



A method to identify hadronic molecules

and its application to $f_0(980)$ and $a_0(980)$

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Key references:

S. Weinberg, Phys. Rev. **130**, 776 (1963); **131**, 440 (1963); **137** B672 (1965).

V. Baru et al., Phys. Lett. B **586** (2004) 53; C.H. et al., arXiv:0707.0262 [hep-ph], PRD in print

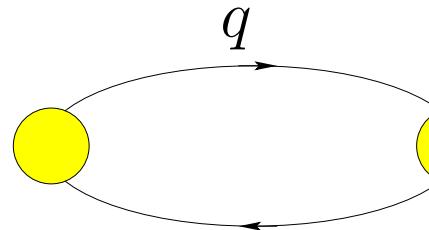
The Idea



Difference between bound states of quarks or hadrons?

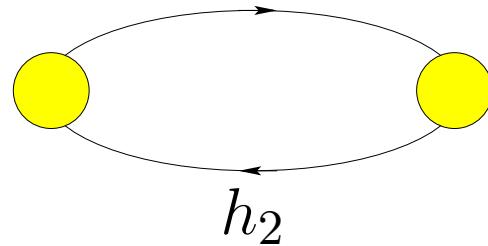
Hadrons can go on-shell → non-analyticities

Quark–
loop:



= (Polynomial in E)

Hadron–
loop:



$$= \begin{cases} i\mu\sqrt{E\mu} + (\text{Pol. in } E); & E > 0 \\ -\mu\sqrt{-E\mu} + (\text{Pol. in } E); & E < 0 \end{cases}$$

Focus on resonances very near thresholds



Weinberg (1963)

Expand in terms of non-interacting quark and meson states

$$|\Psi\rangle = \begin{pmatrix} \cos(\theta)|\psi_0\rangle \\ \sin(\theta)\chi(\mathbf{p})|h_1 h_2\rangle \end{pmatrix},$$

here $|\psi_0\rangle$ = quark state and $|h_1 h_2\rangle$ = two-hadron continuum with $\langle\Psi|\Psi\rangle = 1$ and $\int d^3 p \chi^2 = 1$. Let

$$\mathcal{Z} = |\langle\Psi|\psi_0\rangle|^2 = \cos(\theta)^2$$

Equals probability to find the bare state in the physical state
→ the quantity of interest!

Use Schrödinger equation to fix \mathcal{Z} .

When can we make model-independent statements?

Coupled channels II



The Schrödinger equation reads

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}_{hh}^0 \end{pmatrix},$$

Note: \hat{H}_{hh}^0 contains only meson kinetic terms!

Introducing the transition form factor $\langle \psi_0 | \hat{V} | hh \rangle = f(p^2)$,

$$\frac{\partial}{\partial E} \left(\bullet \text{---} \text{---} \bullet \right) = \frac{1}{Z} - 1 = \tan^2 \theta = \int \frac{f^2(p^2) d^3 p}{(\frac{p^2}{m} + \epsilon)^2} = \frac{\pi^2 m^2 f(0)^2}{\sqrt{m\epsilon}}$$

for **s-waves** and ϵ smaller than any scale of problem; then it depends only on $f(0)$ =effective coupling and binding energy ϵ
 \rightarrow model-independent!

Discussion



We can now define **effective coupling**
for the **scattering amplitude** we get with

$$4\pi^2 m f^2 = g = 4\sqrt{\epsilon/m}(1/\mathcal{Z} - 1)$$

$$\begin{aligned} F_{MM} &= - \frac{g/2}{E + \epsilon + (g/2) \left(\sqrt{m\epsilon} + i\sqrt{mE} \right)} + \dots \\ &= - \left(\frac{1}{16\pi(2m)^2} \right) \frac{g_{\text{eff}}^2}{E + \epsilon} + \dots \quad (\text{rel.-norm.}) \end{aligned}$$

$$\rightarrow \frac{g_{\text{eff}}^2}{4\pi} = \mathcal{Z}8m^2g = 32(1 - \mathcal{Z})m\sqrt{\epsilon m} \leq 32m\sqrt{\epsilon m}$$

Equivalent to, e.g.,

Morgan and Pennington (1991), Törnqvist (1995)

Inelastic resonances



now the scattering amplitude reads

Baru et al. (2004)

$$F_{MM} = -\frac{g/2}{E + \epsilon + (g/2) \left(\sqrt{m\epsilon} + i\sqrt{mE} \right) + i\Gamma_l/2} + \dots$$

where we assume the light threshold far away ...

Then:

- no new momentum/energy dependence
- \mathcal{Z} remains unchanged

and therefore

g_{eff} contains the same structure information



Summary: the **structure information** is hidden in the **effective coupling**, adjusted to experiment,
independent of the phenomenology
used to introduce the pole(s)

Focus on light scalar mesons $f_0(980)$ and $a_0(980)$
poles located very close to the $\bar{K}K$ threshold ($2m_K = 992$ MeV)
→ Binding energy $\epsilon \simeq 10$ MeV IDEAL

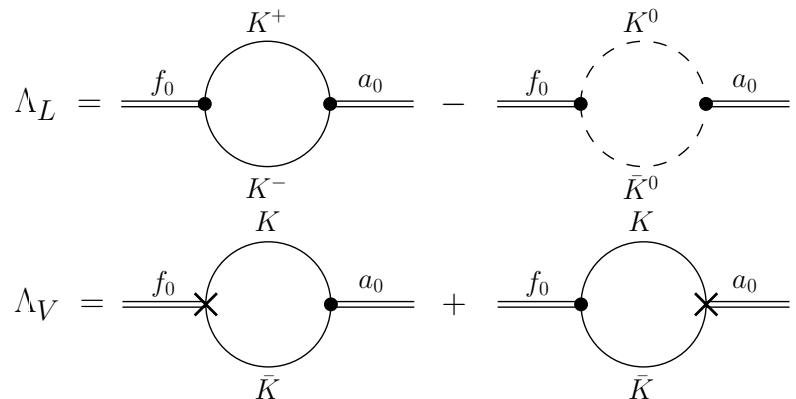
Our analyses for radiative decays $\phi \rightarrow \gamma s$, $s \rightarrow \gamma\gamma$ are
→ consistent with molecular interpretation for f_0 ; talk A. Nefediev
→ less clear for a_0 ;
it either has non-molecular component, or is a virtual state

Are a_0 and f_0 relatives?



To understand degree of relationship of $a_0(980)$ and $f_0(980)$
study a_0 - f_0 mixing

For molecules → with cut



$$\sim \sqrt{(m_d - m_u)/m_s}$$

Achasov et al. (1979)

$$\sim (m_d - m_u)/m_s$$

C.H. et al. (2007)

For compact states in addition → no cut

$$\begin{array}{c} f_0 \\ \hline \times \\ a_0 \end{array}$$

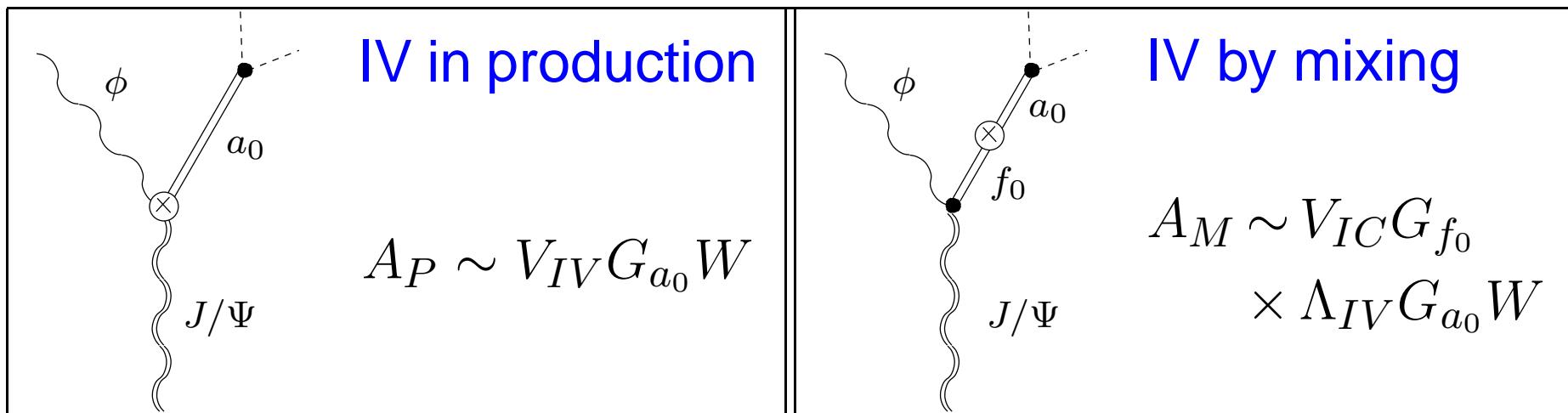
$$\sim (m_d - m_u)/m_s$$

should lead to observable differences!

Why this is observable

Nullexperiment: $J/\Psi \rightarrow \phi\pi^0\eta$

Wu et al. (2007), C.H. et al. (2007)



Ratio, using

C.H. (2004)

$$V_{IV} \sim (m_d - m_u)/m_s V_{IC}, \quad \Lambda_{IV} \sim m_K^2 \sqrt{(m_d - m_u)/m_s},$$
$$G_{f_0}(m_{a_0}^2) \sim (s - m_{f_0}^2 + i m_{f_0} \Gamma_{f_0})^{-1} \sim -i/(m_{f_0} \Gamma_{f_0})$$

$$A_P/A_M = V_{IV}/(V_{IC} G_{f_0} \Lambda_{IV}) \sim \sqrt{\frac{m_d - m_u}{m_s}} \frac{m_f \Gamma_f}{m_K^2} \simeq 4\%$$

Mixing should dominate IV in production



We use the **unitarized chiral approach**; take

$$\mathcal{L}^{(2)} = \frac{f^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle + Z f^4 \langle Q U Q U^\dagger \rangle .$$

Then, using $\epsilon \approx \sqrt{3}/4(m_d - m_u)/m_s$

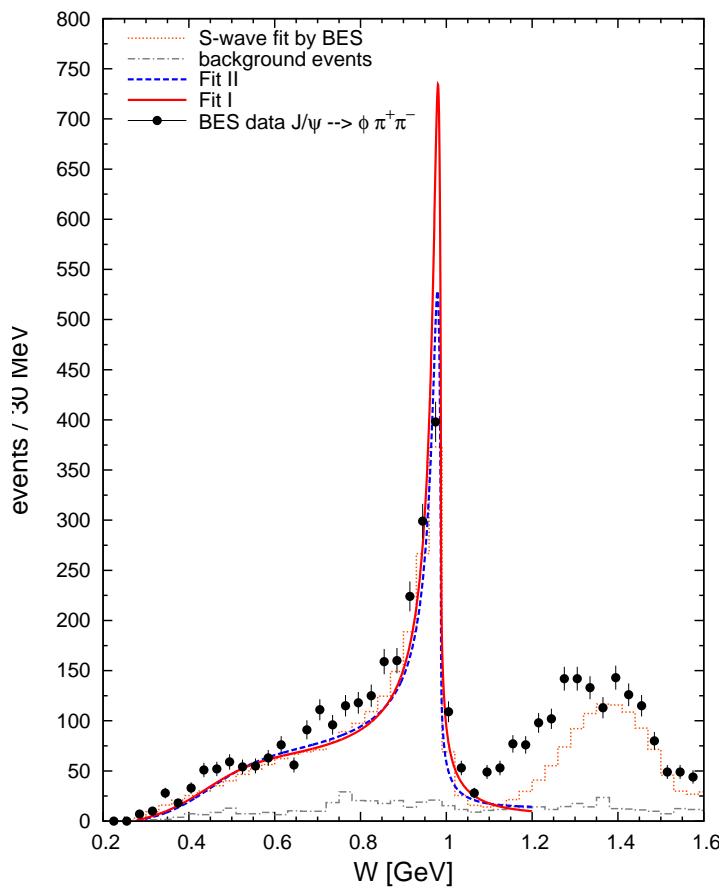
$$\begin{aligned} M_{\pi^+}^2 - M_{\pi^0}^2 &= 2Z e^2 f^2 &= \Delta_\pi \\ M_{K^+}^2 - M_{K^0}^2 &= B(m_u - m_d) + 2Z e^2 f^2 &= \Delta_\pi - \frac{4\epsilon}{\sqrt{3}} (M_K^2 - M_\pi^2) \end{aligned}$$

and

$$\begin{aligned} T(\pi^+ \pi^- \rightarrow K^+ K^-) &= \frac{1}{f_\pi f_K} \left(\frac{s}{4} + \Delta_\pi \right) , \\ T(\pi^0 \pi^0 \rightarrow \pi^0 \eta) &= -\frac{4\epsilon}{3\sqrt{f_\pi^3 f_\eta}} (M_K^2 - M_\pi^2) , \end{aligned}$$

which is unitarized using the **inverse amplitude method**

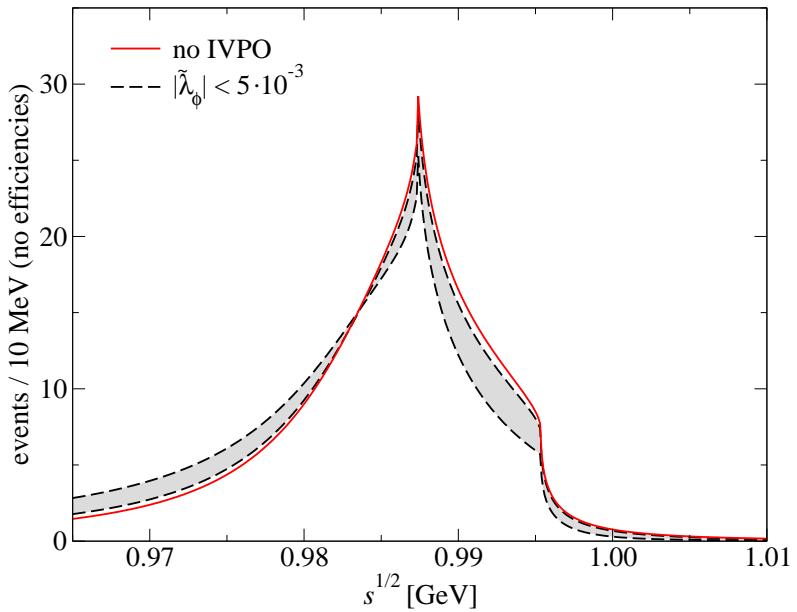
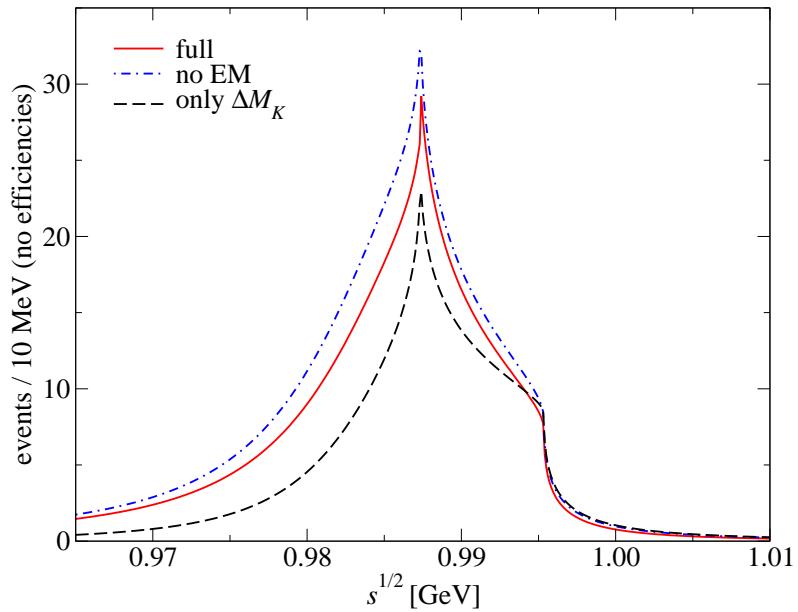
IC Results



- Automatically produces light scalars
Pelaez, Dobado (1993+), Oller et al. (1998+), ...
- Interpreted as non- $\bar{q}q$ states
Pelaez (2004+)
- Consistent with production reactions
Meißner, Oller (2001), Lähde, Meißner (2006)

Allows for consistent inclusion of IV

Results IV



We find:

C.H., B. Kubis, J.R. Pelaez (2007)

- Kaon mass difference dominant effect
 - sensitive to g_{eff}
- Isospin violating vertices significant
- Photon exchange small
- Effect of IV in production small

talk B.S. Zou

see also Wu, Zhao, Zou (2007)



Effective couplings contain important structure information, especially

the larger g_{eff} the larger the molecular component

→ Applied to $f_0(980)$: Predominantly $K\bar{K}$ molecule

talk by A. Nefediev

→ Situation unclear for $a_0(980)$

→ Measurement of $a_0 - f_0$ mixing very important

talk by B.S. Zou