On the gluon content of the η and η' mesons

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HADRON 07
October 8, 2007
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in collab. with Jordi Nadal, JHEP 05 (2007) 6

Purpose: to perform a phenomenological analysis of radiative $V \to P\gamma$ and $P \to V\gamma$ decays, with $V=\rho, K^*, \omega, \phi$ and $P=\pi, K, \eta, \eta'$, aimed at determining the gluonic content of the η and η' wave functions

Outline:

- *Notation*
- *Motivation*
- *A model for $VP\gamma$ $M1$ transitions*
- *Data fitting*
- *Comparison with other approaches*
- *Summary and conclusions*

• *Notation*

We work in a basis consisting of the states

$$|\eta_q\rangle \equiv \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle \qquad |\eta_s\rangle = |s\bar{s}\rangle \qquad |G\rangle \equiv |\text{gluonium}\rangle$$

The physical states η and η' are assumed to be the linear combinations

$$\begin{aligned} |\eta\rangle &= X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|G\rangle \ , \\ |\eta'\rangle &= X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|G\rangle \ , \end{aligned}$$

with $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} + Z^2_{\eta(\eta')} = 1$ and thus $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} \leq 1$

A significant gluonic admixture in a state is possible only if

$$Z^2_{\eta(\eta')} = 1 - X^2_{\eta(\eta')} - Y^2_{\eta(\eta')} > 0$$

Assumptions:

- no mixing with π^0 (isospin symmetry)
- no mixing with η_c states
- no mixing with radial excitations

- *Notation*

In absence of gluonium (standard picture)

$$Z_{\eta(\eta')} \equiv 0$$

$$\begin{aligned} |\eta\rangle &= \cos\phi_P|\eta_q\rangle - \sin\phi_P|\eta_s\rangle \\ |\eta'\rangle &= \sin\phi_P|\eta_q\rangle + \cos\phi_P|\eta_s\rangle \end{aligned}$$

with $X_\eta = Y_{\eta'} \equiv \cos\phi_P$ and $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} = 1$

$$X_{\eta'} = -Y_\eta \equiv \sin\phi_P$$

where ϕ_P is the η-η' mixing angle in the quark-flavour basis related to its octet-singlet analog through

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$$

Similarly, for the vector states ω and ϕ the mixing is given by

$$\begin{aligned} |\omega\rangle &= \cos\phi_V|\omega_q\rangle - \sin\phi_V|\phi_s\rangle \\ |\phi\rangle &= \sin\phi_V|\omega_q\rangle + \cos\phi_V|\phi_s\rangle \end{aligned}$$

where ω_q and ϕ_s are the analog non-strange and strange states of η_q and η_s, respectively.

- *Euler angles*

In presence of gluonium,

glueball-like state
η(1440)?

$$
\begin{aligned}
|\eta\rangle &= X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|G\rangle \\
|\eta'\rangle &= X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|G\rangle \\
|\iota\rangle &= X_\iota|\eta_q\rangle + Y_\iota|\eta_s\rangle + Z_\iota|G\rangle
\end{aligned}
$$

Normalization:

$$
\begin{aligned}
X_\eta^2 + Y_\eta^2 + Z_\eta^2 &= 1 \\
X_{\eta'}^2 + Y_{\eta'}^2 + Z_{\eta'}^2 &= 1 \\
X_\iota^2 + Y_\iota^2 + Z_\iota^2 &= 1
\end{aligned}
$$

Orthogonality:

$$
\begin{aligned}
X_\eta X_{\eta'} + Y_\eta Y_{\eta'} + Z_\eta Z_{\eta'} &= 0 \\
X_\eta X_\iota + Y_\eta Y_\iota + Z_\eta Z_\iota &= 0 \\
X_{\eta'} X_\iota + Y_{\eta'} Y_\iota + Z_{\eta'} Z_\iota &= 0
\end{aligned}
$$

3 independent parameters: ϕ_P, $\phi_{\eta G}$ and $\phi_{\eta' G}$

$$
\begin{pmatrix} \eta \\ \eta' \\ \iota \end{pmatrix} = \begin{pmatrix} c\phi_{\eta\eta'}c\phi_{\eta G} & -s\phi_{\eta\eta'}c\phi_{\eta G} & -s\phi_{\eta G} \\ s\phi_{\eta\eta'}c\phi_{\eta' G} - c\phi_{\eta\eta'}s\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta\eta'}c\phi_{\eta' G} + s\phi_{\eta\eta'}s\phi_{\eta' G}s\phi_{\eta G} & -s\phi_{\eta' G}c\phi_{\eta G} \\ s\phi_{\eta\eta'}s\phi_{\eta' G} + c\phi_{\eta\eta'}c\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta\eta'}s\phi_{\eta' G} - s\phi_{\eta\eta'}c\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta' G}c\phi_{\eta G} \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \\ G \end{pmatrix}
$$

- *Euler angles*

$$X_\eta = \cos\phi_P \cos\phi_{\eta G} \,, \quad X_{\eta'} = \sin\phi_P \cos\phi_{\eta' G} - \cos\phi_P \sin\phi_{\eta G} \sin\phi_{\eta' G} \,,$$

$$Y_\eta = -\sin\phi_P \cos\phi_{\eta G} \,, \quad Y_{\eta'} = \cos\phi_P \cos\phi_{\eta' G} + \sin\phi_P \sin\phi_{\eta G} \sin\phi_{\eta' G} \,,$$

$$Z_\eta = -\sin\phi_{\eta G} \,, \quad Z_{\eta'} = -\sin\phi_{\eta' G} \cos\phi_{\eta G} \,.$$

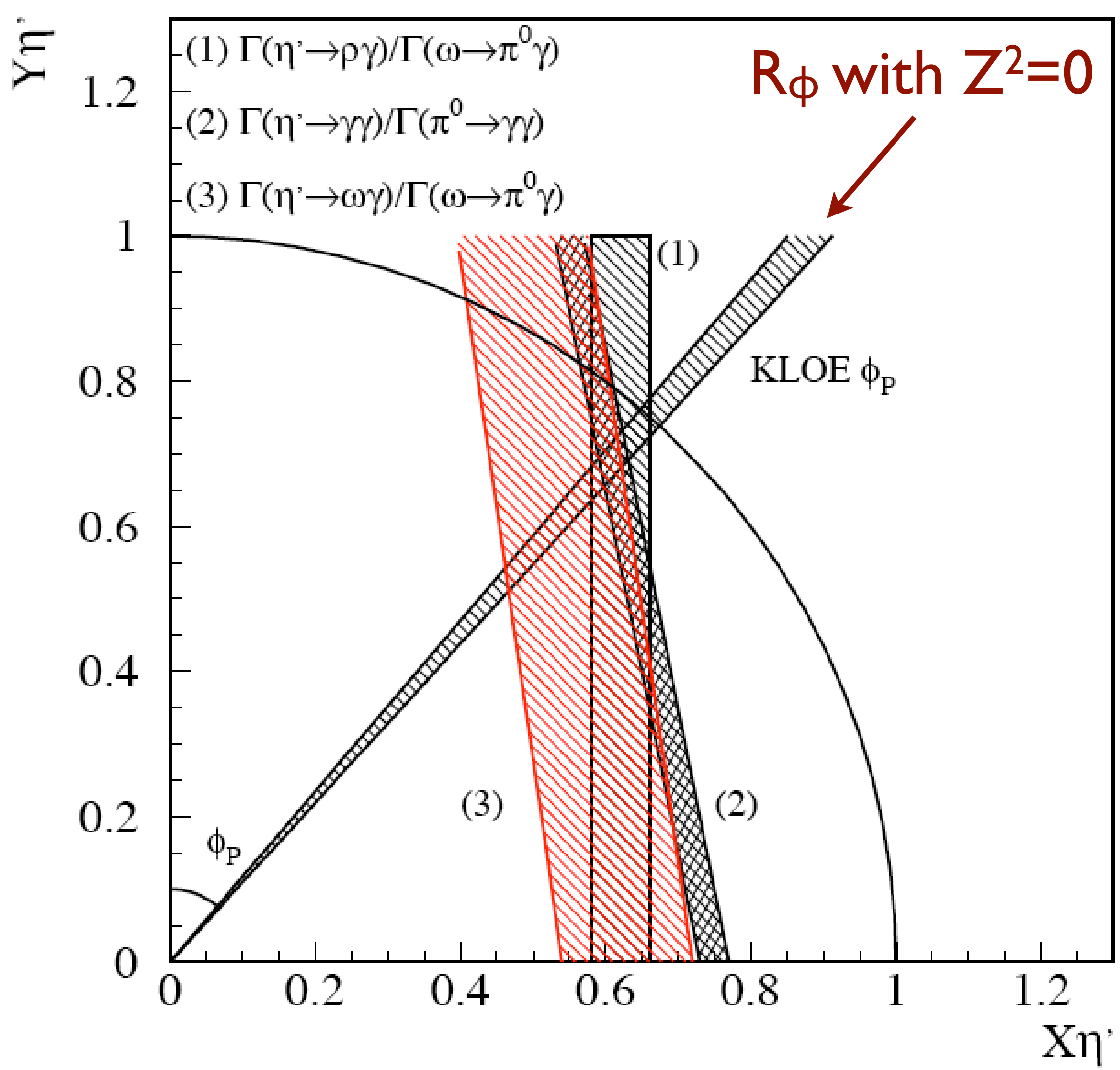
In the limit $\phi_{\eta G}=0$:

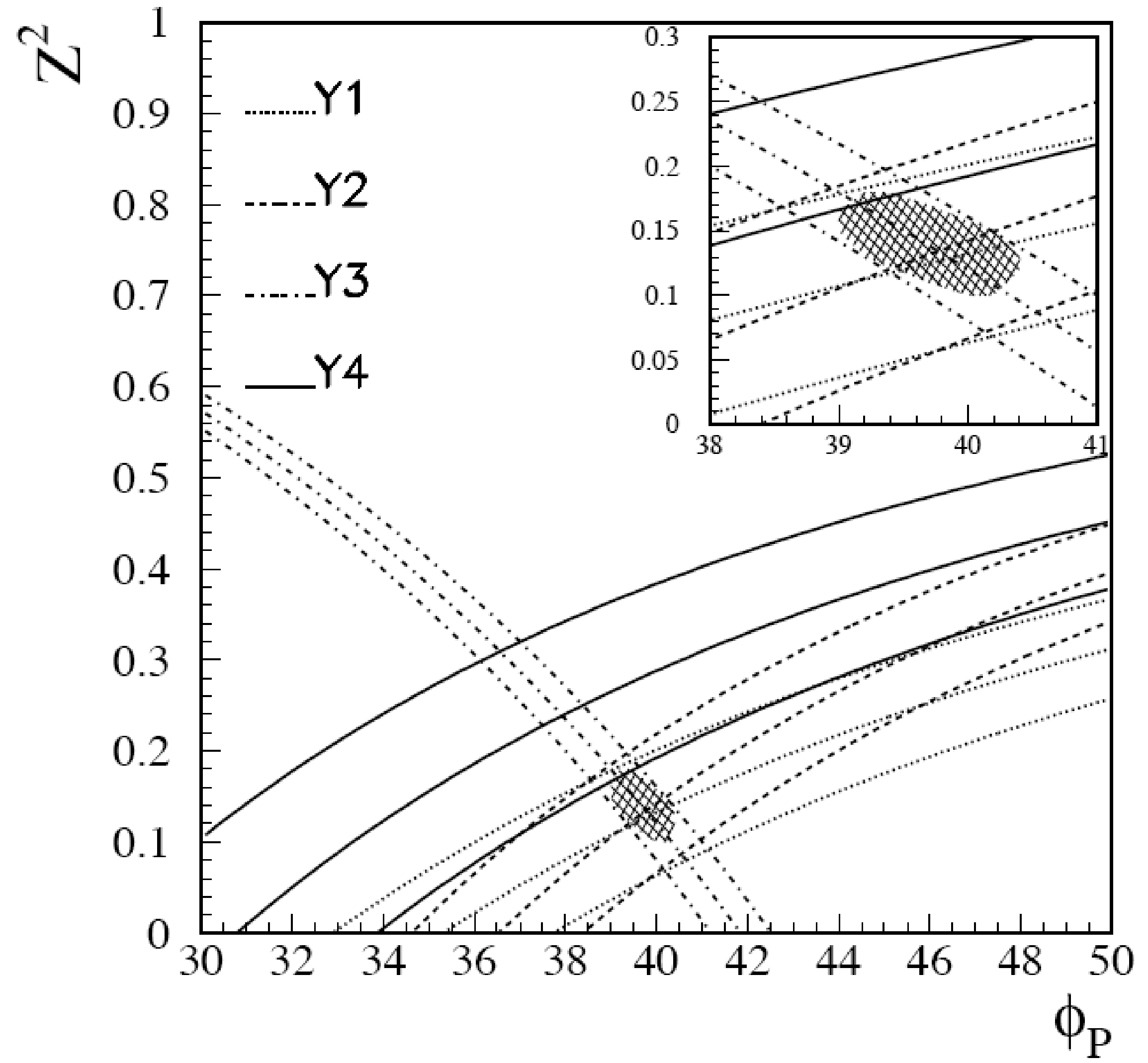
$$X_\eta = \cos\phi_P \,, \qquad Y_\eta = -\sin\phi_P \,, \qquad Z_\eta = 0 \,,$$

$$X_{\eta'} = \sin\phi_P \cos\phi_{\eta' G} \,, \quad Y_{\eta'} = \cos\phi_P \cos\phi_{\eta' G} \,, \quad Z_{\eta'} = -\sin\phi_{\eta' G} \,.$$

- *Motivation*

KLOE Collaboration, Phys. Lett. B648 (2007) 267





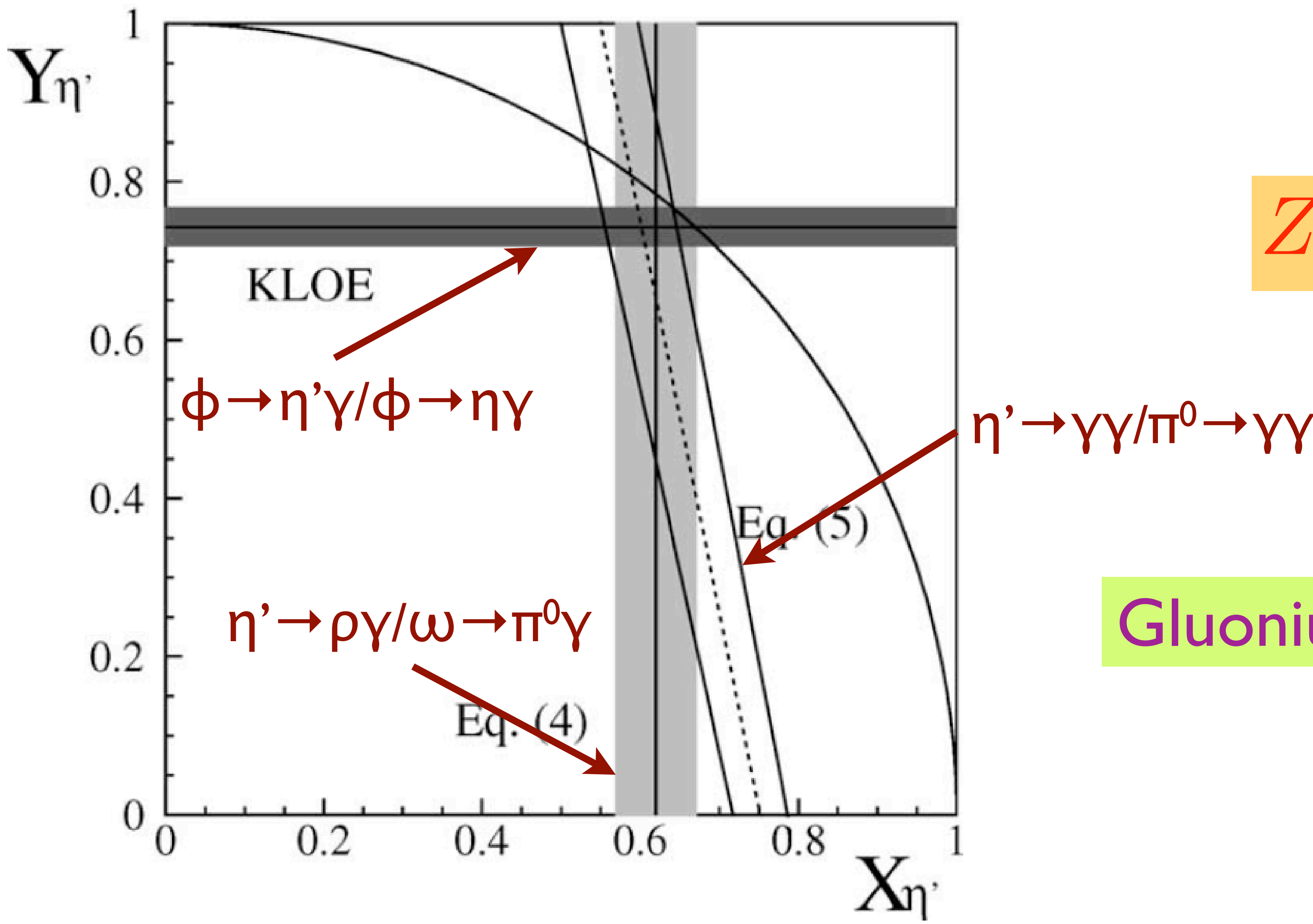
$$\phi_P = (39.7 \pm 0.7)^\circ$$

$$Z^2_{\eta'} = 0.14 \pm 0.04$$

Y1=η'→γγ/π⁰→γγ
Y2=η'→ργ/ω→π⁰γ
Y3=ϕ→η'γ/ϕ→ηγ
Y4=η'→ωγ/ω→π⁰γ

- *Motivation*

KLOE Collaboration, PLB 541 (2002) 45



$$Z^2_{\eta'} = 0.06^{+0.09}_{-0.06}$$

Gluonium fraction below 15%

What are the differences between the two analyses?

- improvement in the precision of the new measurements
- the use of the overlapping parameters relating the pseudoscalar and vector wave functions

• *A model for VPγ M1 transitions*

We will work in a conventional quark model context: P and V are simple quark-antiquark S-wave bound states

→ all these hadrons are thus extended objects with characteristics spatial extensions fixed by their respective P and V wave functions

SU(2) limit → identical spatial extension within each isomultiplet

SU(3) broken → constituent quark masses with $m_s > m$ and different spatial extensions for each isomultiplet

Ingredients of the model:

i) a VPγ magnetic dipole transition proceeding via quark or antiquark spin flip amplitude $\propto \mu_q = e_q/2m_q$

ii) spin-flip V→P conversion amplitude corrected by the relative overlap between the P and V wave functions

iii) OZI-rule reduces considerably the possible transitions and overlaps

U(1)$_A$ anomaly

$$C_\pi \equiv \langle \pi | \omega_q \rangle = \langle \pi | \rho \rangle \qquad C_K \equiv \langle K | K^* \rangle$$

$$C_q \equiv \langle \eta_q | \omega_q \rangle = \langle \eta_q | \rho \rangle \qquad C_s \equiv \langle \eta_s | \phi_s \rangle$$

• *A model for VPγ M1 transitions*

Amplitudes:

$$g_{\rho^0\pi^0\gamma} = g_{\rho^+\pi^+\gamma} = \tfrac{1}{3} g \ , \quad g_{\omega\pi\gamma} = g \cos\phi_V \ , \quad g_{\phi\pi\gamma} = g \sin\phi_V \ ,$$

$$g_{K^{*0}K^0\gamma} = -\tfrac{1}{3} g\, z_K \left(1 + \tfrac{\bar{m}}{m_s}\right) \ , \quad g_{K^{*+}K^+\gamma} = \tfrac{1}{3} g\, z_K \left(2 - \tfrac{\bar{m}}{m_s}\right) \ ,$$

$$g_{\rho\eta\gamma} = g\, z_q\, X_\eta \ , \quad g_{\rho\eta'\gamma} = g\, z_q\, X_{\eta'} \ ,$$

$$g_{\omega\eta\gamma} = \tfrac{1}{3} g \left(z_q\, X_\eta \cos\phi_V + 2 \tfrac{\bar{m}}{m_s} z_s\, Y_\eta \sin\phi_V \right) \ ,$$

$$g_{\omega\eta'\gamma} = \tfrac{1}{3} g \left(z_q\, X_{\eta'} \cos\phi_V + 2 \tfrac{\bar{m}}{m_s} z_s\, Y_{\eta'} \sin\phi_V \right) \ ,$$

$$g_{\phi\eta\gamma} = \tfrac{1}{3} g \left(z_q\, X_\eta \sin\phi_V - 2 \tfrac{\bar{m}}{m_s} z_s\, Y_\eta \cos\phi_V \right) \ ,$$

$$g_{\phi\eta'\gamma} = \tfrac{1}{3} g \left(z_q\, X_{\eta'} \sin\phi_V - 2 \tfrac{\bar{m}}{m_s} z_s\, Y_{\eta'} \cos\phi_V \right) \ ,$$

with $g_{\omega\pi\gamma} = g\ \cos\phi_V = e\, C_\pi \cos\phi_V / \bar{m}$

and $z_q \equiv C_q / C_\pi \ , \quad z_s \equiv C_s / C_\pi \ , \quad z_K \equiv C_K / C_\pi$

$$\Gamma(V \to P\gamma) = \frac{1}{3} \frac{g^2_{VP\gamma}}{4\pi} |\mathbf{p}_\gamma|^3 = \frac{1}{3} \Gamma(P \to V\gamma)$$

• *Data fitting*

The overlapping parameters $z_{q,s}$ and the mixing parameters $X_{\eta(\eta')}$ and $Y_{\eta(\eta')}$ cannot be determined independently

Thus we start assuming $C_q = C_s = C_K = C_\pi = 1$ → $z_q = z_s = z_K = 1$

→ χ^2/d.o.f.=31.2/6 gluonium allowed for η and η'

or χ^2/d.o.f.=45.9/8 gluonium not allowed with $\phi_P=(41.1\pm1.1)^\circ$

Then we leave the overlapping parameters free

Three possibilities:

i) $Z_\eta=Z_{\eta'}=0$ → gluonium not allowed for η or η'

ii) $Z_\eta=0$ → gluonium allowed only for η'

iii) $Z_{\eta'}=0$ → gluonium allowed only for η

i) assuming $Z_\eta=Z_{\eta'}=0$ from the beginning, we get from χ^2/d.o.f.=14.0/7 to

$$g = 0.72 \pm 0.01 \text{ GeV}^{-1} , \quad \phi_P = (41.5 \pm 1.2)^\circ , \quad \phi_V = (3.2 \pm 0.1)^\circ ,$$

$$\frac{m_s}{\bar{m}} = 1.24 \pm 0.07 , \quad z_K = 0.89 \pm 0.03 , \quad z_q = 0.86 \pm 0.03 , \quad z_s = 0.78 \pm 0.05 .$$

χ^2/d.o.f.=4.4/5

• *Data fitting*

ii) assuming $Z_\eta=0$ from the beginning, we get

$$g = 0.72 \pm 0.01\ \mathrm{GeV}^{-1}\ , \quad \frac{m_s}{\bar{m}} = 1.24 \pm 0.07\ , \quad \phi_V = (3.2 \pm 0.1)^\circ\ ,$$

$$\phi_P = (41.4 \pm 1.3)^\circ, \quad |\phi_{\eta' G}| = (12 \pm 13)^\circ,$$

χ^2/d.o.f.=4.2/4

$$z_K = 0.89 \pm 0.03\ , \quad z_q = 0.86 \pm 0.03\ , \quad z_s = 0.79 \pm 0.05\ ,$$

Accepting the absence of gluonium for the η meson, the gluonic content of the η' wave function amounts to $|\phi_{\eta' G}|=(12\pm13)^\circ$ or $(Z_{\eta'})^2=0.04\pm0.09$ and the η-η' mixing angle is found to be $\phi_P=(41.4\pm1.3)^\circ$

Transition	$g^{\mathrm{exp}}_{VP\gamma}$(PDG)	$g^{\mathrm{th}}_{VP\gamma}$(Fit 1)	$g^{\mathrm{th}}_{VP\gamma}$(Fit 2)
$\rho^0 \to \eta\gamma$	0.475 ± 0.024	0.461 ± 0.019	0.464 ± 0.030
$\eta' \to \rho^0\gamma$	0.41 ± 0.03	0.41 ± 0.02	0.40 ± 0.04
$\omega \to \eta\gamma$	0.140 ± 0.007	0.142 ± 0.007	0.143 ± 0.010
$\eta' \to \omega\gamma$	0.139 ± 0.015	0.149 ± 0.006	0.146 ± 0.014
$\phi \to \eta\gamma$	0.209 ± 0.002	0.209 ± 0.018	0.209 ± 0.013
$\phi \to \eta'\gamma$	0.22 ± 0.01	0.22 ± 0.02	0.22 ± 0.02

no gluonium

gluonium

• *Data fitting*

iii) assuming $Z_{\eta'} = 0$ from the beginning, we get

$$g = 0.72 \pm 0.01 \ \mathrm{GeV}^{-1} , \quad \frac{m_s}{\bar{m}} = 1.24 \pm 0.07 , \quad \phi_V = (3.2 \pm 0.1)^\circ ,$$

$$\phi_P = (41.5 \pm 1.3)^\circ , \quad |\phi_{\eta G}| \simeq 0^\circ ,$$

$$z_q = 0.86 \pm 0.04 , \quad z_s = 0.78 \pm 0.06 , \quad z_K = 0.89 \pm 0.03 ,$$

$\chi^2/\mathrm{d.o.f.} = 4.4/4$

Accepting the absence of gluonium for the η' meson, the gluonic content of the η wave function amounts to $|\phi_{\eta G}| \simeq 0^\circ$ or $(Z_\eta)^2 = 0.00 \pm 0.12$ and the η-η' mixing angle is found to be $\phi_P = (41.5 \pm 1.3)^\circ$

The current experimental data on VPγ transitions indicate within our model a negligible gluonic content for the η and η' mesons

• *Data fitting*

Using the latest experimental data on $(\rho,\omega,\phi)\rightarrow\eta\gamma$ (SND) and $\phi\rightarrow\eta'\gamma$ (KLOE), we get

$$\phi_P = (42.7 \pm 0.7)^\circ, \quad z_q = 0.83 \pm 0.03 \ , \quad z_s = 0.79 \pm 0.05 \ , \quad \chi^2/\text{d.o.f.}=4.0/5$$

$$\phi_P = (42.6 \pm 1.1)^\circ, \quad |\phi_{\eta' G}| = (5 \pm 21)^\circ, \quad z_q = 0.83 \pm 0.03 \ , \quad z_s = 0.79 \pm 0.05 \ , \quad \chi^2/\text{d.o.f.}=4.0/4$$

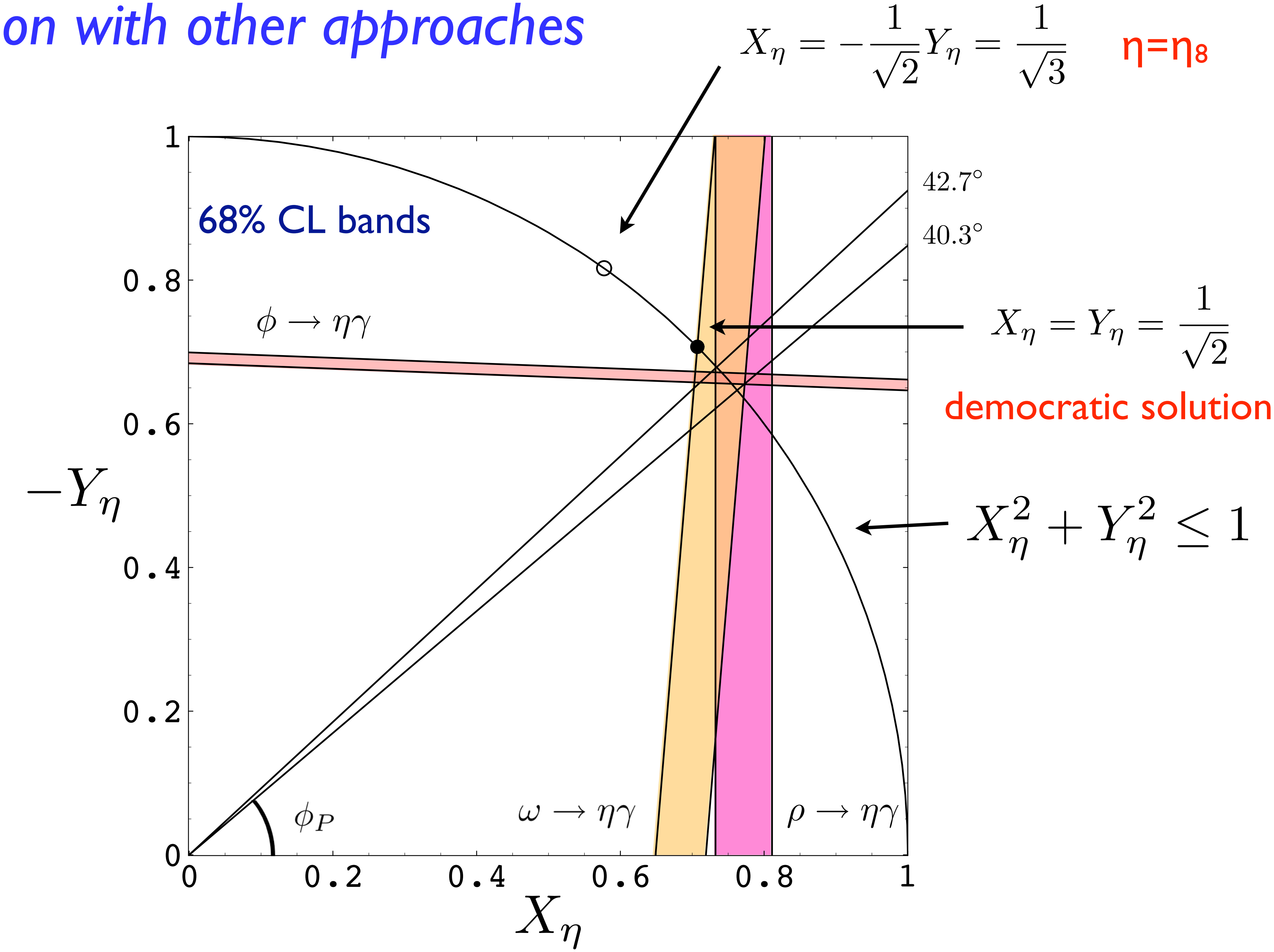
confirmation of the null gluonic content of the η and η' wave functions

no gluonium

gluonium

Transition	$g^{\text{exp}}_{VP\gamma}$(latest)	$g^{\text{th}}_{VP\gamma}$(Fit 3)	$g^{\text{th}}_{VP\gamma}$(Fit 4)
$\rho^0 \to \eta\gamma$	0.429 ± 0.023	0.436 ± 0.017	0.437 ± 0.028
$\eta' \to \rho^0\gamma$	0.41 ± 0.03 (PDG)	0.40 ± 0.02	0.40 ± 0.04
$\omega \to \eta\gamma$	0.136 ± 0.007	0.134 ± 0.006	0.134 ± 0.009
$\eta' \to \omega\gamma$	0.139 ± 0.015 (PDG)	0.146 ± 0.006	0.146 ± 0.013
$\phi \to \eta\gamma$	0.214 ± 0.003	0.214 ± 0.017	0.214 ± 0.012
$\phi \to \eta'\gamma$	0.216 ± 0.005	0.216 ± 0.019	0.216 ± 0.018

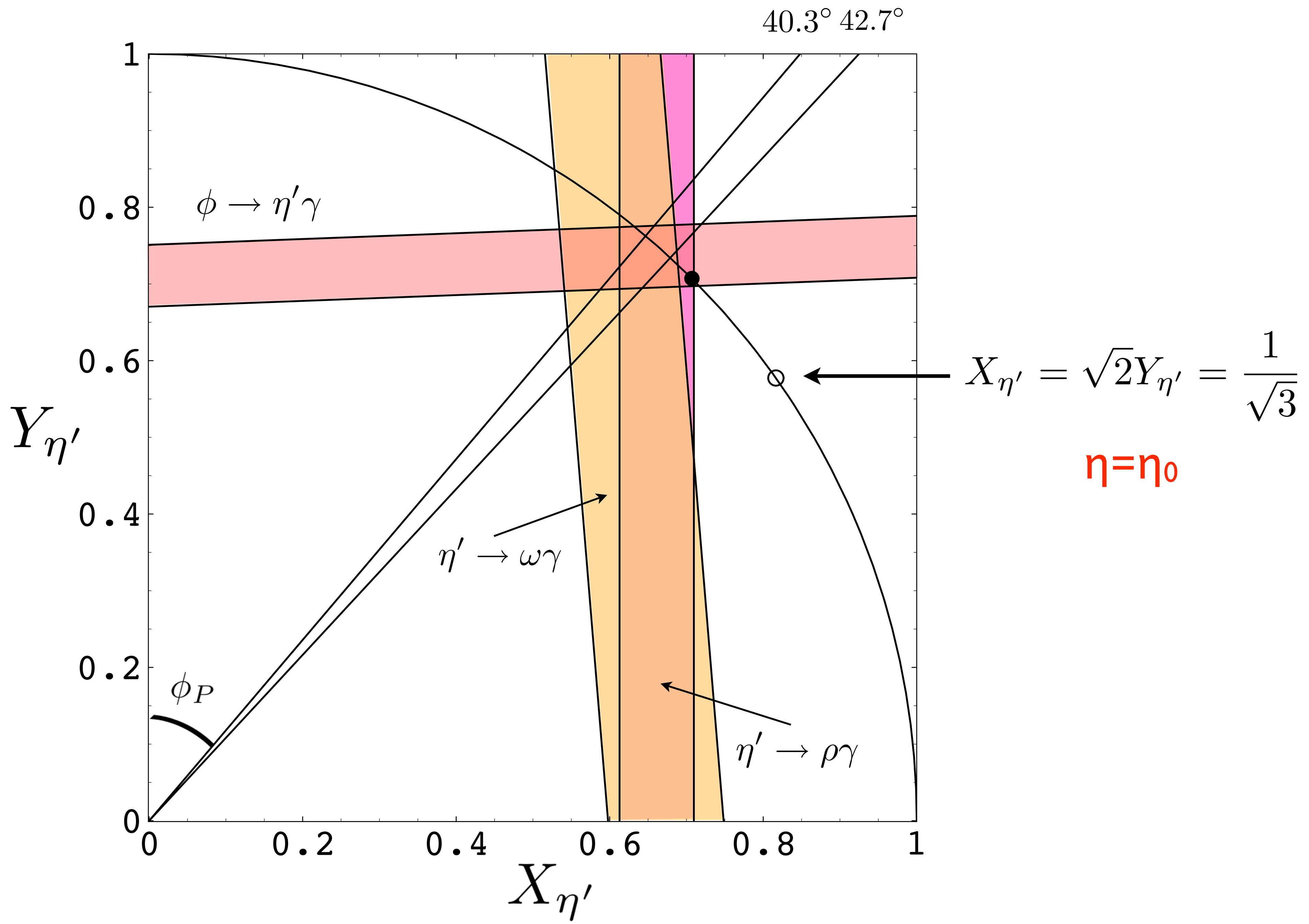
- *Comparison with other approaches*



✔ importance of $\phi \to \eta\gamma$

✔ importance of the slopes (ϕ_V)

- *Comparison with other approaches*

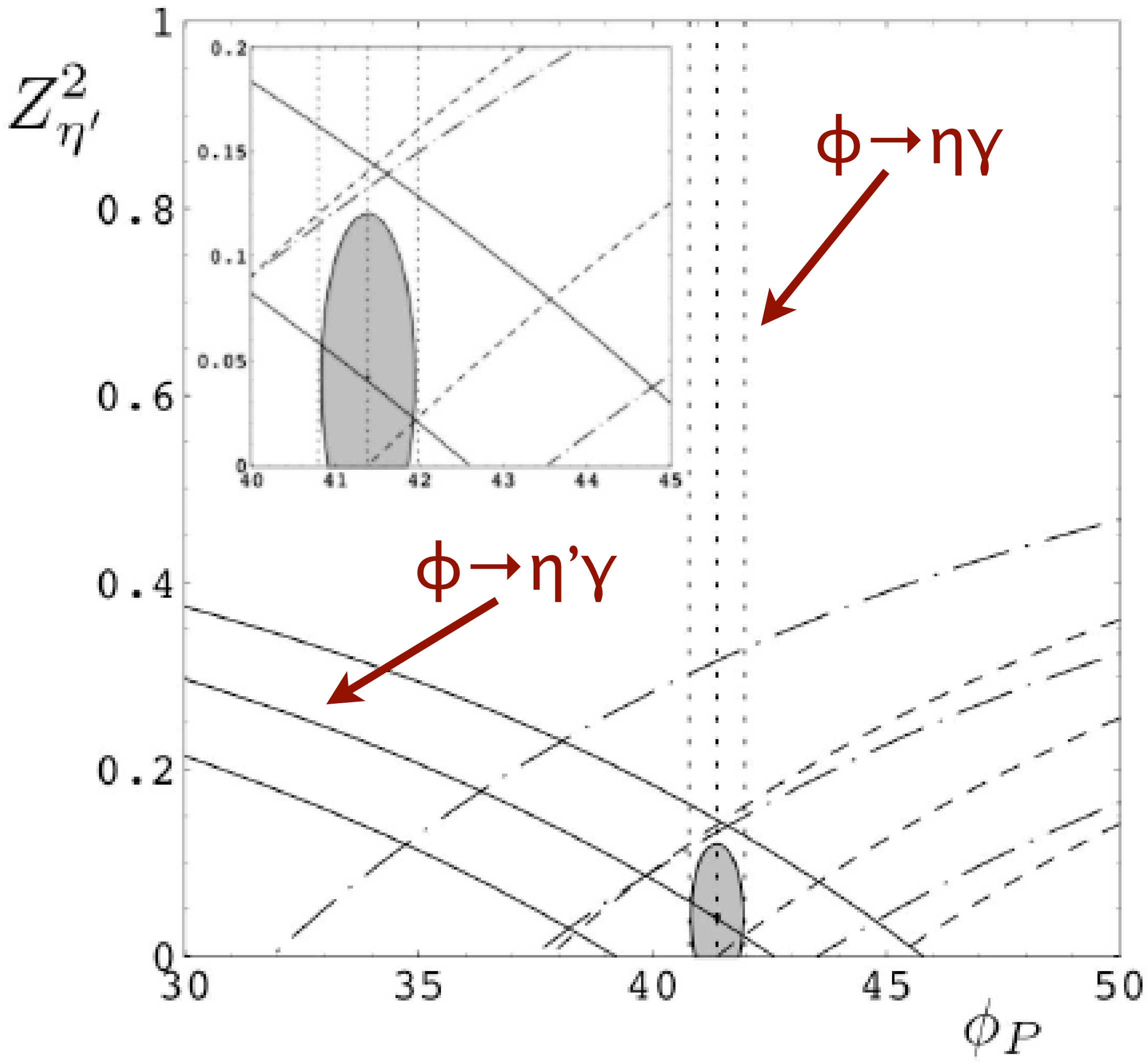


✔ importance of constraining even more $\phi \to \eta'\gamma$

More refined data for this channel will contribute decisively to clarify this issue

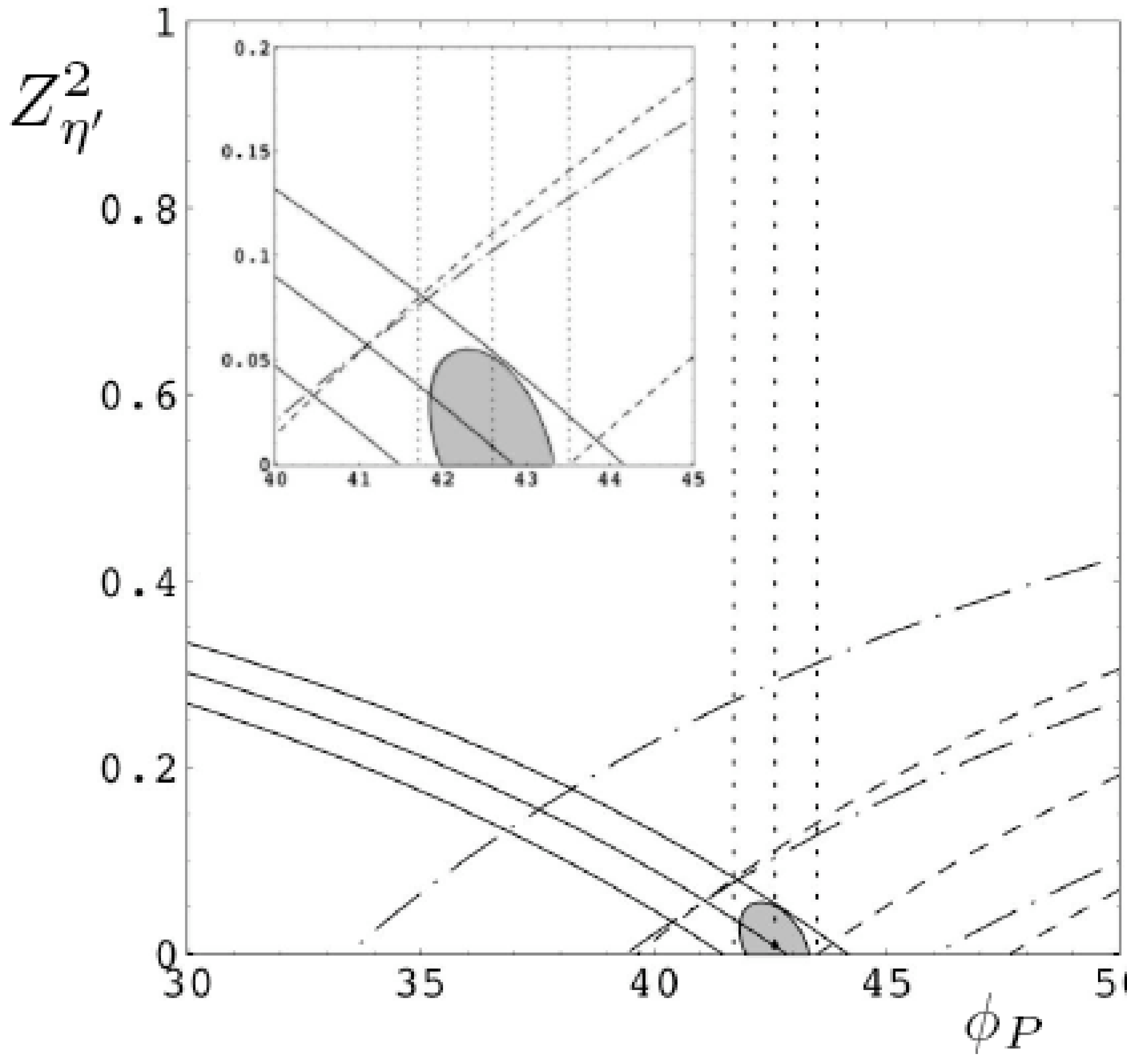
• *Comparison with other approaches*

PDG'06 data



$$(\phi_P, Z^2_{\eta'}) = (41.4^\circ, 0.04)$$

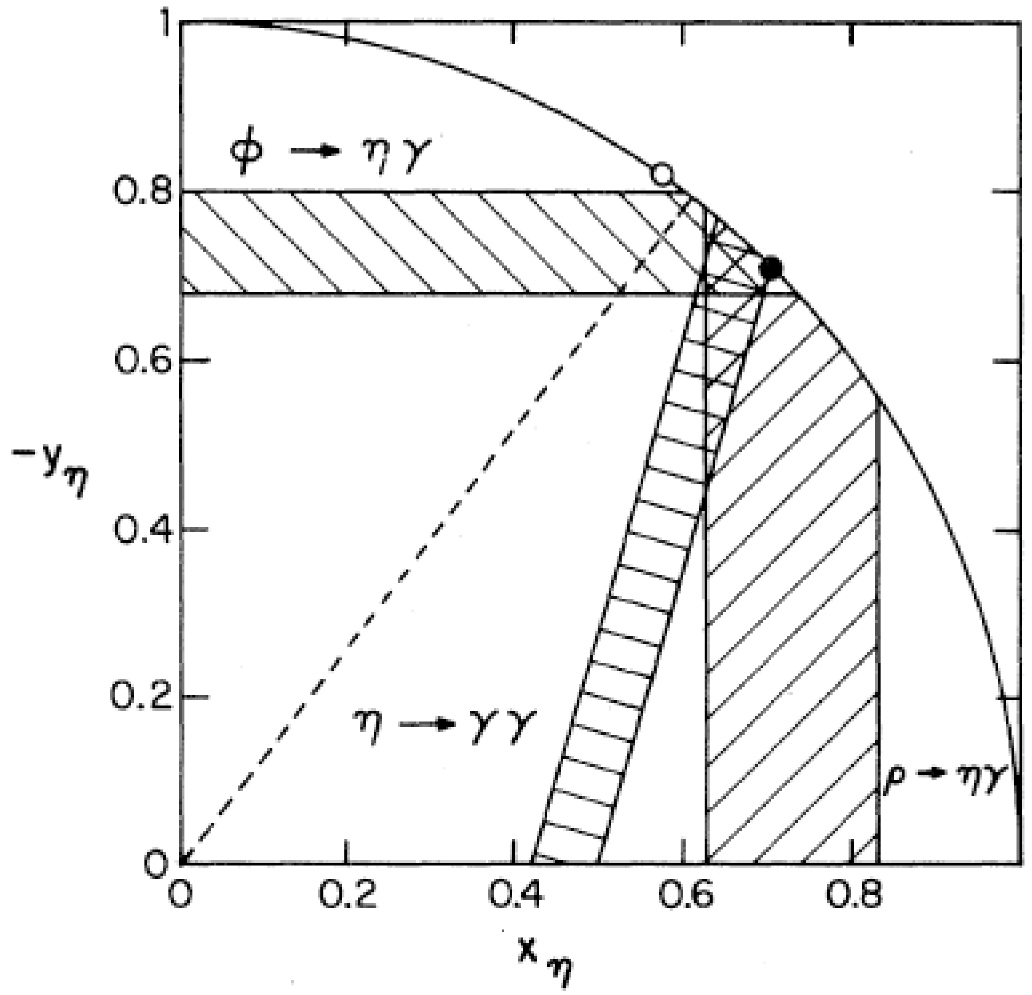
latest data

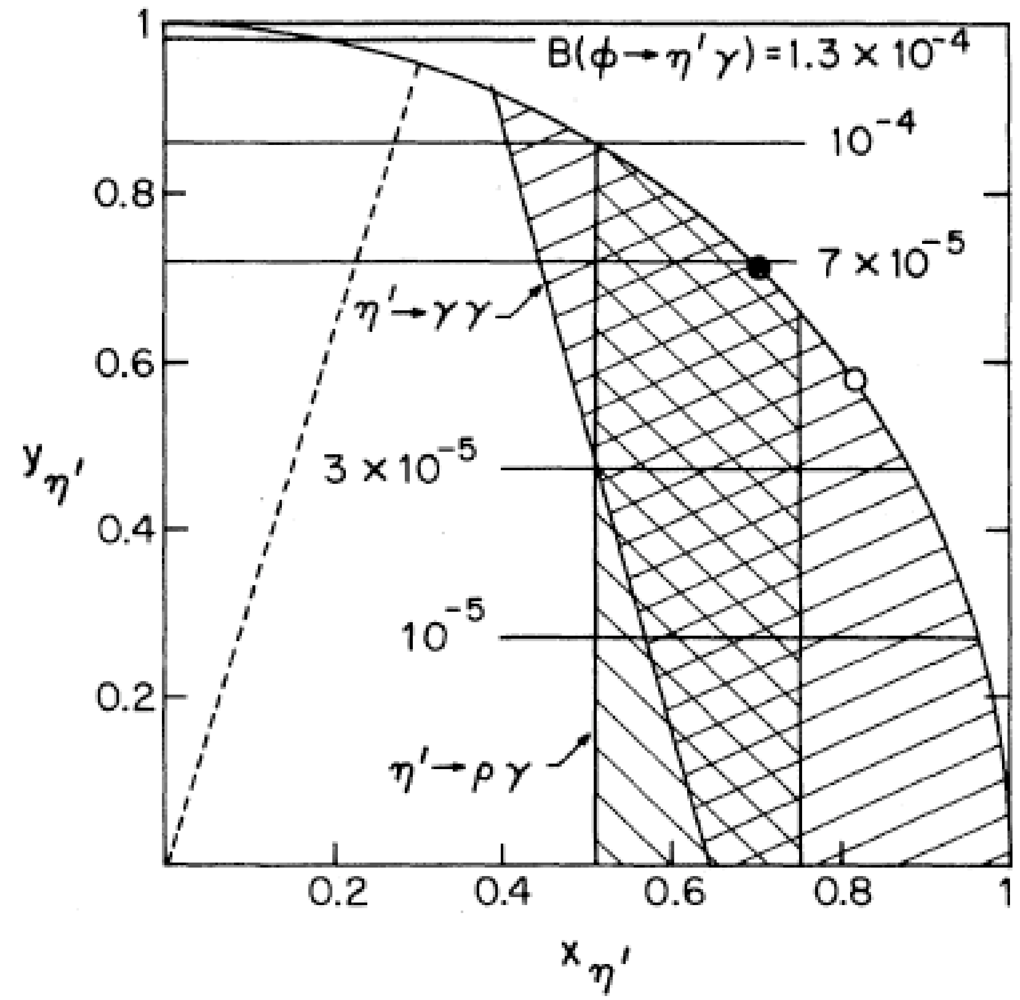


$$(\phi_P, Z^2_{\eta'}) = (42.6^\circ, 0.01)$$

- *Comparison with other approaches*

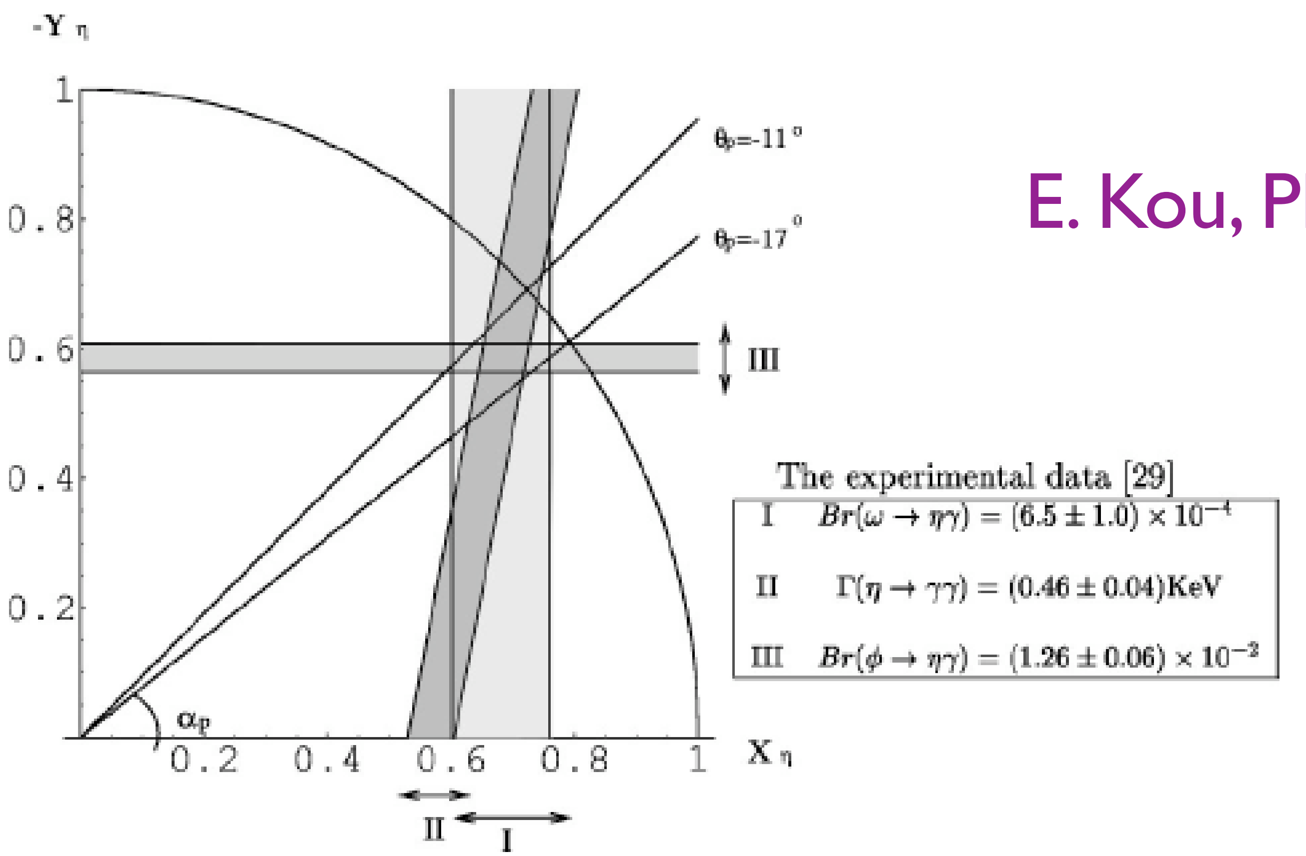
J. L. Rosner, Phys. Rev. D27 (1983) 1101





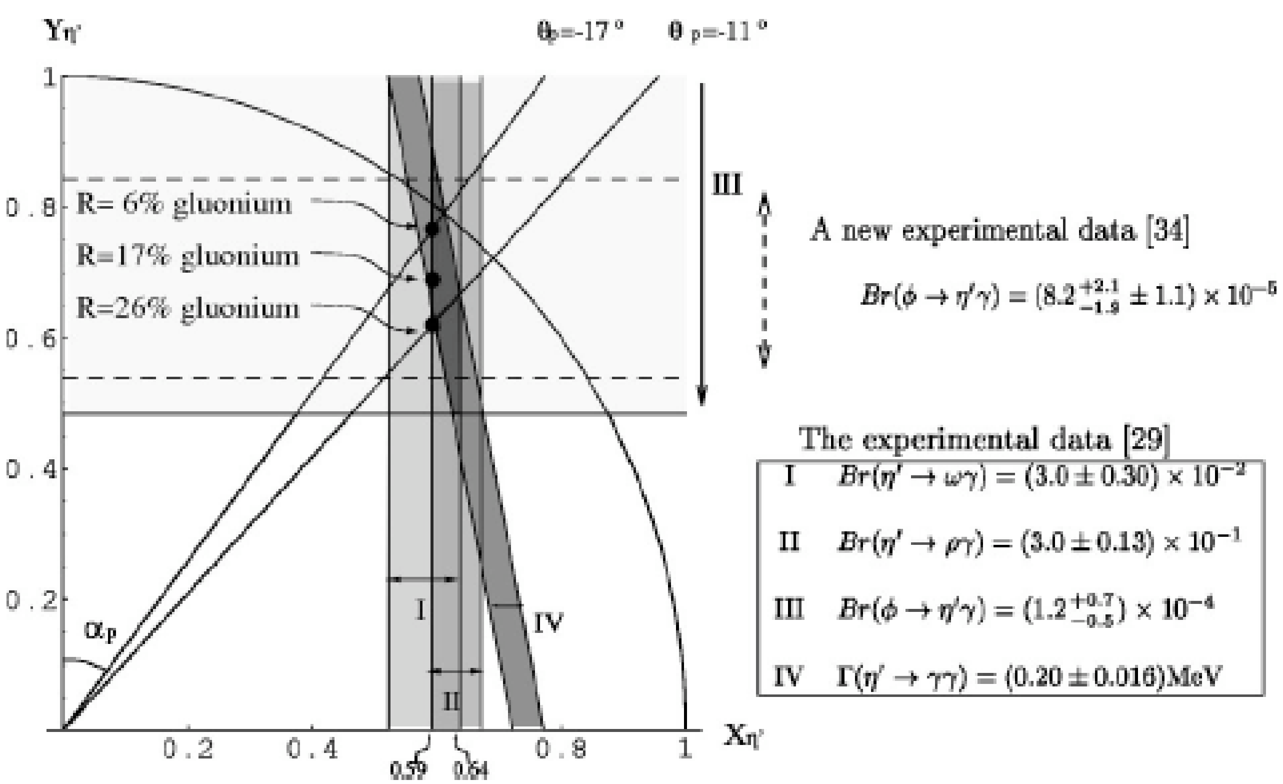
$|Z_\eta| < 0.4$

• *Comparison with other approaches*



E. Kou, Phys. Rev. D63 (2001) 054027

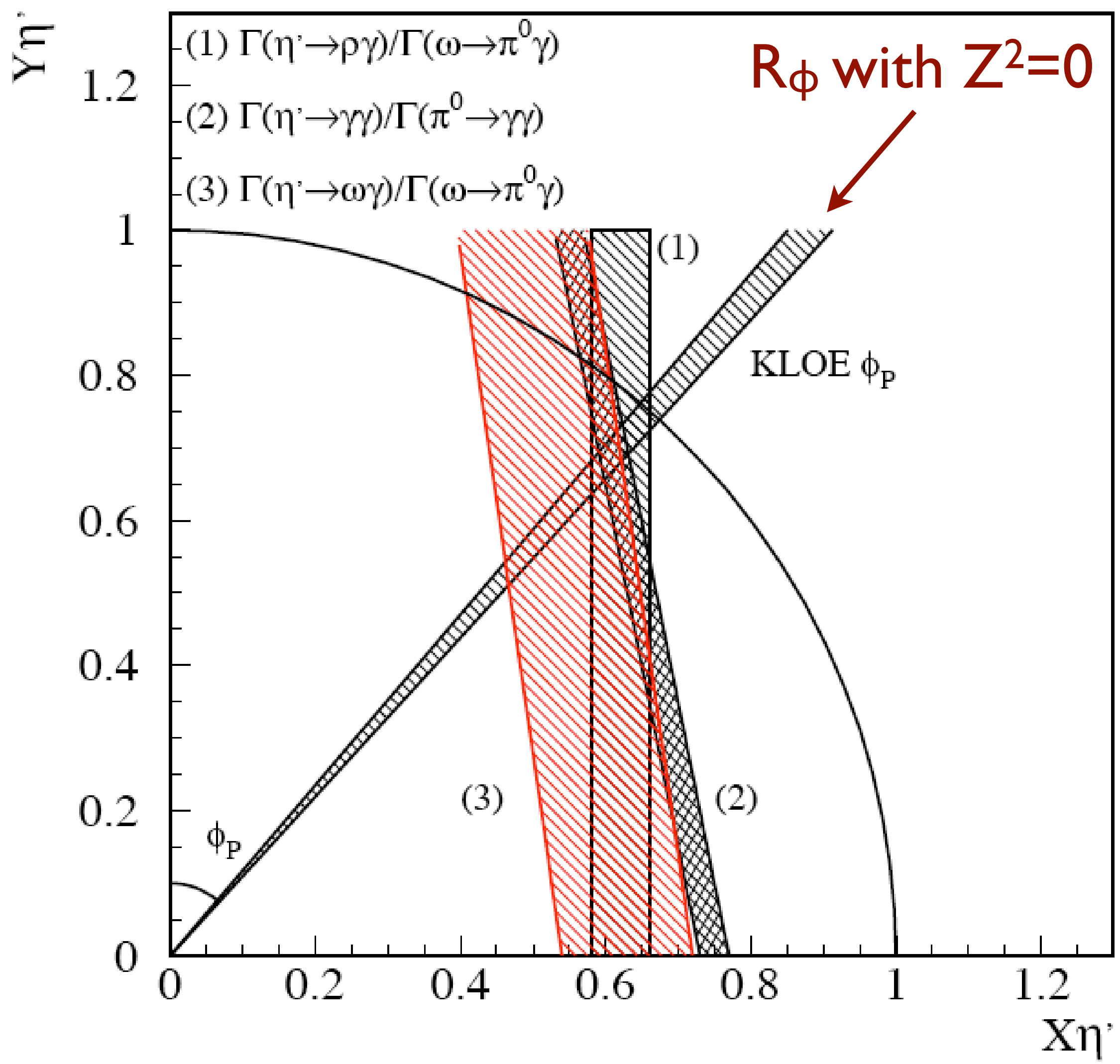
$$R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} = 26\%$$

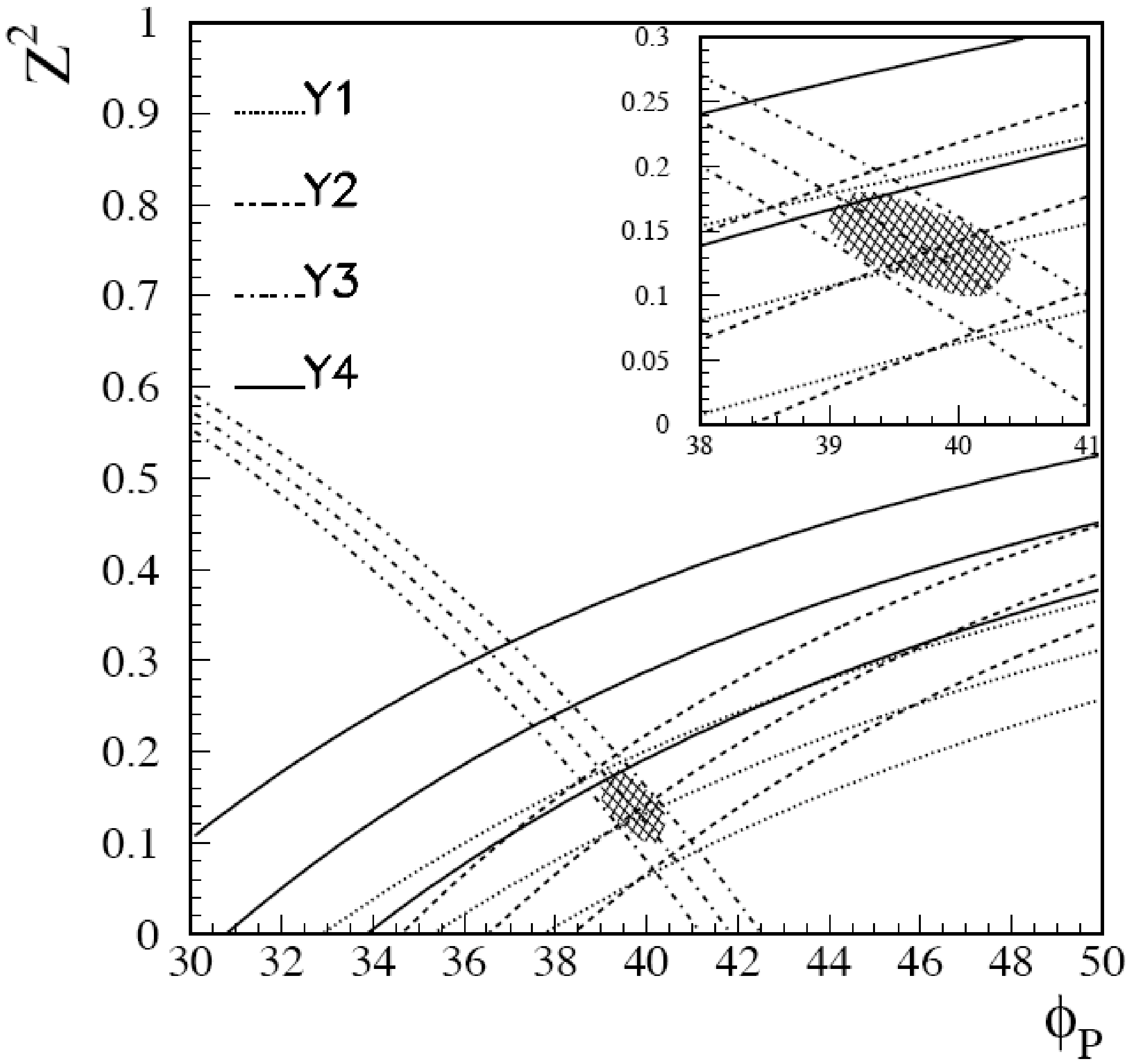


$$R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} = (13 \pm 13)\%$$

• *Comparison with other approaches*

KLOE Collaboration, Phys. Lett. B648 (2007) 267





$$\phi_P = (39.7 \pm 0.7)^\circ$$

$$Z^2_{\eta'} = 0.14 \pm 0.04$$

Y1=η'→γγ/π⁰→γγ
Y2=η'→ργ/ω→π⁰γ
Y3=ϕ→η'γ/ϕ→ηγ
Y4=η'→ωγ/ω→π⁰γ

- *Comparison with other approaches*

$$R_\phi \equiv \frac{\Gamma(\phi \to \eta'\gamma)}{\Gamma(\phi \to \eta\gamma)} = \cot^2\phi_P \cos^2\phi_{\eta' G}\left(1 - \frac{m_s}{\bar{m}}\frac{z_q}{z_s}\frac{\tan\phi_V}{\sin 2\phi_P}\right)^2 \left(\frac{p_{\eta'}}{p_\eta}\right)^3$$ =(4.7±0.6)×10^{-3}

in agreement with (4.8±0.5)×10^{-3} (PDG'06) and (4.77±0.09±0.19)×10^{-3} (KLOE) ✔

• *Summary*

We have performed a phenomenological analysis of radiative $V \to P\gamma$ and $P \to V\gamma$ decays with the purpose of determining the gluon content of the η and η' mesons

The present approach is based on a conventional SU(3) quark model supplemented with two sources of SU(3) breaking, the use of constituent quark masses with $m_s > m$ and the different overlaps between the P and V wave functions

The use of these different overlapping parameters (a specific feature of our analysis) is shown to be of primary importance in order to reach a good agreement

• *Conclusions*

1) The current experimental data on $VP\gamma$ transitions indicate within our model a negligible gluonic content for the η and η' mesons,

$$Z_\eta^2 = 0.00 \pm 0.12 \quad \text{and} \quad Z_{\eta'}^2 = 0.04 \pm 0.09$$

2) Accepting the absence of gluonium for the η meson, the gluonic content of the η' wave function amounts to $|\phi_{\eta' G}| = (12 \pm 13)^\circ$ or $(Z_{\eta'})^2 = 0.04 \pm 0.09$ and the η-η' mixing angle is found to be $\phi_P = (41.4 \pm 1.3)^\circ$

• *Conclusions*

3) Imposing the absence of gluonium for both mesons one finds $\phi_P=(41.5\pm1.2)^\circ$, in agreement with the former result

4) The latest experimental data on $(\rho,\omega,\phi)\to\eta\gamma$ and $\phi\to\eta'\gamma$ decays confirm the null gluonic content of the η and η' wave functions

5) More refined experimental data, particularly for the $\phi\to\eta'\gamma$ channel, will contribute decisively to clarify this issue