Exotica possibility of new observations by BES

Ailin-Zhang
Department of Physics, Shanghai University, China

October 10, 2007
Incompleteness of existed studies of four-quark state and glueball with QCD sum rule approach is analyzed. The masses of the lowest lying scalar and pseudo-scalar glueball were determined with sum rules. The masses of some four-quark states and their first orbital excitations were attempted to be obtained through a combination of the sum rule with the constituent quark model. Exotica possibility of the new observations by BES is discussed.
• QCD sum rules and exotic states
• $0^{++}$ and $0^{--}$ glueballs
• $0^{++}$ and $1^{--}$ tetraquark states
• Exotica possibility of the new observations by BES
• Conclusions and discussions
QCD sum rules and exotic states

♠ QCD (SVZ) sum rules


Nonperturbative method of relating fundamental parameters of QCD Lagrangian and vacuum to parameters of hadrons

QCD & Vacuum \textit{sum rule} Hadron
• Correlator

\[ i \int d^4 x e^{i q x} \langle 0 | T(j_\Gamma(x) j_\Gamma^\dagger(0)) | 0 \rangle \]  

\( j_\Gamma(x) \): interpolating current (local operator)

• Dispersion relation

\[ \Pi_P(q^2) = \frac{1}{\pi} \int ds \frac{\text{Im} \Pi(s)}{s - q^2 - i\epsilon} \]  

Frascati, Oct 10, 2007

Ailin-Zhang, Shanghai University 4/49
\[ i \int d^4 x e^{i q x} T(j_\Gamma(x)j^{\dagger}_\Gamma(0)) = \sum_n C_n^\Gamma(q^2)O_n \] (3)

Hard (coefficients) + soft (condensates)

fundamental parameters of QCD Lagrangian (coefficients) + universal parameters of vacuum (condensates)

Spectral density

\[ Im \Pi(s) \sim \text{bound state (or resonance) + continuum} \]

Duality relation: \( Im \Pi(s) = Im \Pi^{QCD}(s), \ s > s_0 \) (threshold)
• Borel transformation

♠ Interpolating currents (Local operators)

To detect the properties of hadrons, suitable interpolating currents corresponding to the hadrons are employed.

Can local operators detect internal structures in exotic hadrons?

♠ Exotic states

Normal hadron: $q\bar{q}$ Meson; $qqq$ Baryon
Glueball: $GG$, $GGG$ ···
Multi-quark state: $(q\bar{q})(q\bar{q})$, $(qq)(\bar{q}\bar{q})$, $H$, dibaryon, ···
Hybrid: $q\bar{q}g$
The existence of glueball was first mentioned by Fritzsch and Gell-Mann. V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B165**, 67 (1980)

\[ j_s = \alpha_s G^a_{\mu\nu} G^a_{\mu\nu} \]  

\[ m_\sigma = 700 \text{ MeV} \]

\[ m = 1.5 \pm 0.2 \text{ GeV} \]


Moment, \( m = 1.7 \text{ GeV} \)


Gaussian sum rule, Instanton, Double resonance model

Heavier: \( m = 1.4 \text{ GeV} \); Lighter: \( M \approx 1.0 - 1.25 \text{ GeV} \)


Instantons,

\[ m = 1.25 \pm 0.2 \text{ GeV} \]
$\diamondsuit$  $0^{-+}$ glueball


$$j_{ps} = \alpha_s G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a$$  \hspace{1cm} (5)

\[ m = 2 - 2.5 \text{ GeV} \]


\[ m = 2.05 \pm 0.19 \text{ GeV} \]
Gaussian sum rule, Higher-loop perturbative contributions, Instantons

\[ m = 2.65 \pm 0.33 \text{ GeV}, \Gamma < 530 \text{ MeV} \]


Instanton, Topological charge screening effect

\[ m = 2.2 \pm 0.2 \text{ GeV} \]
three gluons glueballs

\[ 0^{++} \text{ scalar} \]


\[ j_{s3g} = g^3 f_{abc} G^{a}_{\mu\nu} G^{b}_{\nu\rho} G^{c}_{\rho\mu} \]  \hspace{1cm} (6)

\[ M_{s3g} = 3.1 \text{ GeV} \]
0−+ pseudoscalar


\[ j_{ps3g} = g^3 f_{abc} \tilde{G}^a_{\mu\nu} \tilde{G}^b_{\nu\rho} \tilde{G}^c_{\rho\mu} \]  \hspace{0.5cm} (7)

\[ M_{ps3g} = 1.9 - 2.7 \text{ GeV} \]
Two gluons glueballs will mix with three gluons glueballs!

Problems:

♣ Can these currents detect the corresponding glueballs?
♣ How to deal with the mixing effect?
Hadron scattering amplitudes: four-quark state
J.L. Rosner, Phys. Rev. Lett. 21, 950 (1968)

- MIT bag model: \((Q^2)(\bar{Q}^2)\)
spectrum, decays

- Color junction model: \(qq\bar{q}\bar{q}\)
spectrum, decay
Hong-Mo Chan and H. Högaasen, Phys. Lett. B72, 121 (1977)
• Potential model: $qq\bar{q}\bar{q}$


• Effective Lagrangian:

● Relativistic quark model:

● QCD sum rules:

- Other works:

N.N. Achasov and V.N. Ivanchenko, Nucl. Phys. B315, 465 (1989);
J.R. Pelaez, Phys. Rev. Lett. 92, 102001 (2004);
I.V. Anikin, B. Pire and O.V. Teryaev, Phys. Lett. B626, 86 (2005);
Review:


Four-quark states

Four-quark state: consists of four quarks and anti-quarks
Intrinsic quarks/anti-quarks may make clusters

Spacial extension of clusters:

\[(qq)(\bar{q}\bar{q}), \quad (q\bar{q})(q\bar{q})\]
Diquark $qq$ cluster: strong correlation between pairs of quarks

The diquark correlation may be most important for the light multi-quark states

Color, Flavor, Spin, et al

Molecule state, Color $8 \otimes 8$ $(q\bar{q})(q\bar{q})$ state

Color, Flavor, Spin, et al
Mixing

The correlations of color, flavor, spin, et al. are inter-related (symmetry constraints)

Color singlet

Crypto-exotic states: Diquark $[qq]$ and anti-diquark $[\bar{q}\bar{q}]$ are in flavor $\bar{3}$ and 3 representation, respectively, and the tetraquark is in $SU(3)_F$ nonet: $3 \otimes \bar{3} = 8 + 1$; $(q\bar{q})(q\bar{q})$ may be in the same flavor representations

The Crypto-exotic four-quark state is in the same flavor representations as those in normal $q\bar{q}$ meson
◇ $(qq)(\bar{q}\bar{q})$ will mix with $(q\bar{q})(q\bar{q})$
◇ Flavor Crypto-exotic four-quark state will mix with $q\bar{q}$

$$|\text{meson} \rangle = |q\bar{q} \rangle + |(qq)(\bar{q}\bar{q}) \rangle + |(q\bar{q})(q\bar{q}) \rangle + \cdots$$

Intrinsic color, flavor configurations could not be distinguished except that some special observable is established
Unfortunately, no such an observable has been definitely set up
♣ Quark dynamics

• Tetraquark state

• Molecule state

• Color $8 \times 8 (q\bar{q})(q\bar{q})$ state: ?

Not very clear!
♦ Interpolating currents

• \((q\bar{q})(q\bar{q})\)


• \((q\bar{q})^2, (qq)(q\bar{q})\)

Thomas Schafer, Phys. Rev. D68, 114017 (2003): ”Instantons and scalar multiquark states: From small to large \(N_c\)”
• \((cq)(\bar{q}\bar{q})\)


• \((cu)(\bar{s}\bar{u})\)

Hungchong Kim and Yongseok Oh, Phys. Rev. **D72**, 074012 (2005): ”\(D_s(2317)\) as a four-quark state in QCD sum rules”

• \((ud)(\bar{s}\bar{s})\)

Hua-Xing Chen, A. Hosaka and Shi-Lin Zhu, Phys. Rev. **D74**, 054001 (2006): ”Exotic tetraquark \(ud\bar{s}\bar{s}\) of \(J^P = 0^+\) in the QCD sum rule”
Leading order in perturbative contribution

Structure in current (operator) picture → Structure in constituent quark picture

Diquark in operator picture → Diquark in constituent quark picture
Problems:

♣ Fierz transformation (re-arrangement)

\[(qq)(\bar{q}\bar{q}) \rightarrow (q\bar{q})(q\bar{q})\]

\[j_{(dq)}^2 = \frac{1}{4} \left\{ (\bar{q}\gamma_5 \bar{\tau}^a q)(\bar{q}\gamma_5 \bar{\tau}^a q) + (\bar{q}\bar{\tau}^a q)(\bar{q}\bar{\tau}^a q) + (\bar{q}\gamma_\mu \bar{\tau}^a q)(\bar{q}\gamma_\mu \bar{\tau}^a q) \right. \]
\[+ \left. (\bar{q}\gamma_5 \gamma_\mu \bar{\tau}^a q)(\bar{q}\gamma_5 \gamma_\mu \bar{\tau}^a q) - \frac{1}{2}(\bar{q}\sigma_{\mu\nu} \bar{\tau}^a q)(\bar{q}\sigma_{\mu\nu} \bar{\tau}^a q) \right\} \]

where \(\bar{\tau}^a = (\bar{\tau}, i)\)

♣ Renormalization group improvement

All these currents will mix with each other under the renormalization
Conclusions:

• In view of the sum rule approach, there is no definite difference among these currents. The internal constituent quark structures of multi-quark state can hardly be detected through the couplings of interpolating currents to hadrons.

• In principle, there is no direct way to turn the operator picture into the constituent quark picture.

• Diquark: meaningful in constituent quark picture; not meaningful in operator picture in the framework of QCD sum rules.
Prescriptions (?)

- Mixing

Interpolating currents: Mixing currents \(((qq)(\bar{q}\bar{q}) + (q\bar{q})(\bar{q}q))\)

Hadrons saturation: Mixture of hadrons

- Constituents

Diquark: SVZ sum rule

Four-quark state: Constituent quark model
Diquark and SVZ sum rule

The diquark picture was applied to weak hadron decays and was postulated to be computed reliably with SVZ sum rules


The diquark current with flavor \((sq)\)

\[
\vec{j}_i(x) = \epsilon_{ijk} s_j^T(x) CO q_k(x) 
\]  

(8)
An updated analysis was performed
Ailin Zhang, Tao Huang and Tom Steele, Phys. Rev. D76, 036004 (2007)

The most "suitable" masses of diquark $m_{qq}$ and $m_{sq}$:

$$m_{qq} \sim 400 \text{ MeV}, \quad s_0 = 1.2 \text{ GeV}^2$$

$$m_{sq} \sim 460 \text{ MeV}, \quad s_0 = 1.2 \text{ GeV}^2$$

The mass scale of diquark is the same as that of the constituent quark

The results obtained here are consistent with the fit of L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer (Phys. Rev. D71, 014028 (2005))

$$m_{[ud]} = 395 \text{ MeV}, \quad m_{[sq]} = 590 \text{ MeV}$$
The diquark masses may be determined with QCD sum rule.

The mass determined by sum rules may be regarded as the constituent diquark masses(?).
**Fig. 1:** $\tau$ dependence of $m_{0+}^2$ for $qq$ good diquark with $s_0 = 1.2$ GeV$^2$. 
Fig. 2: $\tau$ dependence of $m_{0+}^2$ for $sq$ good diquark with $s_0 = 1.2$ GeV$^2$. 
Light tetraquark state


Mass of the $L = 0$ tetraquark state:

$$M \approx 2m_{qq} - 3(\kappa_{qq})^3$$

$(\kappa_{qq})^3 = 103$ MeV, $(\kappa_{sq})^3 = 64$ MeV
Mass of the $L = 1$ excited tetraquark state:

$$M \approx 2m_{qq} - 3\langle \kappa_{qq} \rangle \bar{3} + B'_q \frac{L(L+1)}{2}$$

$m_{qq}$: constituent diquark mass

$L$: the orbital angular momentum between the diquark and the anti-diquark

$$B'_q = \left[ \frac{\alpha_s(m_{qq}^2)}{\alpha_s(m_q^2)} \right]^2 \frac{m_{qq}}{m_q} B_q, \quad B_q = 0.495 \text{ GeV}$$

$B'_q$ are very sensitive to $\Lambda$. 

Frascati, Oct 10, 2007

Ailin-Zhang, Shanghai University 36/49
Neutral tetraquark states consist of "good" diquark: the P-parity and the C-parity are the same \((-1)^L\).

Possible \(J^{PC}\) of neutral tetraquark state: \(0^{++} (L = 0), 1^{--} (L = 1), 2^{++} (L = 2), \ldots\)

The constituent diquark mass is approximately regarded as the mass determined by QCD sum rules.
Our results:

\[
0^{++} \ [qq][\bar{q}q] : \sim 490 \ MeV \\
0^{++} \ [sq][\bar{q}q] : \sim 610 \ MeV \\
0^{++} \ [sq][\bar{s}q] : \sim 730 \ MeV \\
1^{--} \ [qq][\bar{q}q] : \sim 490 + B'_q \ MeV \\
1^{--} \ [sq][\bar{q}q] : \sim 610 + B'_q \ MeV \\
1^{--} \ [sq][\bar{s}q] : \sim 730 + B'_s \ MeV
\]

Flavor dependence is explicit!

Improvement(?): Coulombic + linear confinement

It is reasonable to identify \( f_0(600) \) (or \( \sigma \)), \( f_0(980) \), \( a_0(980) \) and the unconfirmed \( \kappa(800) \) as the \( 0^{++} \) light tetraquark states.
Recently observed by BES (BESII detector: 58 million events sample of $J/\Psi$ decays)

- $p \bar{p}$ enhancement was observed by BES in the radiative decay ($\text{Phys. Rev. Lett. 91, 022001 (2003)}$); PDG

$$J/\Psi \rightarrow \gamma p \bar{p}$$

with $M = 1859^{+3}_{-10}(\text{stat})^{+5}_{-25}(\text{sys})$ (below $2m_p$) and $\Gamma < 30$ MeV if interpreted as a single $0^{-+}$ resonance

$C = +1$, Consistent with either $J^{PC} = 0^{-+}$ or $0^{++}$ quantum number assignment

\[ B^- \rightarrow K^- p\bar{p} \]

BaBar (B. Aubert \textit{et al.}, Phys. Rev. \textbf{D74}: 051101 (2006))

\[ B^0 \rightarrow \bar{D}^0 p\bar{p}, \quad B^0 \rightarrow \bar{D}^{*0} p\bar{p} \]

Final state interaction? Baryonium? Threshold cusp?
• $X(1835)$ was observed by BES in the decay (Phys. Rev. Lett. 95, 262001 (2005); PDG)

$$J/\Psi \rightarrow \gamma \pi^+ \pi^- \eta',$$

with $M = 1833.7 \pm 6.1\,(\text{stat}) \pm 2.7\,(\text{syst})$ MeV and $\Gamma = 67.7 \pm 20.3 \pm 7.7$ MeV

Statistical significance of $7.7\sigma$

Consistent with expectations for the state that produces the strong $p\bar{p}$ mass threshold enhancement

$0^{-+}$ glueball? Baryonium?
• $X(1812)$ was observed by BES in the doubly OZI-suppressed decay (Phys. Rev. Lett. 96, 162002 (2006); PDG)

\[ J/\Psi \rightarrow \gamma \omega \phi \]

with $M = 1812^{+19}_{-26}(\text{stat}) \pm 18(\text{syst})$ MeV, $\Gamma = 105 \pm 20 \pm 28$ MeV

Statistical significance of more than $10\sigma$

Favors $J^P = 0^+$

Rescatterings? Four-quark state?; Glueball? Hybrid?
• $X(1576)$ was observed by BES in the decay (Phys. Rev. Lett. 97, 142002 (2006); PDG)

$$J/\Psi \rightarrow K^+ K^- \pi^0$$

with pole position $1576^{+49}_{-55}(\text{stat})^{+98}_{-91}(\text{syst})$ MeV - $i(409^{+11}_{-12}(\text{stat})^{+32}_{-67}$ MeV

Broad peak

$$J^{PC} = 1^{--}$$

Final state interaction? Tetraquark state?
♣ Possible interpretations
Final state interaction?
Threshold cups?

• Glueballs?

$0^{++}$ or $0^{-+}$

$p\bar{p}$ enhancement

$X(1835)$

$X(1812)$,

Lower mass compared with the expected pure $0^{-+}$ glueball
• Four-quark states?

$0^{++}$ or $0^{-+}$

All of them lie above the $0^{++}$ or $0^{-+}$ tetraquark states

$1^{--}$

$X(1576)$: first orbital excited $1^{--}$ tetraquark state? Orbital excitation of $a_0(980)$ or $f_0(980)$?

Exotic: very large excited energy $\sim 596$ MeV
problems:

A whole family of $1^--$ tetraquark states corresponding to orbital excitations of $f_0(600)$ and $\kappa(800)$ are expected (?)
• The structure of interpolating currents has no direct correspondence to the constituent structure of hadrons

• Masses of diquarks were determined through SVZ sum rule, and masses of tetraquark states were constructed in terms of these diquarks

• The new observations by BES are unlike to be the pure $0^{-+}$ pseudo-scalar glueball, they are unlikely to be the light four-quark states except that $X(1576)$ may be an exotic first orbital excited $(sq)(\bar{s}\bar{q})$ tetraquark state either
Thank you!