Evolution of pion-nucleon transition distribution amplitudes

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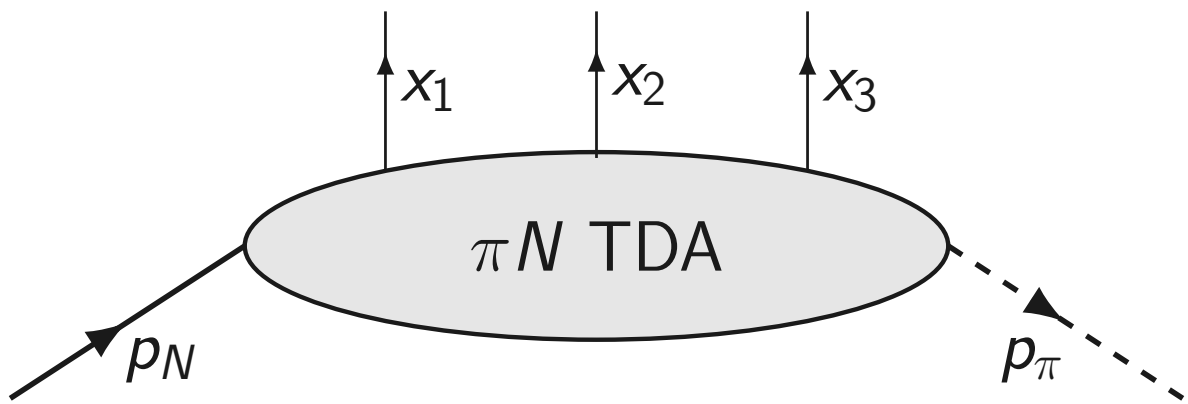
Pion-nucleon TDAs

$$\langle \pi | \, u_{\lambda_1}(z_1) u_{\lambda_2}(z_2) d_{\lambda_3}(z_3) \, | \mathcal{N}, \Lambda \rangle \sim \int [d\mathbf{x}] \, \mathrm{e}^{-iP^+ \sum_j x_j z_j} \, \mathbf{f(x_1, x_2, x_3)}$$

$$\int [d\mathbf{x}] = \int_{-1+\xi}^{+1+\xi} dx_1 \int_{-1+\xi}^{+1+\xi} dx_2 \int_{-1+\xi}^{+1+\xi} dx_3 \, \delta(x_1 + x_2 + x_3 - 2\xi)$$



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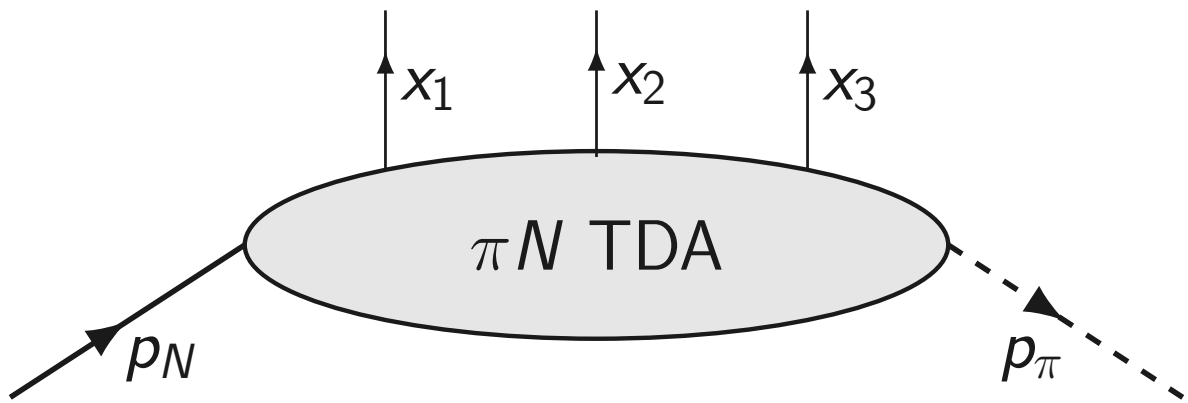
$$\xi = \frac{p_N^+ - p_\pi^+}{p_N^+ + p_\pi^+}$$



Pion-nucleon TDAs

$$\mathbf{T}^{\Lambda}_{\lambda_1\lambda_2\lambda_3}$$

$$\downarrow$$

$$\langle \pi |\, u_{\lambda_1}(z_1) u_{\lambda_2}(z_2) d_{\lambda_3}(z_3)\, | \mathcal{N}, \Lambda \rangle \sim \int [d\mathbf{x}]\, \mathrm{e}^{-iP^+ \sum_j x_j z_j}\, \mathbf{f}(\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3})$$

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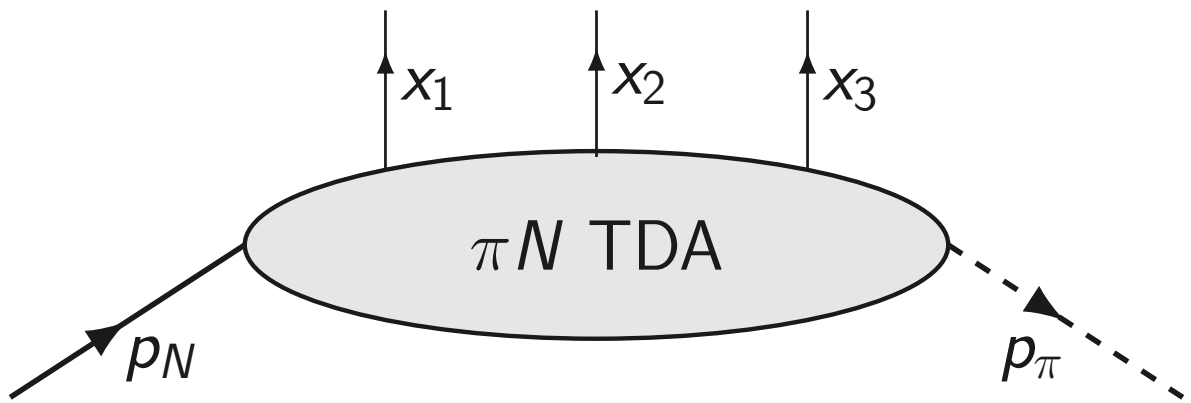
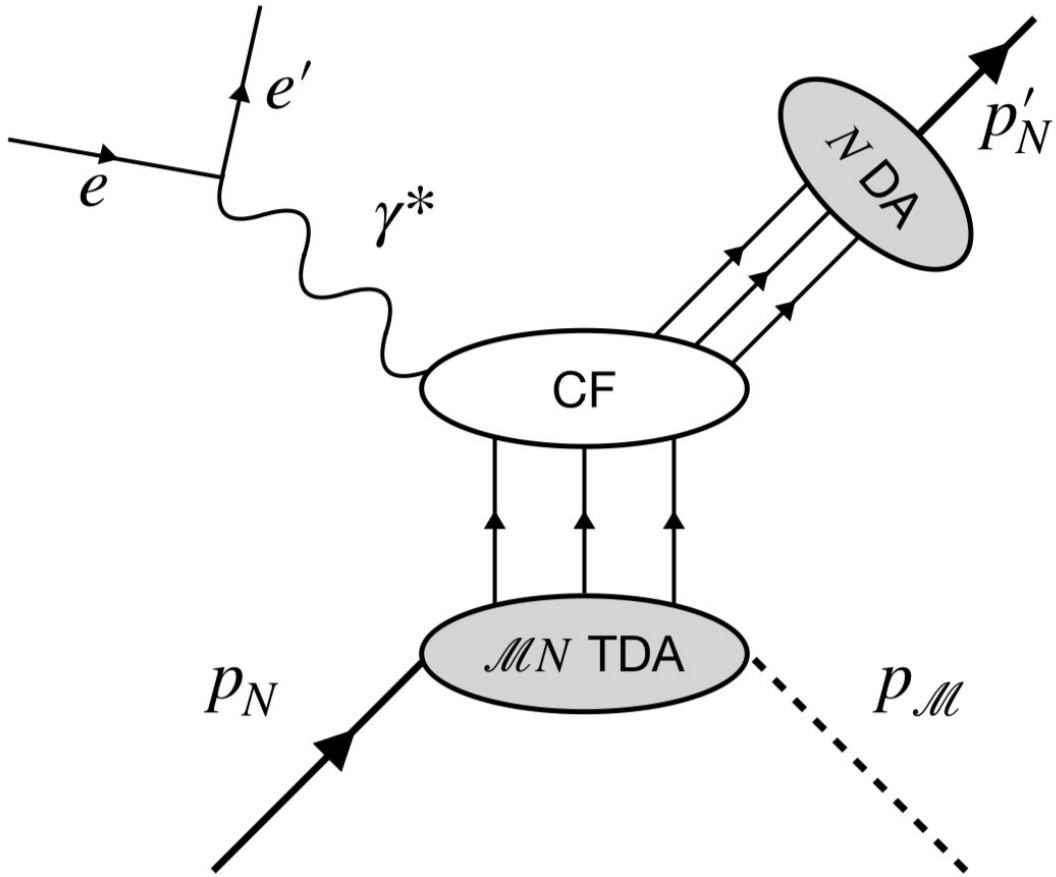
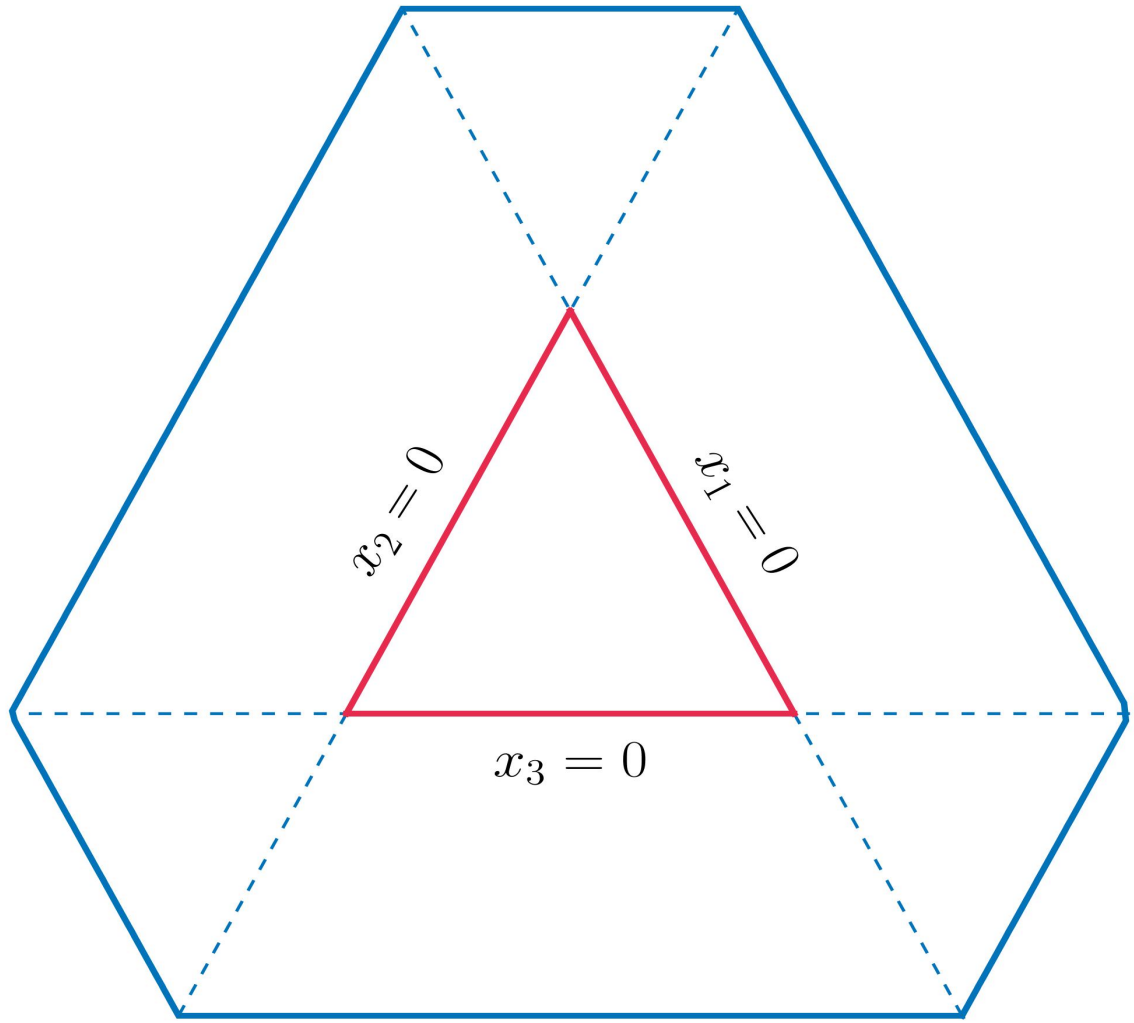


Meson-nucleon TDAs



Hard exclusive meson electroproduction

TDA support



TDA support

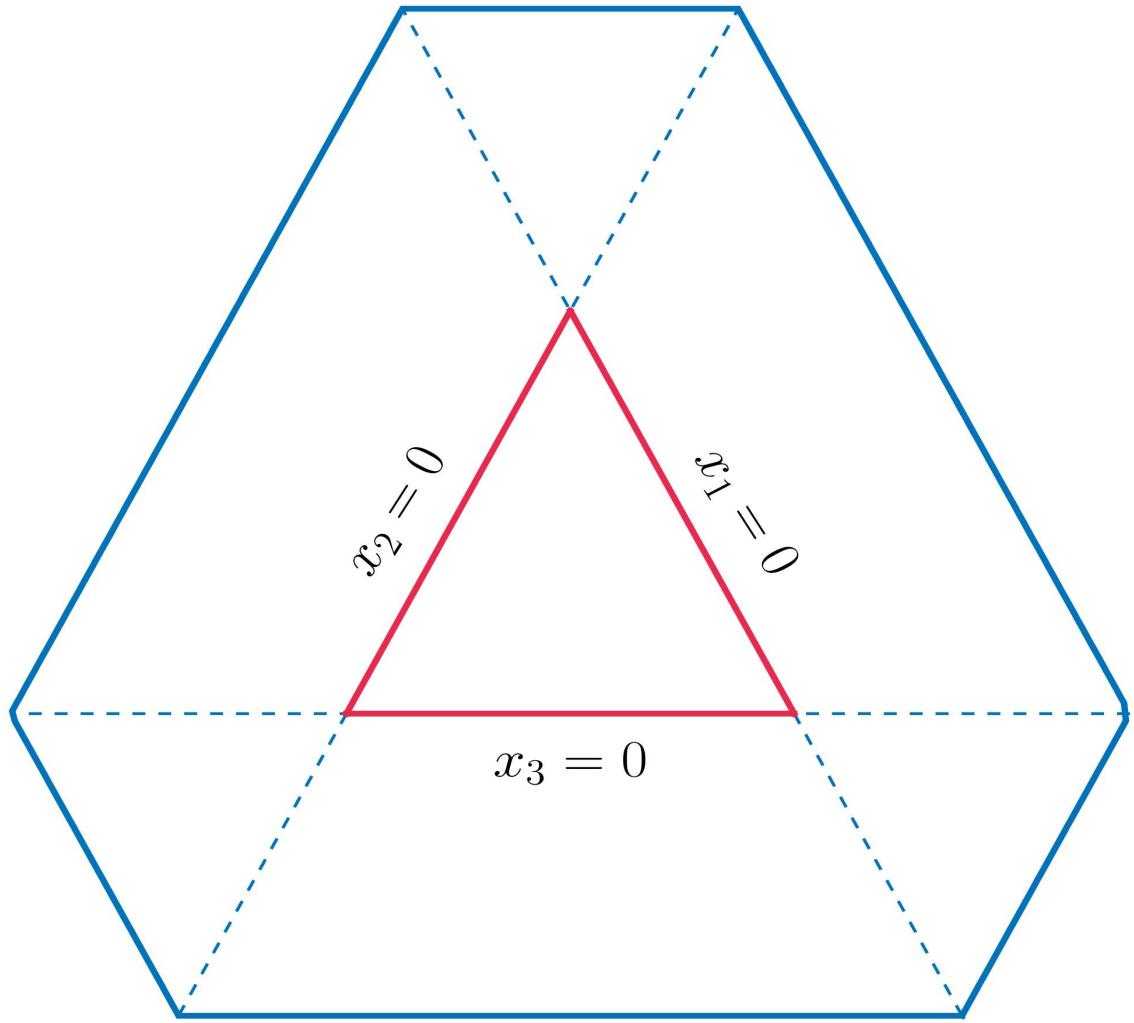
$$w = x_3 - \xi$$

$$w \in [-1, +1]$$

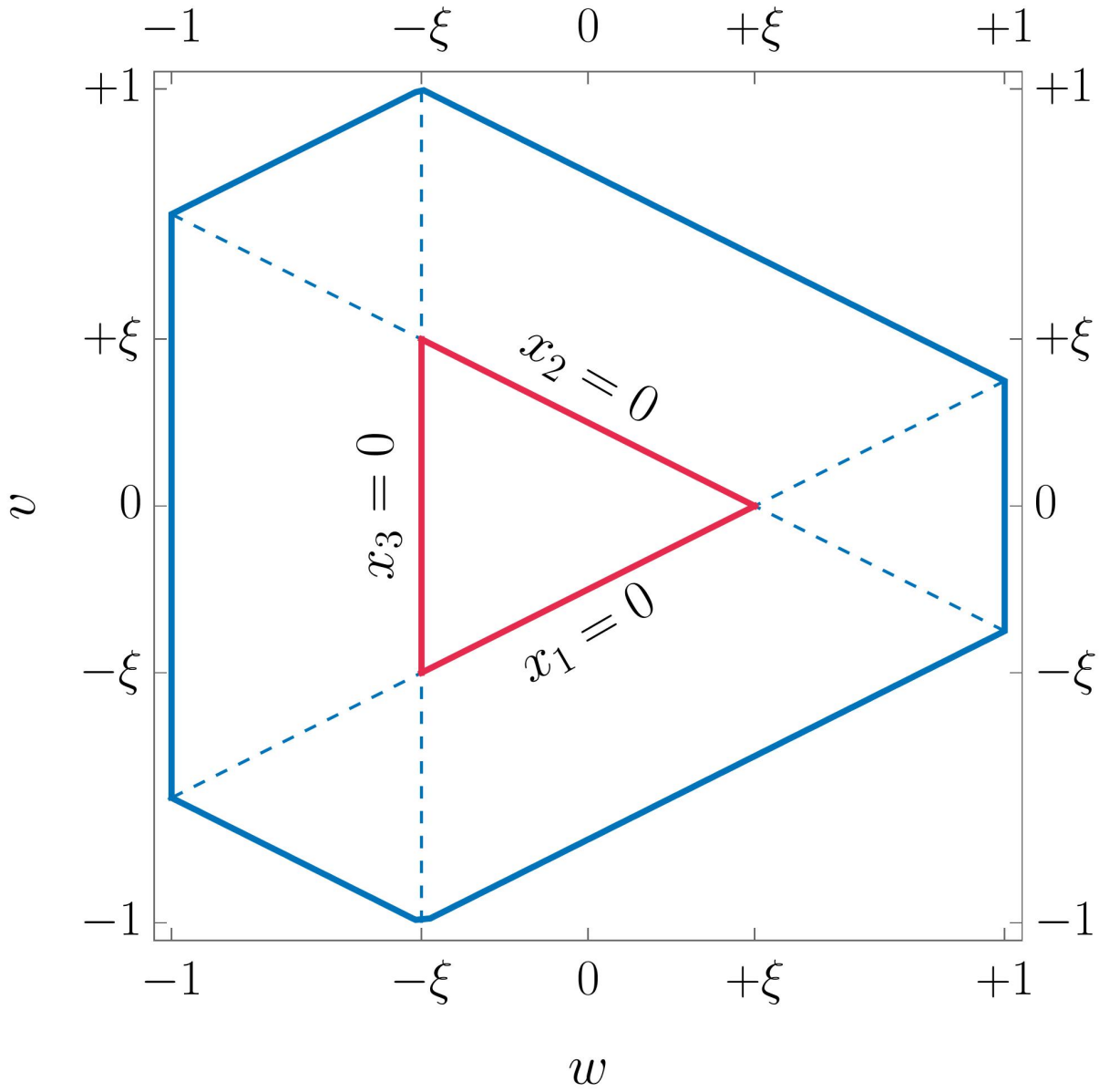
$$v = \frac{x_1 - x_2}{2}$$

$$v \in [-d, +d]$$

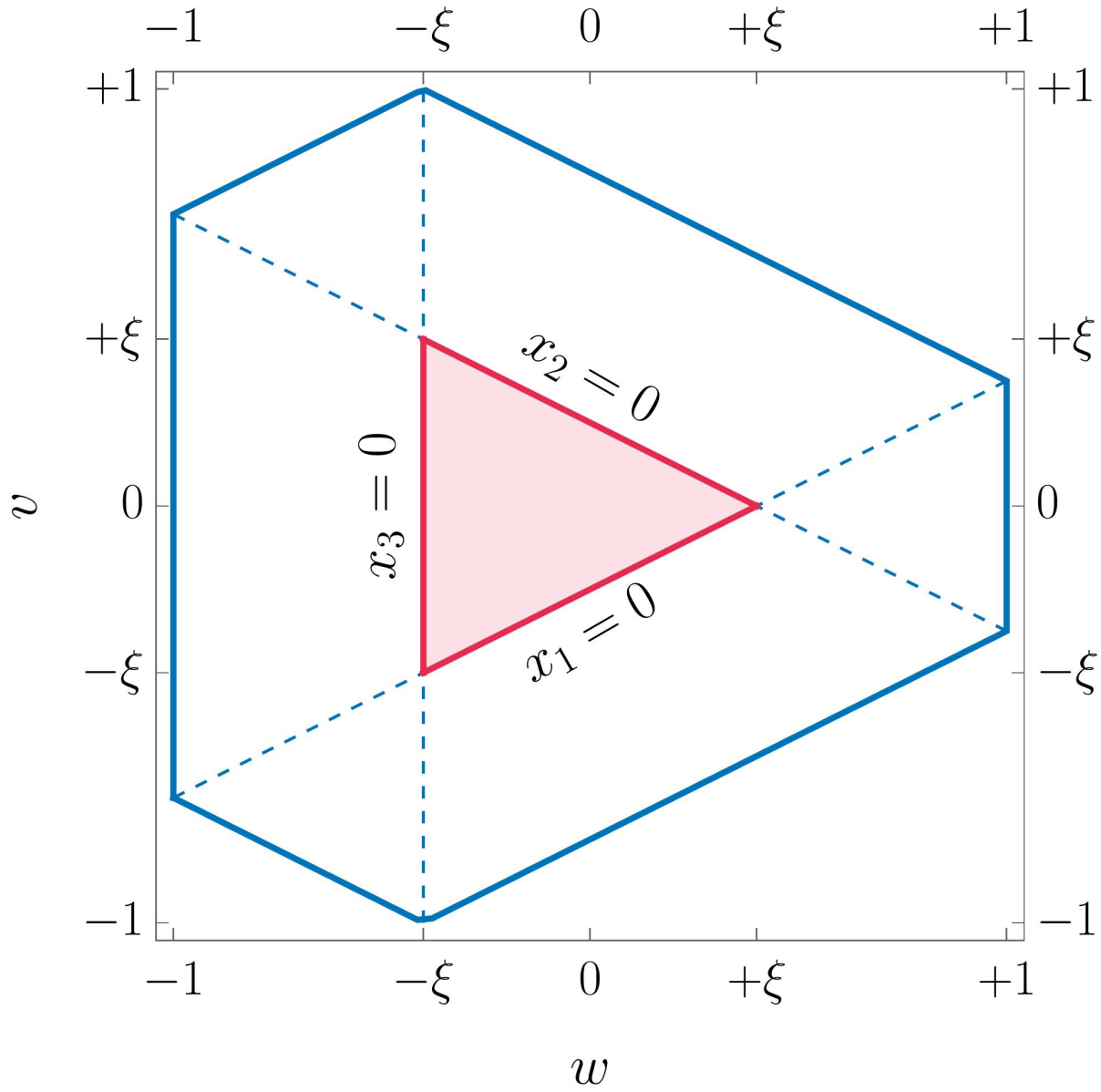
$$d = 1 - \left| \frac{\xi + w}{2} \right|$$



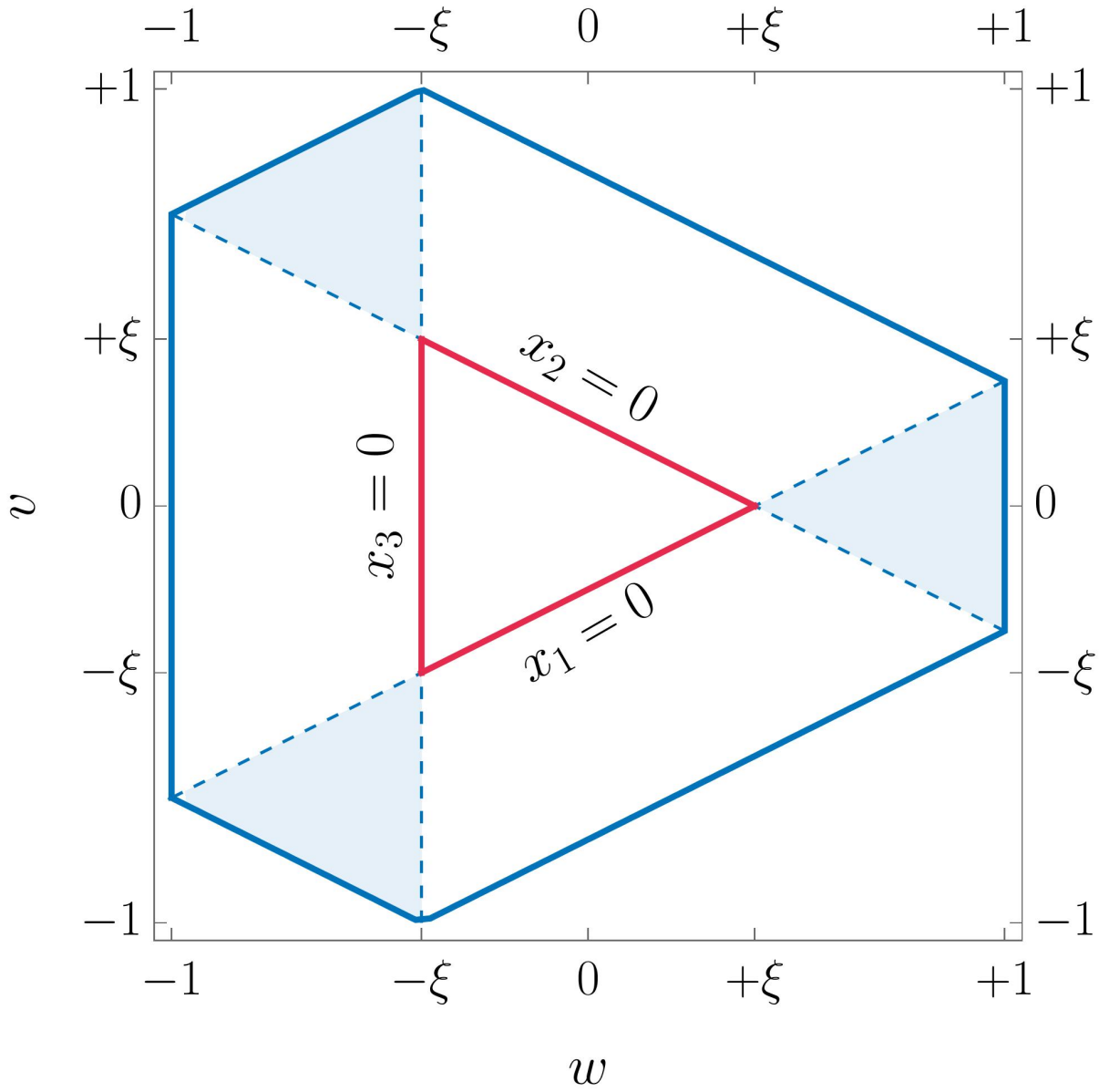
TDA support



TDA support



TDA support



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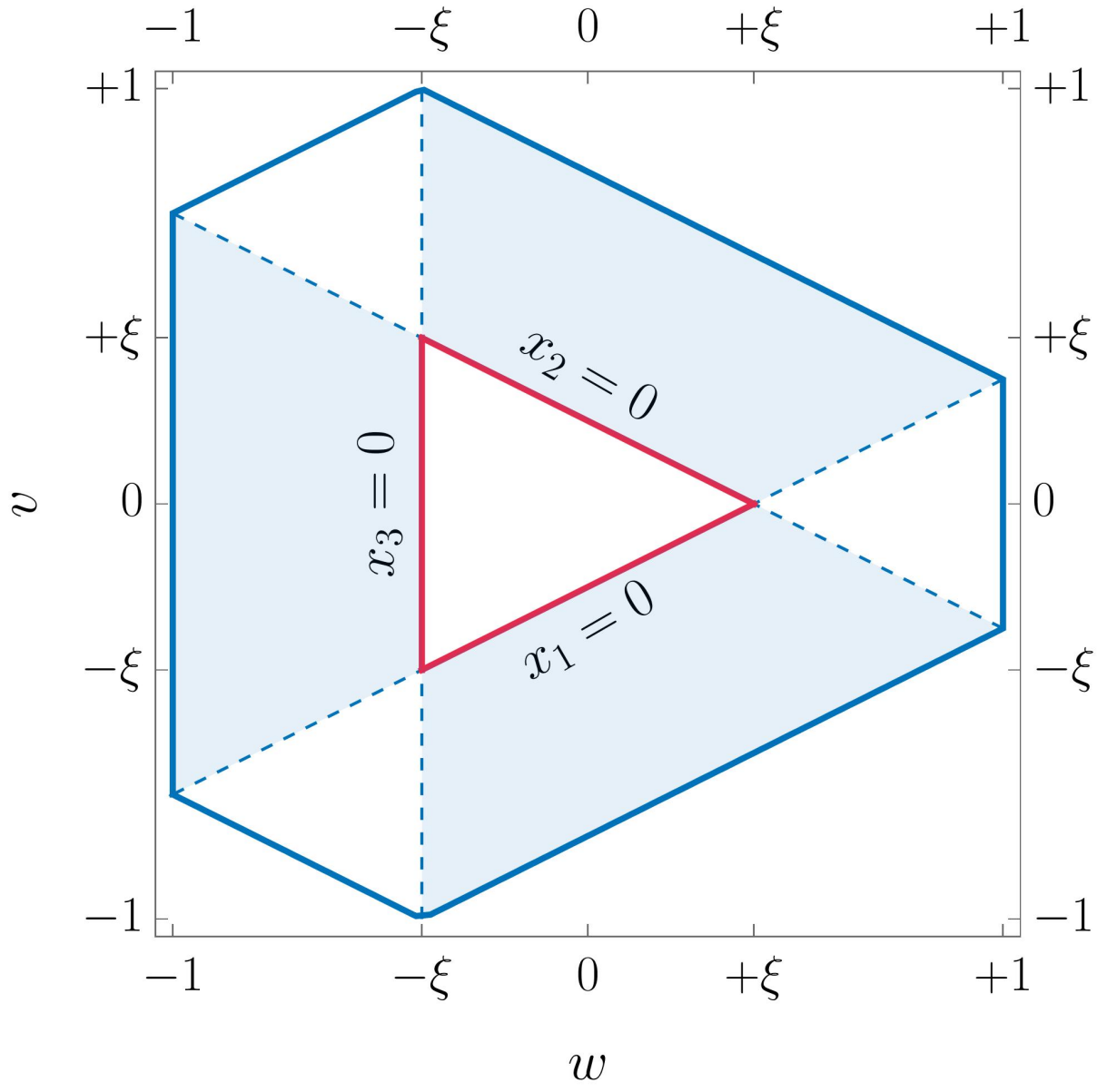

Polynomial expansion

$$T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(w,t;\mu) = \sum_{i,j=0}^{n} c_{ij}(\mu)\, P_i(t)\Pi_j(w)$$

$$c_{ij}(\mu) = \int_{-1}^{1} dw \int_{-1}^{1} dt\, J(w)\, P_i(t)\Pi_j(w)\, T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(w,t;\mu)$$

$$t = v/d,\ t \in [-1,+1] \rightarrow\ J(w) = 1 - \left|\tfrac{\xi+w}{2}\right|$$

Polynomial expansion

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$$P_i(t) = p_i(t)\sqrt{\mu(t)} \qquad \Pi_j(w) = q_j(w)\sqrt{\frac{\eta(w)}{\mathbf{J(w)}}}$$

$$\int_{-1}^{1} dt\, \mu(t)\, p_i(t)p_k(t) = \delta_{ik} \qquad \int_{-1}^{1} dw\, \eta(w)\, q_j(w)q_k(w) = \delta_{jk}$$

Polynomial expansion

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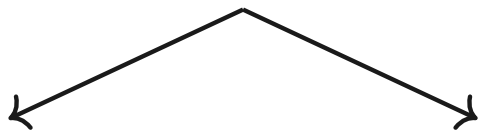

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$$p_n(y), q_n(y) \to C_n^{(3/2)}(y)$$

Evolution equations

$$\mu \frac{\mathrm{d}\, T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(\mathbf{x};\mu)}{d\mu} = -\frac{\alpha_S(\mu)}{2\pi}\left[\left(1+\frac{1}{N_c}\right)\mathbf{H}+\frac{3}{2}C_F\right] T^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(\mathbf{x};\mu)$$

$$\downarrow$$

$$\hat{\mathcal{H}}^{12}+\hat{\mathcal{H}}^{13}+\hat{\mathcal{H}}^{23}-h_{\lambda_1\lambda_2\lambda_3}$$

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$$\hat{\mathcal{H}}^{12}+\hat{\mathcal{H}}^{13}+\hat{\mathcal{H}}^{23}-h_{\lambda_1\lambda_2\lambda_3}$$

- $\underline{\lambda_i=\lambda_j=\lambda_k} \;\rightarrow\; h_{\lambda_i\lambda_j\lambda_k}=0$
- $\underline{\lambda_i=\lambda_j\neq\lambda_k} \;\rightarrow\; h_{\lambda_i\lambda_j\lambda_k}=\mathcal{H}^{+}_{ik}+\mathcal{H}^{+}_{jk}$

Kernel evaluation

- Consider $\hat{\mathcal{H}}^{12} \to x_3 = 2\xi - x_1 - x_2$

$$[\hat{\mathcal{H}}^{12} T](x_1, x_2; \mu) = \int \mathcal{D}x' \, \Phi^{12}(x_1, x_2, x_1', x_2') \, T(x_1', x_2'; \mu)$$

$$\int \mathcal{D}x' = \int_{-1+\xi}^{+1+\xi} dx_1' \int_{-1+\xi}^{+1+\xi} dx_2' \, \delta(x_1 + x_2 - x_1' - x_2')$$

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$$\boxed{\mu \frac{\mathrm{d}\, T(x_1, x_2; \mu)}{d\mu} = -\frac{\alpha_S(\mu)}{2\pi} [\hat{\mathcal{H}}^{12}\ T](x_1, x_2; \mu)}$$

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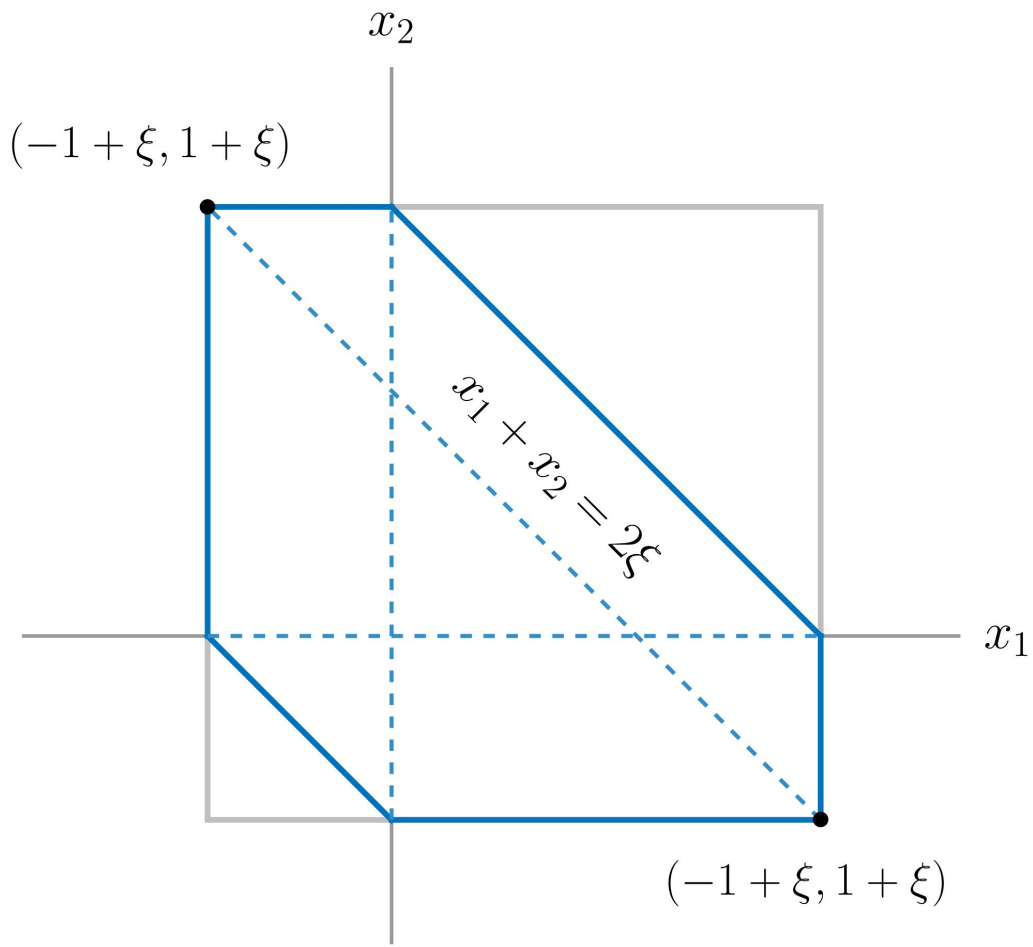
- Notation: $P_i\big(t(x_1, x_2)\big) \Pi_j\big(w(x_1, x_2)\big) \equiv b_{ij}(x_1, x_2)$

Kernel evaluation

- Projection: $\int_\Omega dx_1\, dx_2\, b_{ij}(x_1, x_2)$



Kernel evaluation

- Projection: $\int_\Omega dx_1\, dx_2\, b_{ij}(x_1, x_2)$

$$\int_\Omega dx_1\, dx_2\, b_{ij}(x_1, x_2)\mu \frac{\mathrm{d}\, T(x_1, x_2; \mu)}{d\mu}$$

$$= -\frac{\alpha_S(\mu)}{2\pi} \int_\Omega dx_1\, dx_2\, b_{ij}(x_1, x_2)[\hat{\mathcal{H}}^{12}\, T](x_1, x_2; \mu)$$

Kernel evaluation

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$$\int_\Omega dx_1\, dx_2\, b_{ij}(x_1, x_2)\mu\frac{\mathrm{d}\, T(x_1, x_2; \mu)}{d\mu}$$
$$= -\frac{\alpha_S(\mu)}{2\pi}\int_\Omega dx_1\, dx_2\, b_{ij}(x_1, x_2)[\hat{\mathcal{H}}^{12}\, T](x_1, x_2; \mu)$$

- Definition of $c_{ij}(\mu)$ and expansion of $T(x_1, x_2; \mu)$

$$\mu\frac{\mathrm{d}c_{ij}(\mu)}{d\mu} = -\frac{\alpha_S(\mu)}{2\pi}\int_\Omega dx_1\, dx_2\, b_{ij}(x_1, x_2)\left[\hat{\mathcal{H}}^{12}\sum_{kl} c_{kl}(\mu) b_{kl}(x_1', x_2')\right]$$
$$= -\frac{\alpha_S(\mu)}{2\pi}\sum_{kl} c_{kl}(\mu)\,\mathcal{I}^{12}_{ijkl}$$

Evolved TDA

3D numerical integrals

$$\mu \frac{\mathrm{d}c_{ij}(\mu)}{d\mu} = -\frac{\alpha_S(\mu)}{2\pi} \sum_{kl} c_{kl}(\mu) \left[\left(1 + \frac{1}{N_c}\right) \mathbf{I}_{ijkl} + \frac{3}{2} C_F \, \mathbb{I} \right]$$

evolves the coefficients instead of the TDA

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evolves the coefficients instead of the TDA

evaluate $c_{ij}(\mu_I)\ \forall\, i,j \Rightarrow$ solve for $c_{ij}(\mu_F)$

Evolved TDA

3D numerical integrals

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evaluate $c_{ij}(\mu_I) \; \forall \, i, j \; \Rightarrow$ solve for $c_{ij}(\mu_F)$

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Evolved TDA

WORK IN PROGRESS

$$\mu \frac{\mathrm{d}c_{ij}(\mu)}{d\mu} = -\frac{\alpha_S(\mu)}{2\pi} \sum_{kl} c_{kl}(\mu) \left[\left(1 + \frac{1}{N_c}\right) \mathbf{I}_{ijkl} + \frac{3}{2} C_F \, \mathbb{I} \right]$$

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