Di-hadron fragmentation functions beyond LO

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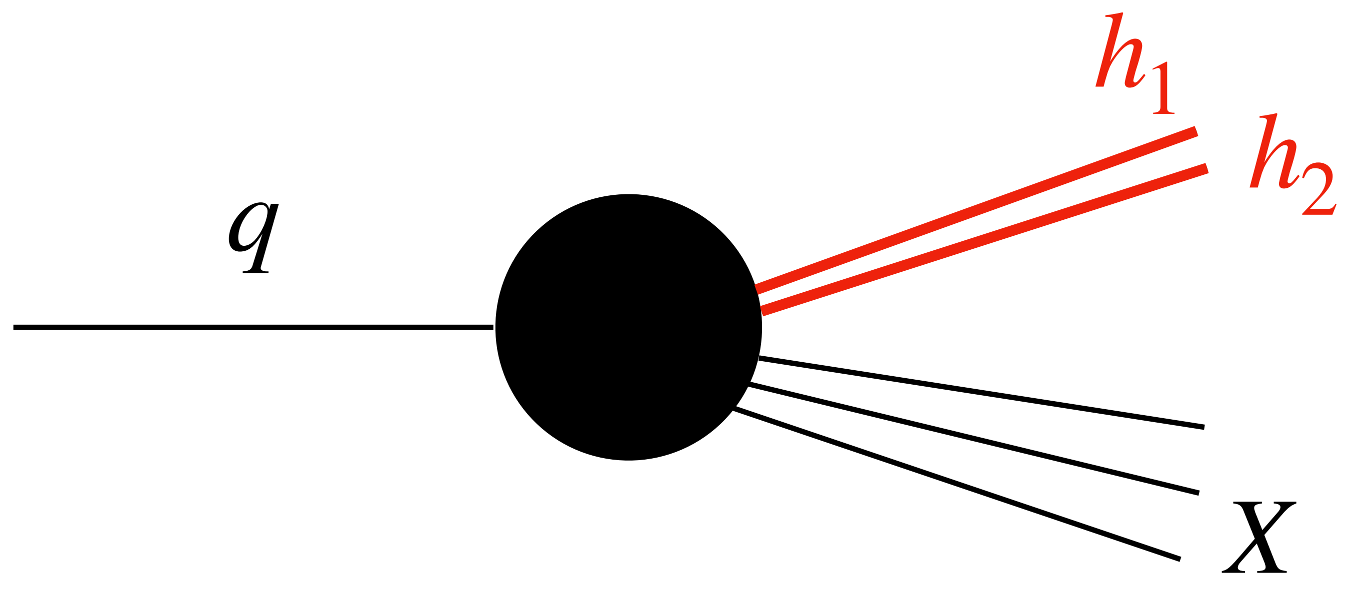
SPRING MAP, Pavia, Italy

Di-Hadron Fragmentation Functions

Non perturbative $q \to h_1 h_2 X$



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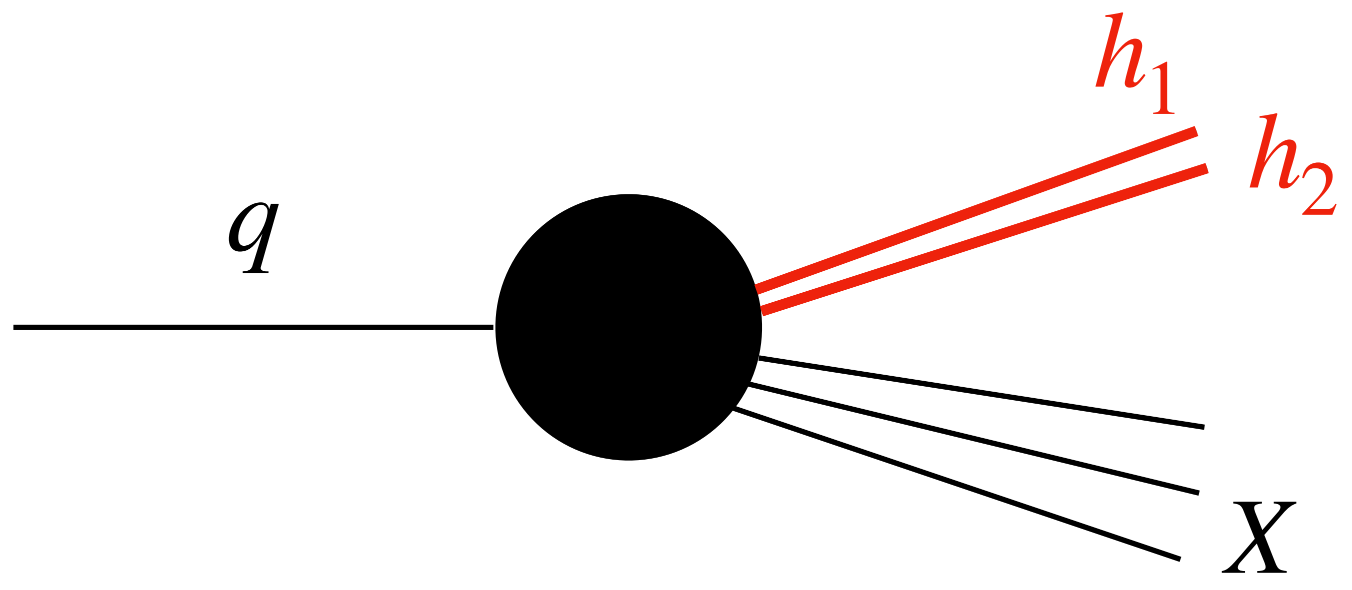

$$z_{1,2} = \frac{2E_{1,2}}{Q}$$

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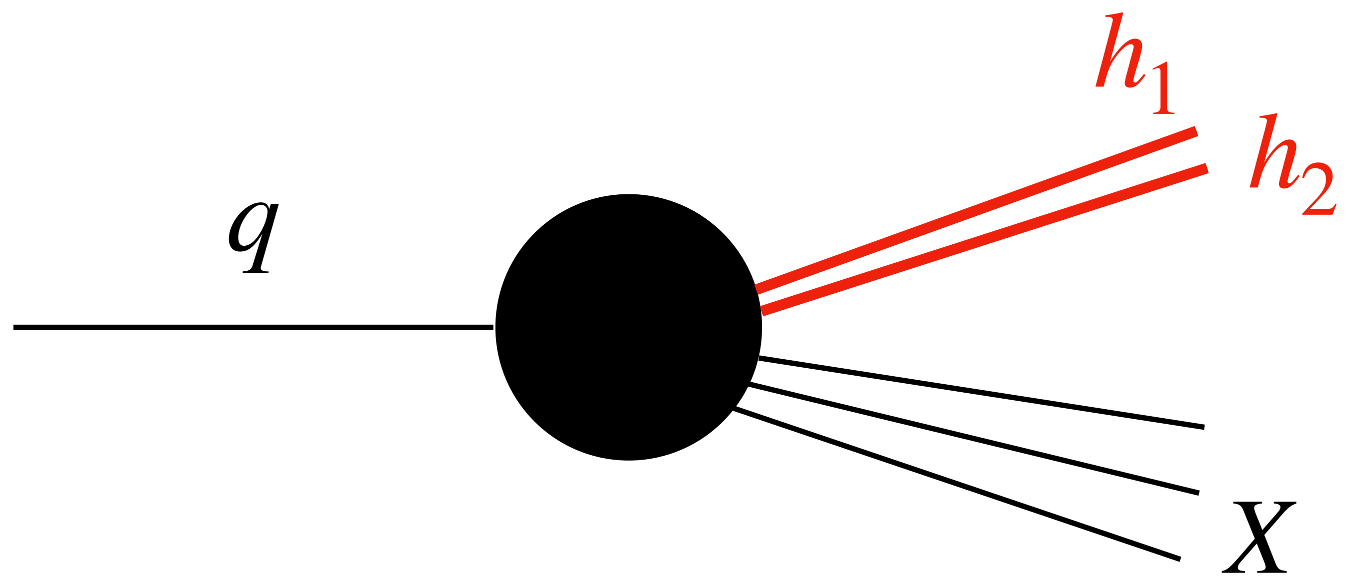

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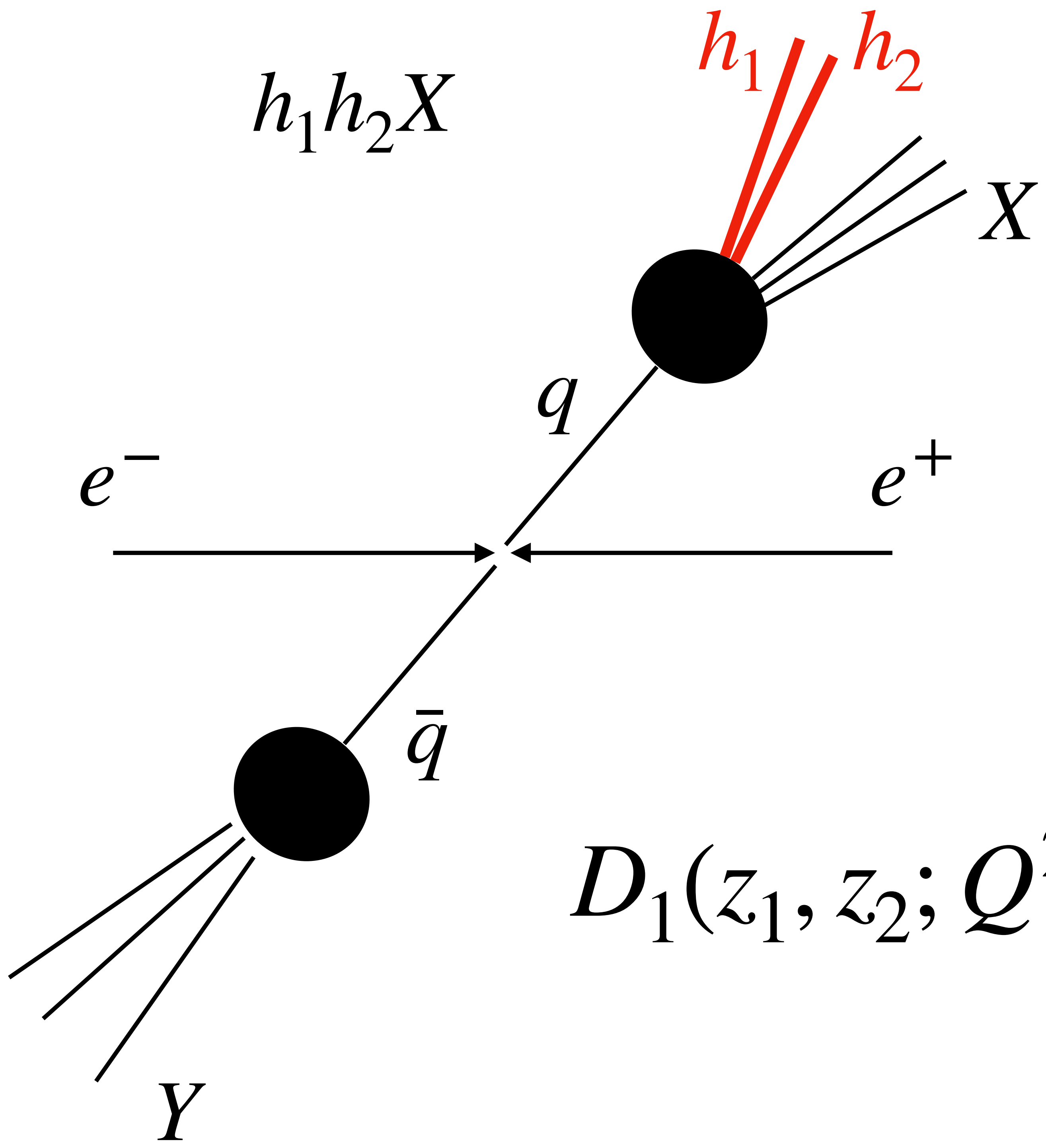
$e^+e^- \to$ $h_1 h_2 X$



$$D_1(z_1, z_2; Q^2)$$

Di-Hadron Fragmentation Functions

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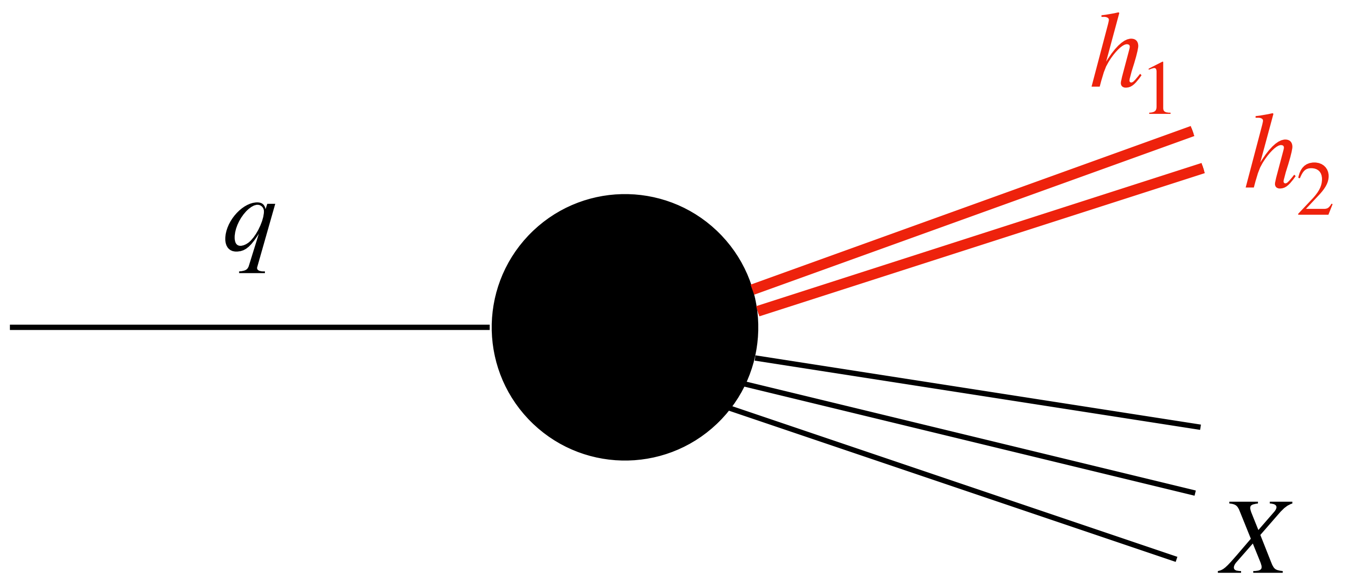

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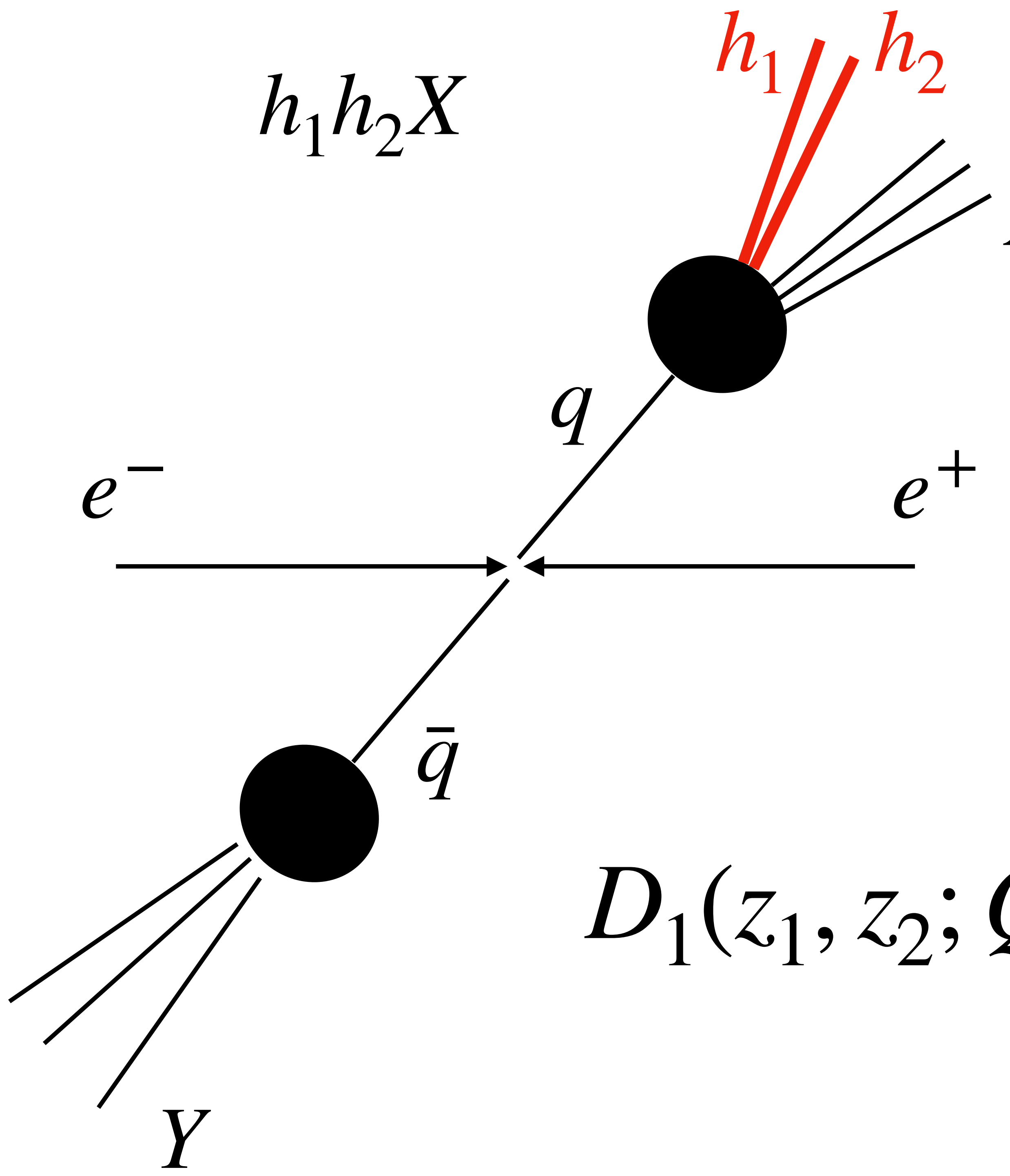
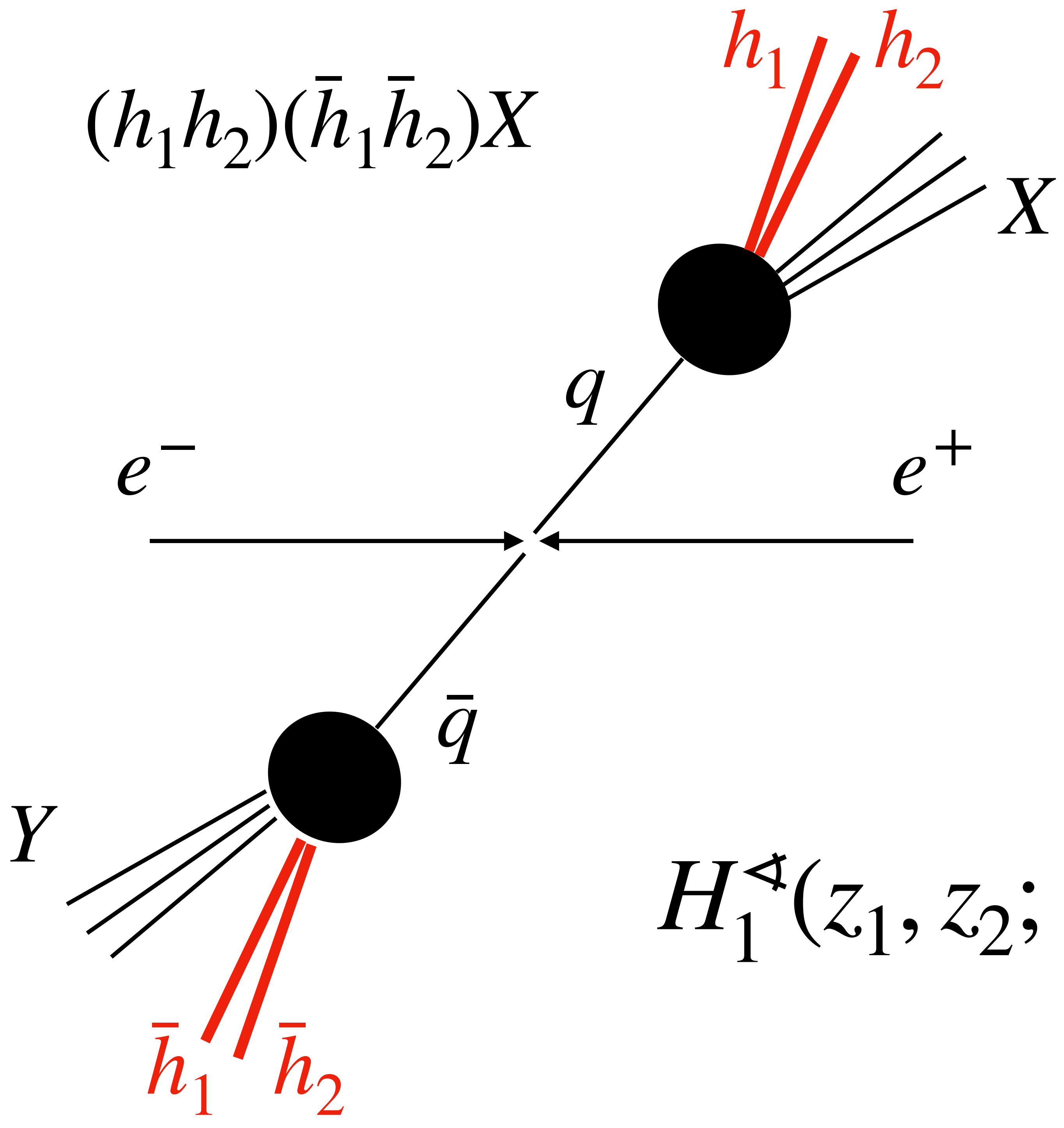
$e^+e^- \to$

$h_1 h_2 X$



$D_1(z_1, z_2; Q^2)$

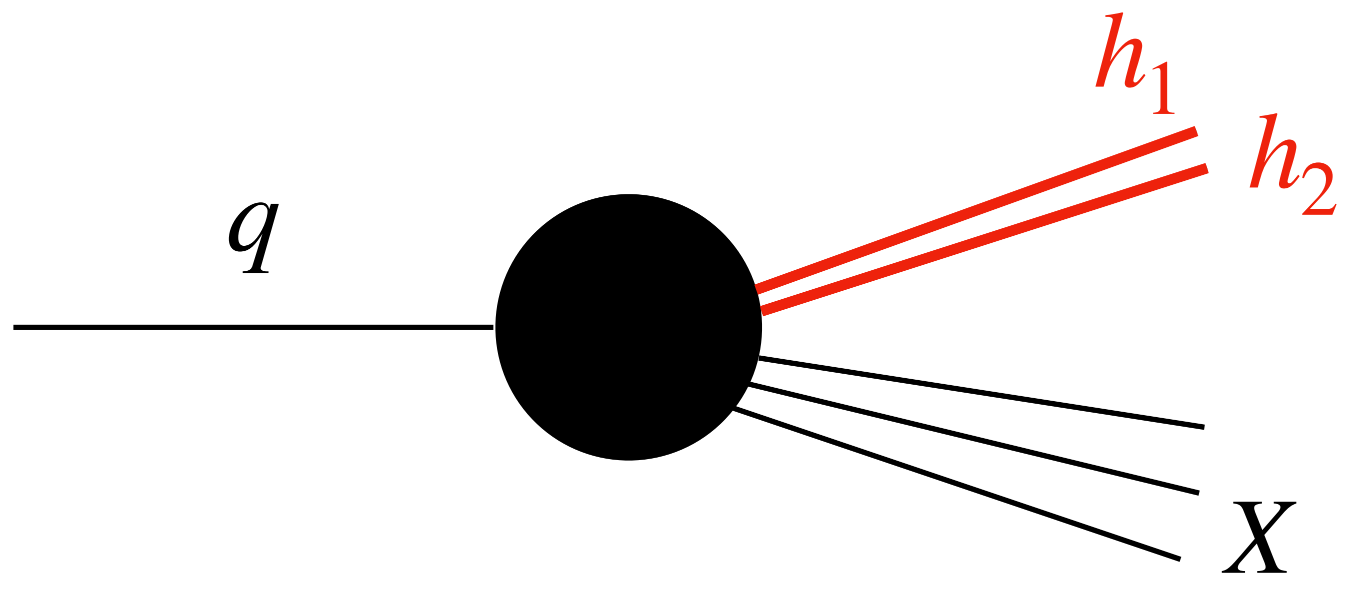
$(h_1 h_2)(\bar{h}_1 \bar{h}_2)X$



$H_1^{\sphericalangle}(z_1, z_2; Q^2)$

Extended Di-Hadron Fragmentation Functions

Non perturbative $q \to h_1 h_2 X$



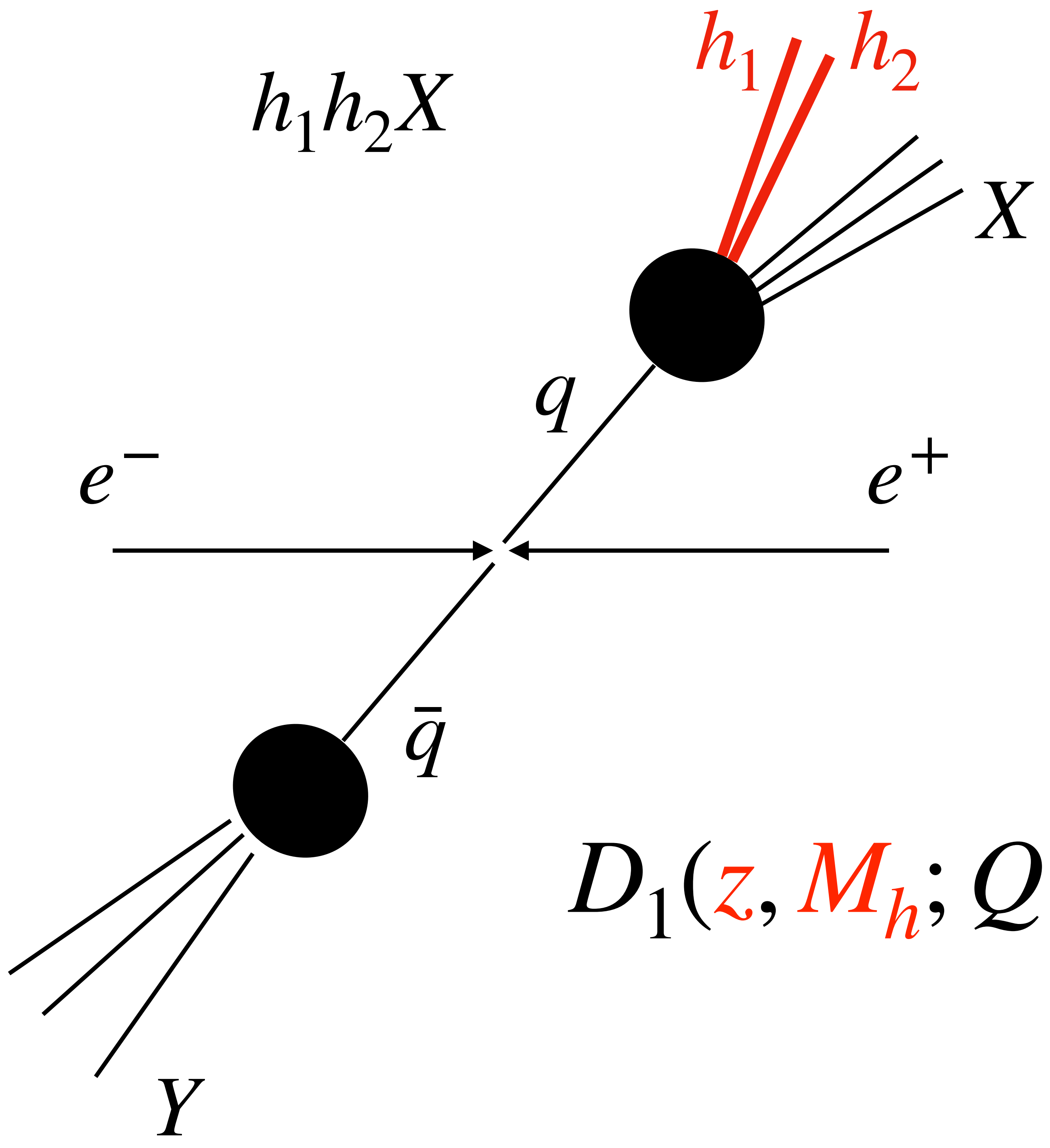
$z = z_1 + z_2$

$P_h = P_{h,1} + P_{h,2}$

$M_h = P_h^2$

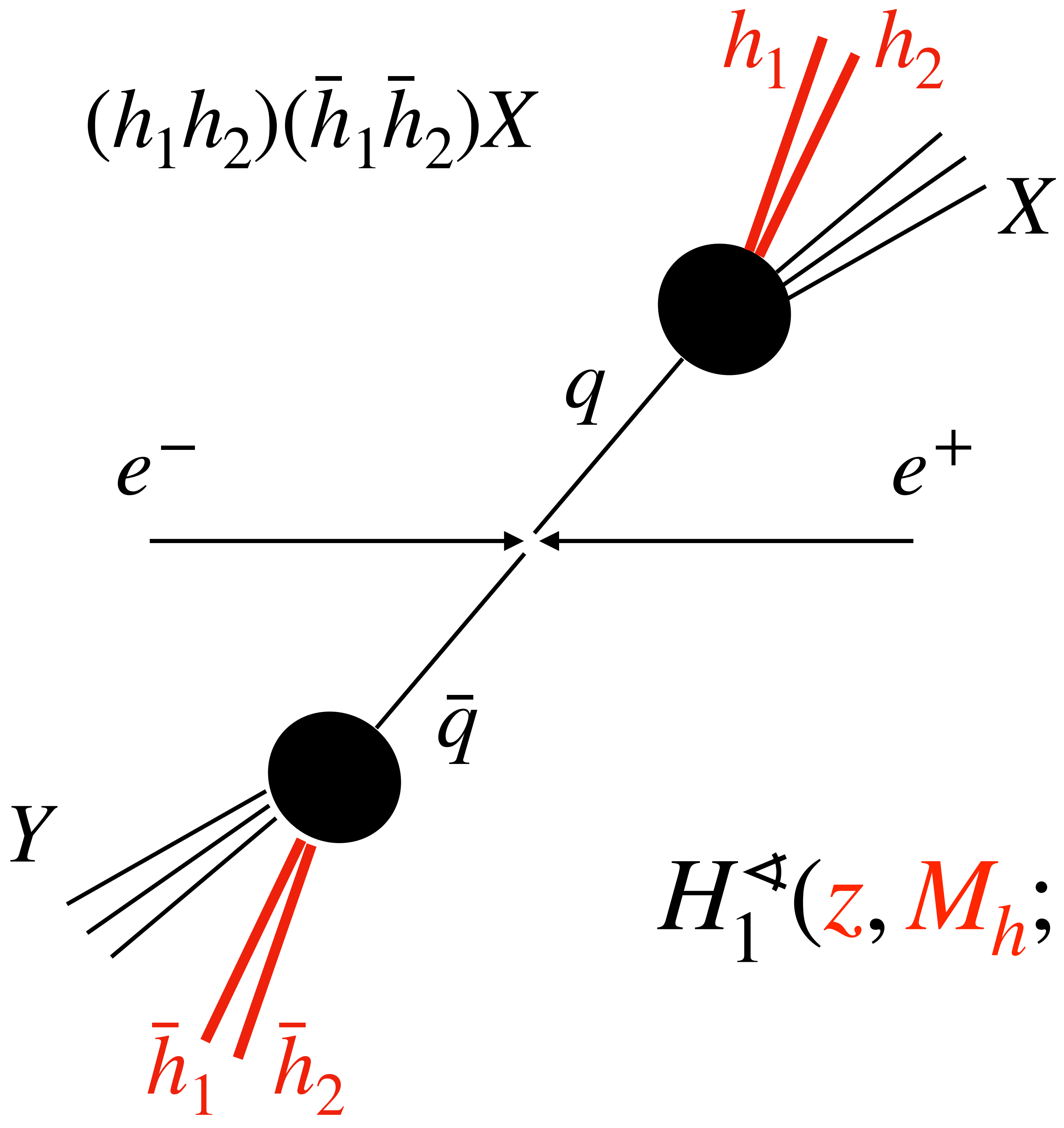
$e^+e^- \to$

$h_1 h_2 X$



$D_1(z, M_h; Q^2)$

$(h_1 h_2)(\bar{h}_1 \bar{h}_2)X$



$H_1^{\sphericalangle}(z, M_h; Q^2)$

Alternative method to extract trasversity PDF

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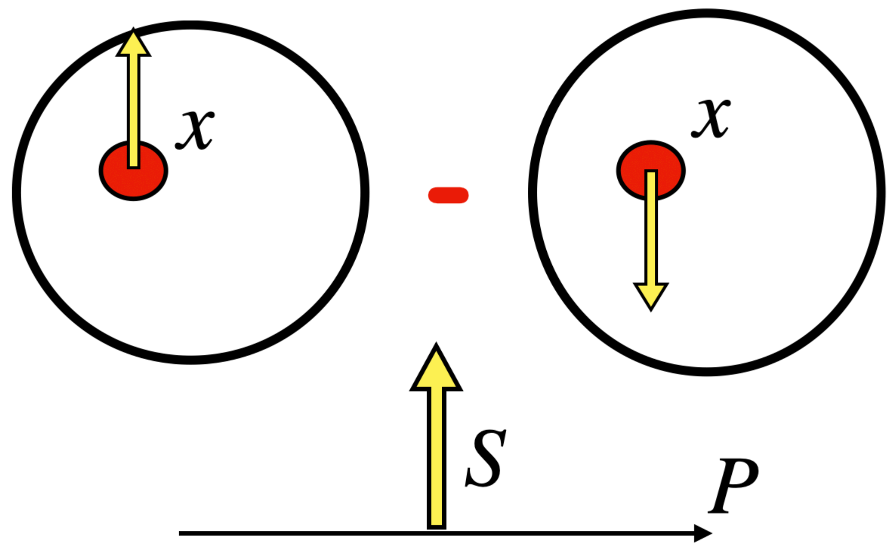
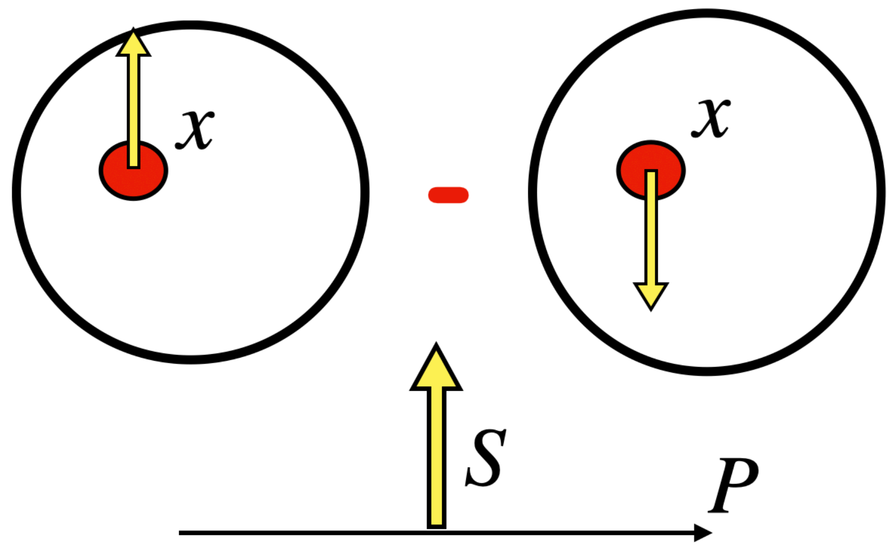


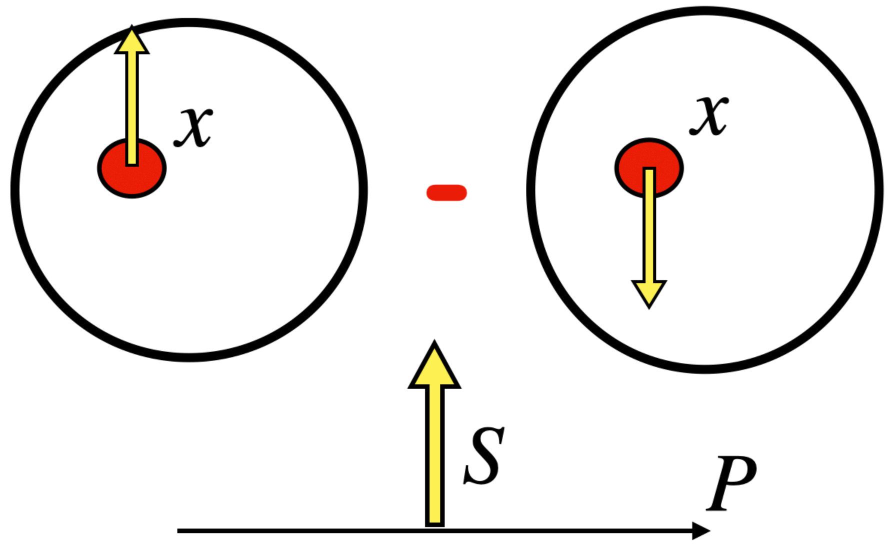
Chiral-odd function

Alternative method to extract trasversity PDF



Chiral-odd function → **It needs to couple to another chiral-odd function**

Alternative method to extract trasversity PDF



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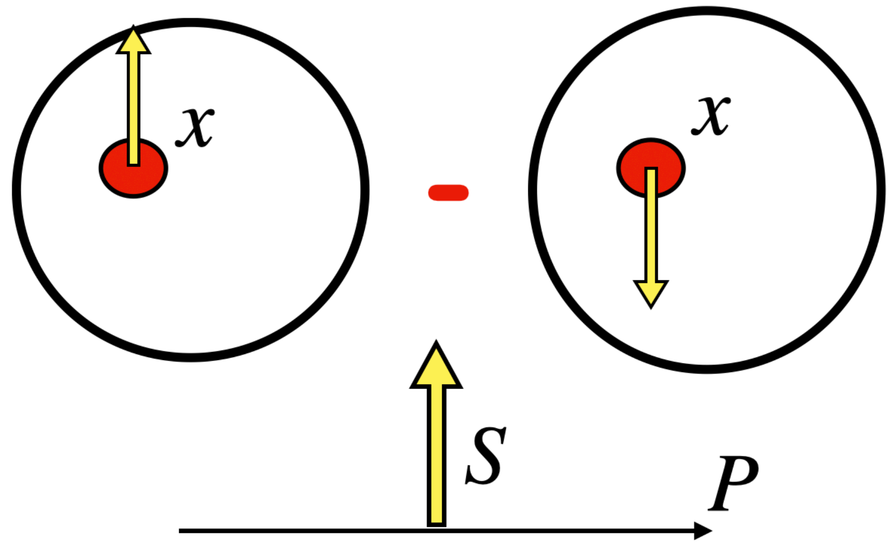

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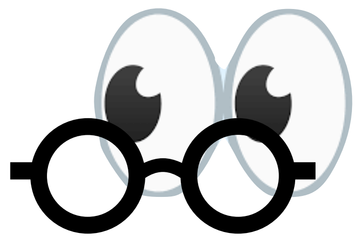

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Ex.: SIDIS

$$h_1^q(x_B, k_T) \otimes H_1^{\perp,q}(z, p_\perp)$$

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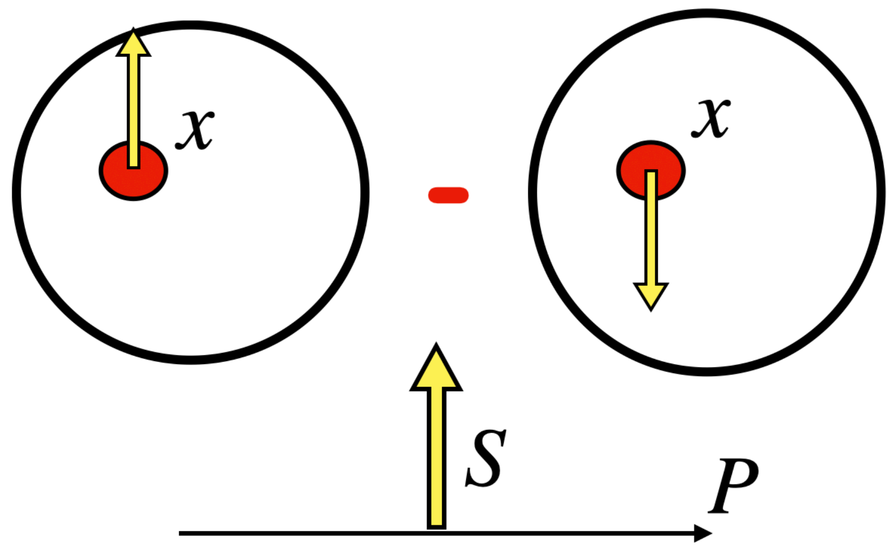

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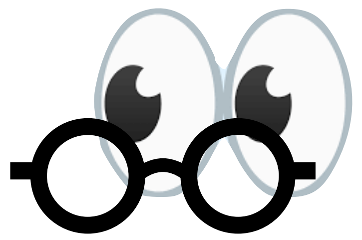

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$\otimes$ convolution over $k_T, p_\perp$

Evolution in the framework of TMD-factorization

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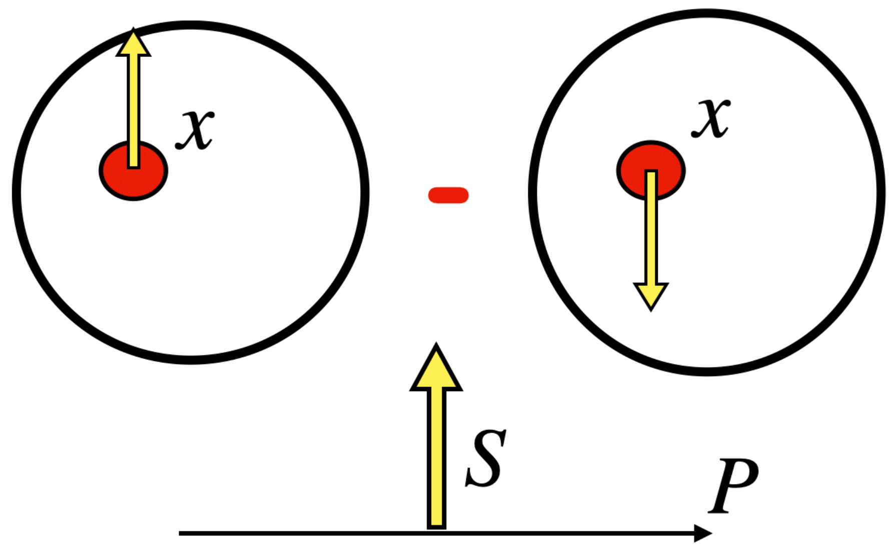

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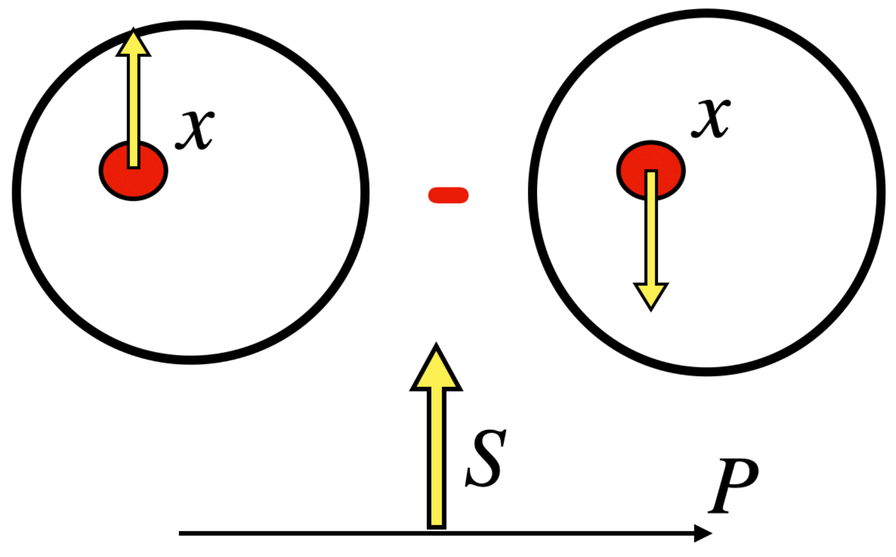

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Evolution in the framework of TMD-factorization

$$h_1^q(x_B) \cdot H_1^{\sphericalangle,q}(z, M_h)$$

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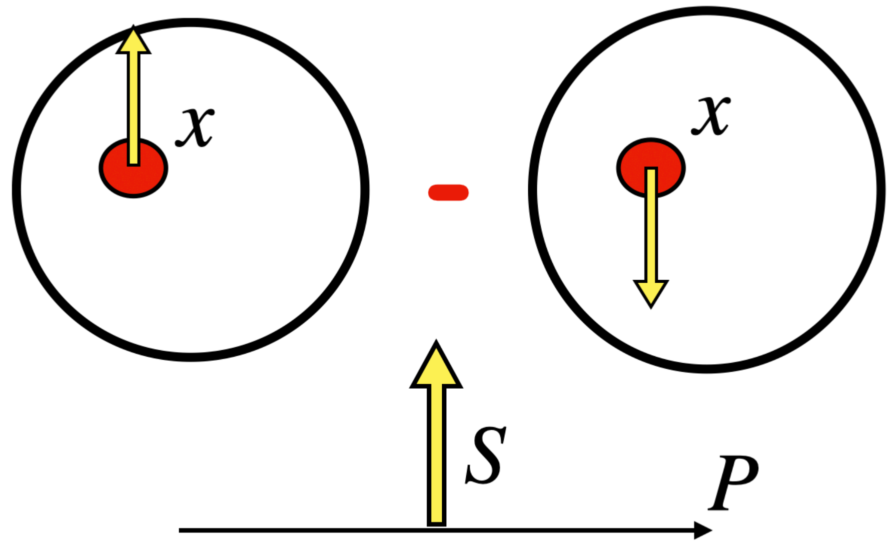

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Di-hadron

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Simple products

Collinear Framework

DGLAP eq. If $M_h \ll Q^2$

$h_1^q H_1^{\sphericalangle,q}$ also in $pp^\uparrow$ collisions

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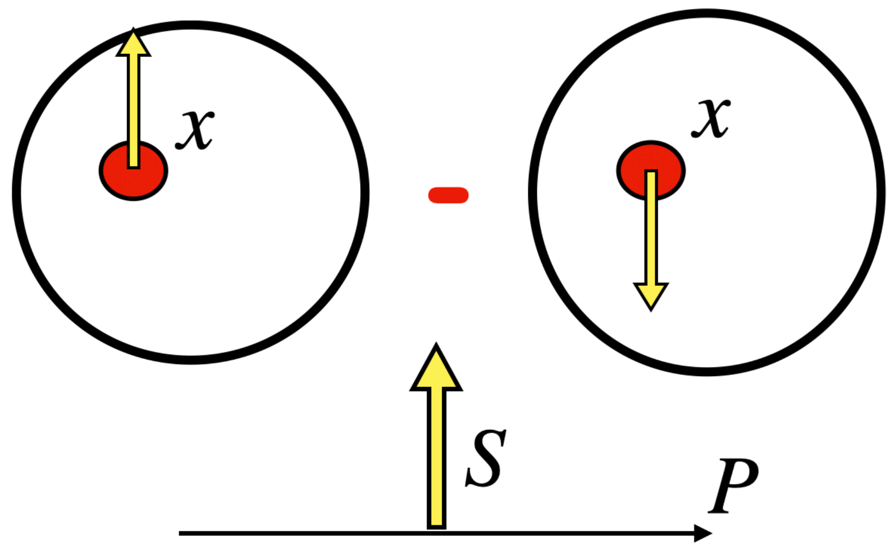

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Evolution in the framework of TMD-factorization

$$A_{UT}^{hh} \sim \frac{\sum_q e_q^2 \cdot h_1^q(x_B) \cdot H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2 f_1^q(x_B) \cdot D_1^q(z, M_h)}$$

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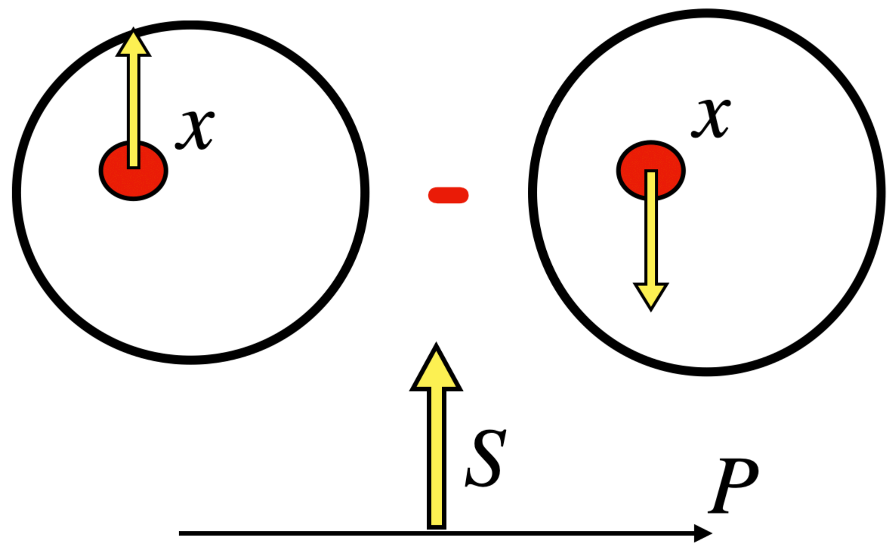

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Di-hadron LO

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Di-hadron FF beyond LO

Unpolarized observable at NLO

$e^+e^- \to \gamma^* \to h_1 h_2 X$

2017 BELLE data of $e^+e^- \to \pi^+\pi^- X$ at $\sqrt{S} = 10.58$ GeV

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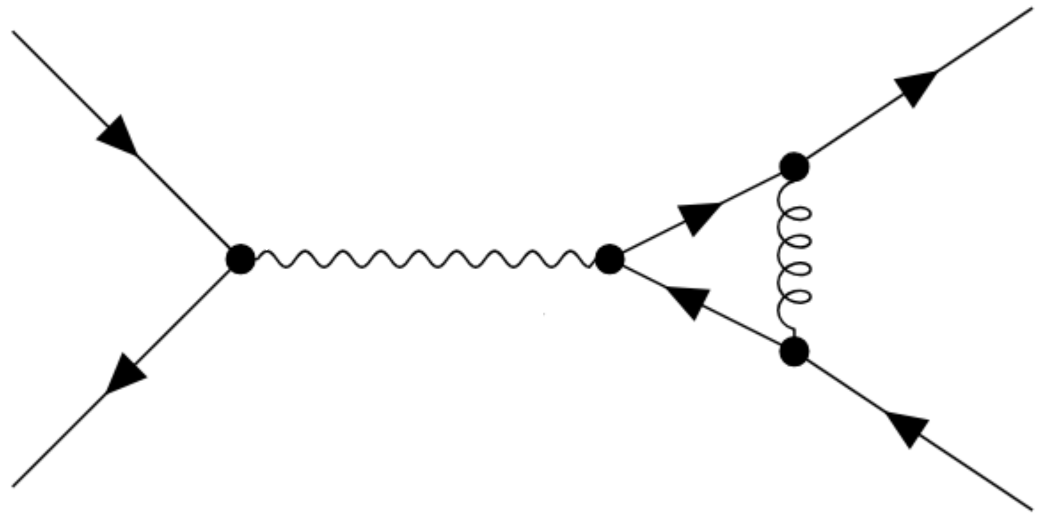
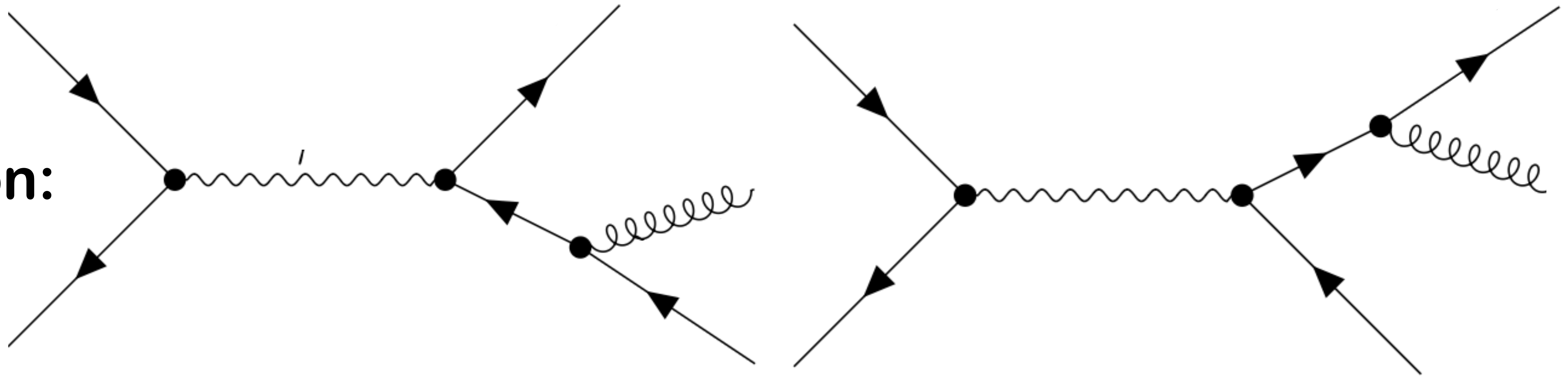
NLO contributions to $e^+e^- \to \gamma^* \to q\,\bar{q}$

Virtual gluon:



Real gluon:



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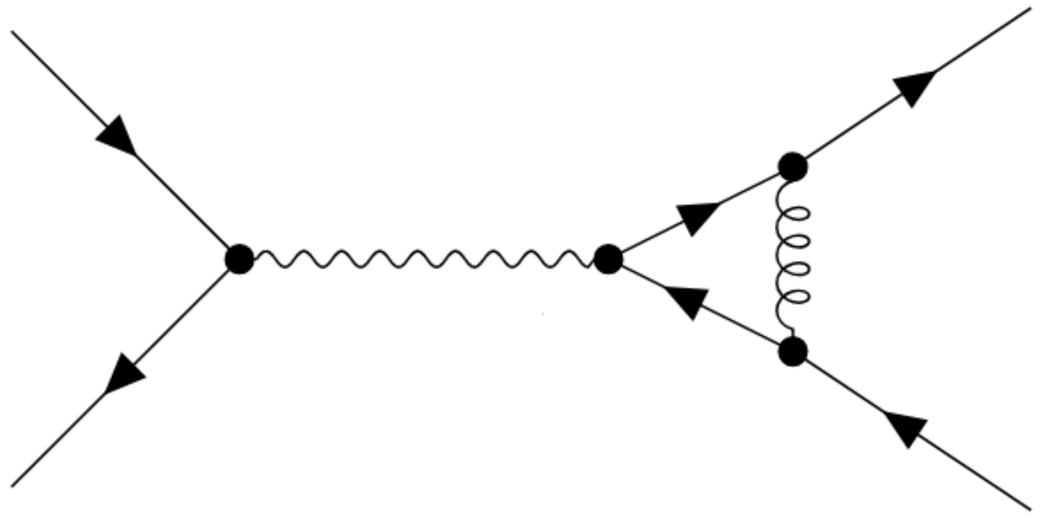

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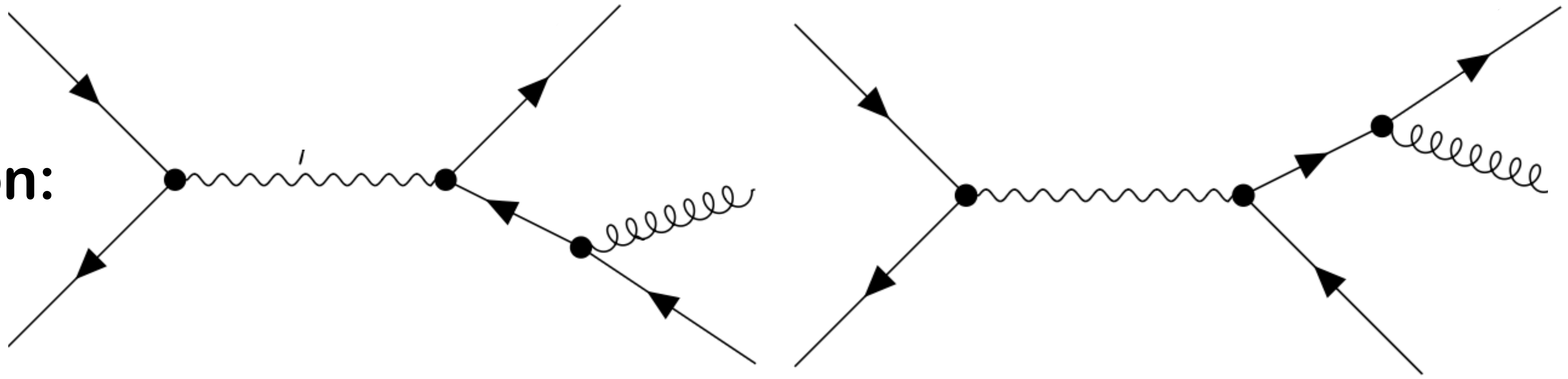
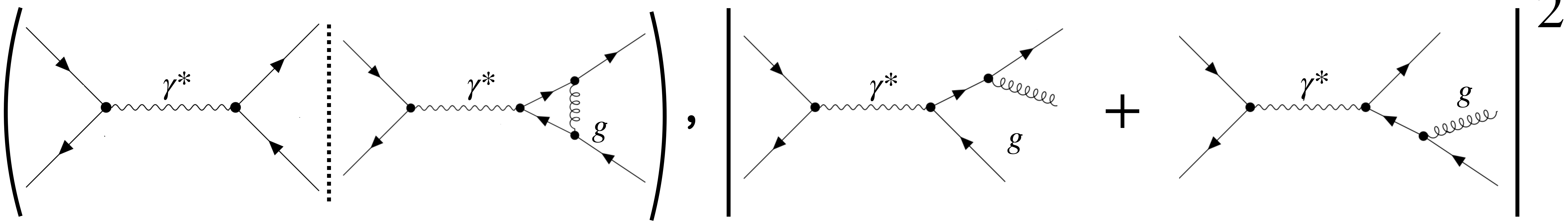
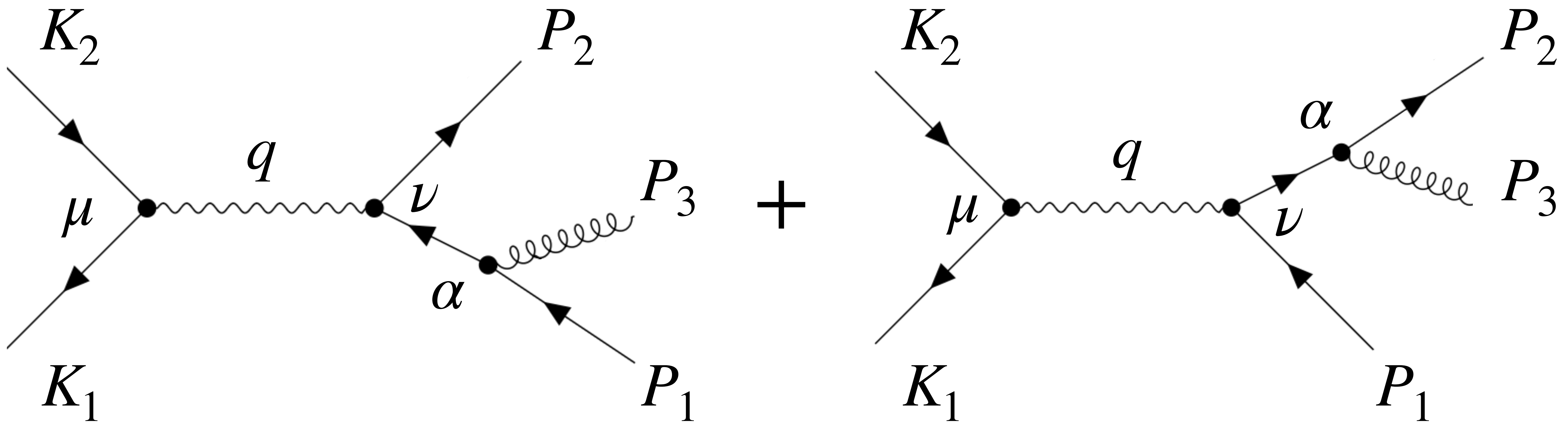


$\mathcal{O}(\alpha_S) \sim$



σ_V σ_R

Calculation



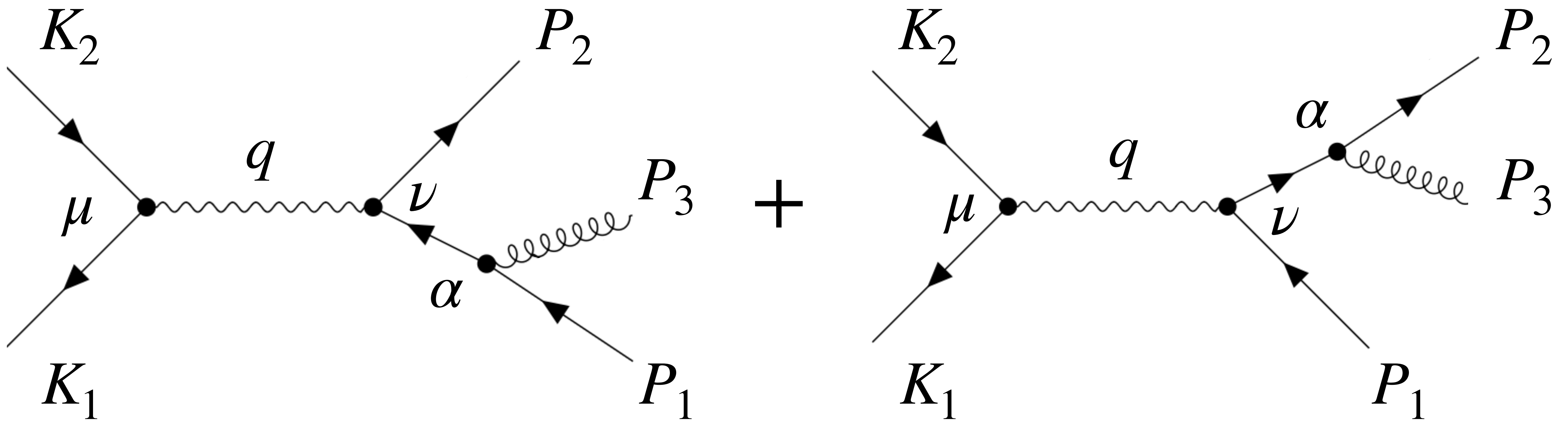
$$\begin{cases} u = \dfrac{P_1 \cdot P_2}{P_1 \cdot q} \\ z = \dfrac{2\, P_1 \cdot q}{Q^2} \end{cases}$$

d-dimensions $\quad |\, iM_R|^2 \sim L_{\mu\nu}\, W^{\mu\nu}$

$$\frac{d\sigma^R}{du\, dz} = \sigma_0\, e_q^2\, \frac{4\alpha_s}{3\pi} \left(\frac{Q^2}{4\pi\mu^2}\right)^{-2\epsilon} \frac{3}{8}\, \frac{(2-2\epsilon)^2}{(3-2\epsilon)}\, \frac{1}{\Gamma(2-2\epsilon)} \times$$

$$u^{-\epsilon}\, z^{-2\epsilon}\, (1-z)^{-1-\epsilon}\, (1-u)^{-1-\epsilon} \left[z^2 + (1+z(u-1))^2 - \epsilon(1-u\,z)^2\right]$$

Calculation



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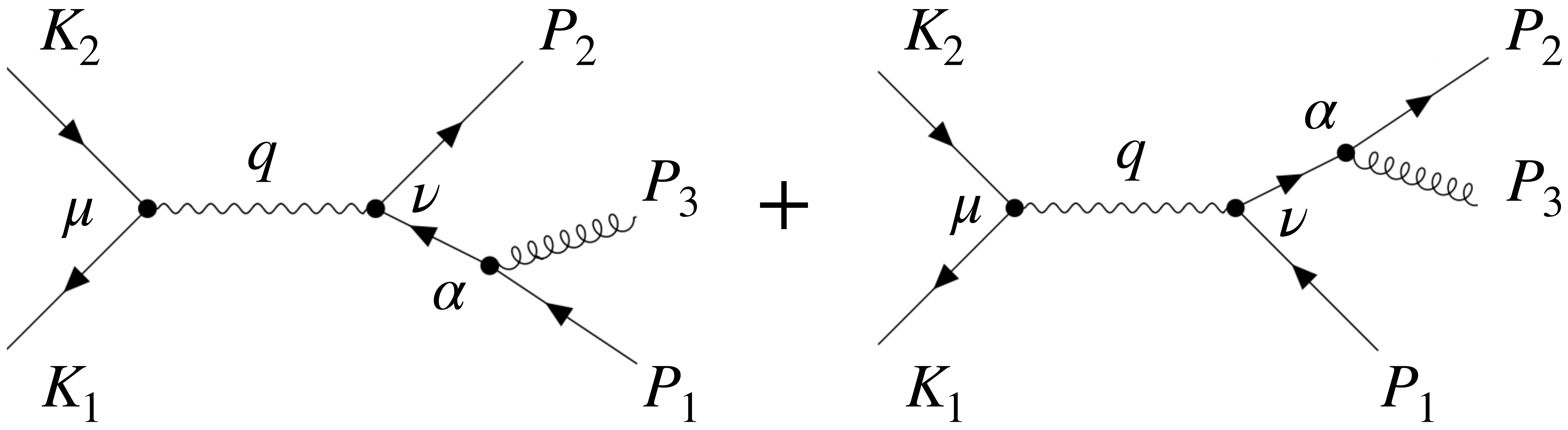
$$\boxed{u^{-\epsilon}\, z^{-2\epsilon}\, (1-z)^{-1-\epsilon}\, (1-u)^{-1-\epsilon} \left[z^2 + (1+z(u-1))^2 - \epsilon(1-u\, z)^2 \right]}$$

$$(1-z)^{-1-\epsilon} = -\frac{1}{\epsilon}\, \delta(1-z) + \frac{1}{(1-z)_+} - \epsilon \left(\frac{\ln(1-z)}{1-z} \right)_+ + \mathcal{O}(\epsilon^2) \qquad \int_0^1 dz\, \frac{f(z)-f(1)}{1-z} = \int_0^1 dz\, \frac{f(z)}{(1-z)_+}$$

Calculation



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$$\frac{d\sigma^R}{du\,dz} = \sigma_0\, e_q^2 \frac{4\alpha_s}{3\pi}\left(\frac{Q^2}{4\pi\mu^2}\right)^{-2\epsilon}(e^{-\gamma_E})^{2\epsilon}\,\frac{3}{8}\,\frac{(2-2\epsilon)^2}{(3-2\epsilon)}\,\frac{1}{(1-2\epsilon)}\left(1-\frac{\pi}{3}\epsilon^2\right)\times$$

$$\times\left\{\frac{1}{\epsilon^2}\Big[\ldots\Big]\delta(1-u)\delta(1-z)+\frac{1}{\epsilon}(\ldots)\left[-\frac{\delta(1-u)}{(1-z)_+}-\frac{\delta(1-z)}{(1-u)_+}\right]+\delta(1-u)(\ldots)+\delta(1-z)(\ldots)+\frac{(\ldots)}{(1-u)_+(1-z)_+}+\mathcal{O}(\epsilon)\right\}$$

$$\sigma^V = -\,\sigma_0\, e_q^2 \frac{4\alpha_s}{3\pi}\left(\frac{Q^2}{4\pi\mu^2}\right)^{-2\epsilon}(e^{-\gamma_E})^{2\epsilon}\left(\frac{1}{\epsilon^2}+\frac{13}{6\epsilon}-\frac{5\pi^2}{4}+\frac{58}{9}+\mathcal{O}(\epsilon)\right)$$

Collinear divergencies

Sum of real and virtual: $\frac{1}{\sigma_0^d}\frac{d\sigma^R}{du\,dz} \quad + \quad \frac{1}{\sigma_0^d}\sigma_V\,\delta(1-z)\delta(1-u)$

$$\frac{1}{\sigma_0^d}\frac{d\sigma}{du\,dz} = e_q^2\frac{\alpha_s}{2\pi}\left(\frac{Q^2}{4\pi\mu^2}\right)^{-\epsilon}(e^{-\gamma_E})^{\epsilon}\left\{-\frac{1}{\epsilon}\left[\delta(1-z)P_{qq}(u)+\delta(1-u)P_{qq}(z)\right]\right\}+$$

$$= + e_q^2\frac{4\alpha_s}{3\pi}\left\{\delta(1-u)\delta(1-z)\left(-4+\frac{\pi^2}{2}\right)+\right.$$
$$+\delta(1-u)\left[\frac{1}{2}(1-z)+\frac{1}{2}(1+z^2)\left(\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{2\ln z}{(1-z)_+}\right)\right]+$$
$$+\delta(1-z)\left[\frac{1}{2}(1-u)+\frac{1}{2}(1+u^2)\left(\left(\frac{\ln(1-u)}{1-u}\right)_+ + \frac{\ln u}{(1-u)_+}\right)\right]+$$
$$\left.+\frac{1}{2}\left(z^2+(1+z(u-1))^2\right)\frac{1}{(1-u)_+(1-z)_+}+\mathcal{O}(\epsilon)\right\}.$$

NLO coefficient

$$\left(\frac{Q^2}{4\pi\mu^2}\right)^{-\epsilon}(e^{-\gamma_E})^{\epsilon} = [1 - \epsilon\ \ln\left(\frac{Q^2}{\mu^2}\right) + \epsilon(\ln(4\pi) - \gamma_E) + \mathcal{O}(\epsilon)]$$

After $\bar{MS}$ subtraction:

$$\frac{1}{\sigma_0^d}\frac{d\sigma}{du\,dz} = e_q^2\frac{\alpha_s}{2\pi}\left[\delta(1-z)P_{qq}(u) + \delta(1-u)P_{qq}(z)\right]\ln\left(\frac{Q^2}{\mu^2}\right) + e_q^2\frac{4}{3}\frac{\alpha_s}{2\pi}\Bigg\{$$

$$+\,\delta(1-u)\delta(1-z)\left(\pi^2 - 8\right)$$

$$+\,\delta(1-u)\left[(1-z) + (1+z^2)\left(\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{2\ln z}{(1-z)_+}\right)\right]$$

$$+\,\delta(1-z)\left[(1-u) + (1+u^2)\left(\left(\frac{\ln(1-u)}{1-u}\right)_+ + \frac{\ln u}{(1-u)_+}\right)\right]$$

$$+\,\left(z^2 + (1 + z(u-1))^2\right)\frac{1}{(1-u)_+(1-z)_+} + \mathcal{O}(\epsilon)\Bigg\}.$$

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$$+e_q^2\frac{4}{3}\frac{\alpha_s}{2\pi}\,\mathrm{NLO}(u,z)$$

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$$\frac{1}{\sigma_0}\frac{d\sigma}{du\,dz\,dt} = \sum_q e_q^2\Bigg\{D_1^{q\to H_1}(z,t)\,D_1^{\bar{q}\to H_2}(u,t)$$

$$+\int_z^1\frac{dz'}{z'}\int_u^1\frac{du'}{u'}\frac{4}{3}\frac{\alpha_s}{2\pi}\,\text{NLO}(u',z')\,D_1^{q\to H_1}\left(\frac{z}{z'},t\right)\,D_1^{\bar{q}\to H_2}\left(\frac{u}{u'},t\right)+q\leftrightarrow\bar{q}$$

$$+\text{gluon terms}\Bigg\}$$

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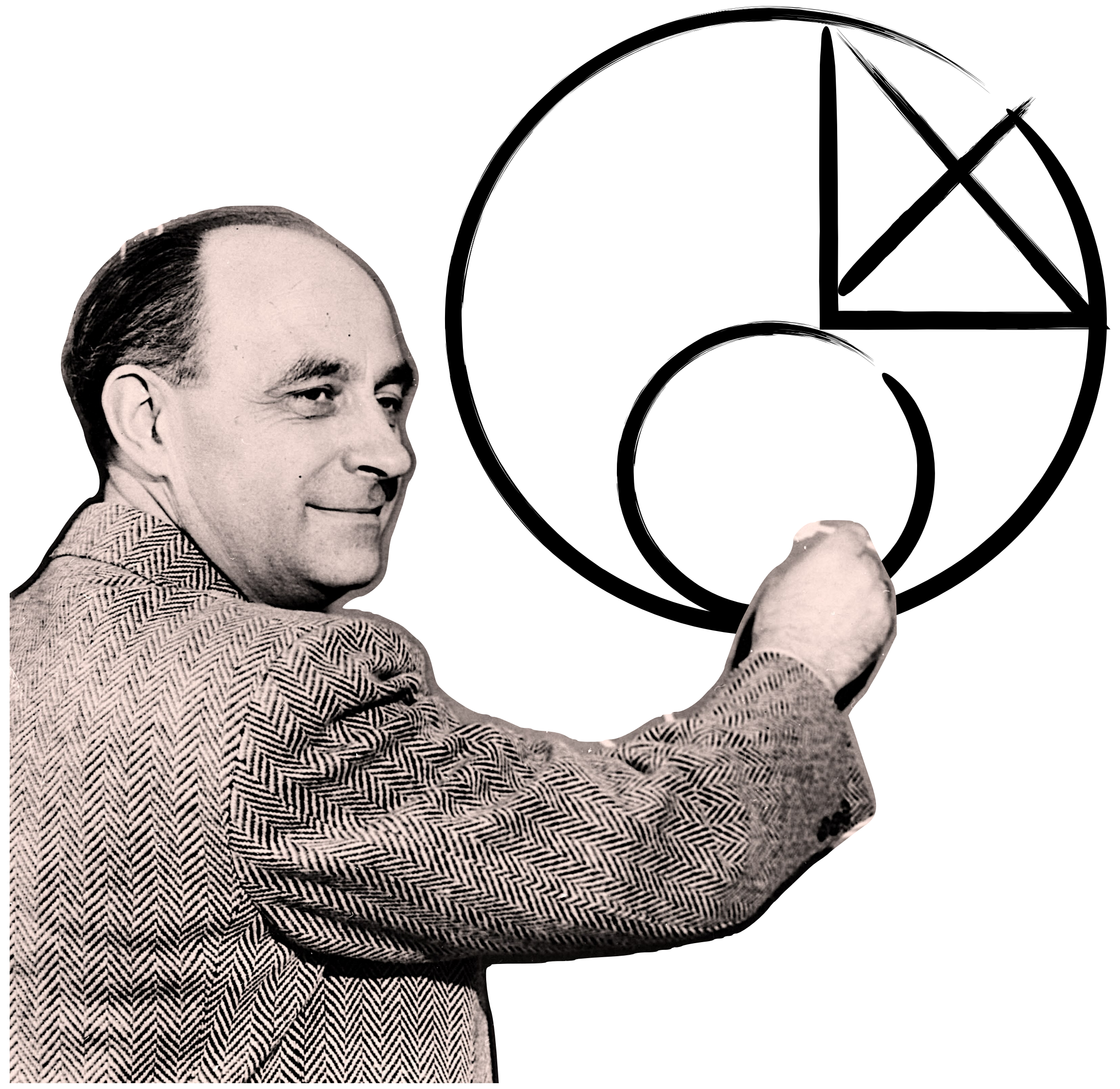
$$\frac{d\sigma}{dz\,dM_h\,dQ^2} = \frac{4\pi\alpha}{Q^2}\sum_q e_q^2\sum_i\int_z^1\frac{dz'}{z'}\,\mathrm{NLO}_i(z')\,D_1^q\left(\frac{z}{z'},M_h;\,Q^2\right)$$

D_1 extraction

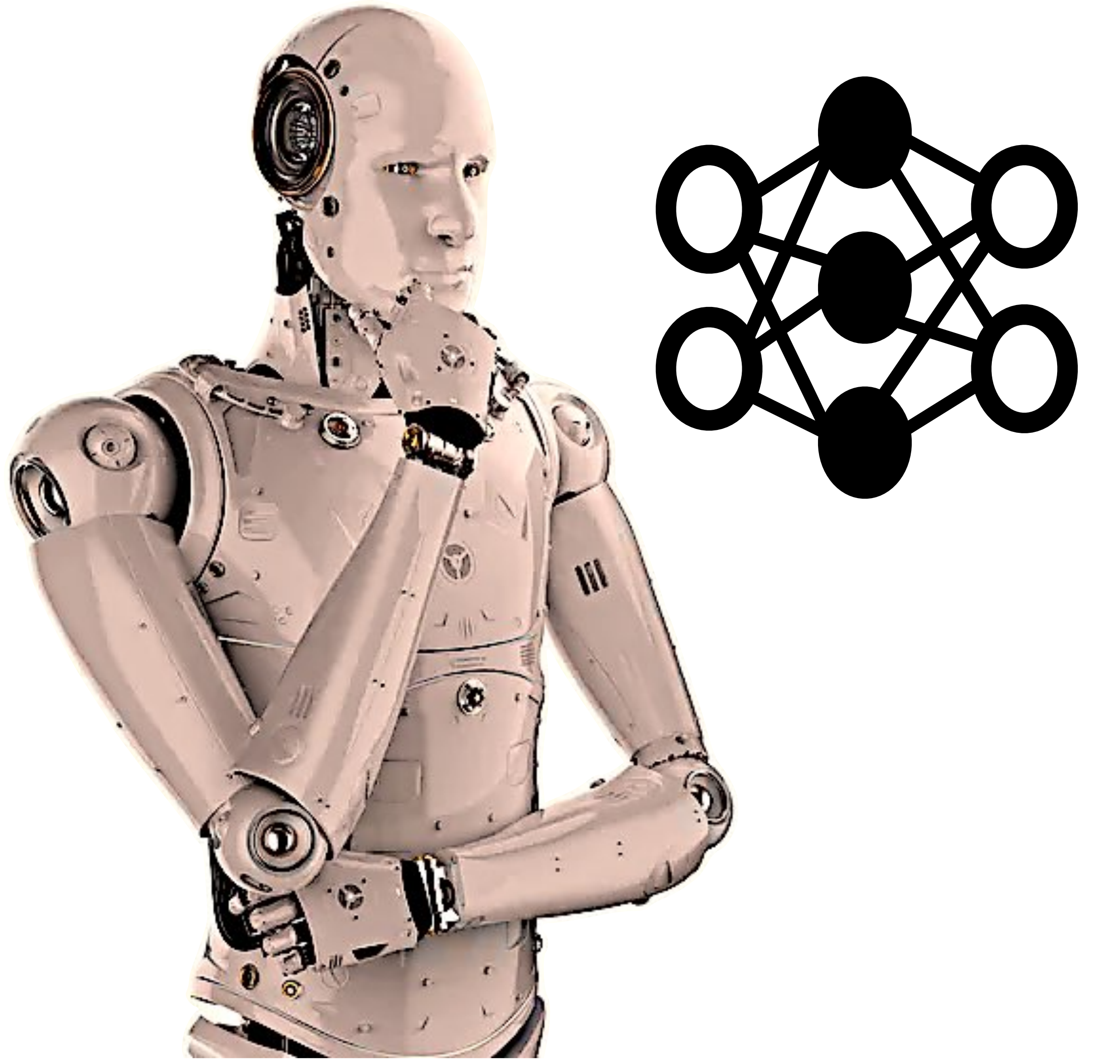
D_1 extraction, GOALS

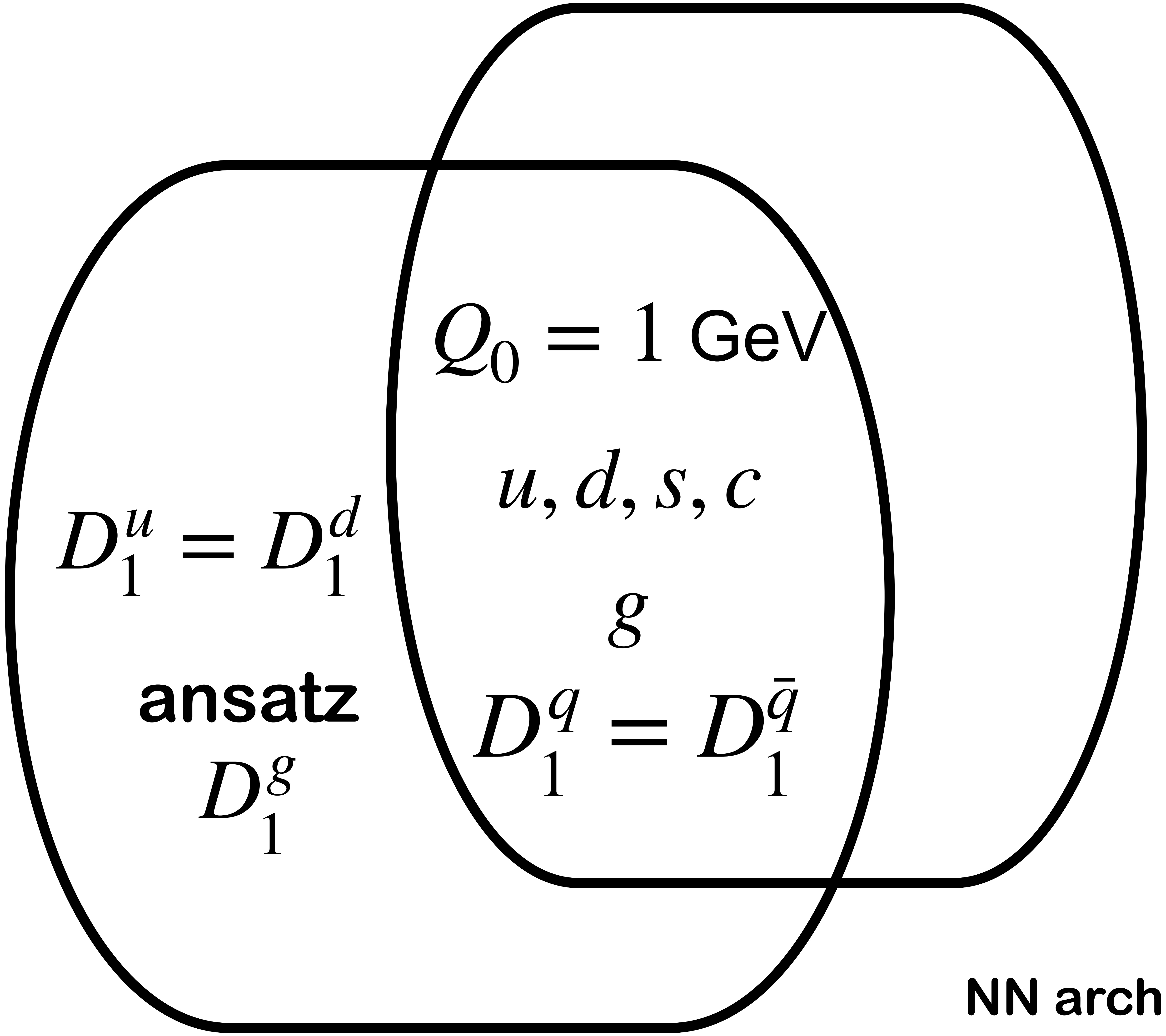
- 2017 BELLE data of $e^+e^- \to \pi^+\pi^- X$ at $\sqrt{S} = 10.58$ GeV
- Use of Monte-Carlo simulation for flavor separation only
- Push the perturbative accuracy up to NNLO
- Explore a Neural Network parameterisation

PHYSICS INFORMED



NEURAL NETWORK





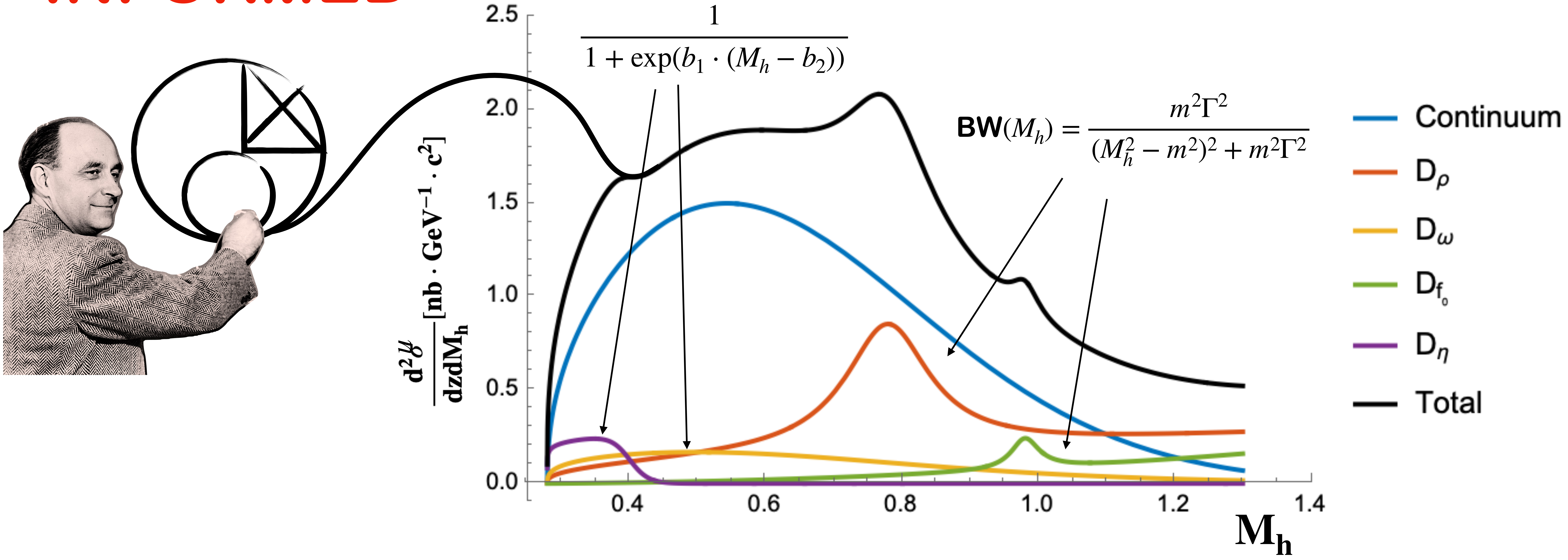
71 par

NN architecture

[2,25,5] ⇝ 205 par

PHYSICS INFORMED

Example for the up quark parameterisation



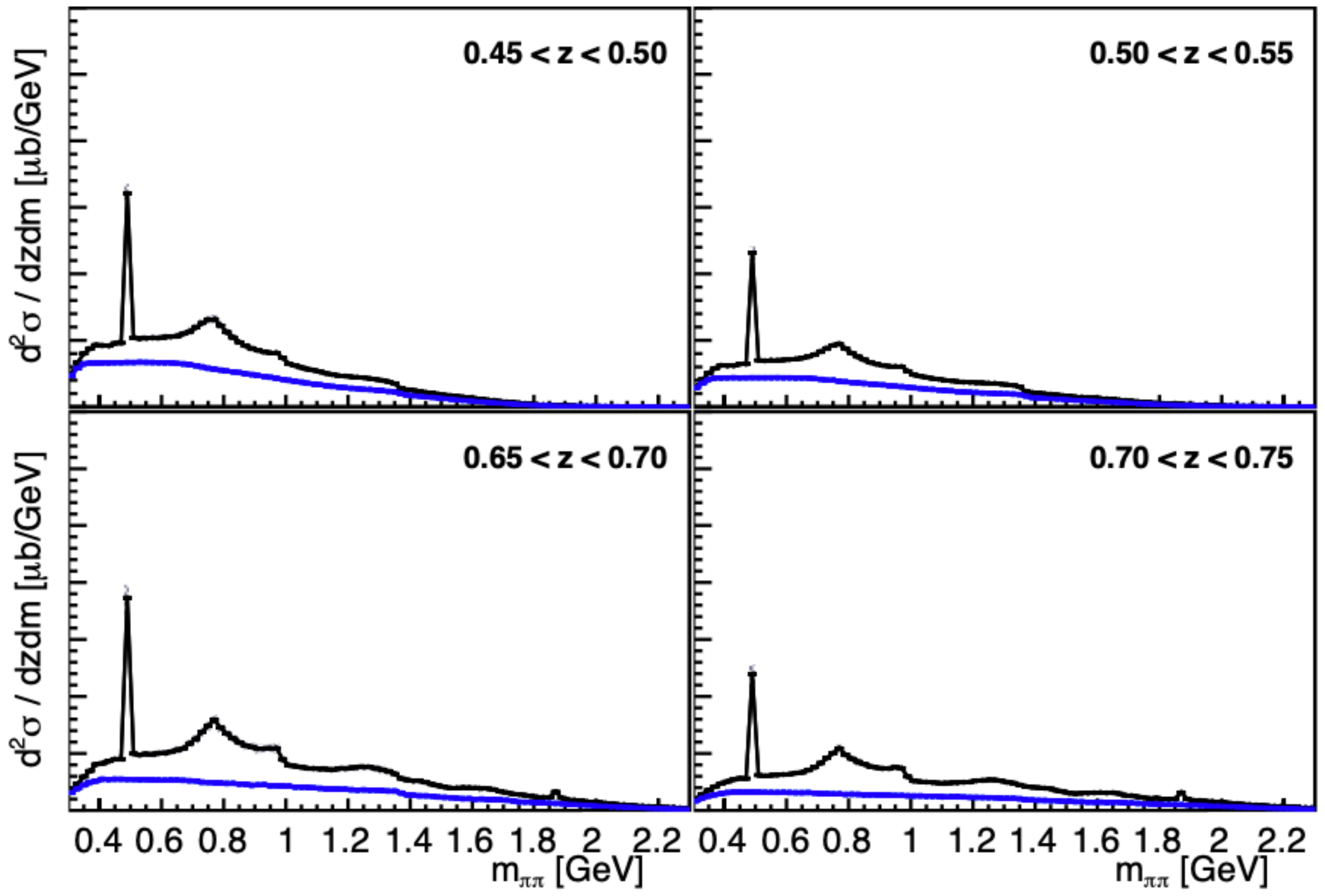
$\mathbf{R}(M_h) = \frac{1}{2}\sqrt{M_h^2 - 4\,m_\pi^2}$ $\qquad z^\alpha(1-z)^\beta$ $\qquad \mathbf{P}(a_1, a_2, a_3, a_4, a_5; z)$

$$= \frac{a_1}{z} + a_2 + a_3 \cdot z \ + a_4 \cdot z^2 + a_5 \cdot z^3$$

Flavour analysis

Physical review D 96 (2017)
R.Siedl et al

2017 BELLE data of $e^+e^- \to \pi^+\pi^- X$
at $\sqrt{S} = 10.58$ GeV



Flavour analysis

Physical review D 96 (2017)
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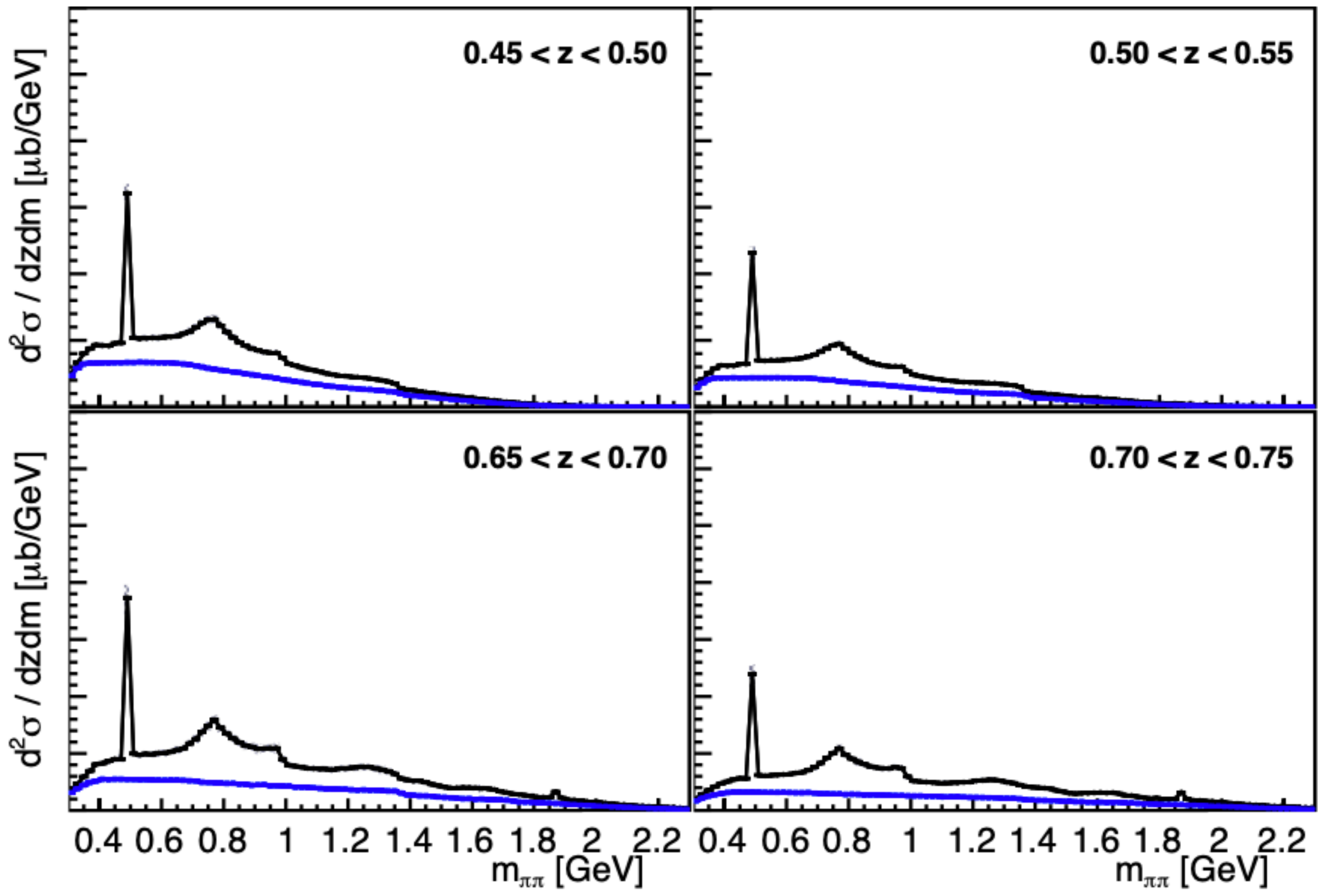
$$\frac{d\sigma}{dzdM_h dQ^2} = \frac{4\pi\alpha^2}{Q^2}\sum_q e_q^2 D_1^q(z, M_h, Q) =$$



Flavour analysis

$$\frac{d\sigma}{dzdM_hdQ^2} = \frac{4\pi\alpha^2}{Q^2}\sum_q e_q^2 D_1^q(z, M_h, Q) = \sum_q \frac{d\sigma^{\mathbf{q}}}{dzdM_hdQ^2}$$

$$= 2\cdot\left(\frac{4\pi\alpha^2}{Q^2} e_{\mathbf{u}}^2 D_1^{\mathbf{u}}(z, M_h, Q) + \frac{4\pi\alpha^2}{Q^2} e_{\mathbf{d}}^2 D_1^{\mathbf{d}}(z, M_h, Q) + \frac{4\pi\alpha^2}{Q^2} e_{\mathbf{s}}^2 D_1^{\mathbf{s}}(z, M_h, Q) + \dots\right)$$

$$= 2\cdot\frac{d\sigma^{exp}}{dzdM_hdQ^2}\cdot\left(R_{MC}^{\mathbf{u}}(z, M_h, Q) + R_{MC}^{\mathbf{d}}(z, M_h, Q) + R_{MC}^{\mathbf{s}}(z, M_h, Q) + R_{MC}^{\mathbf{c}}(z, M_h, Q)\right)$$

$$R^{\mathbf{q}}(z, M_h, Q) = \frac{d\sigma^{\mathbf{q}}}{d\sigma} = \frac{e_{\mathbf{q}}^2 D_1^{\mathbf{q}}(z, Mh, Q)}{\sum_p e_p^2 D_1^p(z, Mh, Q)}$$

accessible from Monte Carlo simulations

$$\mathscr{F}^{\mathbf{q}}(z, M_h; Q^2) = 2\cdot\frac{d\sigma^{exp}}{dzdM_hdQ^2}\cdot R_{MC}^{\mathbf{q}}(z, M_h, Q)$$

$$\mathscr{F}^q$$

$$q = u, d, s, c$$

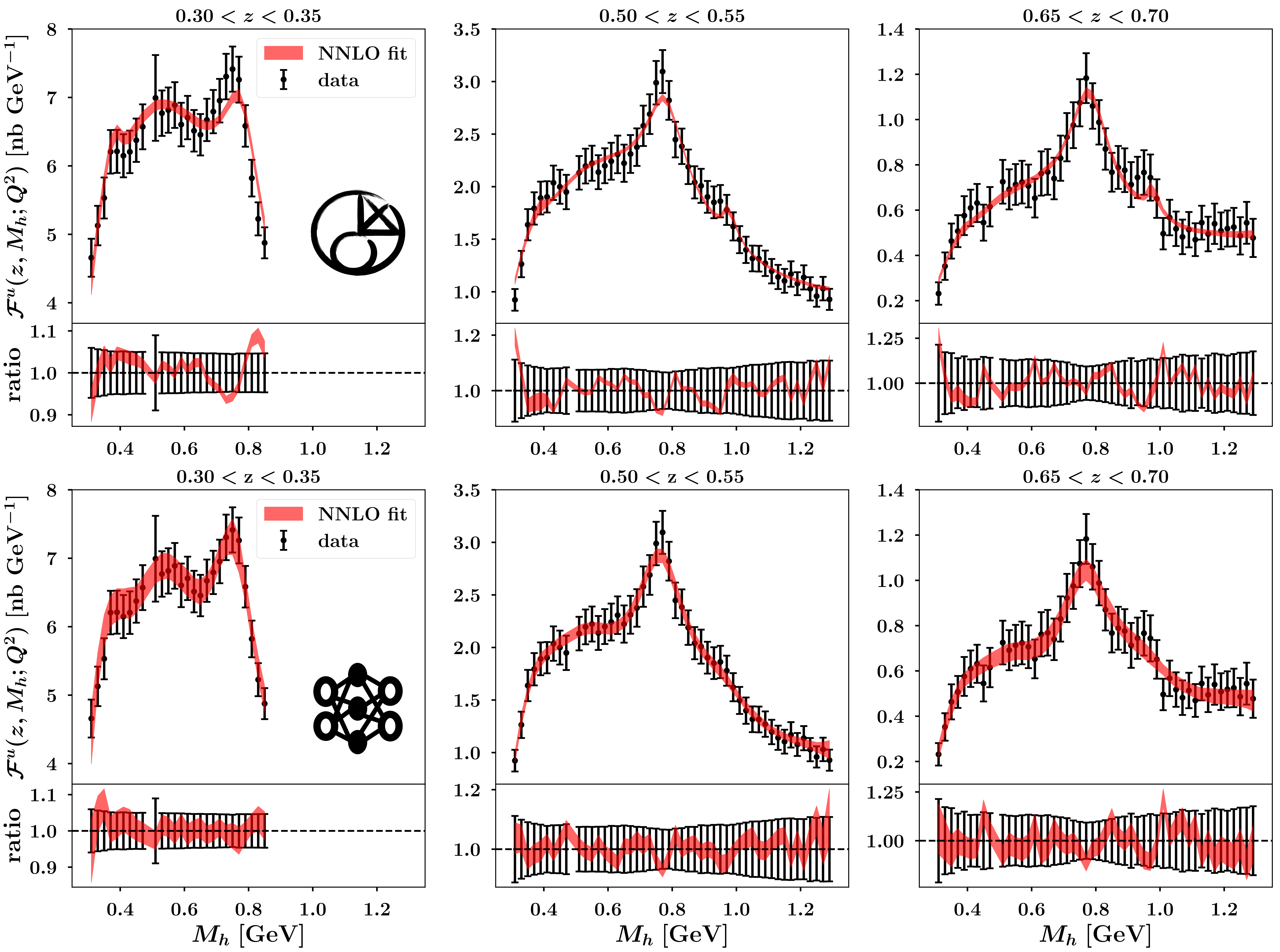
RESULTS

Predictions

vs

NN seems to describe better data

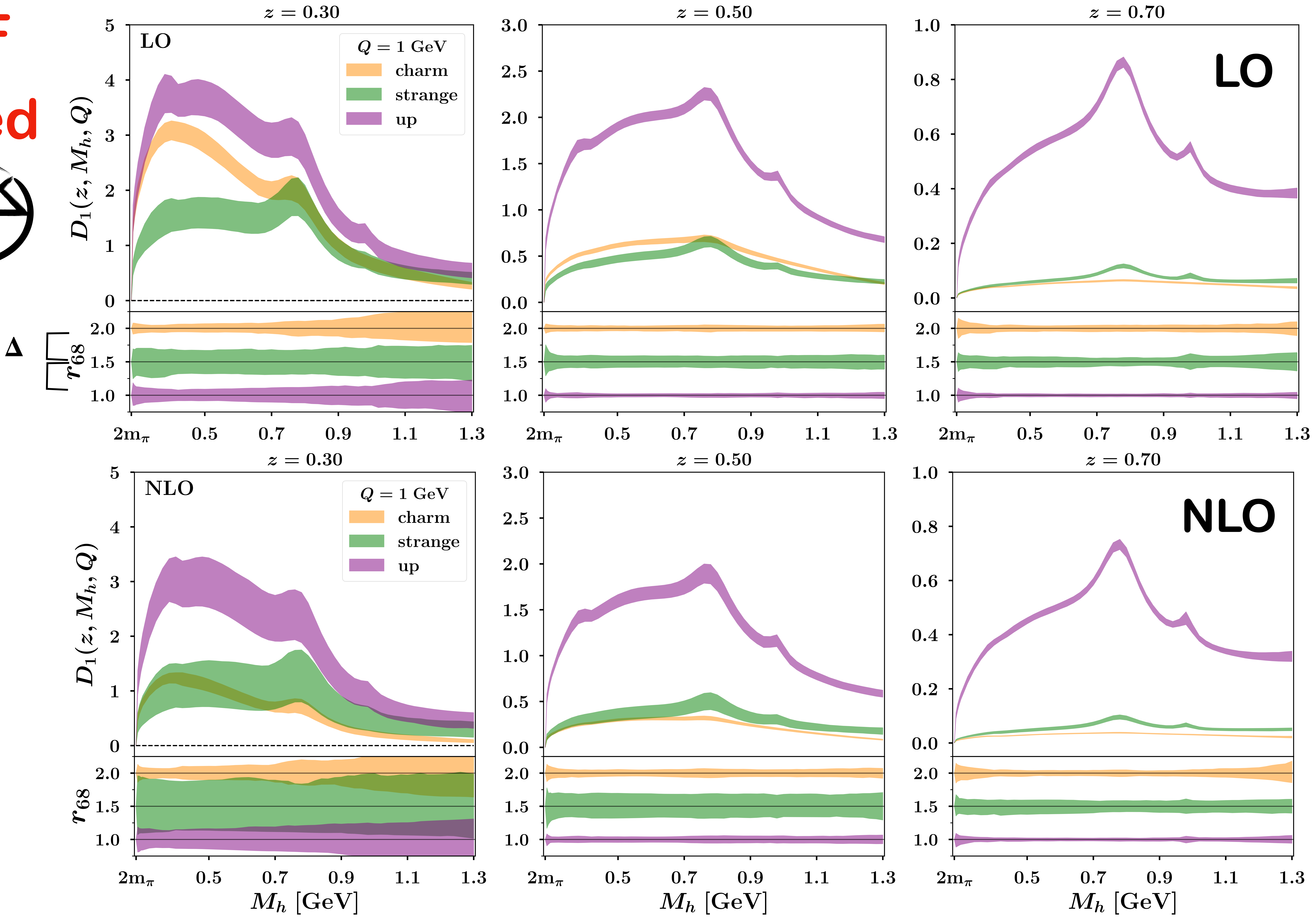
P.I. better captures the resonant structure



DI-HADRON FF
Physics informed

u = d

ratio offset **Δ** = 0.5



DI-HADRON FF
Physics informed

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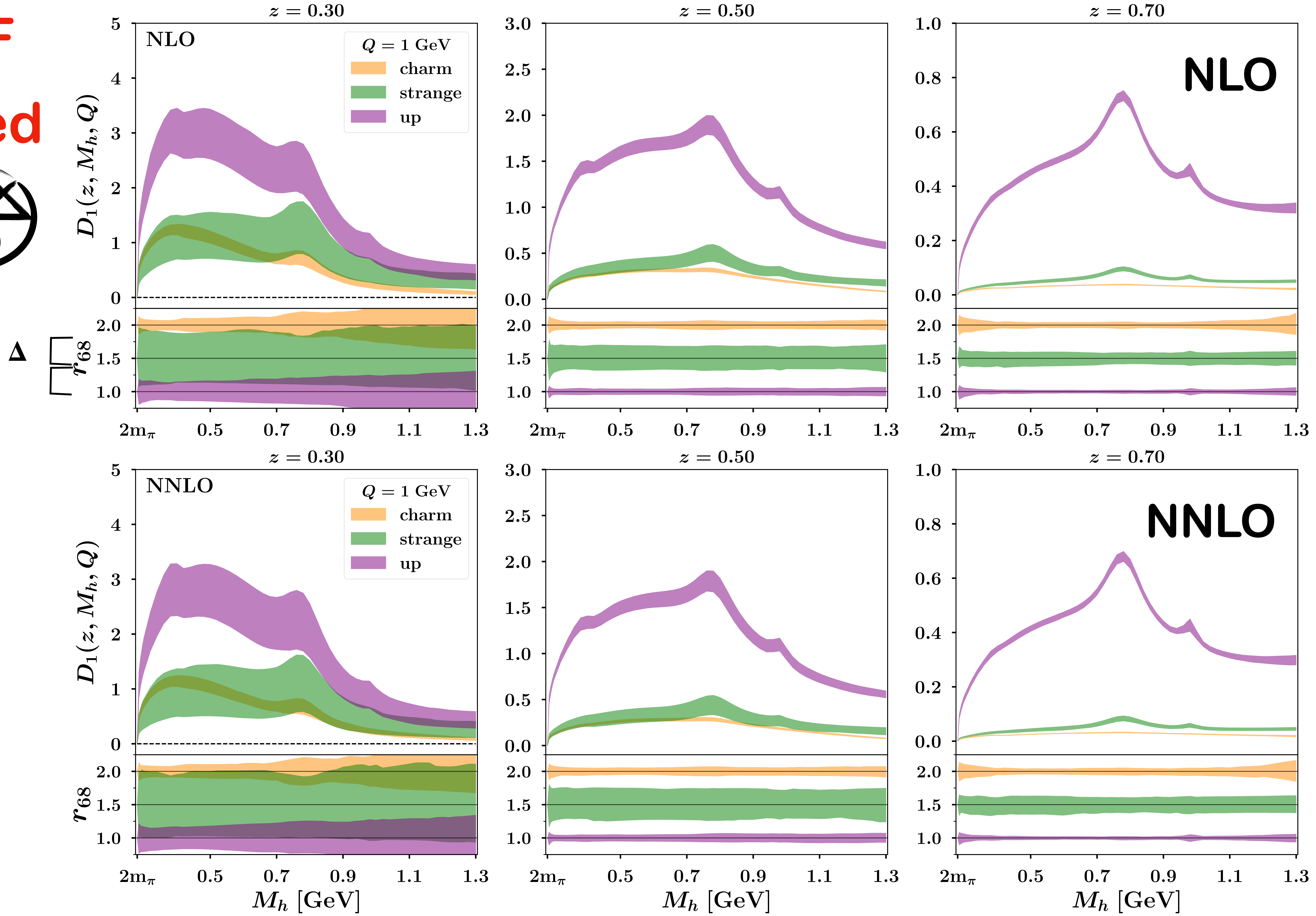
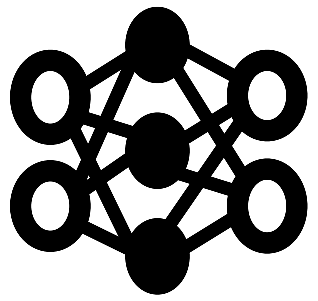
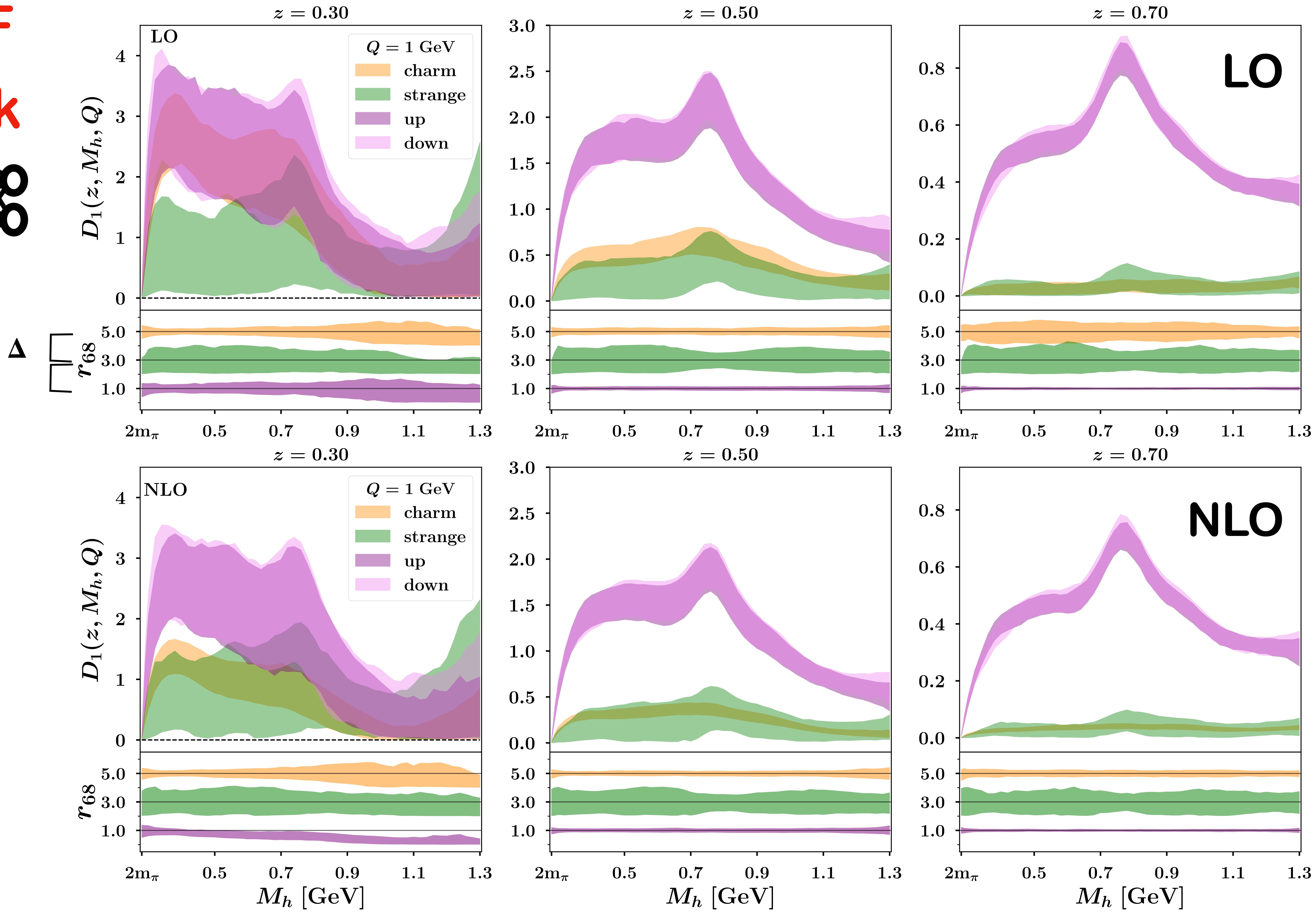


DI-HADRON FF
Neural Network



u ≠ d

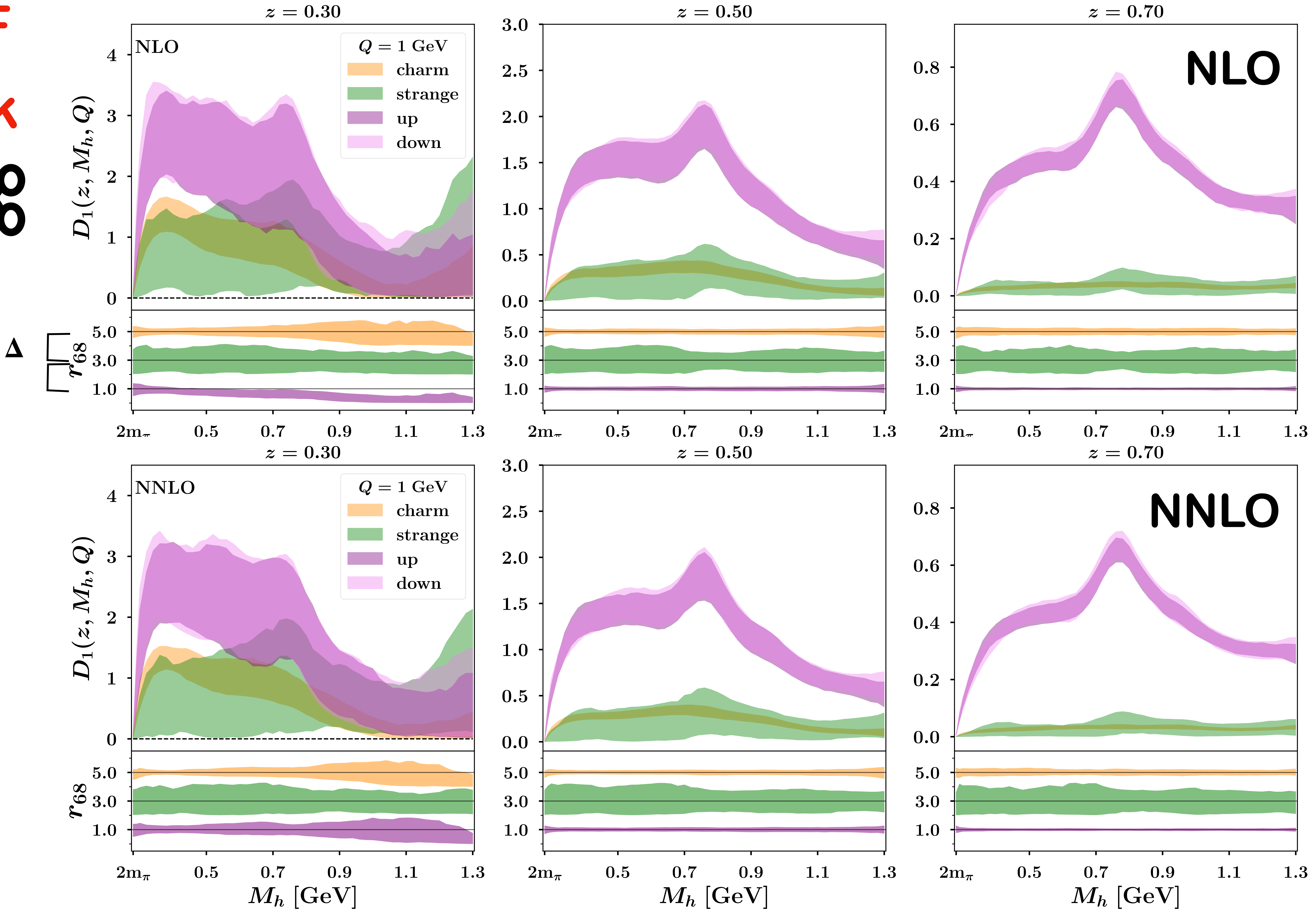
ratio offset **Δ** = 2.5



DI-HADRON FF
Neural Network

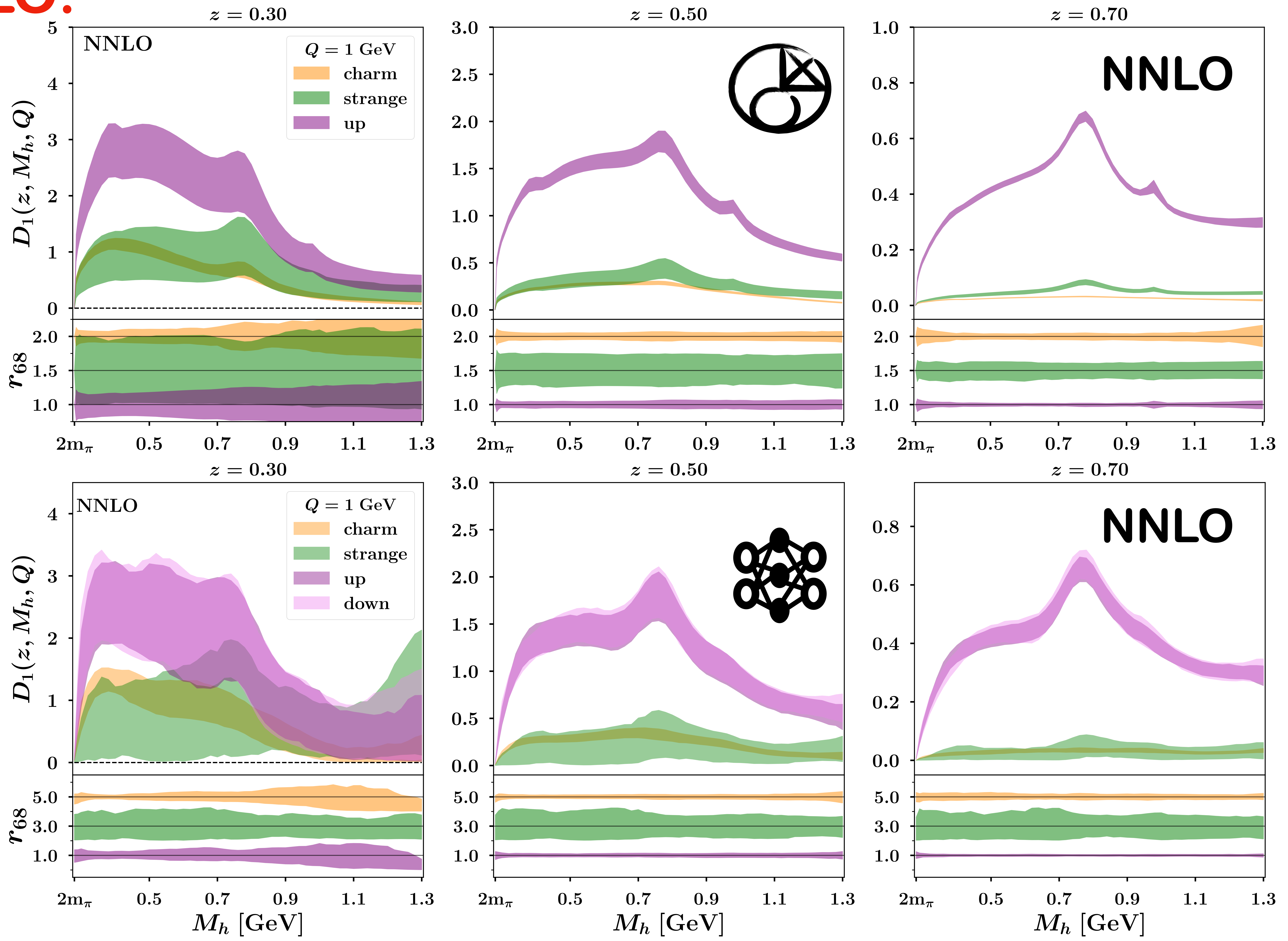
$u \neq d$

ratio offset Δ = 2.5



Comparison at NNLO:

vs



Comparison at NNLO:

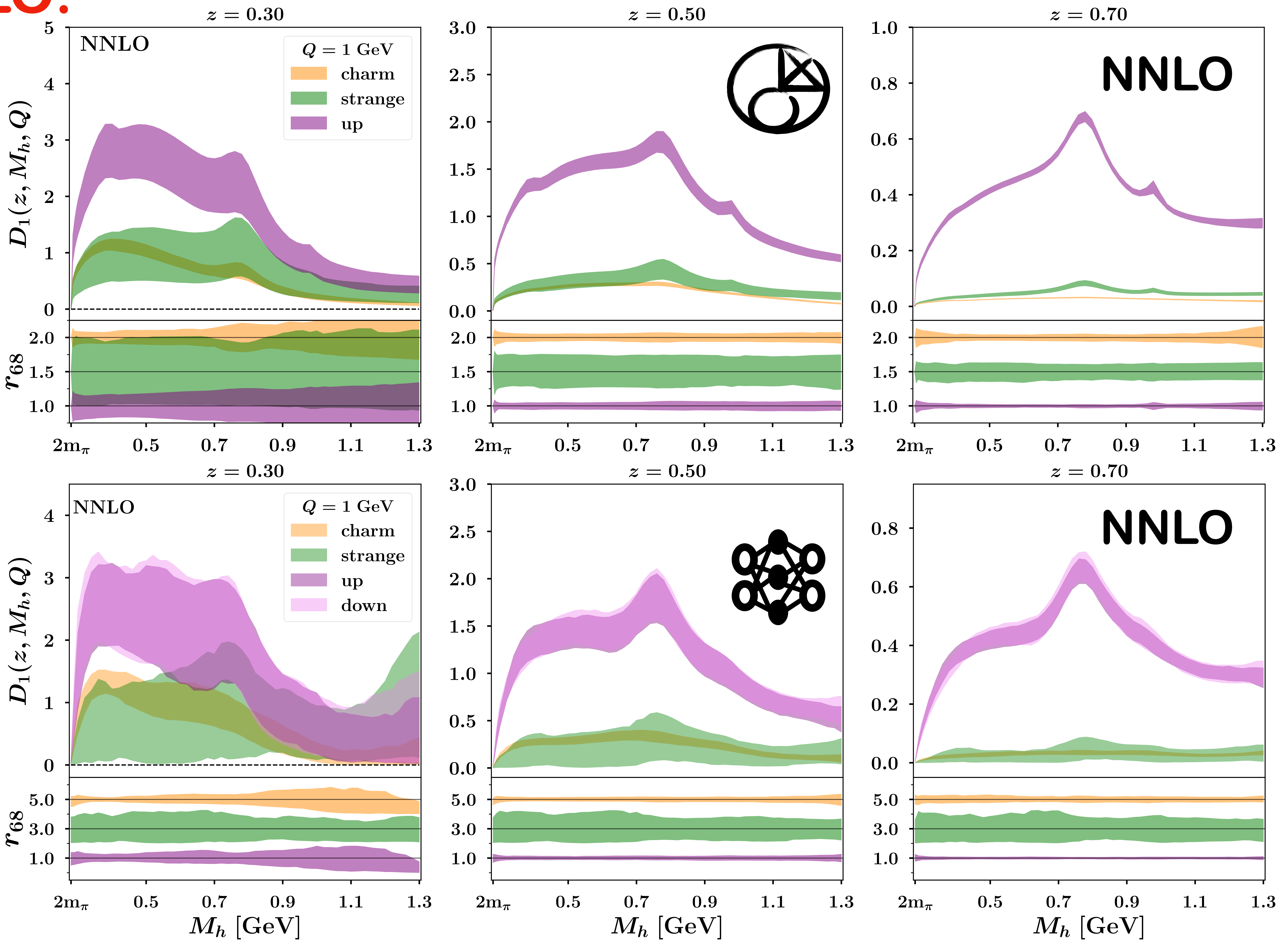
vs

Similar features and compatible results

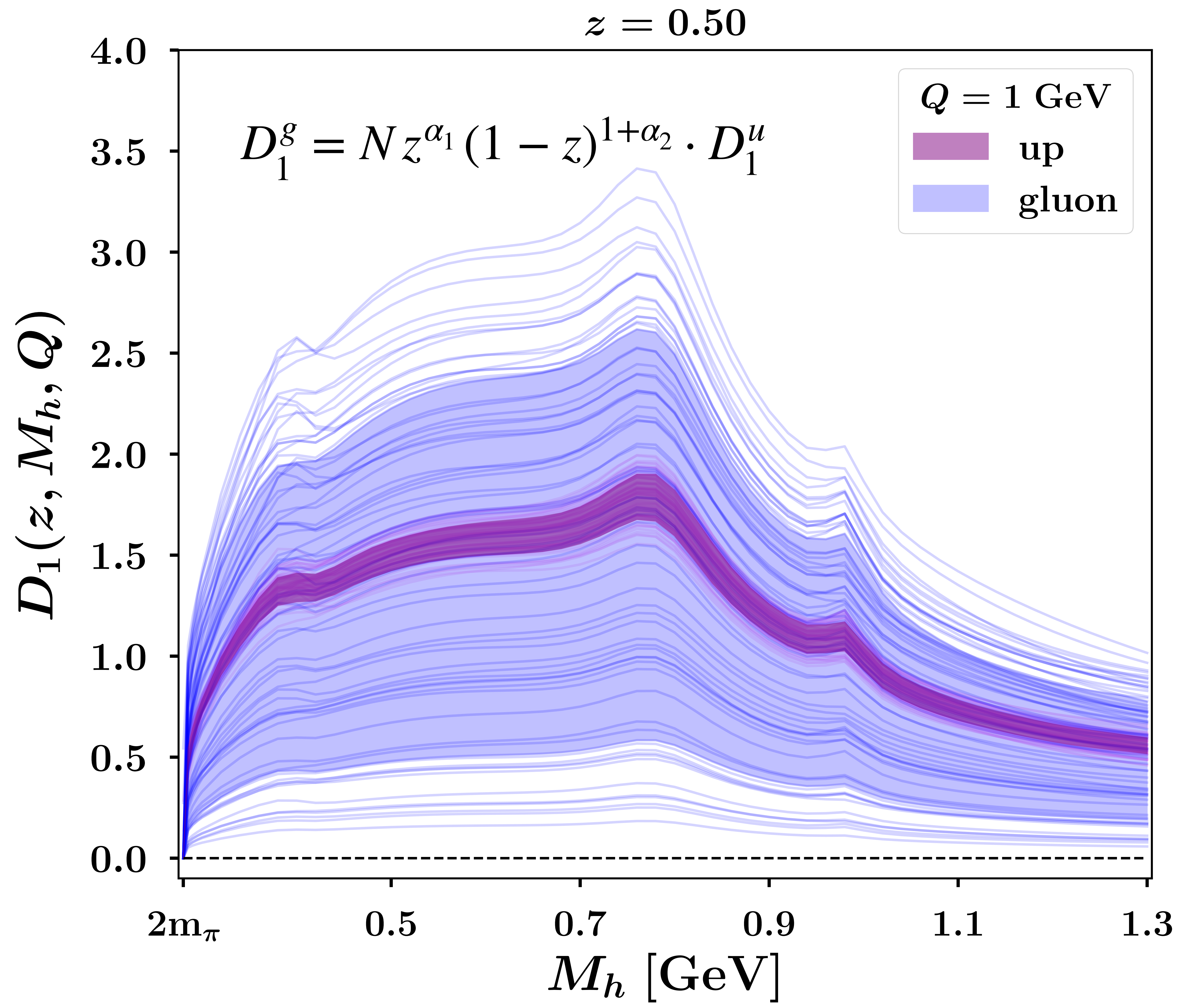
P.I. well reproduce the resonance structure, better than NN

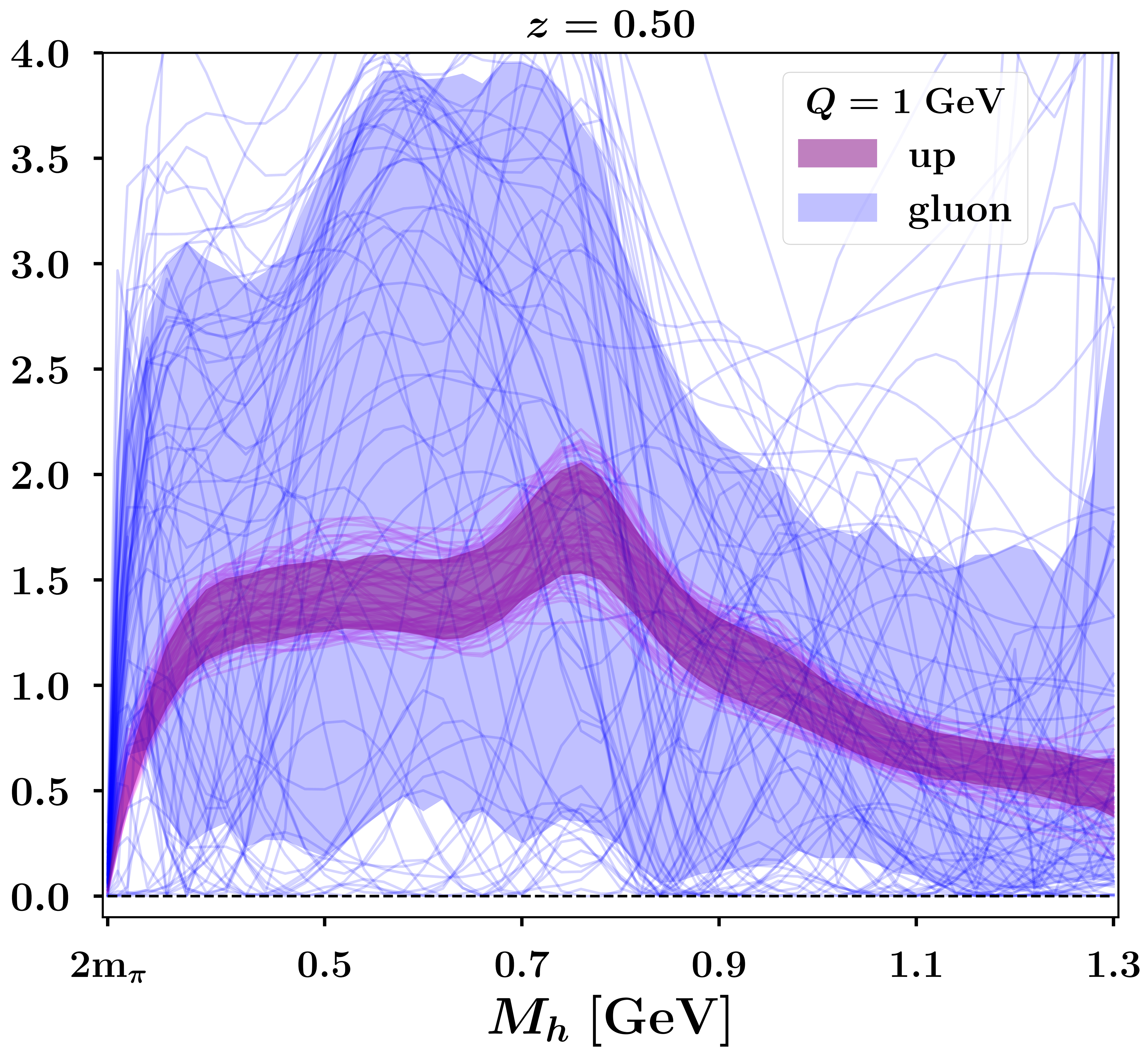
NN has larger uncertainty bands

What about the gluon?



Gluon and up bands at NNLO





Unpolarized

CONCLUSIONS AND OUTLOOKS

- Good agreement between both fits and data, small χ^2

- The different parameterisations allow to grasp aspects of the Di-Hadron not accessible with the single ones

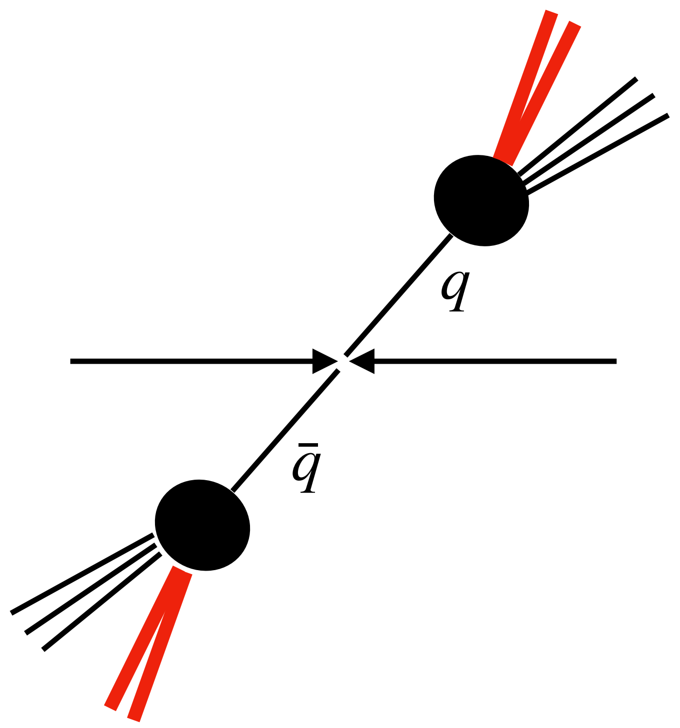
- A hierarchy among quark flavors is clear and stable.
  - Gluon uncostrained with e^+e^- data

  Need for SIDIS, hadron-hadron collisions data

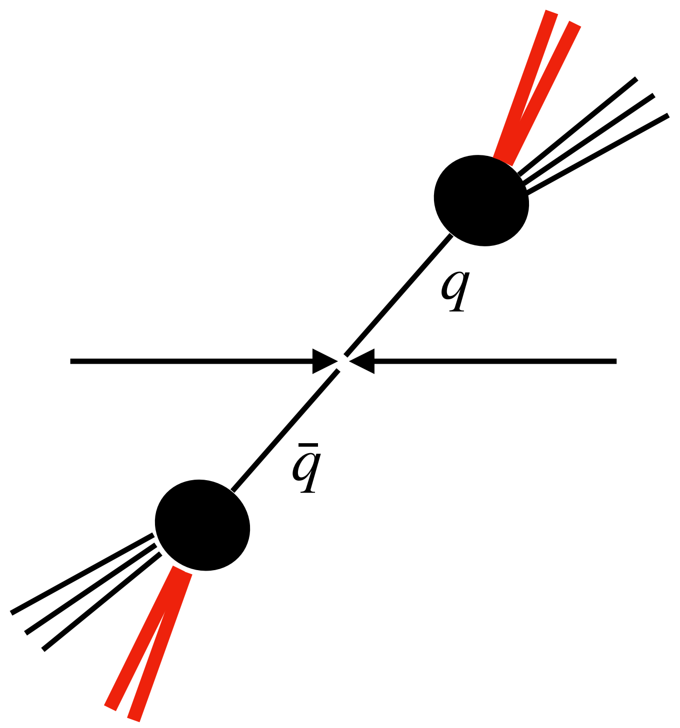
$H_1^{\sphericalangle}$ extraction

$H_1^{\sphericalangle}$ extraction

$$A_{UT}^{hh} \sim \frac{\sum_q e_q^2 \cdot H_1^{\sphericalangle,q}(z, M_h) \cdot H_1^{\sphericalangle,q}(\bar{z}, \bar{M}_h)}{\sum_q e_q^2 \cdot D_1^q(z, M_h) \cdot D_1^q(\bar{z}, \bar{M}_h)}$$



$H_1^{\sphericalangle}$ extraction



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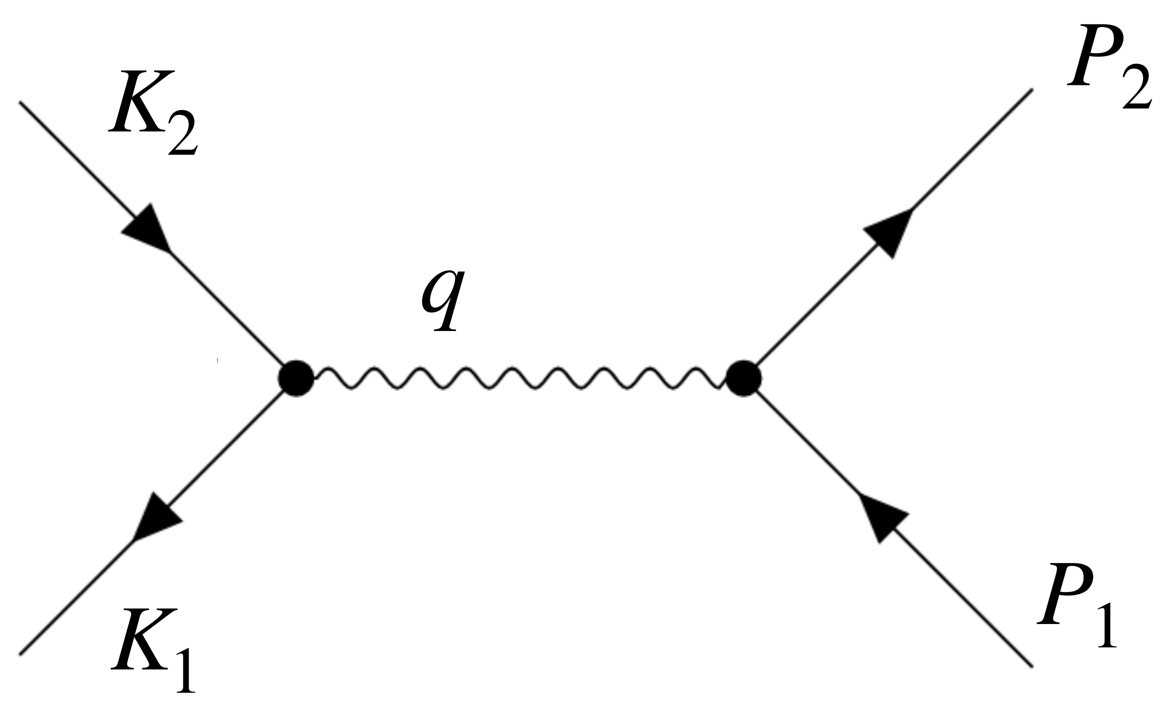
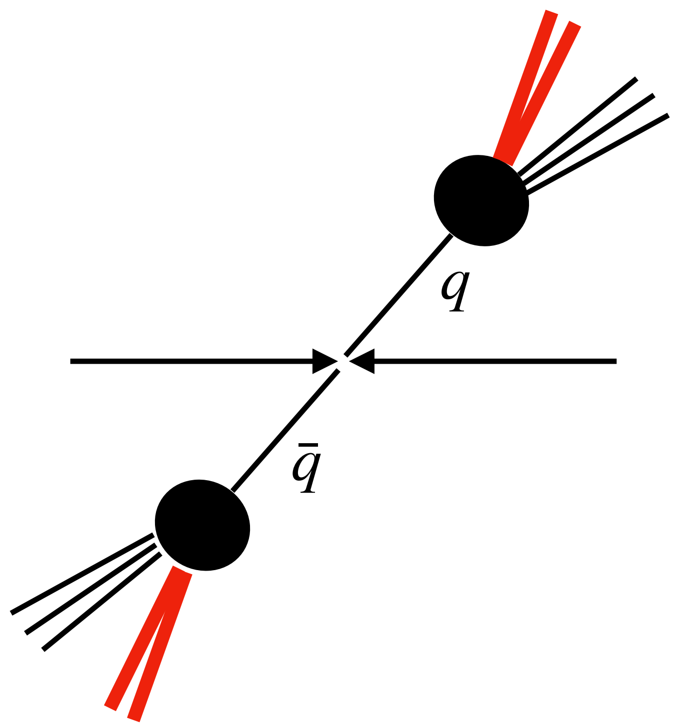


$$\left| \cdots \right|^2 \sim L_{\mu\nu}\left[g_T^{\mu\nu}(S_{T2} \cdot S_{T1}) - S_{T2}^{\mu} S_{T1}^{\nu} - S_{T2}^{\nu} S_{T1}^{\mu}\right] \sim Q^4 \frac{\sin^2(\theta)}{4} \cos(\phi_{S_{T2}} + \phi_{S_{T1}})$$

$H_1^{\sphericalangle}$ extraction



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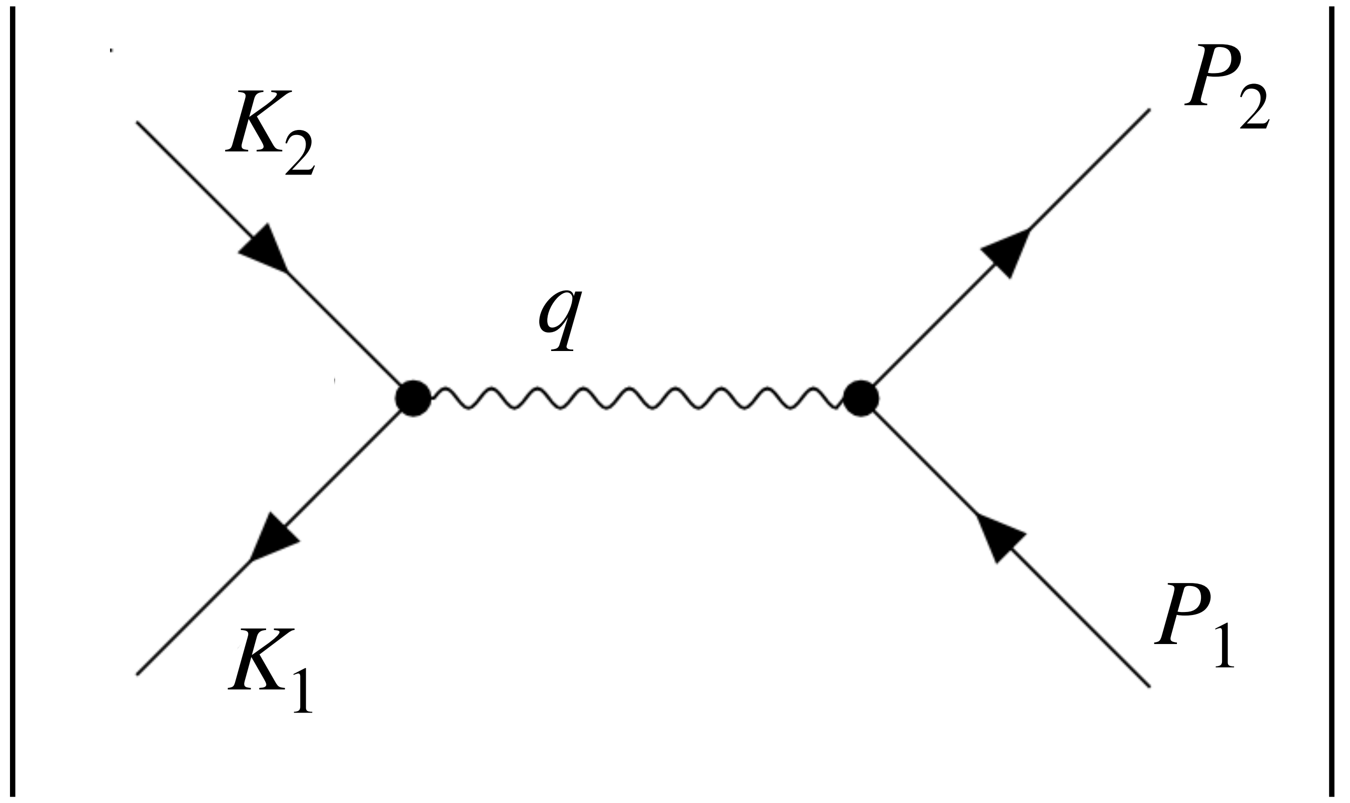

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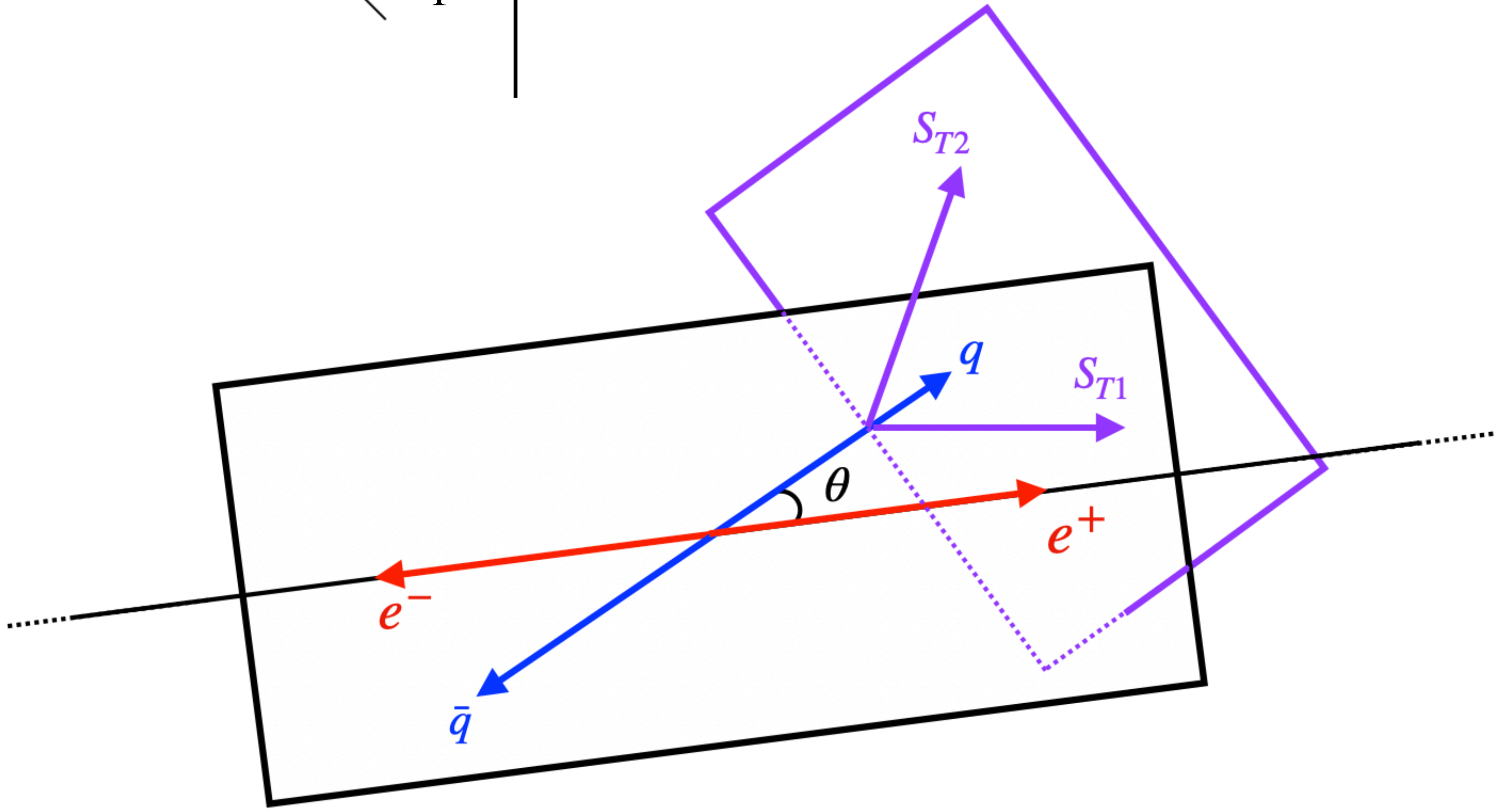
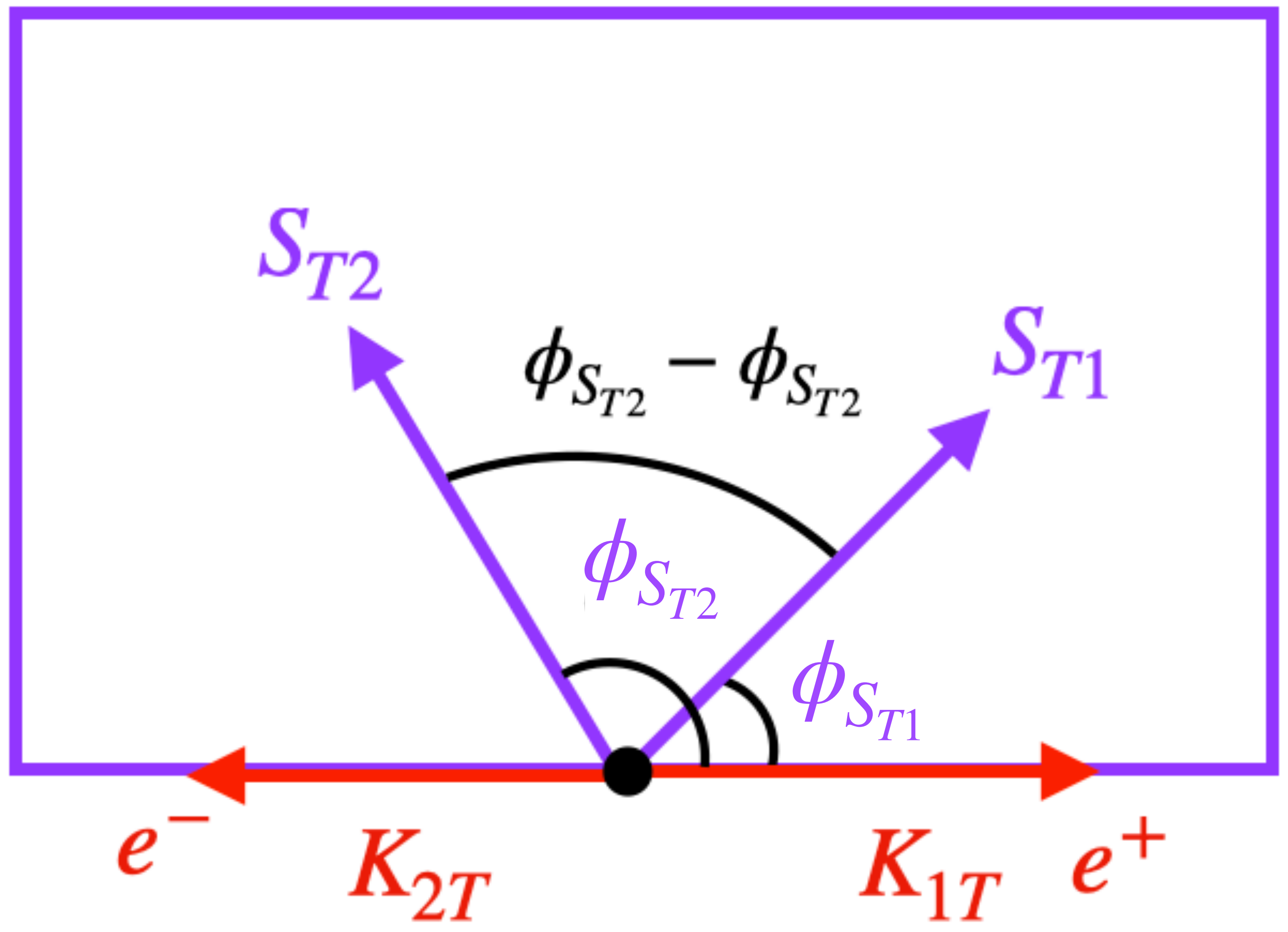




Latest $H_1^{\sphericalangle}$ extraction

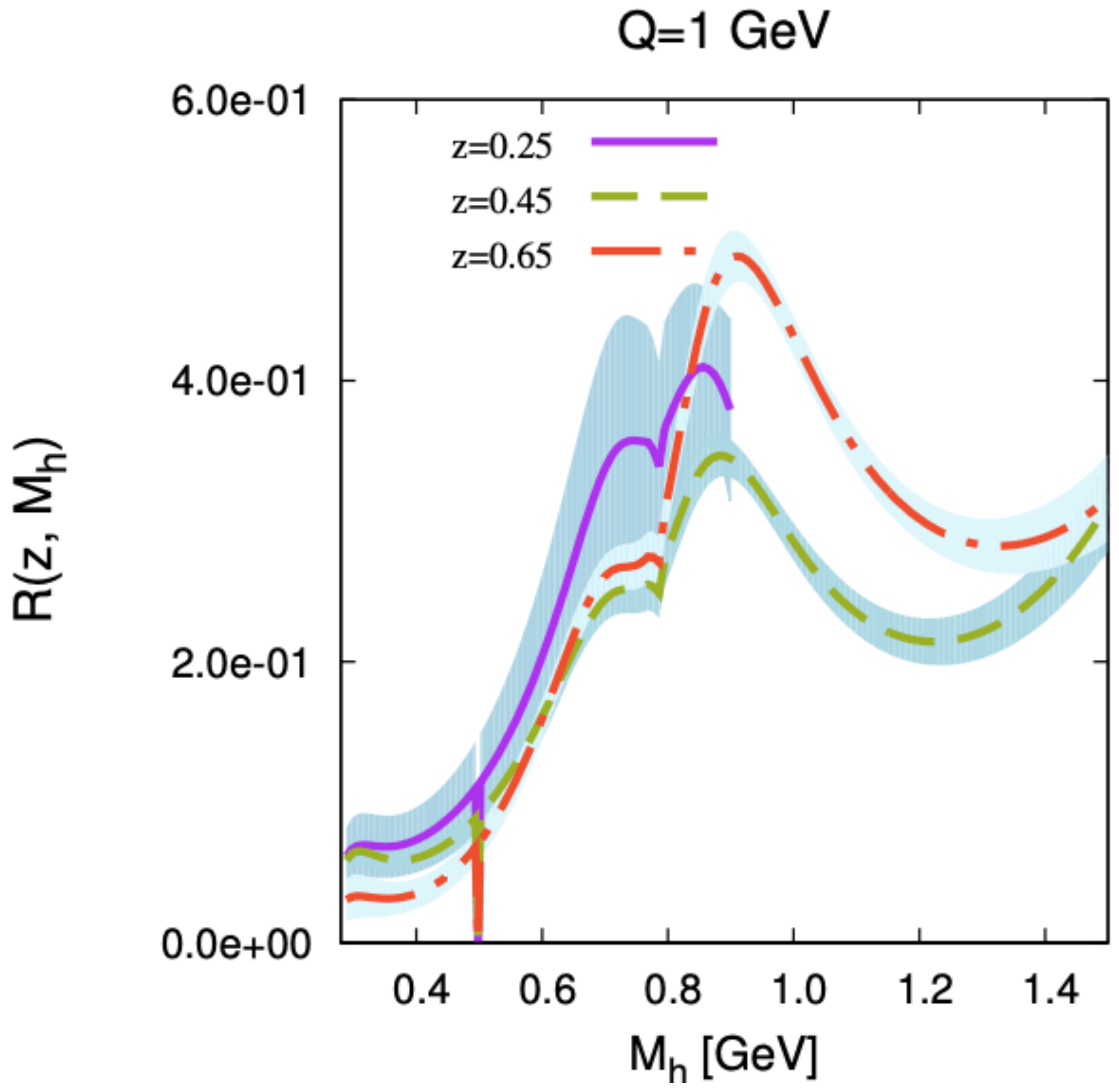
Latest $H_1^{\sphericalangle}$ extraction

Pavia 2012

2011 BELLE data at $\sqrt{S} = 10.58$ GeV of $e^+e^- \to (\pi^+\pi^-)(\bar{\pi}^+\bar{\pi}^-)X$

9 free parameters

LO



$$R(z, M_h) = \frac{|\overrightarrow{p_{h1}} - \overrightarrow{p_{h2}}|}{M_h} \frac{H_1^{\sphericalangle,u}(z, M_h; Q^2)}{D_1^u(z, M_h; Q^2)}$$

Latest $H_1^{\sphericalangle}$ extraction

Pavia 2012

2011 BELLE data at $\sqrt{S} = 10.58$ GeV of $e^+e^- \to (\pi^+\pi^-)(\bar{\pi}^+\bar{\pi}^-)X$

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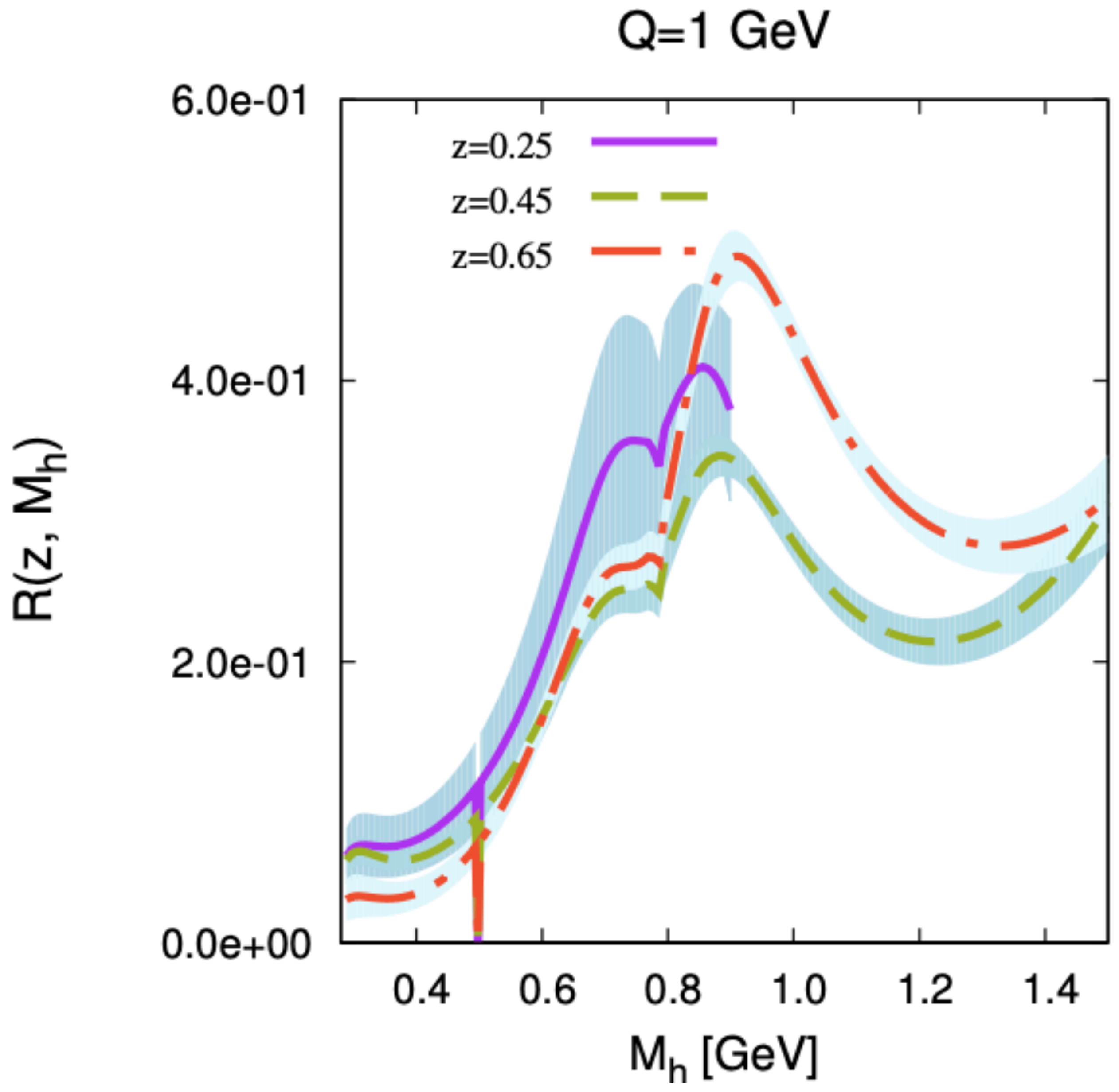

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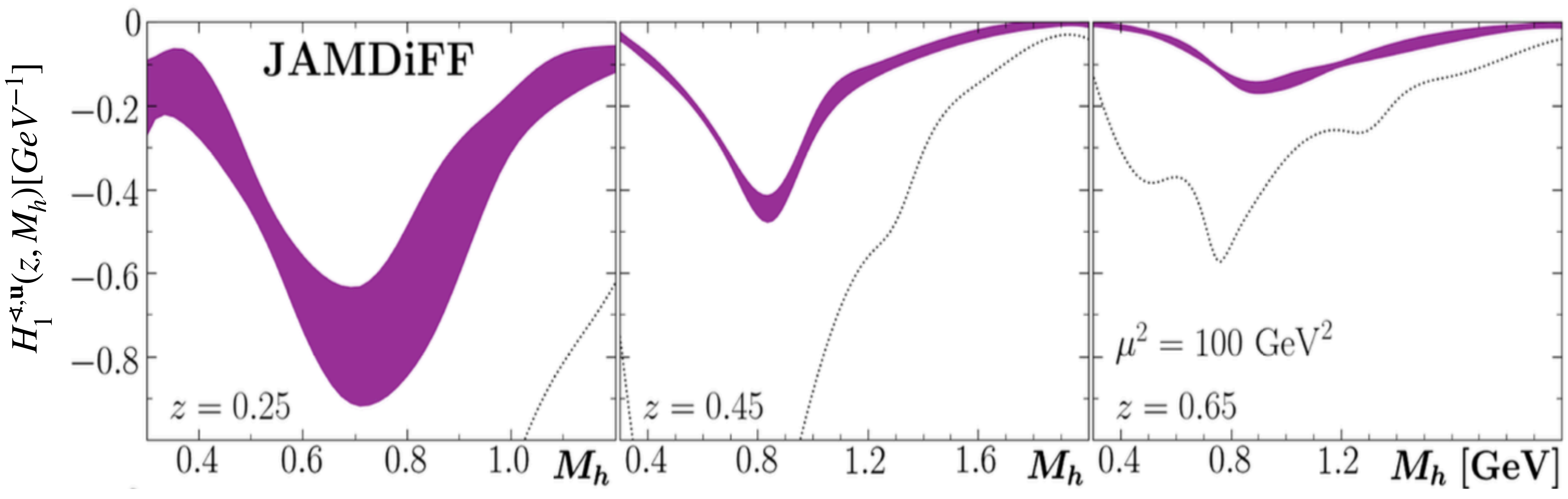
JAM 2024

2011 BELLE data at $\sqrt{S} = 10.58$ GeV of $e^+e^- \to (\pi^+\pi^-)(\bar{\pi}^+\bar{\pi}^-)X$

48 free parameters

LO



Asymmetry at NLO

NLOcalculation in: A.P. Contogouris et al, Phys. Lett. B 334, (1995) ⟶ **Possible inconsistencies between the equations**

We find different results.

Asymmetry at NLO

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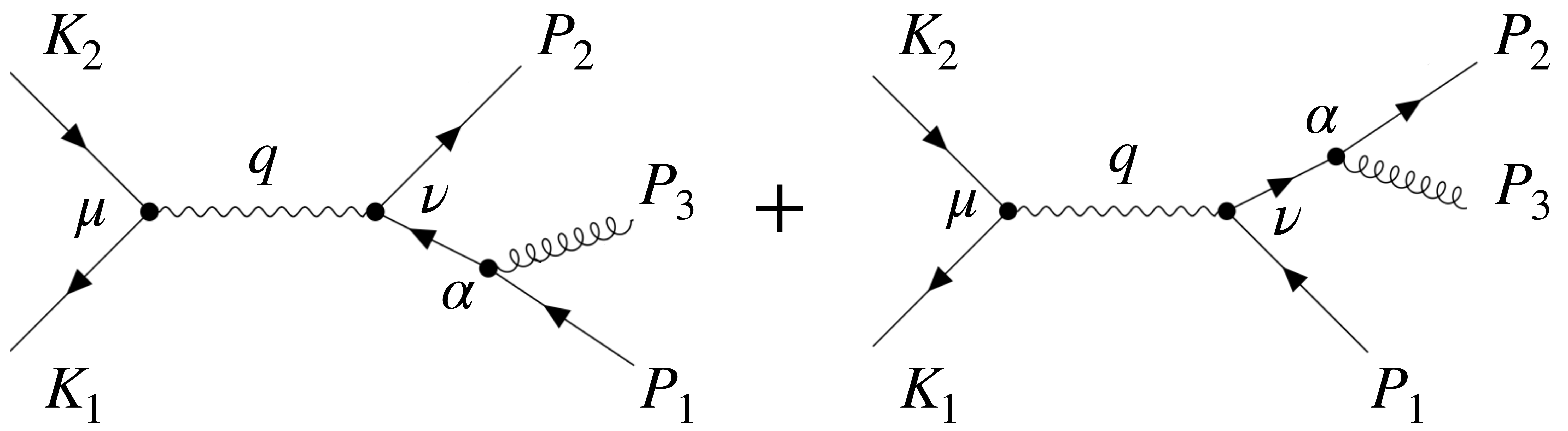

$$u(P_2)\bar{u}(P_2) = \frac{1}{2}\not{P}_2\left(1+\gamma_5\not{S}_{T2}\right)$$

$$v(P_1)\bar{v}(P_1) = \frac{1}{2}\not{P}_1\left(1+\gamma_5\not{S}_{T1}\right)$$

d**-dimensions** $W^{\mu\nu}$ → $P_i^\mu P_j^\nu,\ P_i^\mu S_{T1}^\nu,\ P_i^\mu S_{T2}^\nu,\ S_{T1}^\mu S_{T2}^\nu,\ g^{\mu\nu}$ $\quad i,j = 1,2,3$

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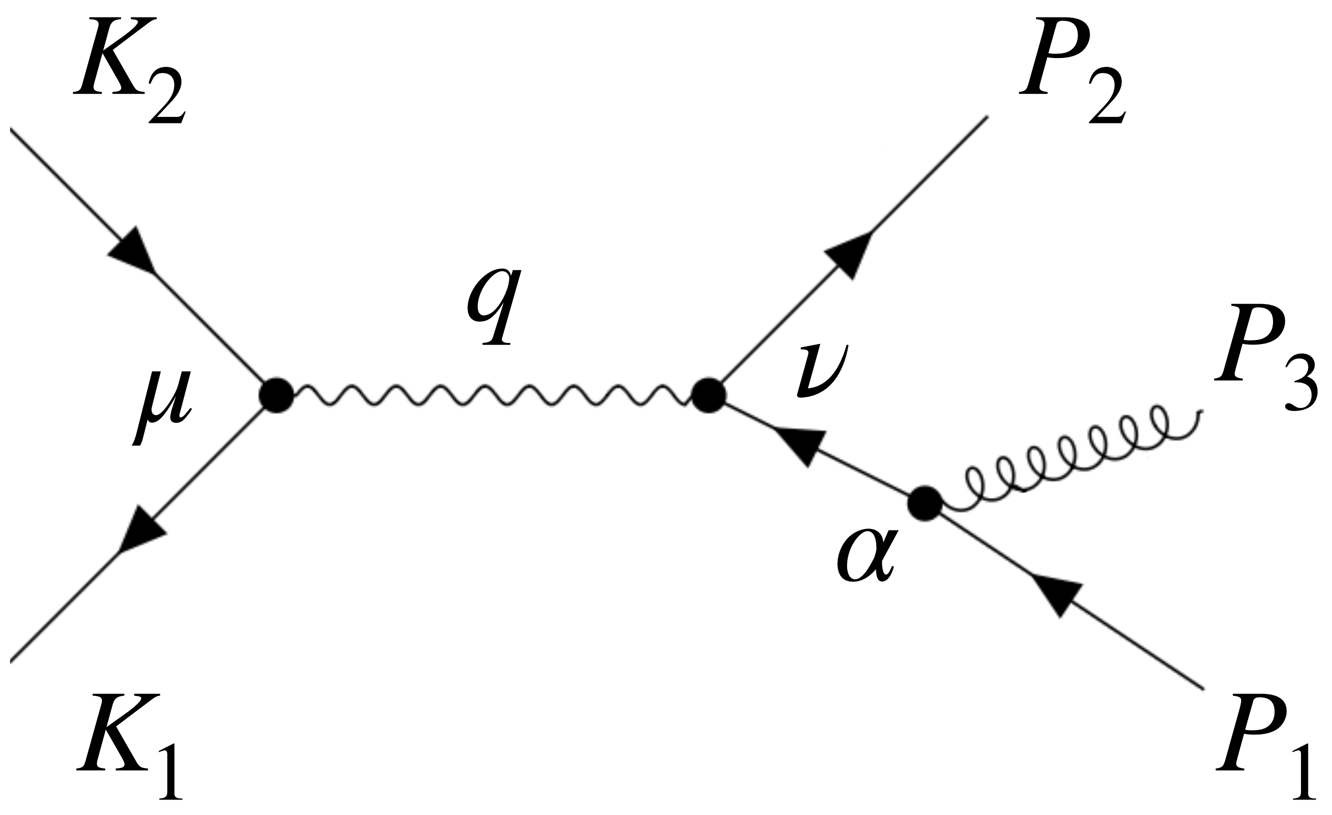

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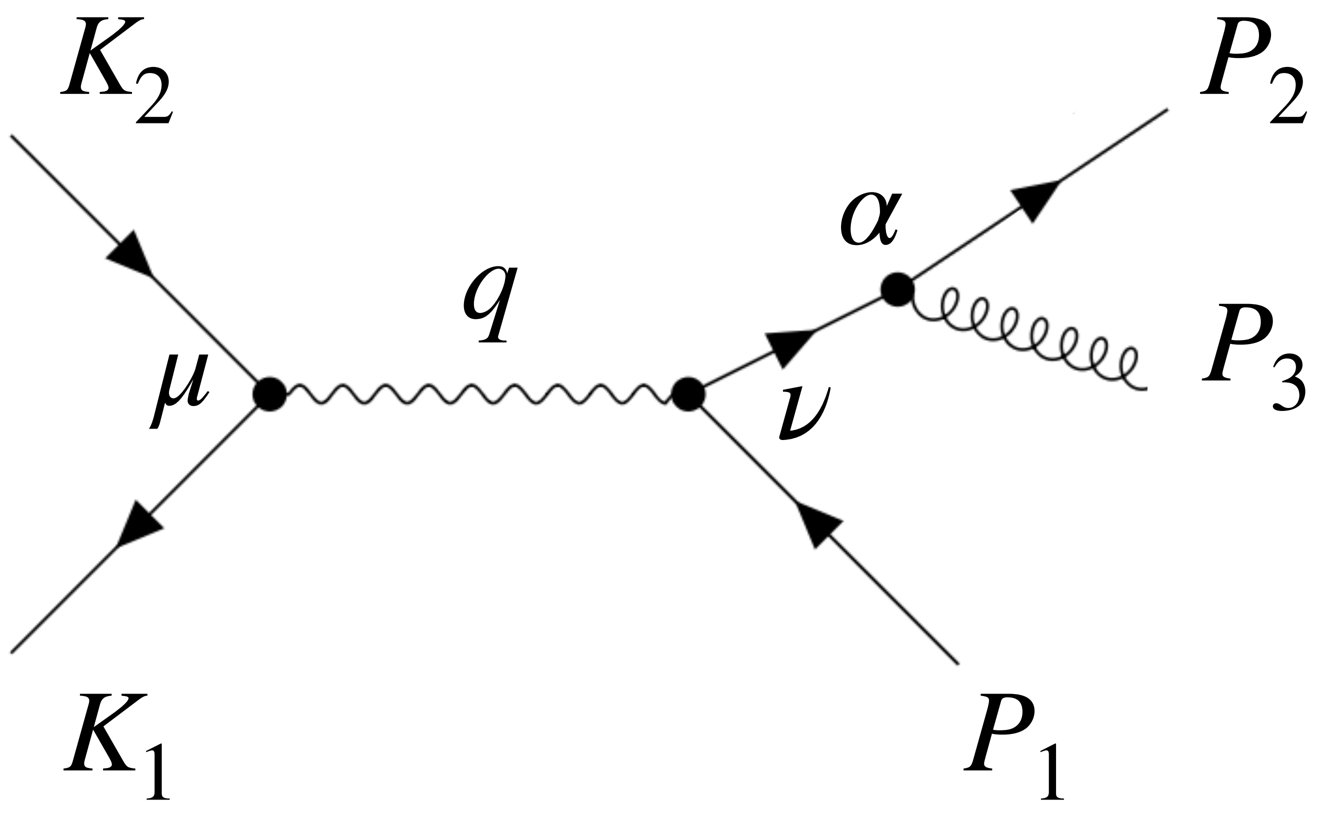

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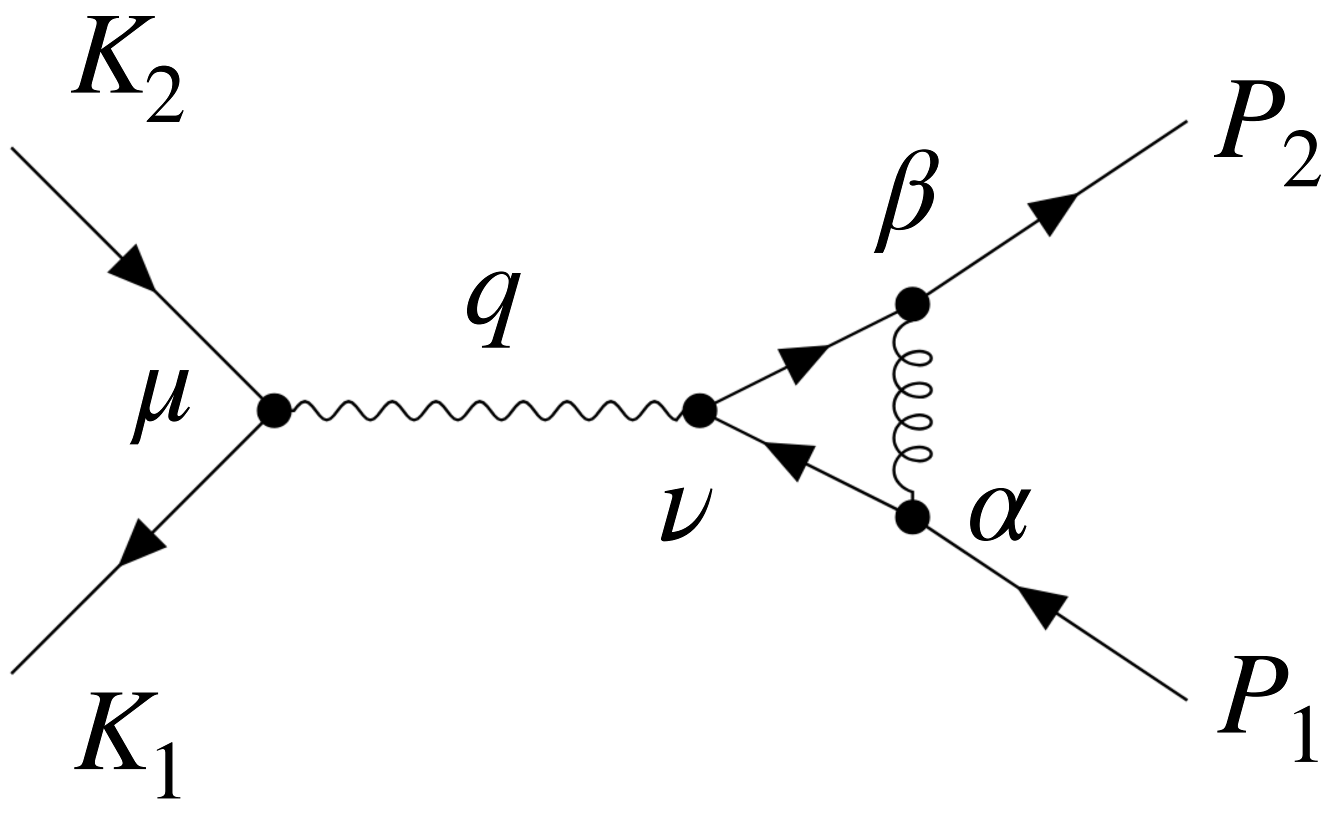
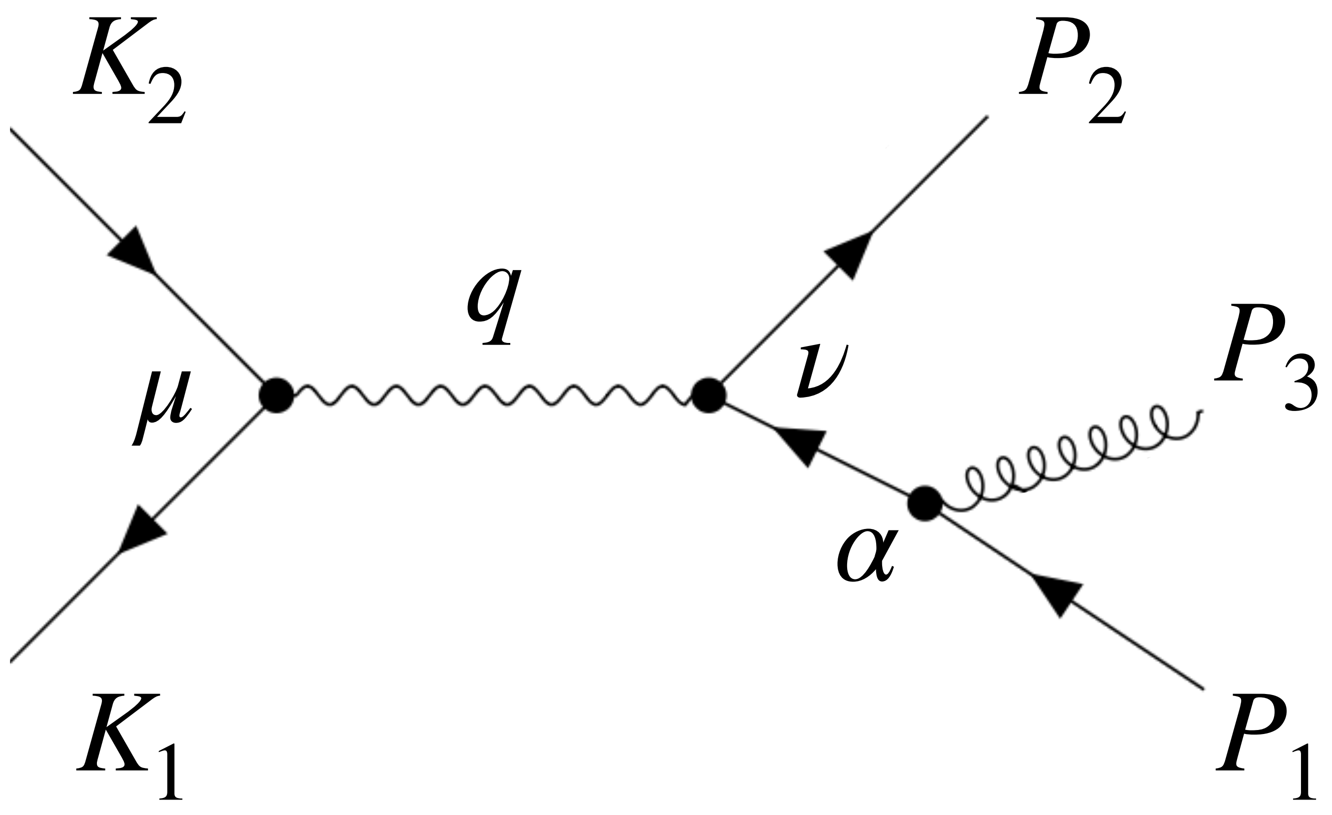


$$\sim 2Re(f(q^2))\frac{1}{4}\sum_{spins}|M_0|^2$$

Asymmetry at NLO

NLOcalculation in: A.P. Contogouris et al, Phys. Lett. B 334, (1995) → **Possible inconsistencies between the equations**

We find different results.



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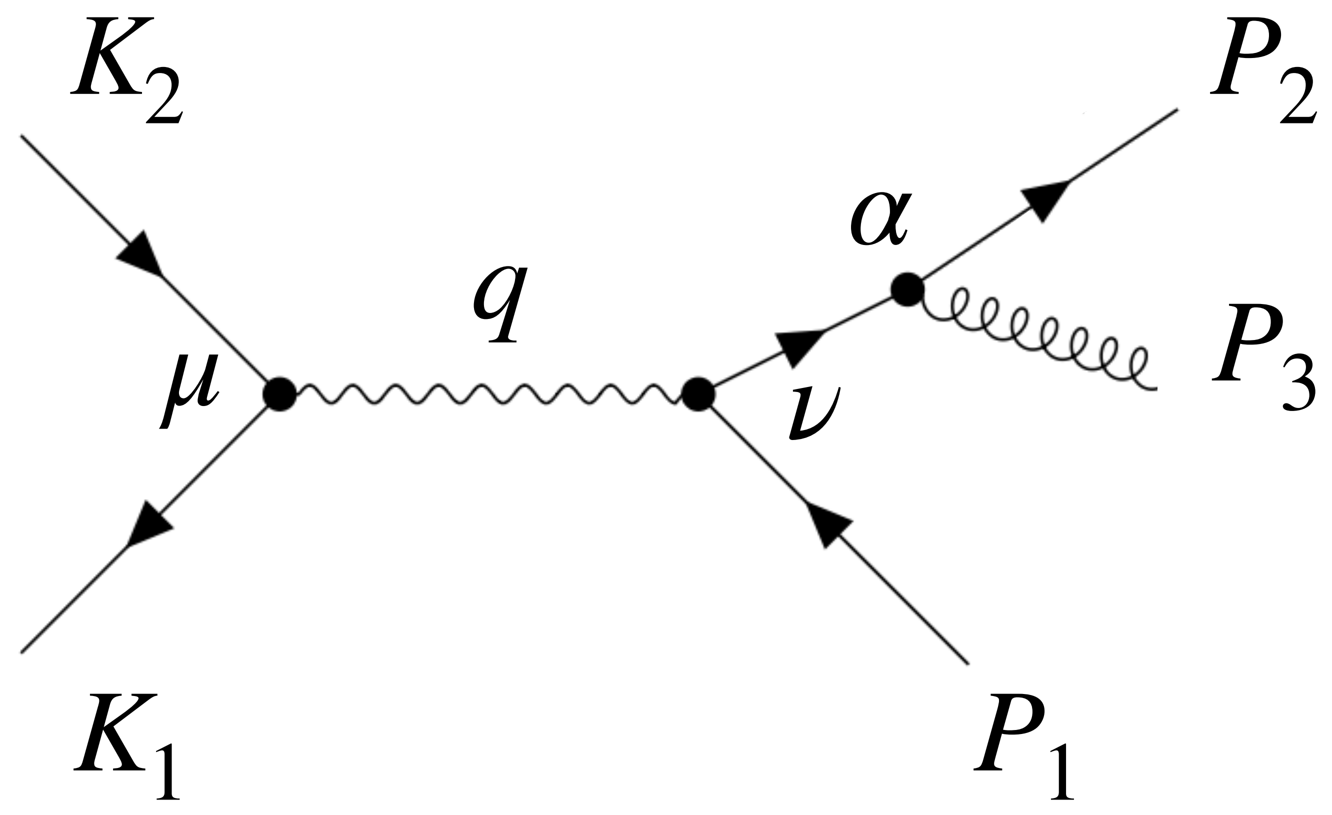

$$u(P_2)\bar{u}(P_2) = \frac{1}{2}\not{P}_2\left(1 + \gamma_5 \not{S}_{T2}\right)$$

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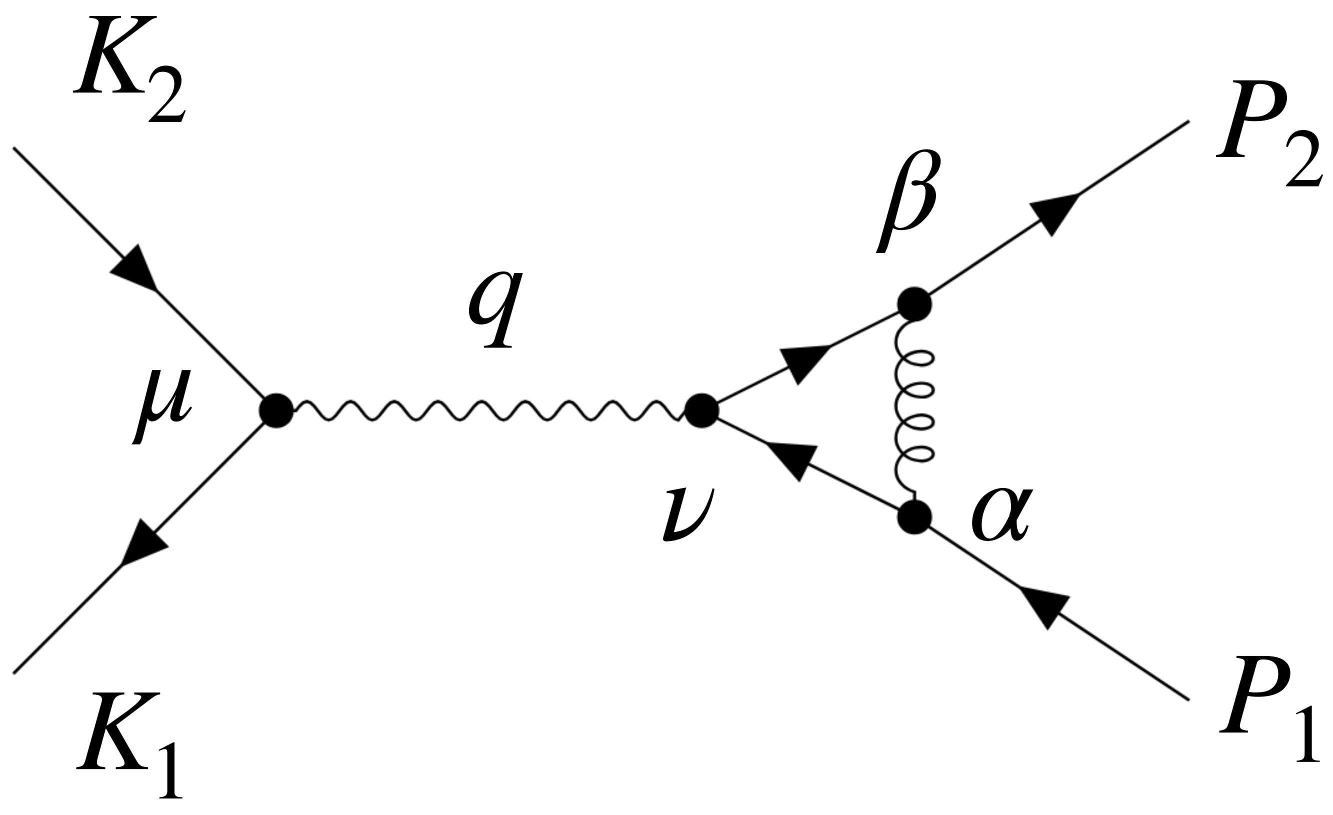

$$\sim 2Re(f(q^2))\,|M_{0,T}|^2$$

$$W^{\mu\nu} \rightsquigarrow P_i^\mu P_j^\nu,\ P_i^\mu S_{T1}^\nu,\ P_i^\mu S_{T2}^\nu,\ S_{T1}^\mu S_{T2}^\nu,\ g^{\mu\nu}$$

$$i, j = 1,2,3$$

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$$i, j = 1,2,3$$

We are integrating over

$$d\Pi_{LIPS} = \frac{d^{d-1}P_1}{(2\pi)^{d-1}} \frac{d^{d-1}P_3}{(2\pi)^{d-1}} (2\pi)^4 \delta^{(d)}(q - P_1 - P_2 - P_3)$$

That allow use to substitute $P_3^\mu = q^\mu - P_1^\mu - P_2^\mu$

$$W^{\mu\nu} \rightsquigarrow P_i^\mu P_j^\nu,\ P_i^\mu S_{T1}^\nu,\ P_i^\mu S_{T2}^\nu,\ S_{T1}^\mu S_{T2}^\nu,\ g^{\mu\nu}$$

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That allow use to substitute $P_3^\mu = q^\mu - P_1^\mu - P_2^\mu$

Great amount of terms, leading do different azimuthal modulations.

$$\left[g_T^{\mu\nu}(S_{T2} \cdot S_{T1}) - S_{T2}^\mu S_{T1}^\nu - S_{T2}^\nu S_{T1}^\mu\right]$$

$$g_T^{\mu\nu} = g^{\mu\nu} - \frac{P_1^\mu P_2^\nu + P_1^\nu P_2^\mu}{P_1 \cdot P_2}$$

$$W^{\mu\nu} \rightsquigarrow P_i^\mu P_j^\nu,\ P_i^\mu S_{T1}^\nu,\ P_i^\mu S_{T2}^\nu,\ S_{T1}^\mu S_{T2}^\nu,\ g^{\mu\nu}$$

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Great amount of terms, leading do different azimuthal modulations.

$$\left[g_T^{\mu\nu}(S_{T2}\cdot S_{T1}) - S_{T2}^\mu S_{T1}^\nu - S_{T2}^\nu S_{T1}^\mu\right]$$

$$g_T^{\mu\nu} = g^{\mu\nu} - \frac{P_1^\mu P_2^\nu + P_1^\nu P_2^\mu}{P_1\cdot P_2}$$

Any contribution of the kind $P_i^\mu S_{Tj}^\nu$ **should be removed.**

Terms proportional to q^μ **vanish after the contraction with** $L_{\mu\nu}$.

$$\sim \int d\Pi_{LIPS}\,\frac{8\,u\,z + 2(D-4)\,(1-u\,z)^2}{z(1-z)(1-u)}\left[g_T^{\mu\nu}(S_{T2}\cdot S_{T1}) - S_{T2}^\mu S_{T1}^\nu - S_{T2}^\nu S_{T1}^\mu\right]$$

$$\frac{1}{\sigma_0^d}\frac{d\sigma^R}{du\,dz} \quad + \quad \frac{1}{\sigma_0^d}\sigma_V\,\delta(1-z)\delta(1-u)$$ **removes non collinear singularities**

After $\bar{M}S$ subtraction:

$$\frac{1}{\sigma_0^T}\frac{d\sigma}{du\,dz} = e_q^2\frac{\alpha_s}{2\pi}\left[\delta(1-z)P_q^T(u) + \delta(1-u)P_q^T(z)\right]\ln\left(\frac{Q^2}{\mu^2}\right)$$

NLO

coefficient

$$+e_q^2\frac{4}{3}\frac{\alpha_s}{2\pi}\Bigg\{(\pi^2-8)\delta(1-u)\delta(1-z)$$
$$+\delta(1-z)\left[(1-u)+2u\left(\left(\frac{\log(1-u)}{1-u}\right)_+ + \frac{\log u}{(1-u)_+}\right)\right]$$
$$+\delta(1-u)\left[(1-z)+2z\left(\left(\frac{\log(1-z)}{1-z}\right)_+ + \frac{2\log z}{(1-z)_+}\right)\right]$$
$$+\frac{2uz}{(1-u)_+(1-z)_+} + O(\epsilon)\Bigg\}.$$

Unpolarized

$$\frac{1}{\sigma_0^d}\frac{d\sigma}{du\,dz} = e_q^2\frac{\alpha_s}{2\pi}\left[\delta(1-z)P_{qq}(u)+\delta(1-u)P_{qq}(z)\right]\ln\left(\frac{Q^2}{\mu^2}\right)$$
$$+\,e_q^2\frac{4}{3}\frac{\alpha_s}{2\pi}\Bigg\{\left(\pi^2-8\right)\delta(1-u)\delta(1-z)$$
$$+\,\delta(1-z)\left[(1-u)+(1+u^2)\left(\left(\frac{\ln(1-u)}{1-u}\right)_+ + \frac{\ln u}{(1-u)_+}\right)\right]$$
$$+\,\delta(1-u)\left[(1-z)+(1+z^2)\left(\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{2\ln z}{(1-z)_+}\right)\right]$$
$$+\left(z^2+(1+z(u-1))^2\right)\frac{1}{(1-u)_+(1-z)_+}+\mathcal{O}(\epsilon)\Bigg\}\,.$$

T polarized

$$\frac{1}{\sigma_0^T}\frac{d\sigma}{du\,dz} = e_q^2\frac{\alpha_s}{2\pi}\left[\delta(1-z)P_q^T(u)+\delta(1-u)P_q^T(z)\right]\ln\left(\frac{Q^2}{\mu^2}\right)$$
$$+e_q^2\frac{4}{3}\frac{\alpha_s}{2\pi}\Bigg\{(\pi^2-8)\delta(1-u)\delta(1-z)$$
$$+\delta(1-z)\left[(1-u)+2u\left(\left(\frac{\log(1-u)}{1-u}\right)_+ + \frac{\log u}{(1-u)_+}\right)\right]$$
$$+\delta(1-u)\left[(1-z)+2z\left(\left(\frac{\log(1-z)}{1-z}\right)_+ + \frac{2\log z}{(1-z)_+}\right)\right]$$
$$+\frac{2uz}{(1-u)_+(1-z)_+}+O(\epsilon)\Bigg\}\,.$$

T polarized

CONCLUSIONS AND OUTLOOKS

- **NLO Artru–Collins asymmetry has been recomputed. We find differences from** A.P. Contogouris et al, Phys. Lett. B 334, (1995).

- **Work in progress for numerical estimations to see the impact of the corrections.**
  Werner code with Vegas.
  Implementation of formulas in Apfel++.

- **Apply the techniques for transversely polarized SIDIS.**
  Needs to check: A.P. Contogouris et al, Phys. Lett. B 365, (1996)

- **Fit of the Artu-Collins asymmetry data to extract at NLO.**

Backup

Why do we need transversity beyond LO?

- **Have a better knowledge of the proton structure.**
  A solid theoretical background is fundamental, especially in view of new data (EIC)

- **Transversity can be used to investigate physics beyond the Standard Model**

Chiral-odd structures do not appear in the SM Tree level Lagrangian

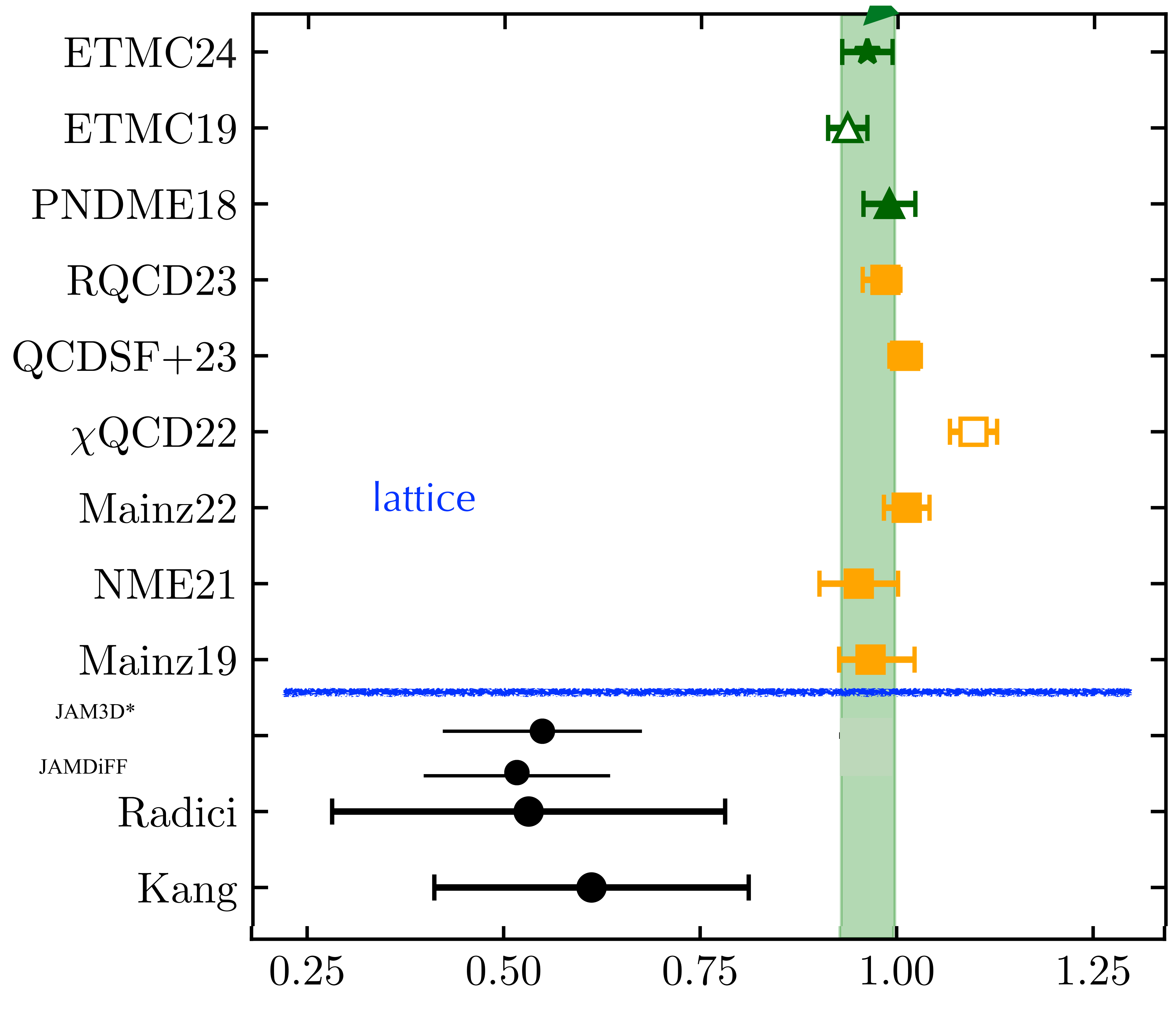
<u>Neutrino β-decay</u>

$$g_T = \delta u - \delta d$$

$$\delta q = \int_0^1 dx\, h_1^q(x)$$

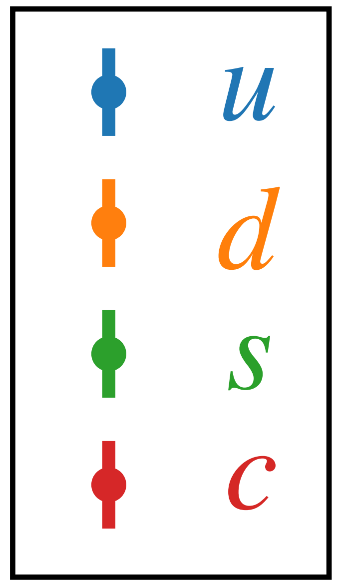
<u>constraints on CP violation</u>

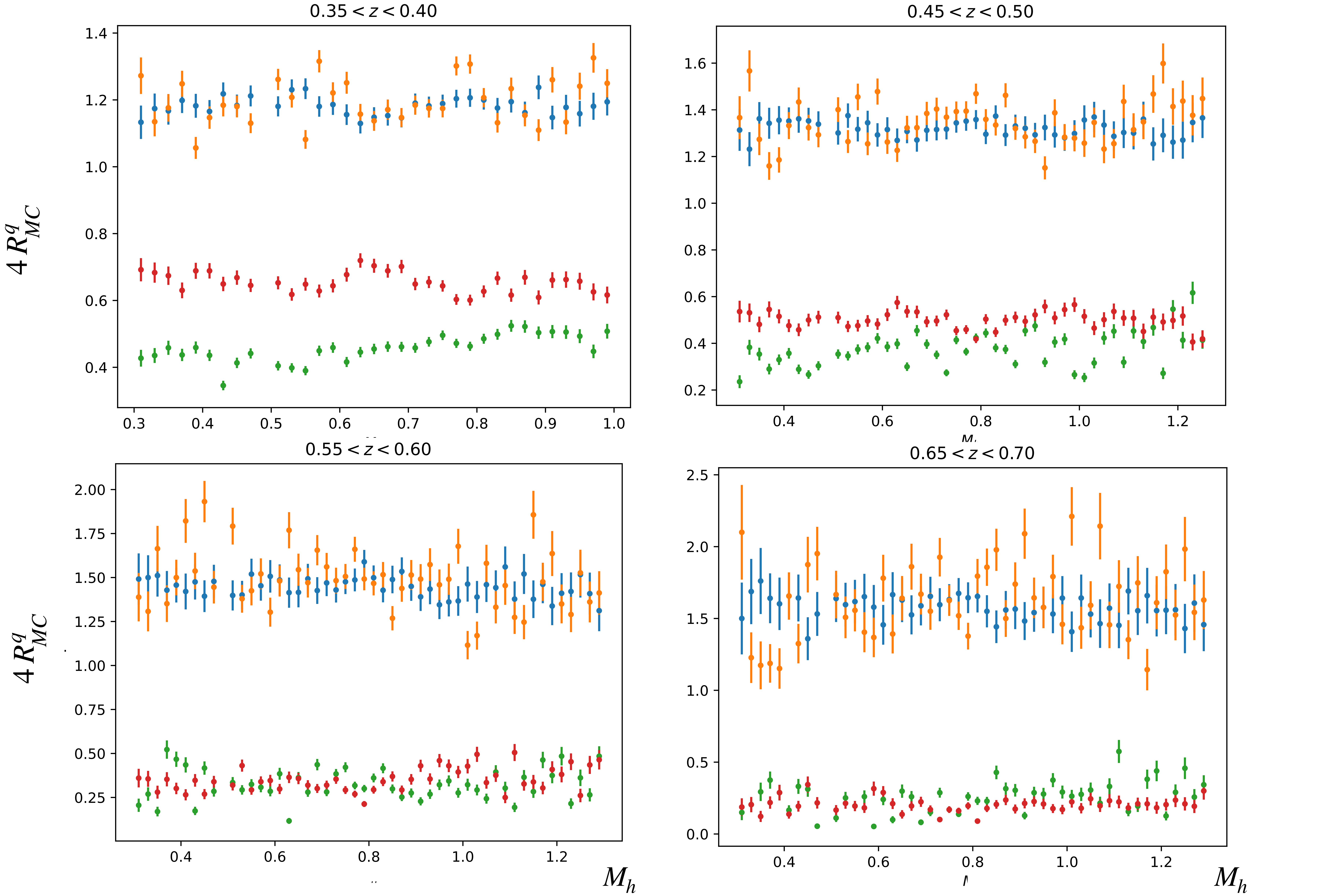
adapted from C. Alexandrou, QCD Evolution 24



$$R^{q}_{MC}(z, M_h, Q)$$

$$\frac{d\sigma}{dz dM_h dQ^2} = 2 \cdot \frac{d\sigma^{exp}}{dz dM_h dQ^2} \cdot \left(R^{\mathbf{u}}_{MC}(z, M_h, Q) + R^{\mathbf{d}}_{MC}(z, M_h, Q) + R^{\mathbf{s}}_{MC}(z, M_h, Q) + R^{\mathbf{c}}_{MC}(z, M_h, Q) \right)$$



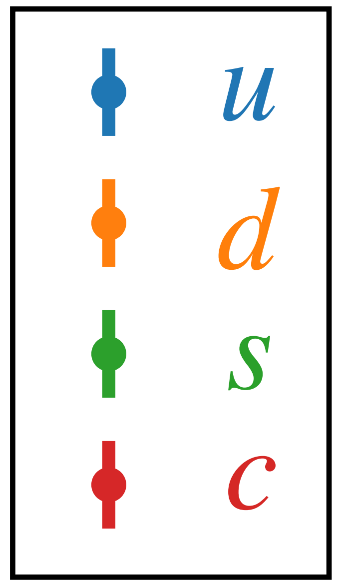


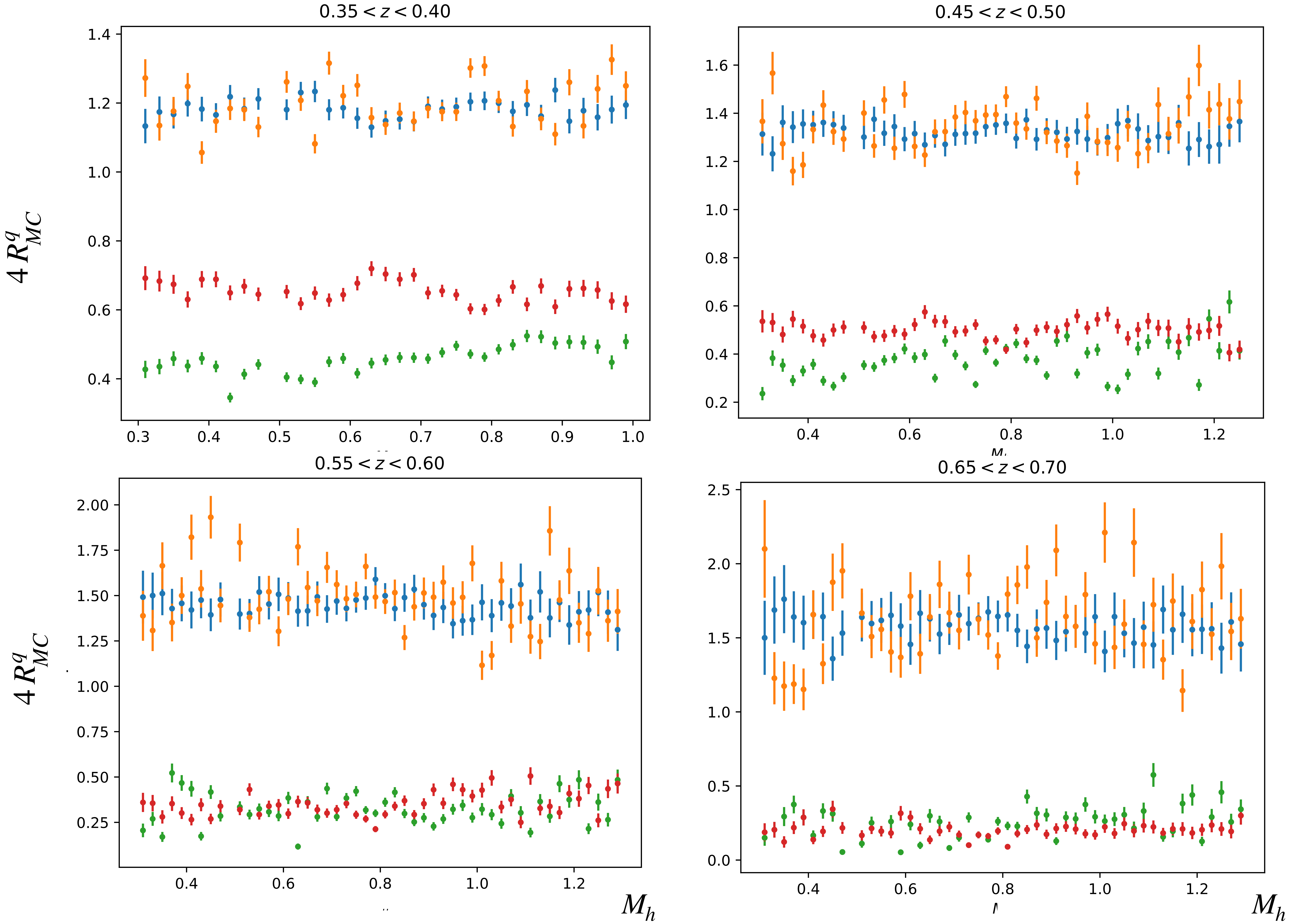
Normalized by the charge and multiplied by 4

$$4 \frac{D_1^{\mathbf{q}}(z, M_h, Q)}{\sum_p e_p^2 D_1^p(z, Mh, Q)}$$

$$R^{q}_{MC}(z, M_h, Q)$$

$$\frac{d\sigma}{dz dM_h dQ^2} = 2 \cdot \frac{d\sigma^{exp}}{dz dM_h dQ^2} \cdot \left(R^{\mathbf{u}}_{MC}(z, M_h, Q) + R^{\mathbf{d}}_{MC}(z, M_h, Q) + R^{\mathbf{s}}_{MC}(z, M_h, Q) + R^{\mathbf{c}}_{MC}(z, M_h, Q) \right)$$





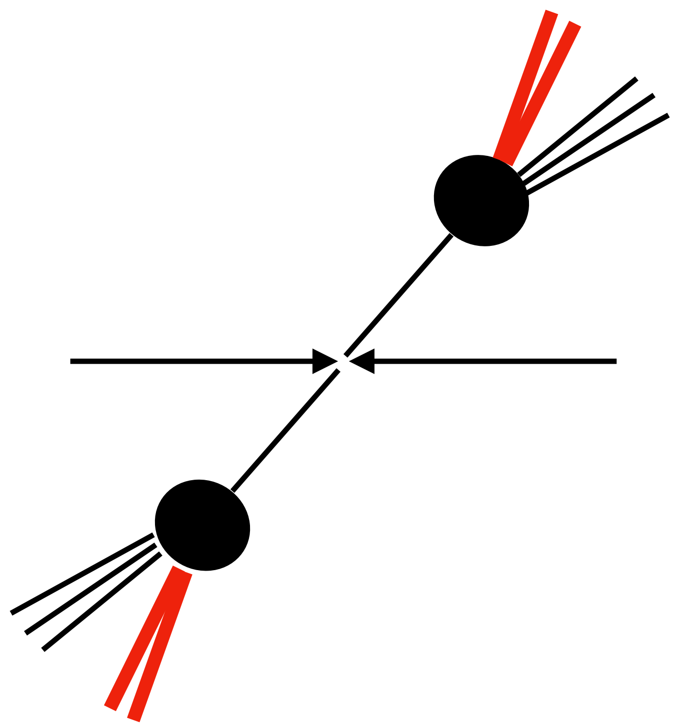
Normalized by the charge and multiplied by 4

$$4\,\frac{D_1^{\mathbf{q}}(z, M_h, Q)}{\sum_p e_p^2\, D_1^p(z, Mh, Q)}$$

$$\mathbf{u} \sim \mathbf{d}$$

$H_1^{\sphericalangle}$ extraction

$$A_{UT}^{hh} \sim \frac{\sum_q e_q^2 \cdot H_1^{\sphericalangle,q}(z, M_h) \cdot H_1^{\sphericalangle,q}(\bar{z}, \bar{M}_h)}{\sum_q e_q^2 \cdot D_1^q(z, M_h) \cdot D_1^q(\bar{z}, \bar{M}_h)}$$



For $\pi^+\pi^-$

Symmetry relations for $H_1^{\sphericalangle}$:

- Isopsin symmetry $H_1^{\sphericalangle,\mathbf{u}\to\pi^+\pi^-} = H_1^{\sphericalangle,\bar{\mathbf{d}}\to\pi^+\pi^-}$, $H_1^{\sphericalangle,\bar{\mathbf{u}}\to\pi^+\pi^-} = H_1^{\sphericalangle,\mathbf{d}\to\pi^+\pi^-}$
- Charge conjugation $H_1^{\sphericalangle,\mathbf{u}\to\pi^+\pi^-} = H_1^{\sphericalangle,\bar{\mathbf{u}}\to\pi^-\pi^+} = H_1^{\sphericalangle,\mathbf{d}\to\pi^-\pi^+} = H_1^{\sphericalangle,\bar{\mathbf{d}}\to\pi^+\pi^-}$

$H_1^{\sphericalangle}$ is proportional to $H_1^{\sphericalangle} \propto (\vec{p}_{h_1} - \vec{p}_{h_2})$, which brings $H_1^{\sphericalangle q\to\pi^+\pi^-} = -H_1^{\sphericalangle q\to\pi^-\pi^+}$

Valence $H_1^{\sphericalangle/,\mathbf{u}} = -H_1^{\sphericalangle/,\mathbf{d}} = -H_1^{\sphericalangle/,\bar{\mathbf{u}}} = H_1^{\sphericalangle/,\bar{\mathbf{d}}}$

Sea: $q = s, c, b$ $H_1^{\sphericalangle/,\mathbf{q}} = -H_1^{\sphericalangle/,\bar{\mathbf{q}}} = -H_1^{\sphericalangle/,\mathbf{q}} = 0$