The large degeneracy of excited hadrons and quark models

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1. Experimental data on very excited hadrons
2. Quark and meson masses in equal time
3. Linear Regge trajectories
4. Search for a principal quantum number with classical orbits

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1. Experimental data on very excited hadrons

The pattern of a large approximate degeneracy of the excited hadron spectra (larger than the chiral restoration degeneracy) is present in the recent experimental report of Bugg.

In: D. V. Bugg, Phys. Rept. 397, 257 (2004) [arXiv:hep-ex/0412045].

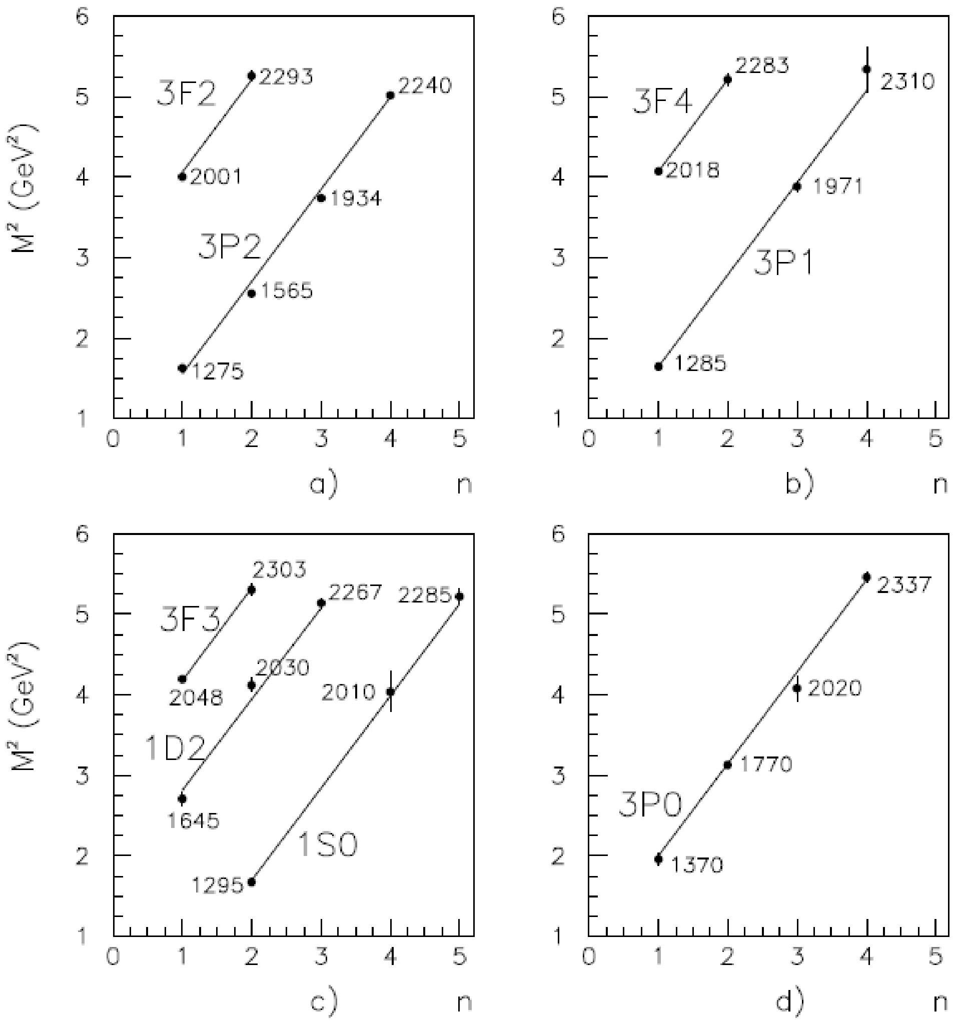


Fig. 1. Trajectories of $I = 0$, $C = +1$ states. Numbers indicate masses in MeV.

radial excitation quantum number

In:
D. V. Bugg, Phys. Rept. 397, 257 (2004) [arXiv:hep-ex/0412045].

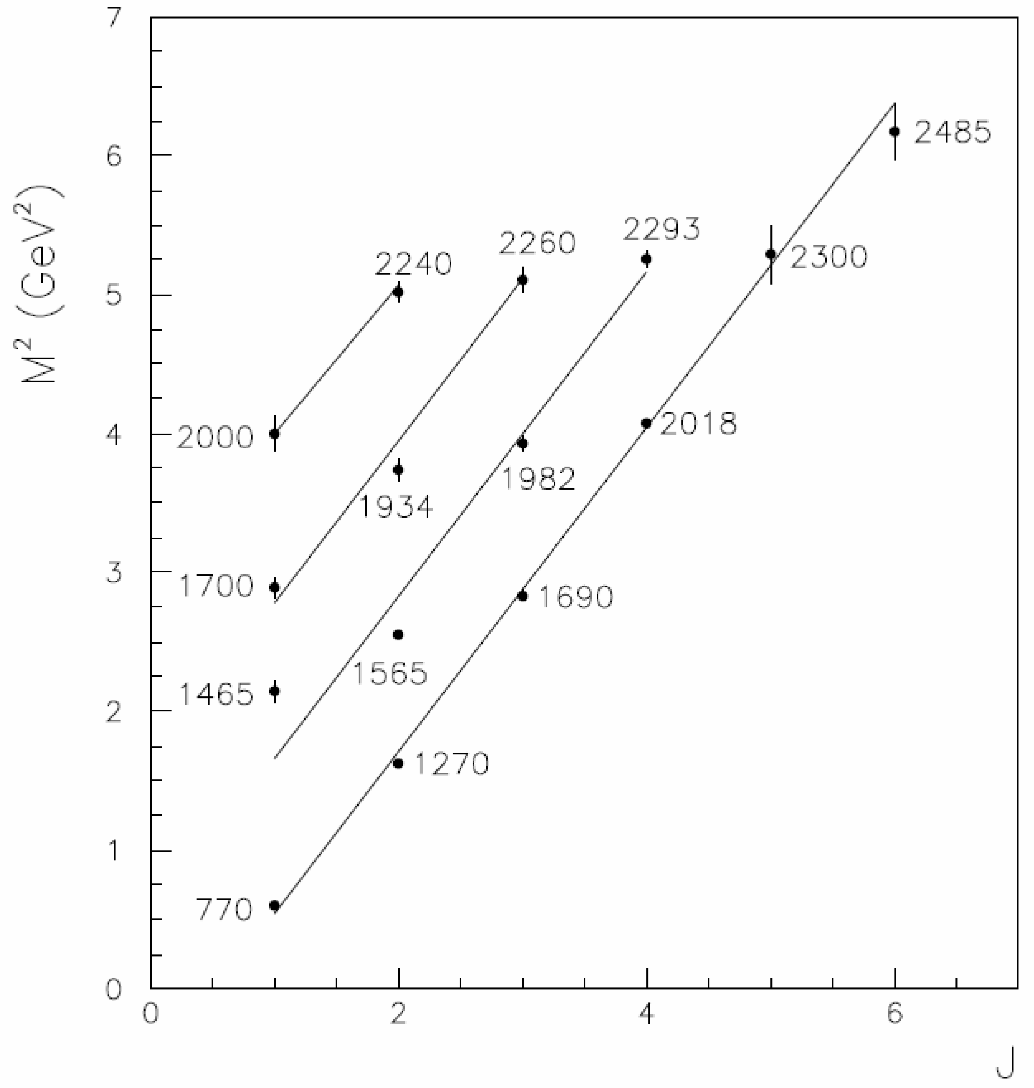


Fig. 56. Regge trajectories.

See also: L. Y.Glozman, Phys. Rept 444, 1 (2007) [arXiv:hep-ph/0701081].

And:
S. S. Afonin, Phys. Lett. B 639, 258 (2006) [arXiv:hep-ph/0603166].

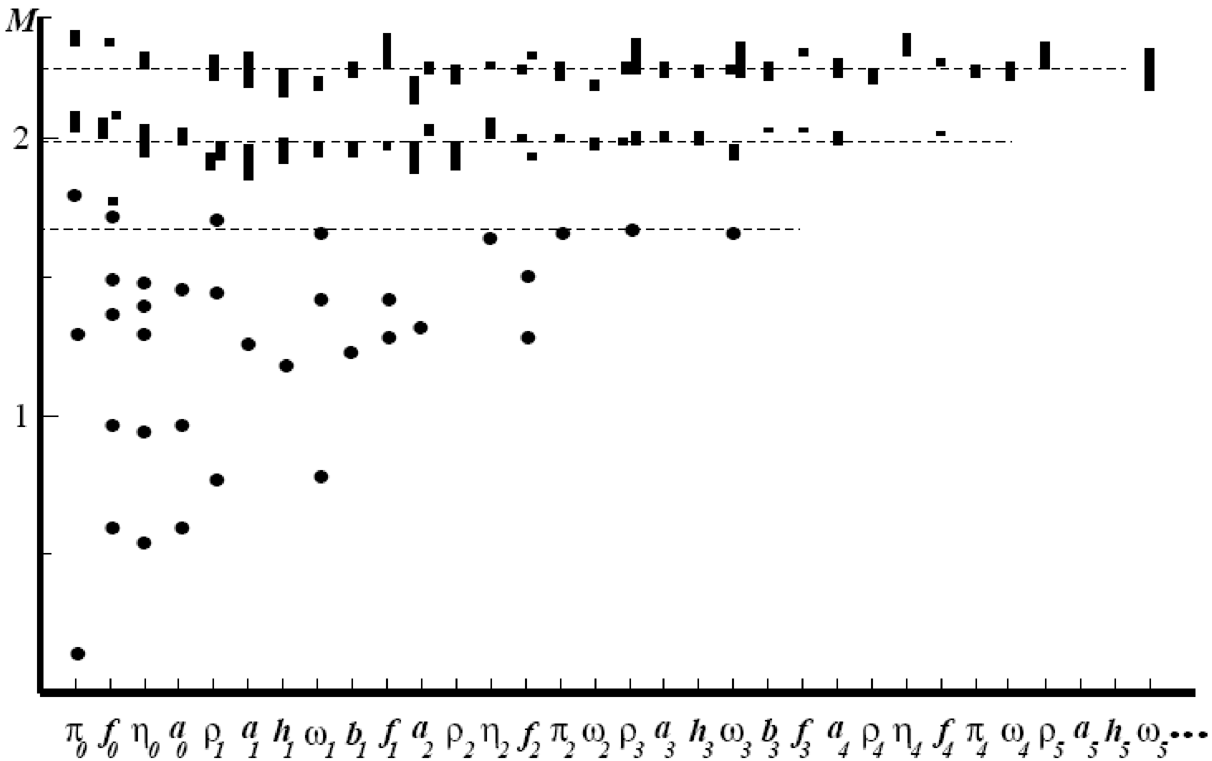


Figure 2. Masses (in GeV) of the well established states from PDG (circles) and new $\bar{n}n$ states from the proton-antiproton annihilation (strips). Note that the well-established states …

2. Quark and meson masses in equal time

Here we try to model this degeneracy with state of the art quark models. We review how the Coulomb Gauge chiral invariant and confining Bethe-Salpeter equation simplifies in the case of very excited quark-antiquark mesons, including angular or radial excitations, to a Salpeter equation with an ultrarelativistic kinetic energy with the spin-independent part of the potential.

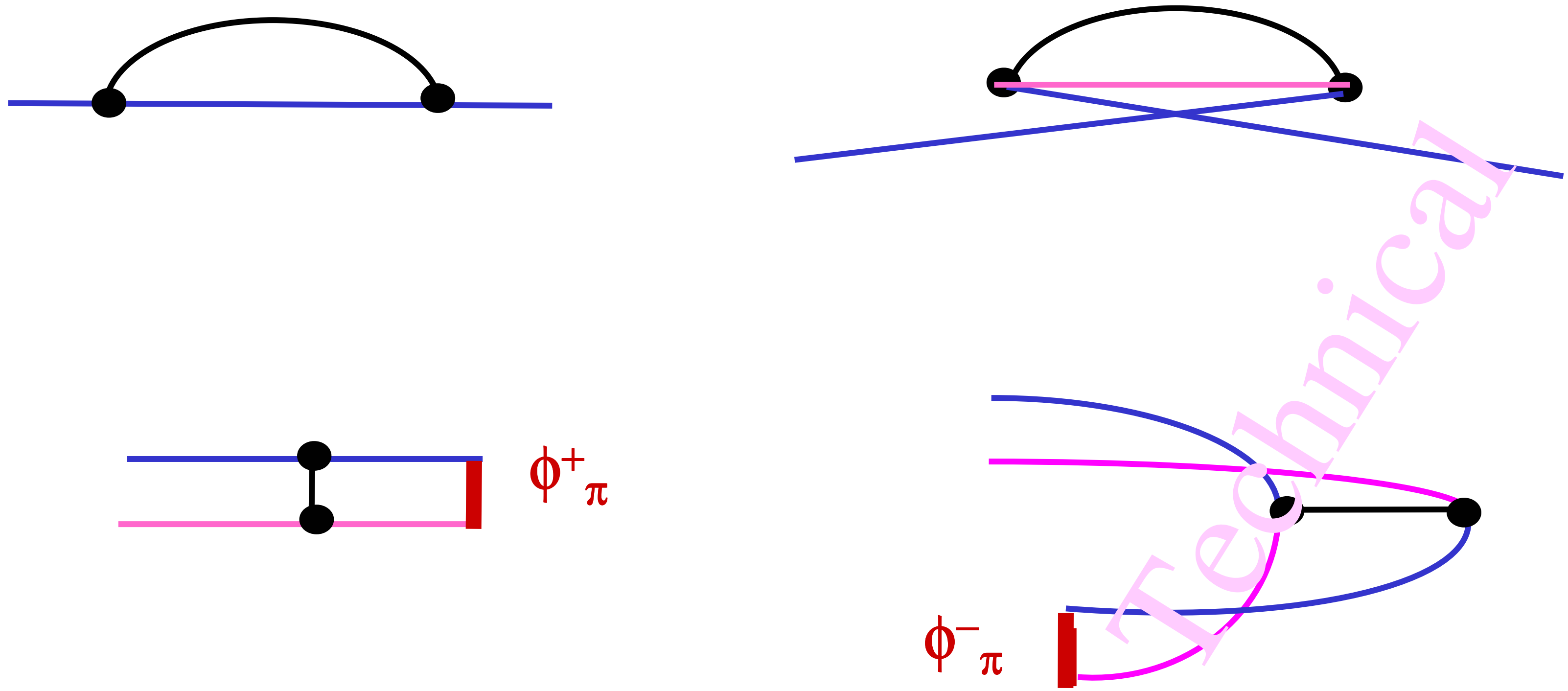
Breakthrough: chiral symmetry and quark boundstates,

Y. le Youanc et al 1983 1984, 1985,

PB, Ribeiro1989 ...Llanes-Estrada, PB 2003 in PRD,

quark

meson



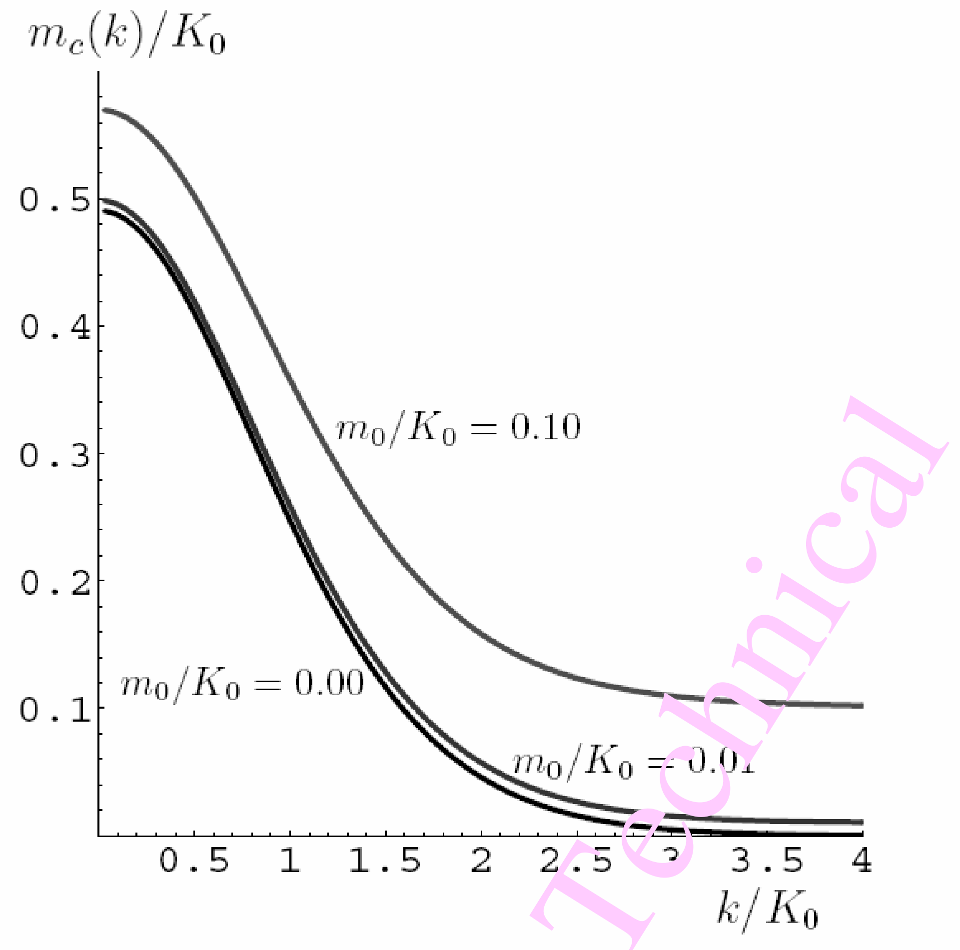
Remark: this also applies to baryons, multiquarks, etc

Quark mass gap equation

$$
\begin{aligned}
0 = u_s^\dagger(k)\Big\{ k\widehat{k}\cdot\vec{\alpha} + m_0\beta - \int \frac{dw'}{2\pi}\frac{d^3k'}{(2\pi)^3} iV(k-k') \\
\sum_{s'}\left[\frac{u(k')_{s'}u^\dagger(k')_{s'}}{w'-E(k')+i\epsilon} - \frac{v(k')_{s'}v^\dagger(k')_{s'}}{-w'-E(k')+i\epsilon}\right]\Big\} v_{s''}(k) \\
E(k) = u_s^\dagger(k)\Big\{ k\widehat{k}\cdot\vec{\alpha} + m_0\beta - \int \frac{dw'}{2\pi}\frac{d^3k'}{(2\pi)^3} iV(k-k') \\
\sum_{s'}\left[\frac{u(k')_{s'}u^\dagger(k')_{s'}}{w'-E(k')+i\epsilon} - \frac{v(k')_{s'}v^\dagger(k')_{s'}}{-w'-E(k')+i\epsilon}\right]\Big\} u_s(k), \quad (4)
\end{aligned}
$$

Meson Bethe-Salpeter equation

$$
\begin{aligned}
\phi^+(k,P) &= \frac{u^\dagger(k_1)\chi(k,P)v(k_2)}{+M(P)-E(k_1)-E(k_2)} \\
\phi^{-t}(k,P) &= \frac{v^\dagger(k_1)\chi(k,P)u(k_2)}{-M(P)-E(k_1)-E(k_2)} \\
\chi(k,P) &= \int \frac{d^3k'}{(2\pi)^3} V(k-k')\left[u(k_1')\phi^+(k',P)v^\dagger(k_2')\right. \\
&\quad \left. +v(k_1')\phi^{-t}(k',P)u^\dagger(k_2')\right] \qquad (6)
\end{aligned}
$$



The constituent quark masses $m_c(k)$, solutions of the mass gap equation, for different current quark masses m_0.

For instance with a quadratic confinement we get

	$V^{++} = V^{--}$
spin-indep.	$-\frac{d^2}{dk^2} + \frac{\mathbf{L}^2}{k^2} + \frac{1}{4}\left({\varphi'_q}^2 + {\varphi'_{\bar q}}^2\right) + \frac{1}{k^2}\left(\mathcal{G}_q + \mathcal{G}_{\bar q}\right) - U$
spin-spin	$\frac{4}{3k^2}\mathcal{G}_q\mathcal{G}_{\bar q}\mathbf{S}_q \cdot \mathbf{S}_{\bar q}$
spin-orbit	$\frac{1}{k^2}\left[\left(\mathcal{G}_q + \mathcal{G}_{\bar q}\right)\left(\mathbf{S}_q + \mathbf{S}_{\bar q}\right) + \left(\mathcal{G}_q - \mathcal{G}_{\bar q}\right)\left(\mathbf{S}_q - \mathbf{S}_{\bar q}\right)\right] \cdot \mathbf{L}$
tensor	$-\frac{2}{k^2}\mathcal{G}_q\mathcal{G}_{\bar q}\left[(\mathbf{S}_q \cdot \hat{k})(\mathbf{S}_{\bar q} \cdot \hat{k}) - \frac{1}{3}\mathbf{S}_q \cdot \mathbf{S}_{\bar q}\right]$
	$V^{+-} = V^{-+}$
spin-indep.	0
spin-spin	$-\frac{4}{3}\left[\frac{1}{2}\varphi'_q\varphi'_{\bar q} + \frac{1}{k^2}\mathcal{C}_q\mathcal{C}_{\bar q}\right]\mathbf{S}_q \cdot \mathbf{S}_{\bar q}$
spin-orbit	0
tensor	$\left[-2\varphi'_q\varphi'_{\bar q} + \frac{2}{k^2}\mathcal{C}_q\mathcal{C}_{\bar q}\right]\left[(\mathbf{S}_q \cdot \hat{k})(\mathbf{S}_{\bar q} \cdot \hat{k}) - \frac{1}{3}\mathbf{S}_q \cdot \mathbf{S}_{\bar q}\right]$

The positive and negative energy spin-independent, spin-spin, spin-orbit and tensor potentials, computed exactly in the framework of the simple density-density harmonic model of eq. (2). $\varphi'(k)$, $\mathcal{C}(k)$ and $\mathcal{G}(k) = 1 - S(k)$ are all functions of the constituent quark(antiquark) mass.

Technical

In the limit where the dynamicalq uark masss is much smaller than the average momentum of the meson, the spin-dependent potentials conspire to remove all explicit dependence in L or S, the equations only depend on J.

Here we show the equations for the cases where j=l+-1 and j=l

$$\left[-\frac{d^2}{dk^2}+2k-\frac{1}{k^2}+\frac{j(j+1)}{k^2}\right]\nu(k) \;=\; M\nu(k)$$

$$\left[-\frac{d^2}{dk^2}+2k-\frac{2}{k^2}+\frac{j(j+1)}{k^2}\right]\nu(k) \;=\; M\nu(k)$$

Technical

3. Linear Regge Trajectories in the ultrarelativisic limit

The resulting meson spectrum is solved, and the excited chiral restoration is recovered, for all mesons with J > 0. Applying the ultrarelativistic simplification to a linear equal-time potential, linear Regge trajectories are obtained, for both angular and radial excitations. The spectrum is also compared with the semi-classical Bohr-Sommerfeld quantization relation. However the excited angular and radial spectra do not coincide exactly.

In the ultrarelativistic limit, the equal-time Bethe-Salpeter boundstate equations can be simplified to a schrodinger-like equation, now for a linear confining potential

$$(2\,p + \sigma\ r)\,\nu = E\nu \ ,$$

where the momentum in the radial equation depends on the angular momentum,

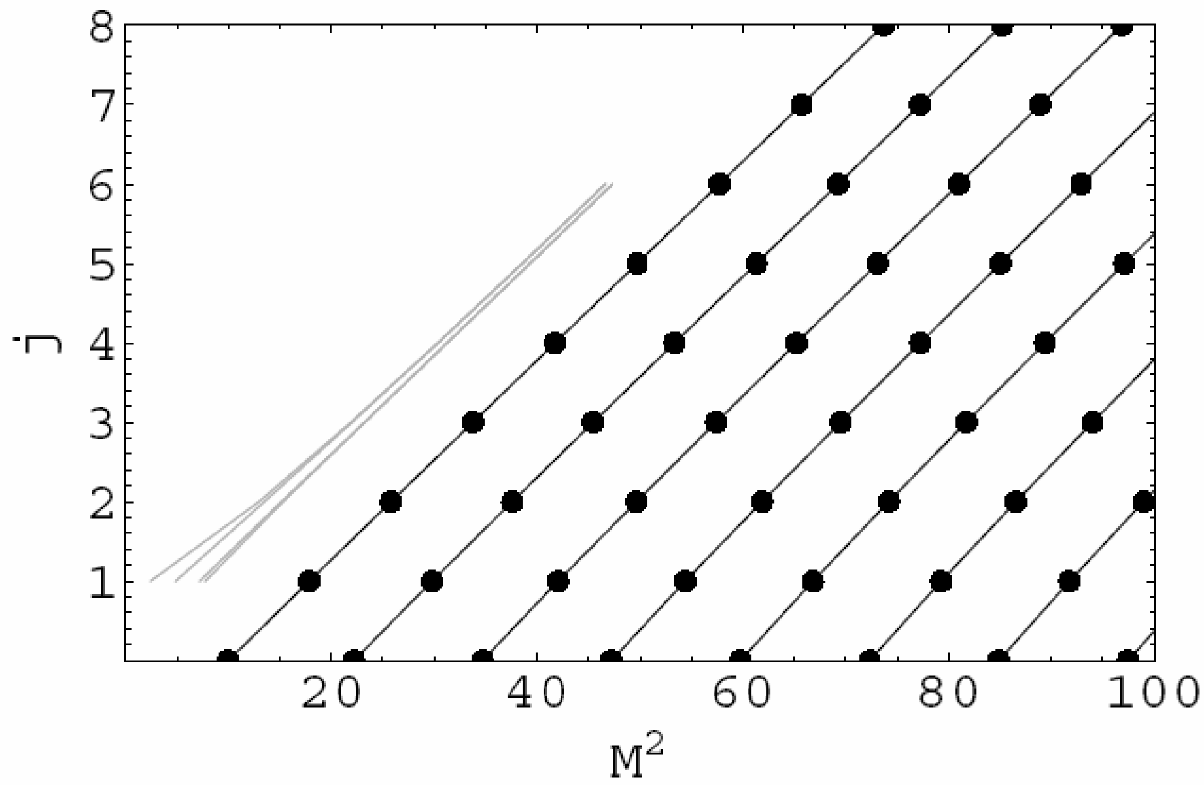
Technical

$$\widehat{p} = \sqrt{-\frac{d^2}{dr^2} + \frac{j(j+1)}{r^2}} \ .$$

In this limit it is easier to get the spectrum of mesons, and to study the Regge trajectories.

The theoretical slope tends to,

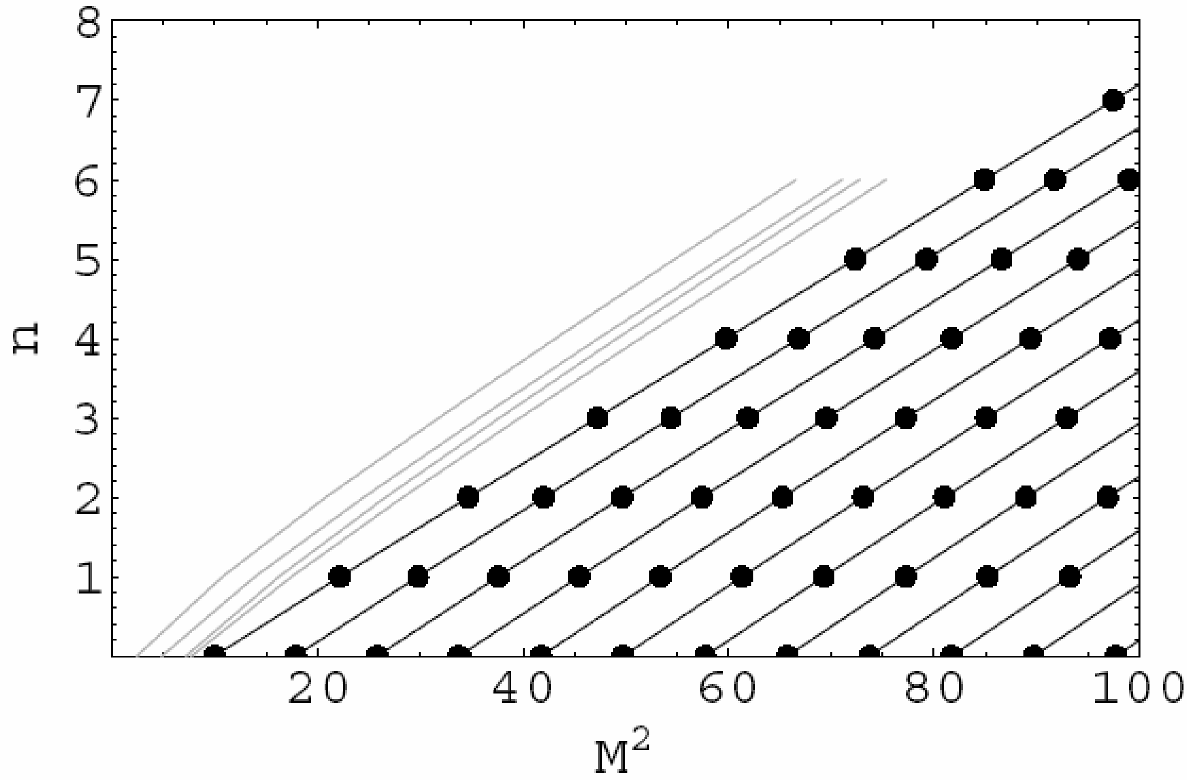
$$\alpha = \frac{1}{8} \, ,$$



We show the quasi-linear Regge trajectories, of j as a function of M^2. Each line corresponds to a fixed radial n, increasing from left to right. The M are the masses of the light-light mesons, in dimensionless units of $\sigma = 1$, computed with the ultra-relativistic equal time chiral degenerate Schrödinger equation (13). In grey we also show the trajectories with $n = 0$ and starting at $j = 1$ of Ref. [10].

The theoretical slope tends to,

$$\beta = \frac{1}{4\pi} ,$$



We show we show the quasi-linear Regge trajectories, of n as a function of M^2. Each line corresponds to a fixed angular j, increasing from left to right. The M are the masses of the light-light mesons, in dimensionless units of $\sigma = 1$, computed with the ultra-relativistic equal time chiral degenerate Schrödinger equation (13). In grey we also show the trajectories with $j = 1$ of Ref. [10].

4. Search for a principal quantum number with classical orbits

We then search, with the classical Bertrand theorem, for central potentials producing always classical closed orbits with the ultrarelativistic kinetic energy. We find that no such potential exists, and this implies that no exact larger degeneracy can be obtained in our equal-time framework, with a single principal quantum number comparable to the non-relativistic Coulomb or harmonic oscillator potentials. Nevertheless we find plausible that the large experimental approximate degeneracy will be modelled in the future by quark models beyond the present state of the art.

Defining,

$$u = 1/r$$

the energy E and angular momentum L conservation imply,

$$\frac{d^2}{d\theta^2}u + u = J(u)$$

$$J(u) = -\frac{E - V(1/u)}{c^2 L^2}\frac{d}{du}V(1/u)$$

We find that classical closed orbits imply that β is an integer,

$$\beta^2 = 1 - \frac{d}{du}J\Big|_{u=u_0} \quad ,$$

However with 2 mechanical constants E and L this cannot be always the case

To conclude, with equal time potentials, we get

[hep-ph/0703114] to be published in **PRD (2007)**,

- **chiral symmetry restoration for excited mesons** ☺
- **linear trajectories in the ultrarelativistic limit for a linear potential** ☺
- **but no principal quantum number, no large degeneracy** ☹

This remains an open and interesting problem!

Next step: extend the quark model to recover the principal quantum number, either with strings or with retardation