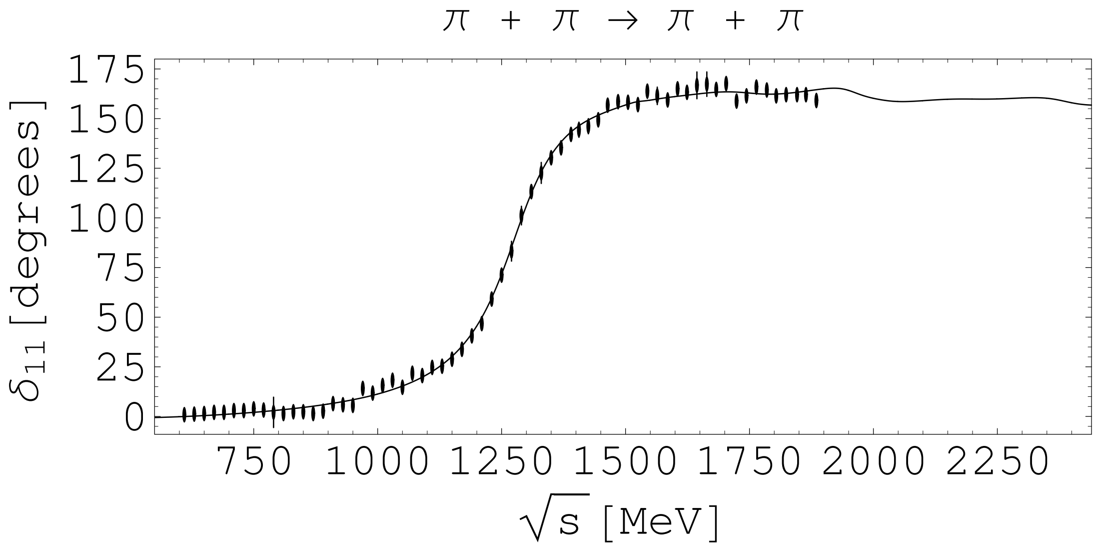
State	M	f_{r1}	f_{r2}	f_{r3}	f_{r4}	Γ_{tot}
$f_2(1270)$	1275.3±1.8	470.8±5.4	201.5±11.4	90.4±4.76	22.4±4.6	≈212
$f_2(1430)$	1450.8±18.7	128.3±45.9	562.3±142	32.7±18.4	8.2±65	>230
$f_2'(1525)$	1535±8.6	28.6±8.3	253.8±78	92.6±11.5	41.6±160	>49
$f_2(1565)$	1601.4±27.5	75.5±19.4	315±48.6	388.9±27.7	127±199	>170
$f_2(1730)$	1723.4±5.7	78.8±43	289.5±62.4	460.3±54.6	107.6±76.7	>182
$f_2(1810)$	1761.8±15.3	129.5±14.4	259±30.7	469.7±22.5	90.3±90	>177
$f_2(1960)$	1962.8±29.3	132.6±22.4	333±61.3	319±42.6	65.4±94	>119
$f_2(2000)$	2017±21.6	143.5±23.3	614±92.6	58.8±24	450.4±221	>299
$f_2(2240)$	2207±44.8	136.4±32.2	551±149	375±114	166.8±104	>222
$f_2(2410)$	2429±31.6	177±47.2	411±196.9	4.5±70.8	460.8±209	>170

For the background : $\alpha_{11} = -0.07805$, $\alpha_{12} = 0.03445$, $\alpha_{10} = -0.2295$, $\beta_1 = -0.0715$, $\gamma_1 = -0.04165$, $\beta_2 = -0.981$, $\gamma_2 = 0.736$, $\beta_3 = -0.5309$, $\gamma_3 = 0.8223$.

Table 4: The resonance parameters for eleven states.

State	M	f_{r1}	f_{r2}	f_{r3}	f_{r4}	Γ_{tot}
$f_2(1270)$	1276.3±1.8	468.9±5.5	201.6±11.6	89.9±4.79	7.2±4.6	≈210.5
$f_2(1430)$	1450.5±18.8	128.3±45.9	562.3±144	32.7±18.6	8.2±63	>230
$f_2'(1525)$	1534.7±8.6	28.5±8.5	253.9±79	89.5±12.5	51.6±155	>49.5
$f_2(1565)$	1601.5±27.9	75.5±19.6	315±50.6	388.9±28.6	127±190	>170
$f_2(1730)$	1719.8±6.2	78.8±43	289.5±62.6	460.3±545.	108.6±76.	>182.4
$f_2(1810)$	1760±17.6	129.5±14.8	259±32.	469.7±25.2	90.3±89.5	>177.6
$f_2(1960)$	1962.2±29.8	132.6±23.3	331±61.5	319±42.8	62.4±91.3	>118.6
$f_2(2000)$	2006±22.7	155.7±24.4	169.5±95.3	60.4±26.7	574.8±211	>193
$f_2(2020)$	2027±25.6	50.4±24.8	441±196.7	58±50.8	128±190	>107
$f_2(2240)$	2202±45.4	133.4±32.6	545±150.4	381±116	168.8±103	>222
$f_2(2410)$	2387±33.3	175±48.3	395±197.7	24.5±68.5	462.8±211	>168

The background parameters are: $\alpha_{11} = -0.0755$, $\alpha_{12} = 0.0225$, $\alpha_{10} = -0.2344$, $\beta_1 = -0.0782$, $\gamma_1 = -0.05215$, $\beta_2 = -0.985$, $\gamma_2 = 0.7494$, $\beta_3 = -0.5162$, $\gamma_3 = 0.786$.



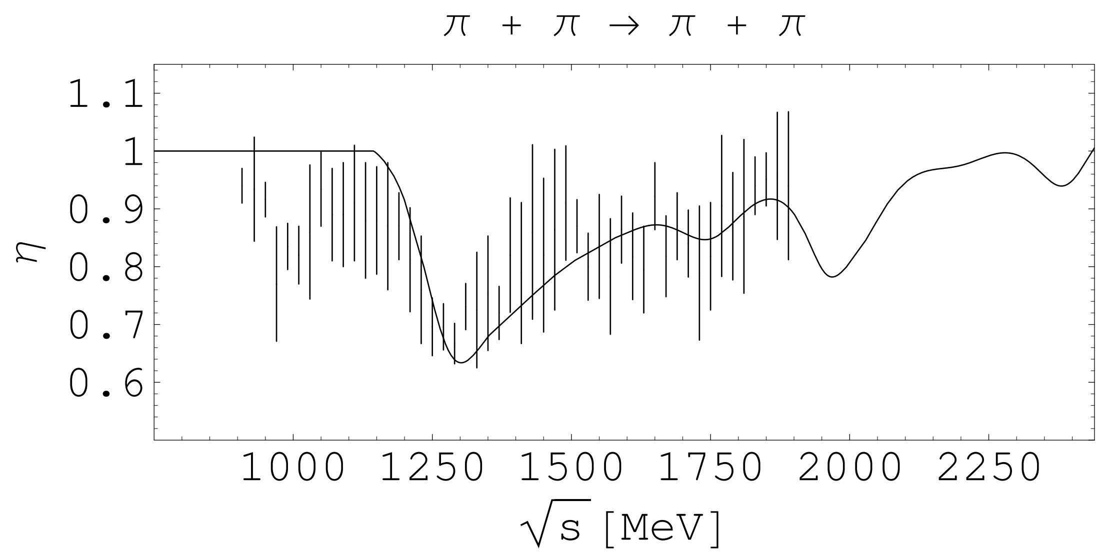


Figure 4: The phase shift and module of the $\pi\pi$-scattering D-wave matrix element.

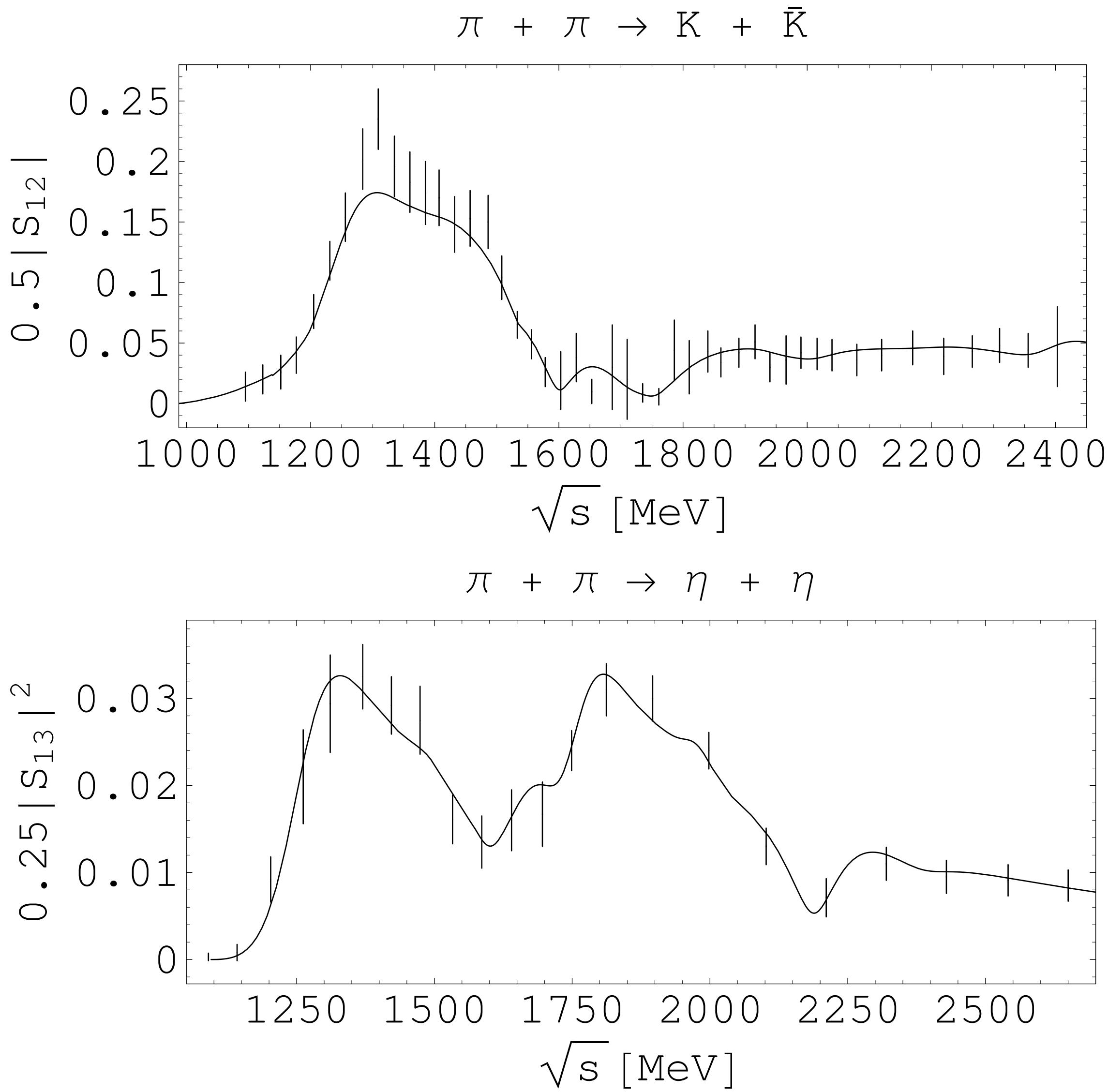


Figure 5: The squared modules of the $\pi\pi \to K\overline{K}$ (upper figure) and $\pi\pi \to \eta\eta$ (lower figure) D-wave matrix elements.

Discussion and conclusions

- **In the combined model-independent analysis of data on the $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ processes in the channel with $I^G J^{PC} = 0^+0^{++}$, an additional confirmation of the σ-meson with mass 869 MeV is obtained. This mass value rather accords with prediction ($m_\sigma \approx m_\rho$) on the basis of mended symmetry by S. Weinberg (*PRL* ***65****, 1177 (1990)*). Evidence of the existence of the σ-meson have been given also in works:** *V.V.Anisovich* et al., *NP Proc.Suppl. A* ***56****, 270 (1997); V.V.Anisovich* et al., *PR D* ***58****, 111503 (1998); N.A.Törnqvist, M.Roos, PRL* ***76****, 1575 (1996); S.Ishida* et al., *Progr.Theor.Phys.* ***95****, 745 (1996); M.Svec, PR D* ***53****, 2343 (1996); R.Kamiński* et al., *EPJ C* ***9****, 141 (1999); Yu.S.Surovtsev* et al., *PR D* ***63****, 054024 (2001); L.Li, B.-S.Zou, G.lie Li, PR D* ***63****, 074003 (2001)*.

- **Indication for $f_0(980)$ to be the $\eta\eta$ bound state is obtained. From point of view of quark structure, this is the 4-quark state. Maybe, this is consistent somehow with arguments of (***N.N.Achasov, NP A* ***675***, *279c (2000); M.N.Achasov* et al., *PL B* **438**, *441 (1998); ibid.* **440**, *442 (1998)*) **in favour of the 4-quark nature of $f_0(980)$.**

- **The $f_0(1370)$ and $f_0(1710)$ have the dominant $s\bar{s}$ component. Conclusion about the $f_0(1370)$ quite well agrees with the one of work of Crystal Barrel Collaboration (***C.Amsler* et al., *PL B* ***355***, *425 (1995)*) **where the $f_0(1370)$ is identified as $\eta\eta$ resonance in the $\pi^0\eta\eta$ final state of the $\bar{p}p$ annihilation at rest. Conclusion about the $f_0(1710)$ is quite consistent with the experimental facts that this state is observed in $\gamma\gamma \to K_S K_S$ (***S.Braccini, Proc. Workshop on Hadron Spectroscopy, Frascati Phys. Series* ***XV***, *53 (1999)*) **and not observed in $\gamma\gamma \to \pi^+\pi^-$ (***R.Barate* et al., *PL B* ***472***, *189 (2000)*).

- **As to the $f_0(1500)$, we suppose that it is practically the eighth component of octet mixed with a glueball being dominant in this state. Its biggest width among enclosing states tells also in behalf of its glueball nature (***V.V.Anisovich et al., NP Proc.Suppl. A**56**, 270 (1997)***).**

- **We propose a following assignment of scalar mesons below 1.9 GeV to lower nonets, when excluding the $f_0(980)$ as the $\eta\eta$ bound state. The lowest nonet: the isovector $a_0(980)$, the isodoublet $K_0^*(900)$, and $f_0(600)$ and $f_0(1370)$ as mixtures of the 8th component of octet and the SU(3) singlet.**

  The Gell-Mann–Okubo (GM-O) formula

$$3m_{f_8}^2 = 4m_{K_0^*}^2 - m_{a_0}^2$$

  gives $m_{f_8} = 880$ MeV.
  Our result: $m_\sigma = 869 \pm 14$ MeV.

In relation for masses of nonet

$$m_\sigma + m_{f_0(1370)} = 2m_{K_0^*}$$

the left-hand side is about 26 % bigger than the right-hand one.

For the next nonet (of radial excitations) we find: $a_0(1450)$, $K_0^*(1450)$, and $f_0(1500)$ and $f_0(1710)$ as mixture of the eighth component of octet and the SU(3) singlet, the $f_0(1500)$ being mixed with a glueball which is dominant in this state. From the GM-O formula, $m_{f_8} \approx 1450$ MeV. In formula $m_{f_0(1500)} + m_{f_0(1710)} = 2m_{K_0^*(1450)}$ the left-hand side is about 12 % bigger than the right-hand one.

This assignment moves a number of questions and does not put the new ones. Now an adequate mixing scheme should be found.

- **As to the tensor sector, we carried out two analysis – without and with the $f_2(2020)$. We do not obtain $f_2(1640)$, $f_2(1910)$, $f_2(2150)$ and $f_2(2010)$, however, we see $f_2(1450)$ and $f_2(1730)$ which are related to the statistically-valued experimental points.**

- **Usually one assigns to the first tensor nonet the states $f_2(1270)$ and $f_2'(1525)$.**

  To the second nonet, one could assign $f_2(1601)$ and $f_2(1767)$ though for now the isodoublet member is not discovered. If $a_2(1730)$ is the isovector of this octet and if $f_2(1601)$ is almost its eighth component, then, on the basis of the GM-O formula, we would expect this isodoublet mass at about 1.635 GeV. Then the relation for masses of nonet would be well fulfilled.

There is an experiment (*V.M.Karnaukhov, C.Coca, V.I.Moroz, Yad.Fiz. **63, 652 (2000)*) in which, in the mode $K_s^0\pi^+\pi^-$, one has observed the strange isodoublet with yet indefinite remaining quantum numbers and with mass 1629 ± 7 MeV. The tensor isodoublet should have that decay mode. We think that the discovered state in the indicated experiment might be the tensor isodoublet of the second nonet.**

- **The states $f_2(1963)$ and $f_2(2207)$ together with the isodoublet $K_2^*(1980)$ could be put into the third nonet. Then in the relation for masses of nonet**

$$M_{f_2(1963)} + M_{f_2(2207)} = 2M_{K_2^*(1980)},$$

the left-hand side is only 5.3 % bigger than the right-hand one.

If one consider $f_2(1963)$ as the eighth component of octet, then the GM-O formula

$$M_{a_2} = 4M_{K_2^*(1980)} - 3M_{f_2(1963)}$$

gives $M_{a_2} = 2031$ MeV. This value coincides with the one (2030 MeV) for a_2-meson obtained on the basis of the recent data (*A.V.Anisovich et al., PL B **452**, 173 (1999); ibid., **452**, 187 (1999); ibid., **517**, 261 (2001).***). This state is interpreted as a second radial excitation of the 1^-2^{++}-state on the basis of consideration of the a_2 trajectory on the (n, M^2) plane (n is the radial quantum number of the $q\bar{q}$ state).** *V.V.Anisovich et al.. IJMP A **20**, 6327 (2005)*.

- **As to $f_2(2000)$, the presence of the $f_2(2020)$ in the analysis with eleven resonances helps to interpret $f_2(2000)$ as the glueball.**

In the case of ten resonances, the ratio of the $\pi\pi$ and $\eta\eta$ widths is in the limits obtained in Ref.(*V.V.Anisovich et al., IJMP A **20, 6327 (2005)*) for the tensor glueball on the basis of the 1/N-expantion rules. However, the $K\overline{K}$ width is too large for the glueball. Namely for investigation of this question, we have carried out the additional analysis with the consideration of the state $f_2(2020)$. At practically the same description of processes with the consideration of eleven resonances as in the case of ten, their parameters have varied not much, except for the ones for $f_2(2000)$ and $f_2(2410)$. Mass of the latter has decreased by about 30 MeV. As to $f_2(2000)$, its $K\overline{K}$ width has changed significantly. Now all the obtained ratios of the partial widths are in the limits corresponding to the glueball. However, there is not demonstrated clearly the glueball property of accumulating the widths of the enclosing states (*V.V.Anisovich et al., NP Proc.Suppl. A**56**, 270 (1997)*).**

The question of interpretation of the $f_2(2020)$ and $f_2(2410)$ is open.

- **Finally we have $f_2(1450)$ and $f_2(1730)$ which are neither $q\bar{q}$ states and nor glueballs. Since one predicts that masses of the lightest $q\bar{q}g$ hybrids are bigger than the ones of lightest glueballs, maybe, these states are the 4-quark ones. Of course, assumption of this possibility presupposes an existence of the scalar 4-quark states at lower energies which are not seen in analysis. One can think that these states are a part of the background in view of their very large widths.**