Sensitivity study UPDATE

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Our work so far: β-decay spectrum

We have worked on the discrete part of the spectrum: we have been able to confirm fig. 5 from ArXiv: 2504.13259, in SUDDEN scheme.

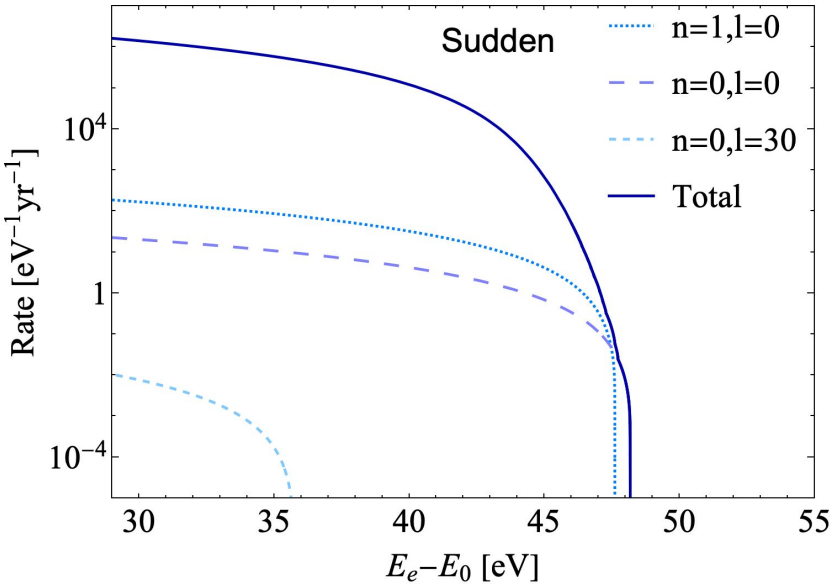

Figure: SUDDEN scheme

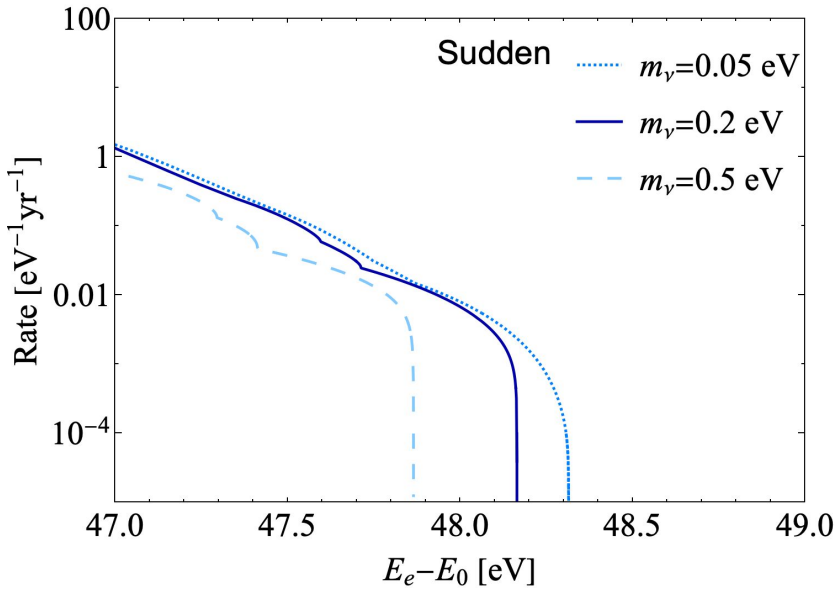

Figure: SUDDEN scheme

Some questions:

- We can't figure out the actual x-axis in fig. 5
- Q-value is a bit smaller than common end-point energy for atomic tritium (18526.6 eV vs 18592.01 eV)

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Fermi Function findings (I)

We have implemented a first version of the Fermi function, using its analytical definition by Konopinsky, applied to the atomic case:

$$F_{\mathrm{rel}}(Z,W) = 4(2p_e\sqrt{\frac{5}{3}}R_{ch})^{2\gamma-2}\frac{|\Gamma(\gamma+i\eta)|^2}{|\Gamma(2\gamma+1)|^2}e^{\pi\eta}$$

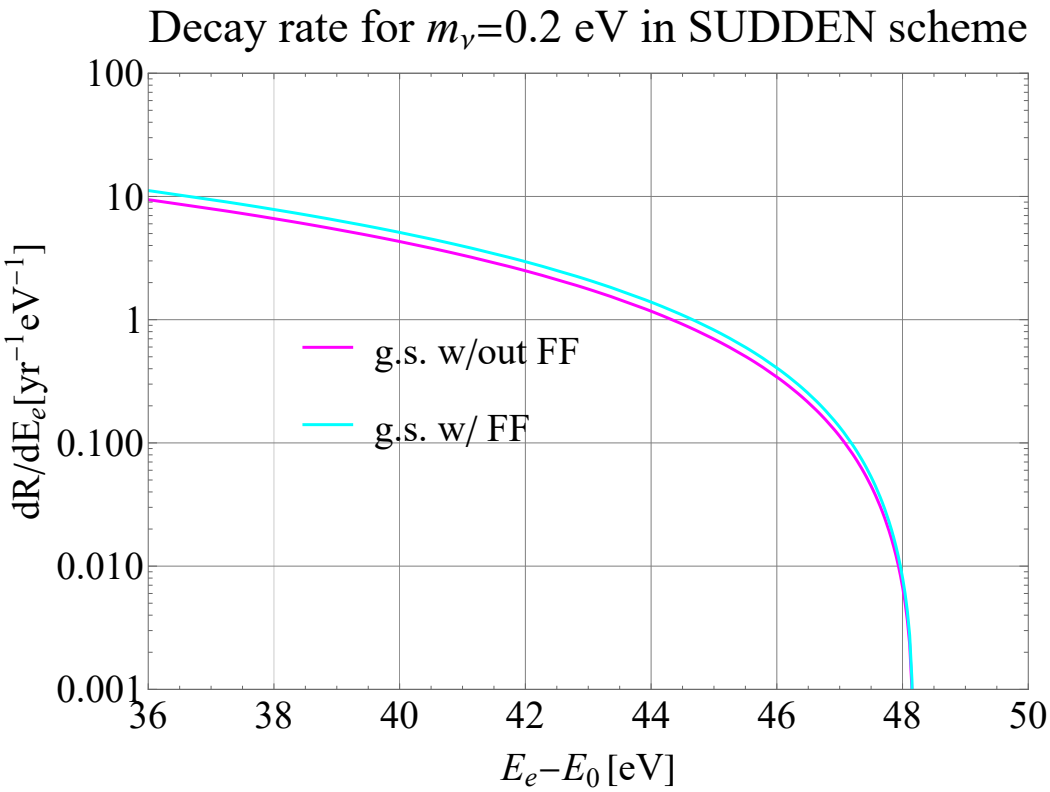

Figure: Impact of FF.

Fermi Function findings (II)

We have computed the difference (%) between the differential rate with and without the Fermi Function, in the case of Sudden regime for $n = l = 0$ and $m_\nu = 0.2$ eV.
For $E_e = Q$, the difference is

$$\simeq 18.5\%$$

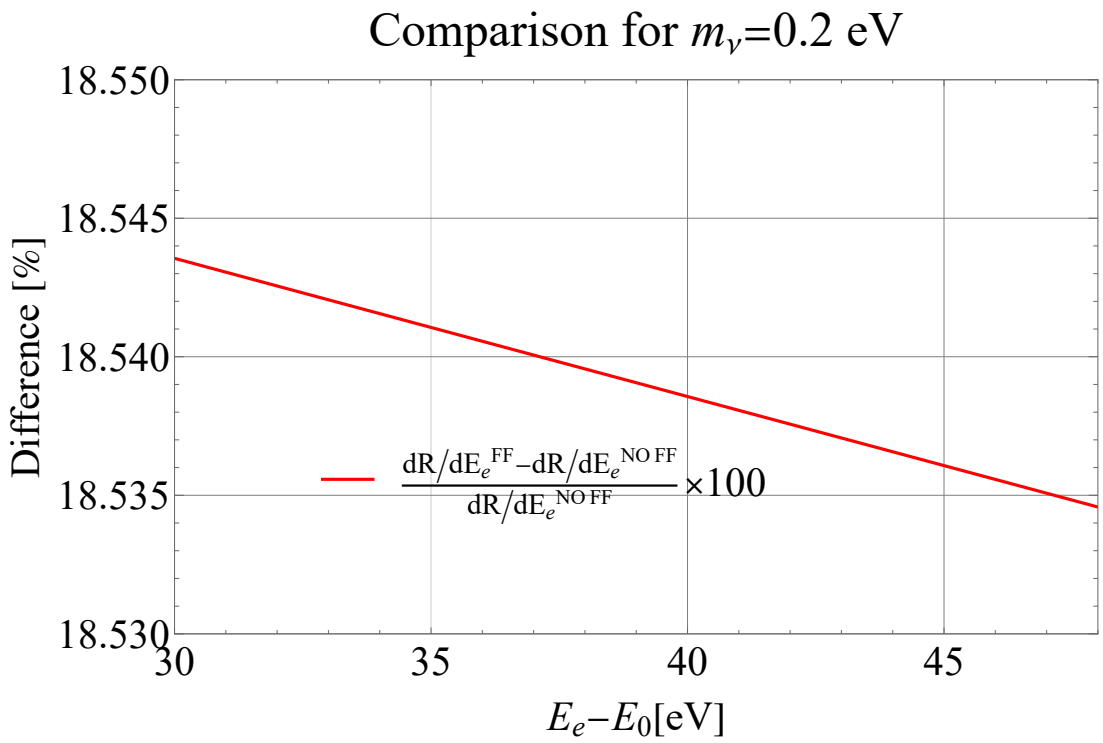

Figure: Impact of FF in the spectrum.

→ If you can give us the potential well, we can compute the Fermi function exactly(!)

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Corrective Factors

In the atomic case, the differential rate still has to be multiplied by a number of corrective factors, to take into accounts minor effects:

- Radiative corrections $G(W, W_0)$;
- Screening corrections $S(Z, W)$;
- Recoil effects $R(W, W_0)$;
- Finite structure of the nucleus $L(Z, W)$ and $C(Z, W)$;
- Recoiling Coulomb field $Q(Z, W, W_0)$;
- Orbital-electron interactions $I(Z, W)$.

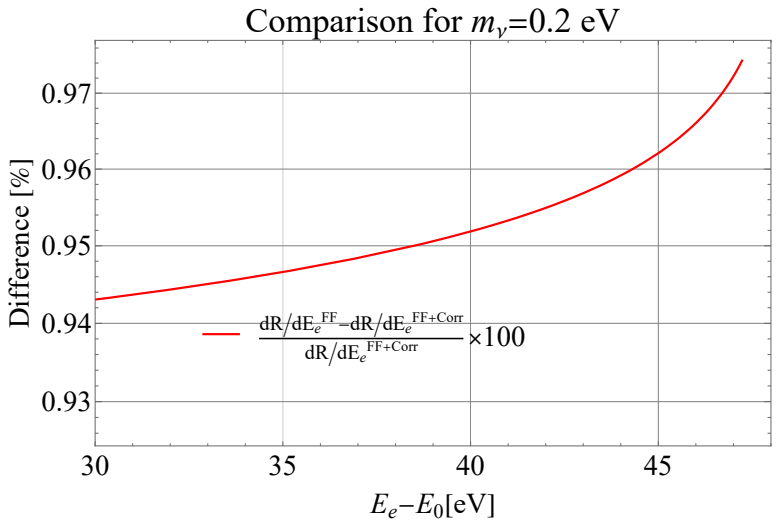

Figure: Difference between DR w/ and w/out corrective factors (still including FF).

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Impact of Corrections on rate

- Radiative corrections are the most impactful near the endpoint, causing a depletion of the β spectrum;
- All other corrections are nearly constant in the last 30 eV, so they only cause a normalization shift;
- Some terms should be treated with a little more care in the case of tritiated graphene (eg. which screening potential to use?)

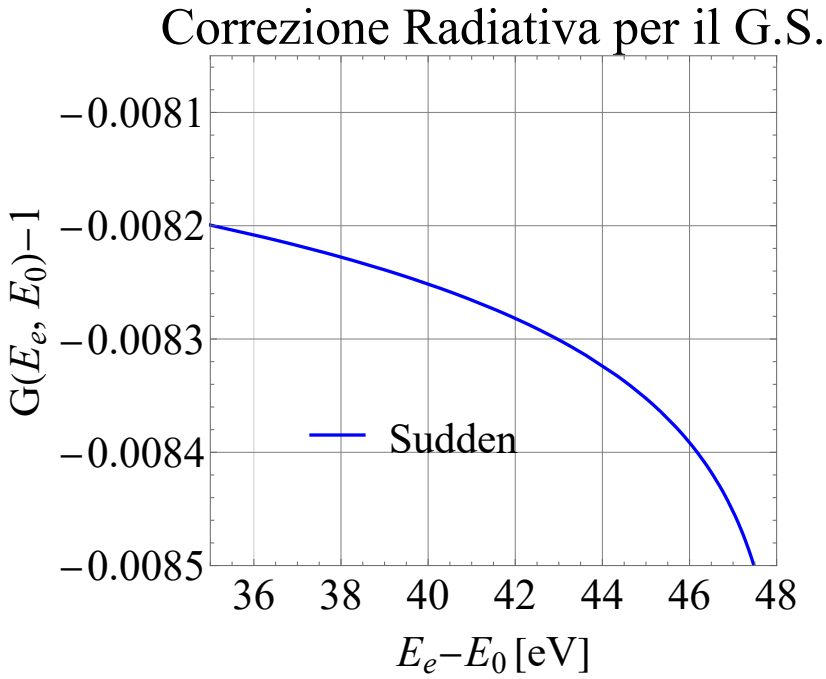

Figure: Radiative corrections for g.s. in sudden regime.

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Next steps

- Compute exact FF in graphene potential;
- Semi-sudden scheme;
- Continouus spectrum;
- Sensitivity curve update