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CONSTRUCTION OF LORENTZ INVARIANT AMPLITUDES FROM REST FRAME WAVE FUNCTIONS IN HQET – APPLICATION TO ISGUR-WISE FUNCTION

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Abstract

Succeeding in predicting/reproducing the heavy-light mesons by our semi-relativistic quark potential model, we examine a method how to calculate the semileptonic weak form factors out of the rest frame wave functions of heavy mesons and numerically calculate the dynamical $1/m_Q$ corrections to those for the process $\bar{B} \to D^{(*)}\ell\nu$ based on our model for heavy mesons. It is shown that nonvanishing expressions for $\rho_1(\omega) = \rho_2(\omega)$ and $\rho_3(\omega) = \rho_4(\omega) = 0$ are obtained in a special Lorentz frame, where $\rho_i(\omega)$ are the parameters used in the Neubert-Rieckert decomposition of form factors. Various values of form factors are estimated, which are compatible with recent experimental data as well as other theoretical calculations.

1 Introduction

Discovery of narrow meson states $D_{s0}(2317)$ by BaBar and $D_{s1}(2460)$ by CLEO and the following confirmation by Belle has driven many theorists to explain these states since the former study of these states using quark potential model seems to fail to reproduce these mass values. More recent experiments have found many other heavy-light mesons, broad $D_0^*(2308)$ and $D_1'(2427)$ mesons by Belle collaboration, which have the quantum numbers $j^P = 0^+$ and 1^+; narrow B and B_s states of $\ell = 1$, $B_1(5720)$, $B_2^*(5745)$, and $B_{s2}^*(5839)$ by CDF and D0 whose decay widths are also narrow because of their decay through the D-waves; and seemingly radial excitations ($n = 2$) of 0^+ $D_{s0}(2860)$ state by BaBar and 1^- $Ds^*(2715)$ state by Belle. Furthermore $c\bar{c}$ quarkonium-like states have been discovered one after another, $X(3872)$, $X(3940)$, $Y(3940)$, $Z(3930)$, and $Y(4260)$. These mesons have been successfully predicted and/or reproduced by our formulation [1, 2, 3], except for the quarkonium-like states. What we need to do next is to show that our approach can also give a method to calculate scattering amplitudes and decay widths using the rest frame wave functions.

Recently there appears a paper [5] in which the authors use the "relativistic" formulation to calculate masses of heavy mesons and apply it to calculate the Isgur-Wise functions for semileptonic B decays. Although this paper adopts a different approach from ours, they present how to calculate higher order corrections in $1/m_Q$ to form factors, which is one of the same purposes as we intend to do in this paper. In our former paper [1], we have developed a semi-relativistic formulation how to calculate the meson masses and wave functions for heavy-light $Q\bar{q}$ system introducing phenomenological dynamics. Meson wave functions obtained thereby and expanded in $1/m_Q$ can be used in principle to calculate ordinary form factors or Isgur-Wise functions and their higher order corrections in $1/m_Q$ for semileptonic weak or other decay processes. However what we have obtained are wave functions in the rest frame so that we need to develop a method to obtain Lorentz invariant amplitudes or Lorentz-boosted wave functions.

The problem how to construct a Lorentz-boosted wave function is in that there is ambiguity to determine space-time coordinates of two composite particles from information of one bound state, in this case a heavy meson. We study four cases of reference frames for composite particles and then give a prescription how to calculate matrix elements of currents using the rest frame wave functions. We give form factors in the zeroth order for four reference frames, show that they agree with each other in the HQET limit, and give those in the first order for just one special reference frame. The lowest order calculation in $1/m_Q$ gives the numerical value of the slope for the Isgur-Wise function at the origin and the semi-leptonic weak form factors are calculated up to the first order in $1/m_Q$. It is derived that *there are no dynamical contributions to the form factors*, i.e., the first order corrections to the wave functions do not contribute to the form factors. Studying $\bar{B} \to D\ell\nu$ and $\bar{B} \to D^*\ell\nu$ processes, physical quantities related to the CKM matrix elements are obtained. The summary and discussions are given for this paper. [1]

2 Results

As an example of this formulation, we have calculated the semi-leptonic weak form factors up to the first order in $1/m_Q$. What we have found in this paper is

1. The Isgur-Wise function has the following form up to the first order in $1/m_Q$ and in $(\omega - 1)$.

   $$\xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3}\bar{\Lambda}^2 \left\langle r^2 \right\rangle\right)(\omega - 1)\,, \qquad \bar{\Lambda} = \bar{\Lambda}_u = \lim_{m_Q \to \infty} \tilde{E}_D = \lim_{m_Q \to \infty} \tilde{E}_B,$$

   and hence

   $$\xi(1) = 1, \qquad \xi'(1) = -\frac{1}{2} - \frac{1}{3}\bar{\Lambda}^2 \left\langle r^2 \right\rangle.$$

   Here since $\bar{\Lambda}$ depends only on light quark mass and we treat only heavy mesons D, D^*, and B which include only u and d light quarks with $m_u = m_d$, the subscript of $\bar{\Lambda}_u$ expresses this fact.

2. We find that there is no contribution from correction terms in the first order of m_Q for the rest frame wave functions to the six form factors. That is, some terms do not contribute to the physical quantities.

3. The first order corrections are derived from phase factors of the wave functions and also given by kinetic terms and there are no contributions from the first order corrections to the wave functions. That is, in the terminology of Neubert and Rieckert in [7],

   $$\rho_1(\omega) \quad = \quad \rho_2(\omega) = -\frac{1}{3} C^1 \bar{\Lambda} \left\langle r^2 \right\rangle (\omega - 1)\,, \quad \rho_3(\omega) = \rho_4(\omega) = 0.$$

4. We have calculated the values for the form factor $\mathcal{F}(\omega)$ at the zero recoil and/or their first derivatives up to the first order in $1/m_Q$ as

   $$\mathcal{F}_D(1) = 1.07, \qquad \mathcal{F}_D{}'(1) = -0.875,$$
   $$\mathcal{F}_{D^*}(1) = \xi(1) = 1, \qquad \mathcal{F}_{D^*}{}'(1) = -1.09,$$
   $$R_1(1) = 1.45, \quad R_1(1) = -0.222, \quad R_2(1) = 0.942, \quad R_2'(1) = 0.0286,$$

   the first equations are obtained by analyzing $\bar{B} \to D\ell\nu$ process and the second by $\bar{B} \to D^*\ell\nu$. These values are consistent with experimental data as well as other theoretical estimates listed in Tables I and II of [5].

[1] This paper is based on the recent paper [6].

5. The above values can be used to estimate the CKM matrix element $|V_{cb}|$ which we have obtained as

$$|V_{cb}| = 0.0387 \pm 0.0060,$$

for the $\bar{B} \to D\ell\nu$ process and

$$|V_{cb}| = 0.0380 \pm 0.0021,$$

for the $\bar{B} \to D^*\ell\nu$ process. These values are consistent with the value in PDG [8)]

$$|V_{cb}| = 0.0409 \pm 0.0018 \quad \text{(exclusive)}.$$

Here to give theoretical predictions for $|V_{cb}|$ we have neglected theoretical undertainties although experimental errors are taken into account.

We have developed a method to obtain the relativistically invariant results using the rest frame wave functions, and to do this the four different Lorentz-boosted frames are adopted to check the validity of our results. The same form is obtained for the Isgur-Wise function in all four cases up to the zeroth order in $1/m_Q$ and the first order in $(\omega - 1)$, however their first order corrections in $1/m_Q$ are not the same for all four cases. Only the case, written as 2-ii) in [6)], i.e., in the Breit frame with $t' = x'^0 = y'^0$ both for $\bar{B}$ and $D^{(*)}$, gives consistent results with the relativistic ones given by [7)].

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