On the gluon content of the $\eta$ and $\eta'$ mesons

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in collab. with Jordi Nadal, JHEP 05 (2007) 6
Purpose: to perform a phenomenological analysis of radiative $V \rightarrow P \gamma$ and $P \rightarrow V \gamma$ decays, with $V = \rho, K^*, \omega, \phi$ and $P = \pi, K, \eta, \eta'$, aimed at determining the gluonic content of the $\eta$ and $\eta'$ wave functions.

Outline:

- **Notation**
- **Motivation**
- **A model for $VP\gamma$ M1 transitions**
- **Data fitting**
- **Comparison with other approaches**
- **Summary and conclusions**
• **Notation**

We work in a basis consisting of the states

\[
|\eta_q\rangle \equiv \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle \quad |\eta_s\rangle = |s\bar{s}\rangle \quad |G\rangle \equiv |\text{gluonium}\rangle
\]

The physical states $\eta$ and $\eta'$ are assumed to be the linear combinations

\[
|\eta\rangle = X_\eta |\eta_q\rangle + Y_\eta |\eta_s\rangle + Z_\eta |G\rangle, \\
|\eta'\rangle = X_{\eta'} |\eta_q\rangle + Y_{\eta'} |\eta_s\rangle + Z_{\eta'} |G\rangle,
\]

with \( X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 + Z_{\eta(\eta')}^2 = 1 \) and thus \( X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 \leq 1 \)

A significant gluonic admixture in a state is possible only if

\[
Z_{\eta(\eta')}^2 = 1 - X_{\eta(\eta')}^2 - Y_{\eta(\eta')}^2 > 0
\]

**Assumptions:**

- no mixing with $\pi^0$ (isospin symmetry)
- no mixing with $\eta_c$ states
- no mixing with radial excitations
**Notation**

In absence of gluonium (standard picture)

\[
\begin{align*}
Z_{\eta(\eta')} & \equiv 0 \\
|\eta\rangle & = \cos \phi_P |\eta_q\rangle - \sin \phi_P |\eta_s\rangle \\
|\eta'\rangle & = \sin \phi_P |\eta_q\rangle + \cos \phi_P |\eta_s\rangle
\end{align*}
\]

with

\[
\begin{align*}
X_\eta & = Y_{\eta'} \equiv \cos \phi_P \\
X_{\eta'} & = -Y_\eta \equiv \sin \phi_P
\end{align*}
\]

and

\[
X_{\eta(\eta')^2} + Y_{\eta(\eta')}^2 = 1
\]

where \( \phi_P \) is the \( \eta-\eta' \) mixing angle in the quark-flavour basis related to its octet-singlet analog through

\[
\theta_P = \phi_P - \arctan \sqrt{2} \simeq \phi_P - 54.7^\circ
\]

Similarly, for the vector states \( \omega \) and \( \phi \) the mixing is given by

\[
\begin{align*}
|\omega\rangle & = \cos \phi_V |\omega_q\rangle - \sin \phi_V |\phi_s\rangle \\
|\phi\rangle & = \sin \phi_V |\omega_q\rangle + \cos \phi_V |\phi_s\rangle
\end{align*}
\]

where \( \omega_q \) and \( \phi_s \) are the analog non-strange and strange states of \( \eta_q \) and \( \eta_s \), respectively.
• Euler angles

In presence of gluonium,

\[ |\eta\rangle = X_{\eta} |\eta_q\rangle + Y_{\eta} |\eta_s\rangle + Z_{\eta} |G\rangle \]
\[ |\eta'\rangle = X_{\eta'} |\eta_q\rangle + Y_{\eta'} |\eta_s\rangle + Z_{\eta'} |G\rangle \]
\[ |\iota\rangle = X_{\iota} |\eta_q\rangle + Y_{\iota} |\eta_s\rangle + Z_{\iota} |G\rangle \]

Normalization:

\[ X_{\eta}^2 + Y_{\eta}^2 + Z_{\eta}^2 = 1 \]
\[ X_{\eta'}^2 + Y_{\eta'}^2 + Z_{\eta'}^2 = 1 \]
\[ X_{\iota}^2 + Y_{\iota}^2 + Z_{\iota}^2 = 1 \]

Orthogonality:

\[ X_{\eta} X_{\eta'} + Y_{\eta} Y_{\eta'} + Z_{\eta} Z_{\eta'} = 0 \]
\[ X_{\eta} X_{\iota} + Y_{\eta} Y_{\iota} + Z_{\eta} Z_{\iota} = 0 \]
\[ X_{\eta'} X_{\iota} + Y_{\eta'} Y_{\iota} + Z_{\eta'} Z_{\iota} = 0 \]

Normalization:

\[ \begin{pmatrix} \eta \\ \eta' \\ \iota \end{pmatrix} = \begin{pmatrix} c\phi_{\eta\eta'} c\phi_{\eta G} \\ s\phi_{\eta\eta'} c\phi_{\eta' G} - c\phi_{\eta\eta'} s\phi_{\eta' G} s\phi_{\eta G} \\ s\phi_{\eta\eta'} s\phi_{\eta' G} + c\phi_{\eta\eta'} c\phi_{\eta' G} s\phi_{\eta G} \end{pmatrix} \begin{pmatrix} -s\phi_{\eta\eta'} c\phi_{\eta G} \\ c\phi_{\eta\eta'} c\phi_{\eta' G} + s\phi_{\eta\eta'} s\phi_{\eta' G} s\phi_{\eta G} \\ c\phi_{\eta\eta'} s\phi_{\eta' G} - s\phi_{\eta\eta'} c\phi_{\eta' G} s\phi_{\eta G} \end{pmatrix} \begin{pmatrix} -s\phi_{\eta G} \\ s\phi_{\eta' G} c\phi_{\eta G} \\ c\phi_{\eta' G} c\phi_{\eta G} \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \\ G \end{pmatrix} \]

3 independent parameters: \( \phi_{\eta G}, \phi_{\eta' G} \text{ and } \phi_{\eta' G} \)
• Euler angles

\[
X_\eta = \cos \phi_P \cos \phi_{\eta G} , \quad X_{\eta'} = \sin \phi_P \cos \phi_{\eta' G} - \cos \phi_P \sin \phi_{\eta G} \sin \phi_{\eta' G} , \\
Y_\eta = - \sin \phi_P \cos \phi_{\eta G} , \quad Y_{\eta'} = \cos \phi_P \cos \phi_{\eta' G} + \sin \phi_P \sin \phi_{\eta G} \sin \phi_{\eta' G} , \\
Z_\eta = - \sin \phi_{\eta G} , \quad Z_{\eta'} = - \sin \phi_{\eta' G} \cos \phi_{\eta G} .
\]

In the limit \(\phi_{\eta G} = 0\):

\[
X_\eta = \cos \phi_P , \quad Y_\eta = - \sin \phi_P , \quad Z_\eta = 0 , \\
X_{\eta'} = \sin \phi_P \cos \phi_{\eta' G} , \quad Y_{\eta'} = \cos \phi_P \cos \phi_{\eta' G} , \quad Z_{\eta'} = - \sin \phi_{\eta' G} .
\]
• **Motivation**


\[ \phi_P = (39.7 \pm 0.7)^\circ \]

\[ Z^2_{\eta'} = 0.14 \pm 0.04 \]
**Motivation**

What are the differences between the two analyses?

- improvement in the precision of the new measurements
- the use of the overlapping parameters relating the pseudoscalar and vector wave functions

\[ Z_{\eta'}^2 = 0.06^{+0.09}_{-0.06} \]

Gluonium fraction below 15%
A model for VPγ M1 transitions

We will work in a conventional quark model context: P and V are simple quark-antiquark S-wave bound states.

All these hadrons are thus extended objects with characteristics spatial extensions fixed by their respective P and V wave functions.

**SU(2) limit**

- Identical spatial extension within each isomultiplet.

**SU(3) broken**

- Constituent quark masses with \( m_s > m \) and different spatial extensions for each isomultiplet.

**Ingredients of the model:**

i) A VPγ magnetic dipole transition proceeding via quark or antiquark spin flip amplitude \( \propto \mu_q = e_q/2m_q \)

ii) Spin-flip \( V \rightarrow P \) conversion amplitude corrected by the relative overlap between the P and V wave functions.

iii) OZI-rule reduces considerably the possible transitions and overlaps.

**U(1)A anomaly**

\[
C_\pi \equiv \langle \pi | \omega_q \rangle = \langle \pi | \rho \rangle \quad C_K \equiv \langle K | K^* \rangle \\
C_q \equiv \langle \eta_q | \omega_q \rangle = \langle \eta_q | \rho \rangle \quad C_s \equiv \langle \eta_s | \phi_s \rangle
\]
A model for VP\gamma M1 transitions

Amplitudes:

\[ g_{\rho^0\pi^0\gamma} = g_{\rho^+\pi^+\gamma} = \frac{1}{3} g, \quad g_{\omega\pi\gamma} = g \cos \phi_V, \quad g_{\phi\pi\gamma} = g \sin \phi_V, \]
\[ g_{K^*0K^0\gamma} = -\frac{1}{3} g z_K \left(1 + \frac{\bar{m}}{m_s}\right), \quad g_{K^*+K^+\gamma} = \frac{1}{3} g z_K \left(2 - \frac{\bar{m}}{m_s}\right), \]
\[ g_{\rho\eta\gamma} = g z_q X_\eta, \quad g_{\rho\eta'\gamma} = g z_q X'_\eta, \]
\[ g_{\omega\eta\gamma} = \frac{1}{3} g \left(z_q X_\eta \cos \phi_V + 2 \frac{\bar{m}}{m_s} z_s Y_\eta \sin \phi_V\right), \]
\[ g_{\omega\eta'\gamma} = \frac{1}{3} g \left(z_q X'_\eta \cos \phi_V + 2 \frac{\bar{m}}{m_s} z_s Y'_\eta \sin \phi_V\right), \]
\[ g_{\phi\eta\gamma} = \frac{1}{3} g \left(z_q X_\eta \sin \phi_V - 2 \frac{\bar{m}}{m_s} z_s Y_\eta \cos \phi_V\right), \]
\[ g_{\phi\eta'\gamma} = \frac{1}{3} g \left(z_q X'_\eta \sin \phi_V - 2 \frac{\bar{m}}{m_s} z_s Y'_\eta \cos \phi_V\right), \]

with \( g_{\omega\pi\gamma} = g \cos \phi_V = e C_\pi \cos \phi_V / \bar{m} \)
and \( z_q \equiv C_q / C_\pi \), \( z_s \equiv C_s / C_\pi \), \( z_K \equiv C_K / C_\pi \)

\[ \Gamma(V \to P\gamma) = \frac{1}{3} \frac{g_{V\gamma}^2}{4\pi} |P_\gamma|^3 = \frac{1}{3} \Gamma(P \to V\gamma) \]
**Data fitting**

The overlapping parameters $z_{q,s}$ and the mixing parameters $X_{\eta(\eta')}$ and $Y_{\eta(\eta')}$ cannot be determined independently.

Thus we start assuming $C_q = C_s = C_K = C_\pi = 1 \implies z_q = z_s = z_K = 1$.

$\chi^2/d.o.f. = 31.2/6$ gluonium allowed for $\eta$ and $\eta'$

or $\chi^2/d.o.f. = 45.9/8$ gluonium not allowed with $\phi_p = (41.1 \pm 1.1)^\circ$.

Then we leave the overlapping parameters free.

Three possibilities:

i) $Z_\eta = Z_{\eta'} = 0$ gluonium not allowed for $\eta$ or $\eta'$

ii) $Z_\eta = 0$ gluonium allowed only for $\eta'$

iii) $Z_{\eta'} = 0$ gluonium allowed only for $\eta$

i) assuming $Z_\eta = Z_{\eta'} = 0$ from the beginning, we get from $\chi^2/d.o.f. = 14.0/7$ to

\[ g = 0.72 \pm 0.01 \text{ GeV}^{-1}, \quad \phi_p = (41.5 \pm 1.2)^\circ, \quad \phi_V = (3.2 \pm 0.1)^\circ, \]

\[ \frac{m_s}{m} = 1.24 \pm 0.07, \quad z_K = 0.89 \pm 0.03, \quad z_q = 0.86 \pm 0.03, \quad z_s = 0.78 \pm 0.05 \]


**Data fitting**

ii) assuming $Z_\eta=0$ from the beginning, we get

\[
g = 0.72 \pm 0.01 \text{ GeV}^{-1}, \quad \frac{m_s}{m} = 1.24 \pm 0.07, \quad \phi_V = (3.2 \pm 0.1)^\circ, \quad \phi_P = (41.4 \pm 1.3)^\circ, \quad |\phi_{\eta'G}| = (12 \pm 13)^\circ, \quad \chi^2/\text{d.o.f.}=4.2/4
\]

Accepting the absence of gluonium for the $\eta$ meson, the gluonic content of the $\eta'$ wave function amounts to $|\phi_{\eta'G}|=(12\pm13)^\circ$ or $(Z_\eta)^2=0.04\pm0.09$ and the $\eta$-$\eta'$ mixing angle is found to be $\phi_P=(41.4\pm1.3)^\circ$.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$g_{VP\gamma}^{\text{exp}}$ (PDG)</th>
<th>$g_{VP\gamma}^{\text{th}}$ (Fit 1)</th>
<th>$g_{VP\gamma}^{\text{th}}$ (Fit 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^0 \to \eta\gamma$</td>
<td>$0.475 \pm 0.024$</td>
<td>$0.461 \pm 0.019$</td>
<td>$0.464 \pm 0.030$</td>
</tr>
<tr>
<td>$\eta' \to \rho^0\gamma$</td>
<td>$0.41 \pm 0.03$</td>
<td>$0.41 \pm 0.02$</td>
<td>$0.40 \pm 0.04$</td>
</tr>
<tr>
<td>$\omega \to \eta\gamma$</td>
<td>$0.140 \pm 0.007$</td>
<td>$0.142 \pm 0.007$</td>
<td>$0.143 \pm 0.010$</td>
</tr>
<tr>
<td>$\eta' \to \omega\gamma$</td>
<td>$0.139 \pm 0.015$</td>
<td>$0.149 \pm 0.006$</td>
<td>$0.146 \pm 0.014$</td>
</tr>
<tr>
<td>$\phi \to \eta\gamma$</td>
<td>$0.209 \pm 0.002$</td>
<td>$0.209 \pm 0.018$</td>
<td>$0.209 \pm 0.013$</td>
</tr>
<tr>
<td>$\phi \to \eta'\gamma$</td>
<td>$0.22 \pm 0.01$</td>
<td>$0.22 \pm 0.02$</td>
<td>$0.22 \pm 0.02$</td>
</tr>
</tbody>
</table>
• **Data fitting**

iii) assuming $Z_{\eta'} = 0$ from the beginning, we get

$$g = 0.72 \pm 0.01\ \text{GeV}^{-1}, \quad \frac{m_s}{m} = 1.24 \pm 0.07, \quad \phi_V = (3.2 \pm 0.1)\degree,$$

$$\phi_P = (41.5 \pm 1.3)\degree, \quad |\phi_{\eta G}| \approx 0\degree, \quad \chi^2/\text{d.o.f.} = 4.4/4$$

$$z_q = 0.86 \pm 0.04, \quad z_s = 0.78 \pm 0.06, \quad z_K = 0.89 \pm 0.03,$$

Accepting the absence of gluonium for the $\eta'$ meson, the gluonic content of the $\eta$ wave function amounts to $|\phi_{\eta G}| \approx 0\degree$ or $(Z_{\eta})^2 = 0.00 \pm 0.12$ and the $\eta$-$\eta'$ mixing angle is found to be $\phi_P = (41.5 \pm 1.3)\degree$.

The current experimental data on $VP\gamma$ transitions indicate within our model a negligible gluonic content for the $\eta$ and $\eta'$ mesons.
• Data fitting

Using the latest experimental data on $(\rho, \omega, \phi) \rightarrow \eta \gamma$ (SND) and $\phi \rightarrow \eta' \gamma$ (KLOE), we get

$$\phi_P = (42.7 \pm 0.7)\degree, \quad z_q = 0.83 \pm 0.03, \quad z_s = 0.79 \pm 0.05, \quad \chi^2/{\text{d.o.f.}} = 4.0/5$$

$$\phi_P = (42.6 \pm 1.1)\degree, \quad |\phi_{\eta'G}| = (5 \pm 21)\degree, \quad z_q = 0.83 \pm 0.03, \quad z_s = 0.79 \pm 0.05, \quad \chi^2/{\text{d.o.f.}} = 4.0/4$$

confirmation of the null gluonic content of the $\eta$ and $\eta'$ wave functions

<table>
<thead>
<tr>
<th>Transition</th>
<th>$g_{V \gamma}^{\exp}$ (latest)</th>
<th>$g_{V \gamma}^{\text{th}}$ (Fit 3)</th>
<th>$g_{V \gamma}^{\text{th}}$ (Fit 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^0 \rightarrow \eta \gamma$</td>
<td>$0.429 \pm 0.023$</td>
<td>$0.436 \pm 0.017$</td>
<td>$0.437 \pm 0.028$</td>
</tr>
<tr>
<td>$\eta' \rightarrow \rho^0 \gamma$</td>
<td>$0.41 \pm 0.03$ (PDG)</td>
<td>$0.40 \pm 0.02$</td>
<td>$0.40 \pm 0.04$</td>
</tr>
<tr>
<td>$\omega \rightarrow \eta \gamma$</td>
<td>$0.136 \pm 0.007$</td>
<td>$0.134 \pm 0.006$</td>
<td>$0.134 \pm 0.009$</td>
</tr>
<tr>
<td>$\eta' \rightarrow \omega \gamma$</td>
<td>$0.139 \pm 0.015$ (PDG)</td>
<td>$0.146 \pm 0.006$</td>
<td>$0.146 \pm 0.013$</td>
</tr>
<tr>
<td>$\phi \rightarrow \eta \gamma$</td>
<td>$0.214 \pm 0.003$</td>
<td>$0.214 \pm 0.017$</td>
<td>$0.214 \pm 0.012$</td>
</tr>
<tr>
<td>$\phi \rightarrow \eta' \gamma$</td>
<td>$0.216 \pm 0.005$</td>
<td>$0.216 \pm 0.019$</td>
<td>$0.216 \pm 0.018$</td>
</tr>
</tbody>
</table>

no gluonium

confirmation of the null gluonic content of the $\eta$ and $\eta'$ wave functions

gluonium
Comparison with other approaches

\[ X_\eta = -\frac{1}{\sqrt{2}} Y_\eta = \frac{1}{\sqrt{3}} \]

η=η^8

68% CL bands

\[ X_\eta = Y_\eta = \frac{1}{\sqrt{2}} \]

democratic solution

\[ X_\eta^2 + Y_\eta^2 \leq 1 \]

✓ importance of \( \phi \rightarrow \eta \gamma \)
✓ importance of the slopes (\( \phi \_v \))
• *Comparison with other approaches*

![Graph showing various decay channels]

- $\phi \rightarrow \eta'\gamma$
- $\eta' \rightarrow \omega\gamma$
- $\eta' \rightarrow \rho\gamma$
- $X_{\eta'} = \sqrt{2}Y_{\eta'} = \frac{1}{\sqrt{3}}$
- $\eta = \eta_0$

- **Importance of constraining even more** $\phi \rightarrow \eta'\gamma$

- More refined data for this channel will **contribute decisively** to clarify this issue
• *Comparison with other approaches*

**PDG’06 data**

\[(\phi_P, Z_{\eta'}^2) = (42.6^\circ, 0.01)\]

**Latest data**

\[(\phi_P, Z_{\eta'}^2) = (41.4^\circ, 0.04)\]
• **Comparison with other approaches**


\[ |Z_\eta| < 0.4 \]
• Comparison with other approaches


\[ R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} = 26\% \]

\[ R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} = (13 \pm 13)\% \]
• Comparison with other approaches

$\phi_P = (39.7 \pm 0.7)^\circ$

$Z^2_{\eta'} = 0.14 \pm 0.04$

$\gamma_1 = \eta' \rightarrow \gamma \gamma / \pi^0 \rightarrow \gamma \gamma$
$\gamma_2 = \eta' \rightarrow \rho \gamma / \omega \rightarrow \pi^0 \gamma$
$\gamma_3 = \phi \rightarrow \eta' \gamma / \phi \rightarrow \eta \gamma$
$\gamma_4 = \eta' \rightarrow \omega \gamma / \omega \rightarrow \pi^0 \gamma$
• Comparison with other approaches

\[ R_{\phi} \equiv \frac{\Gamma(\phi \rightarrow \eta'\gamma)}{\Gamma(\phi \rightarrow \eta\gamma)} = \cot^2 \phi_P \cos^2 \phi_{\eta'} G \left( 1 - \frac{m_s}{m_z} \frac{z_q \tan \phi_V}{z_s \sin 2\phi_P} \right)^2 \left( \frac{p_{\eta'}}{p_{\eta}} \right)^3 \]

= \left( 4.7 \pm 0.6 \right) \times 10^{-3}

in agreement with \left( 4.8 \pm 0.5 \right) \times 10^{-3} (PDG'06) and \left( 4.77 \pm 0.09 \pm 0.19 \right) \times 10^{-3} (KLOE)  

✓
• **Summary**

We have performed a phenomenological analysis of radiative $V \rightarrow P \gamma$ and $P \rightarrow V \gamma$ decays with the purpose of determining the gluon content of the $\eta$ and $\eta'$ mesons.

The present approach is based on a conventional SU(3) quark model supplemented with two sources of SU(3) breaking, the use of constituent quark masses with $m_s > m$ and the different overlaps between the $P$ and $V$ wave functions.

The use of these different overlapping parameters (a specific feature of our analysis) is shown to be of primary importance in order to reach a good agreement.

• **Conclusions**

1) The current experimental data on $VP\gamma$ transitions indicate within our model a negligible gluonic content for the $\eta$ and $\eta'$ mesons,

$$Z_{\eta}^2 = 0.00 \pm 0.12 \quad \text{and} \quad Z_{\eta'}^2 = 0.04 \pm 0.09$$

2) Accepting the absence of gluonium for the $\eta$ meson, the gluonic content of the $\eta'$ wave function amounts to $|\Phi_{\eta'G}|=(12\pm13)^{\circ}$ or $(Z_{\eta'})^2=0.04\pm0.09$ and the $\eta$-$\eta'$ mixing angle is found to be $\phi_P=(41.4\pm1.3)^{\circ}$.
• **Conclusions**

3) **Imposing** the **absence of gluonium** for both mesons one finds $\phi_p = (41.5 \pm 1.2)^\circ$, in agreement with the former result.

4) The **latest experimental data** on $(\rho, \omega, \phi) \to \eta \gamma$ and $\phi \to \eta' \gamma$ decays confirm the null gluonic content of the $\eta$ and $\eta'$ wave functions.

5) More **refined experimental data**, particularly for the $\phi \to \eta' \gamma$ channel, will **contribute decisively** to clarify this issue.