# Matching relations for gluon TMDs up to one-loop accuracy

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# Outline

1) Introduction

2) LO and NLO Results

3) Mass corrections

4) Conclusions

# Introduction

# Matching relations

In the small-$b$ regime:

$$f_i(x,b) = \sum_n \alpha_s^n \sum_j \left[C_{ij}^{(n)} \otimes f_j\right](x) \; + \; O(b^2)$$

TMD

Matching Coefficient

PDF

# Matching relations

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Constraints on TMD functional form → Parameterization $f_i(x, b) \propto \sum_j \left( C_{ij} \otimes f_j \right)(x)$

Predictions for TMD observables

# Known Examples (Quark TMDs)

$$g_1(x,b) = \int_0^1 du \int dy\, \delta(x - uy)\, \delta(1-u)\, g_1(y) = g_1(x)$$

Helicity TMD    Mellin Convolution    Matching coefficient    Helicity PDF

# Known Examples (Quark TMDs)

$$g_1(x, b) = \int_0^1 du \int dy\, \delta(x - uy)\, \delta(1 - u)\, g_1(y) = g_1(x)$$

$$g_1(x, b) = 4a_s C_F \Gamma(-\epsilon) \left(\mu^2 \boldsymbol{b}^2\right)^{\epsilon} \int_0^1 du \int dy\, \delta(x - uy) \left( \frac{2}{(1-u)_+} - 1 - u + (1 - u)\epsilon \right) g_1(y)$$

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$$g_{1T}(x,b) = \int_0^1 du \int dy\, \delta(x-uy)\, u\left(y g_1(y) + y\mathcal{T}_g(y)\right)$$

Worm-gear T TMD

WW approximation

Twist-3 part

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$$g_{1T}(x,b) = \int_0^1 du \int dy\, \delta(x-uy)\, u\left(y g_1(y) + y \mathscr{T}_g(y)\right)$$

$$f_{1T}^{\perp}(x,b) = \pm \int_0^1 du \int dy\, \delta(x-uy)\, \delta(1-u)\, \pi\, T(-y,0,y) = \pm\, \pi\, T(-x,0,x)$$

# Actual Status of Matching Relations

Quark TMDs

| Distribution | Tw2 | Tw3 | Accuracy |
|---|---|---|---|
| $f_1$ | $f_1\,, f_g$ | - | $\mathrm{N^3LO}$ |
| $g_1$ | $g_1\,, \Delta f_g$ | - | $\mathrm{N^3LO}$ |
| $g_{1T}$ | $g_1\,, \Delta f_g$ | $\mathcal{T}_g\,, \mathcal{G}$ | NLO |
| $h_1$ | $h_1$ | - | $\mathrm{N^3LO}$ |
| $h_{1L}$ | $h_1$ | $\mathcal{T}_h$ | NLO |
| $h_{1T}^{\perp}$ | - | $\mathcal{T}_h$ | LO |
| $f_{1T}^{\perp}$ | - | $T\,, \mathcal{G}$ | NLO |
| $h_1^{\perp}$ | - | $E$ | NLO |

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| $g_{1T}$ | $g_1\,,\Delta f_g$ | $\mathcal{T}_g\,,\mathcal{G}$ | NLO |
| $h_1$ | $h_1$ | - | N$^3$LO |
| $h_{1L}$ | $h_1$ | $\mathcal{T}_h$ | NLO |
| $h_{1T}^{\perp}$ | - | $\mathcal{T}_h$ | LO |
| $f_{1T}^{\perp}$ | - | $T\,,\mathcal{G}$ | NLO |
| $h_1^{\perp}$ | - | $E$ | NLO |

## Gluon TMDs

| Distribution | Tw2 | Tw3 | Accuracy |
|---|---|---|---|
| $f_1^g$ | $f_g\,,f_1$ | - | N$^3$LO |
| $h_1^{\perp g}$ | $f_g\,,f_1$ | - | N$^3$LO |
| $g_{1L}^g$ | $\Delta f_g\,,g_1$ | | N$^3$LO |
| $g_{1T}^g$ | | | |
| $f_{1T}^{\perp g}$ | - | | |
| $h_{1T}^g$ | - | | |
| $h_{1L}^{\perp g}$ | | | |
| $h_{1T}^{\perp g}$ | | | |

# LO and NLO Results

# Tree level computation

$$G^{\mu\nu}(x,b) = \sum_{n=0}^{\infty} \frac{1}{n!} b_{\mu_1} \dots b_{\mu_n} \left[\partial^{\mu_1} \dots \partial^{\mu_n} G^{\mu\nu}(x,b)\right]_{|b=0}$$

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Issue: each operator needs to be treated separately

Does a unified treatment exist?

*Moos, Vladimirov, JHEP 12 (2020) 145*

*Rodini, Alvaro, Pasquini, PLB 845 (2023) 138163*

# Example of results: unpolarized

$$f_1^g(x, b) = f_g(x)$$

$$+ \sum_{k=1}^{\infty} \frac{1}{k!(k-1)!} \left( \frac{x^2 M^2 b^2}{4} \right)^k \int_0^1 du \int dy\, \delta(x - uy) \left( \frac{1-u}{u} \right)^{k-1} f_g(y)$$

Leading Term

Mass series

# Example of results: Sivers

$$f_{1T}^{\perp,g}(x,b) = \mp \frac{2\pi}{x}\left(2F_2^+ + F_4^+\right)(-x,0,x)$$

$$\mp \sum_{k=1}^{\infty} \frac{1}{(k-1)!(k+1)!}\left(\frac{x^2M^2b^2}{4}\right)^k \int_0^1 du \int dy\, \delta(x-uy)\, G_k(u)\, \frac{2\pi}{y}\left(2F_2^+ + F_4^+\right)(-y,0,y)$$

Leading Term

Mass series

T-oddness

# Example of results: worm-gear T

$$g_{1T}^{\perp,g}(x,b) = -\int_0^1 du \int dy\, \delta(x-uy)\, u \left( y\Delta f_g(y) + \frac{\mathcal{F}(y)}{y} - \frac{\mathcal{T}(y)}{y} \right)$$

WW approximation

Twist 3 gluon PDF

Twist 3 quark PDF

# Summary tree-level

6 TMDs match at tree-level up to twist 3 PDFs

$$\left\{f_1^g, g_{1L}^g\right\}, \left\{g_{1T}^{\perp,g}\right\}, \left\{f_{1T}^{\perp,g}, h_{1T}^g\right\}, \left\{h_{1L}^{\perp,g}\right\}$$

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$h_1^{\perp,g}$: NLO matching

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$h_1^{\perp,g}$: NLO matching

$h_{1T}^{\perp,g}$: twist 4 and 5 PDFs

# NLO: Parton-in-parton

$$\Phi_i(x, b) \overset{NLO}{=} a_s \int_0^1 du \int dy \, \delta(x - uy) \sum_j C_{ij}(u) \, \Phi_j(y)$$

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$$C_{ij} \Longleftrightarrow \langle p_j\,, s_j | F^{\mu+}\, \Gamma^i_{\mu\nu}\, F^{\nu+} | p_j\,, s_j \rangle$$

$\Gamma_{\mu\nu}$ proper projector

$p_j\,, s_j$ gluon/quark final state

# Main idea

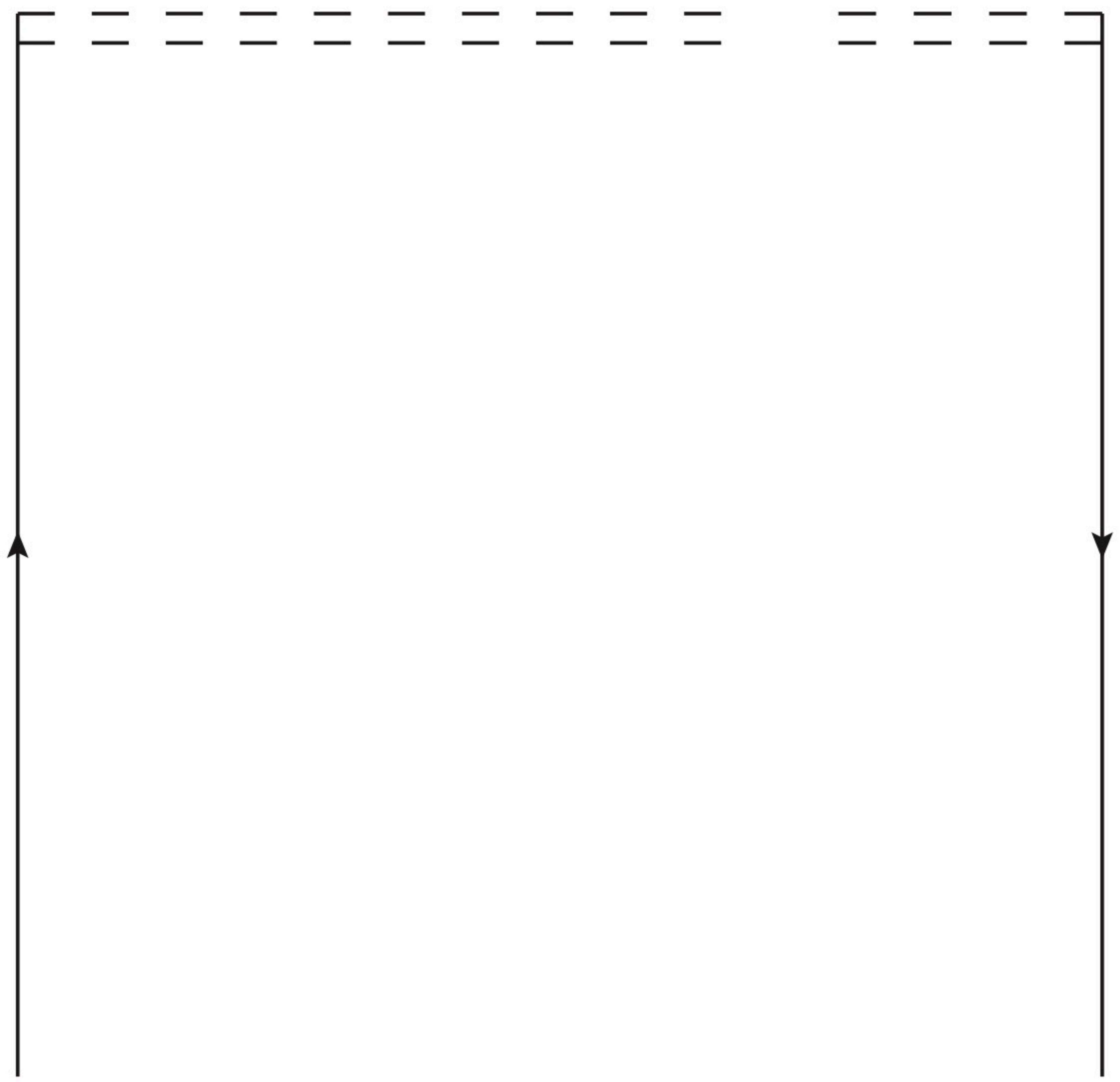
Question: can we extend to higher-twist? and how?

$$p_j^\mu = (p^+, 0, \mathbf{0}) \Longrightarrow p_j^\mu = \left(p^+, \frac{\boldsymbol{p_T}^2}{2p^+}, \boldsymbol{p_T}\right)$$

$\left(\boldsymbol{b} \cdot \boldsymbol{p_T}\right)^n$ associated to higher-twist terms

# Hint: quark worm-gears (tree-level)

$$C_q^{L/L}(u) = e^{ib\cdot p_T}\,\delta(1-u) = (1 + ib\cdot p_T)\,\delta(1-u)$$



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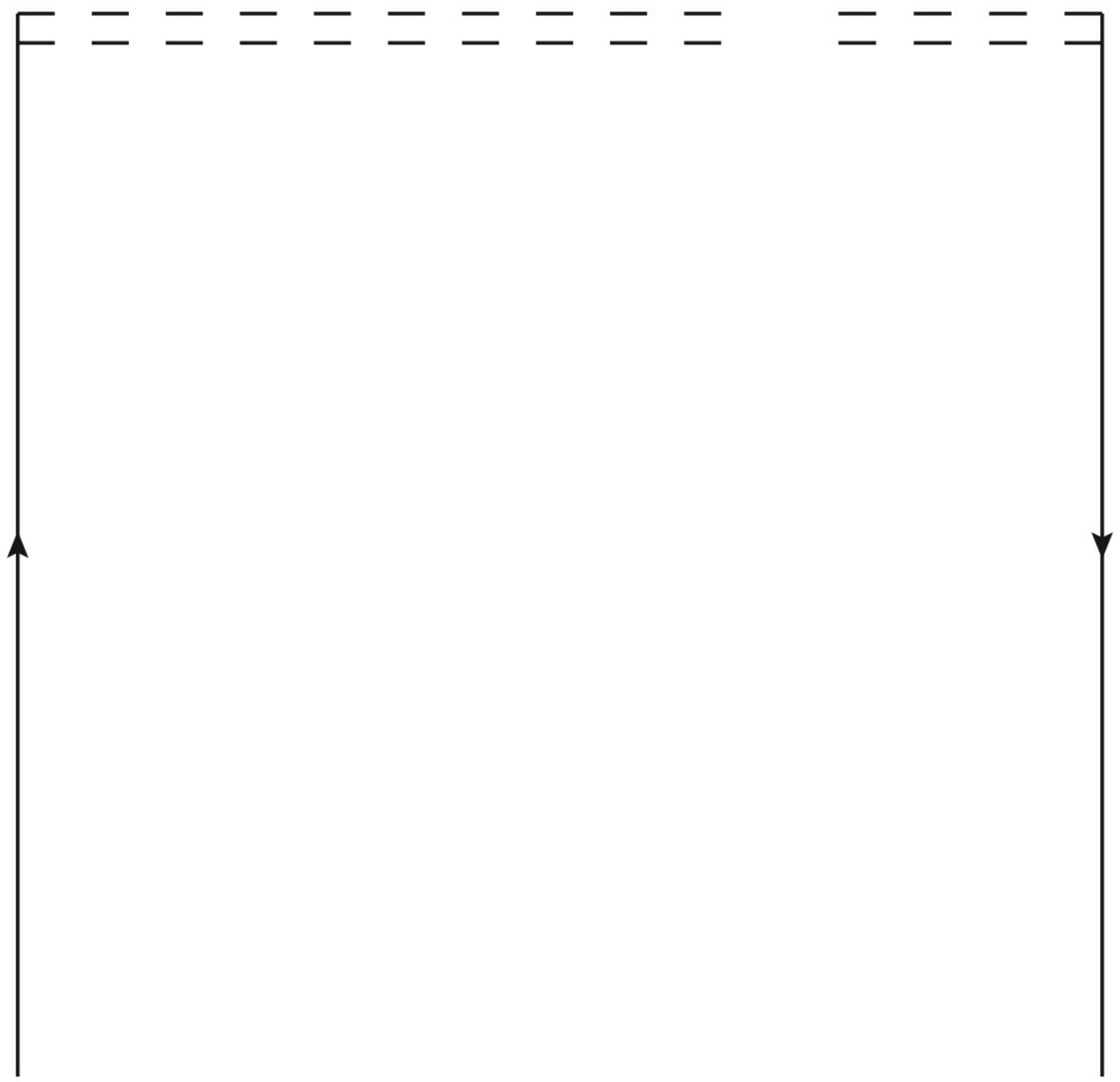
Guess: replacements

$$1 \to s_L\, g_1(y)$$

$$i(b\cdot p_T) \to iM(b\cdot S)\, y\, g_T^q(y)$$



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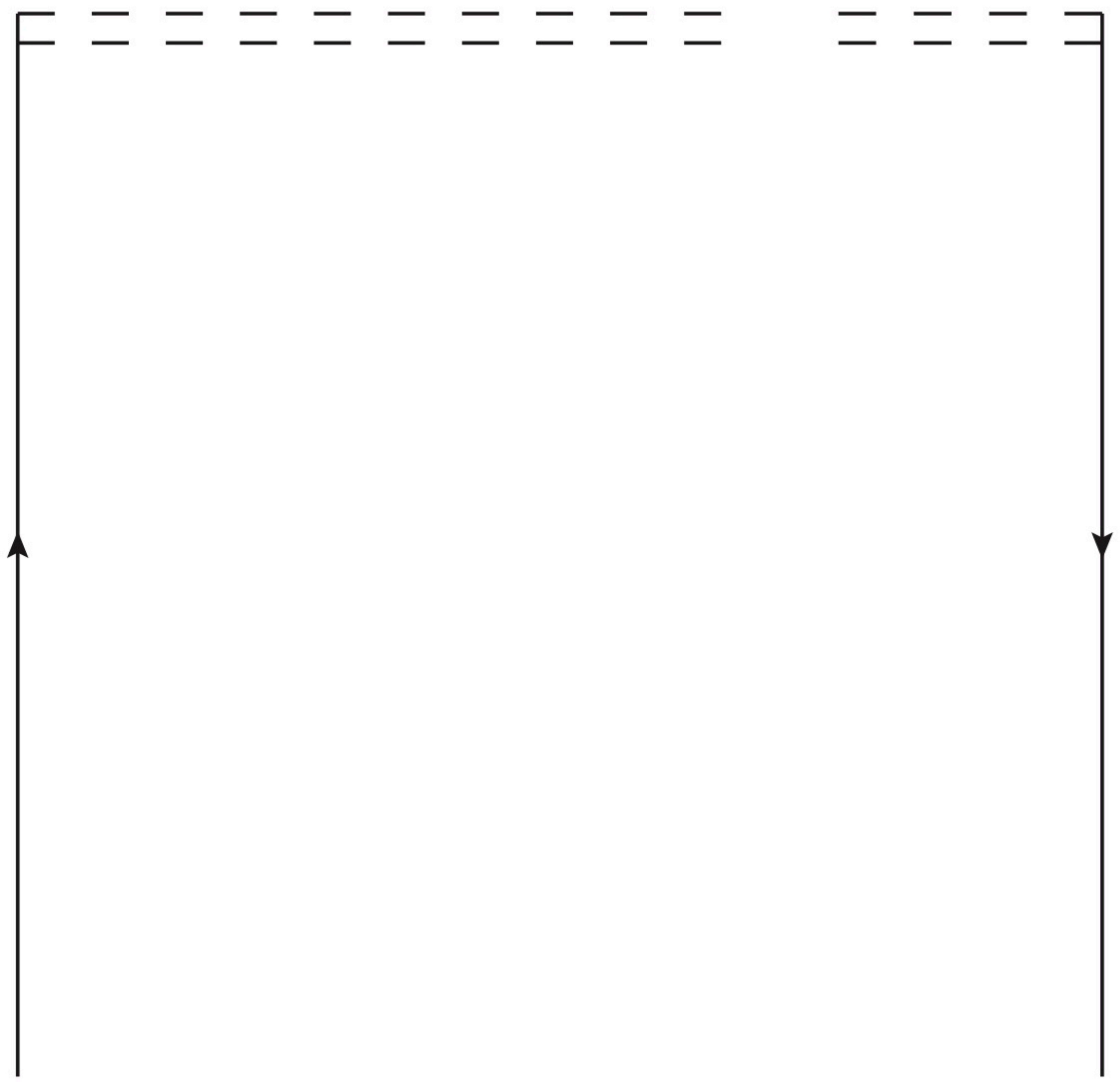

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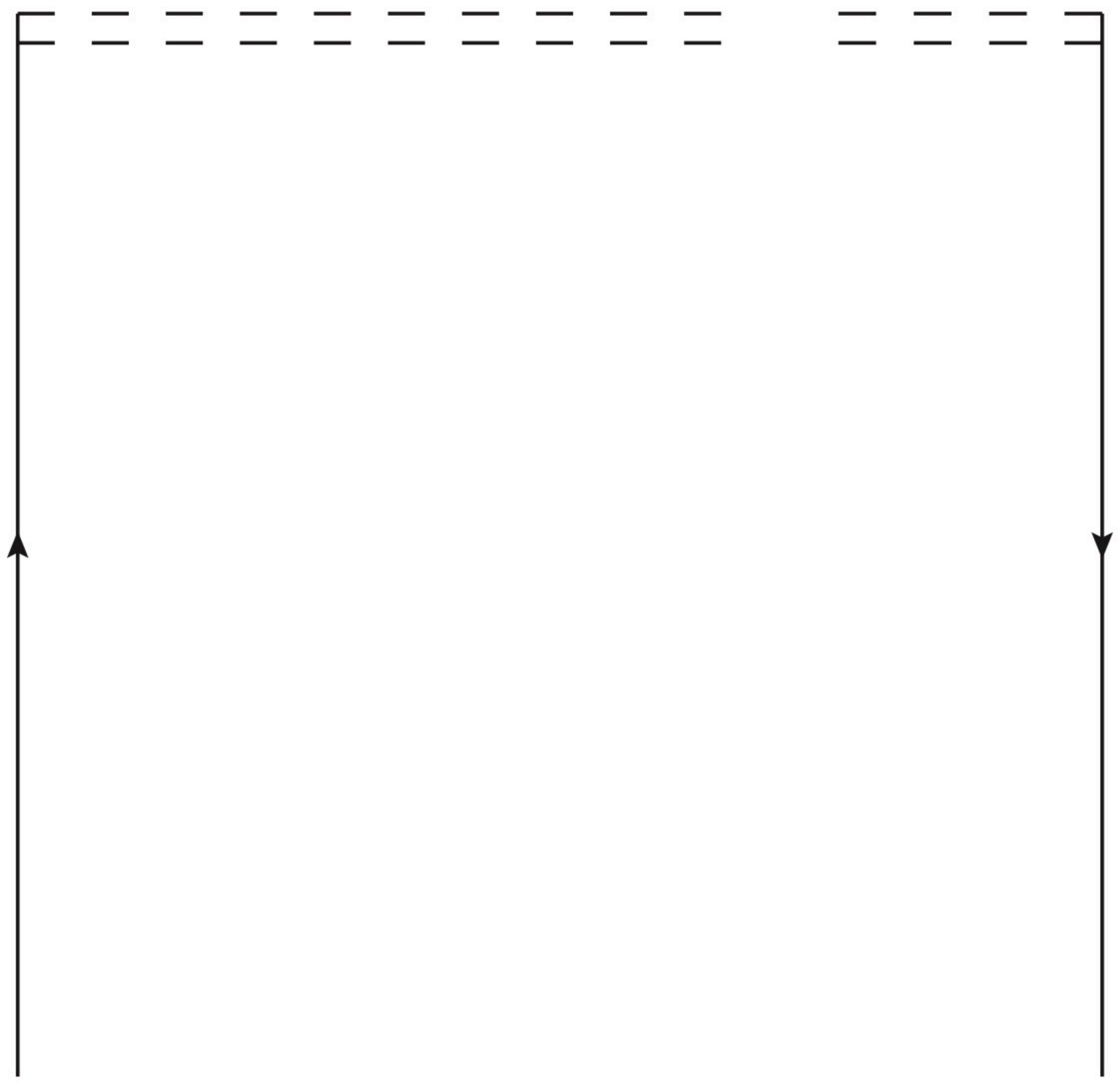
$$\int_0^1 du \int dy\, \delta(x - uy)$$

Comparison with the correlator



# Formalization

Guess: $i\,p_T^\mu \rightarrow$ tree-level matching

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$$p_T^\mu \to -\,i\,\partial_T^\mu(0) \Longrightarrow e^{ib\cdot p_T} \to e^{b\cdot\partial_T(0)}$$

$$\Phi_i(x,b) = \int_0^1 du \int dy\, \delta(x-uy)\, \sum_j C_{ij}(u)\, \left(e^{b\cdot\partial_T(0)}\, \Phi_j(y,b)\right)$$

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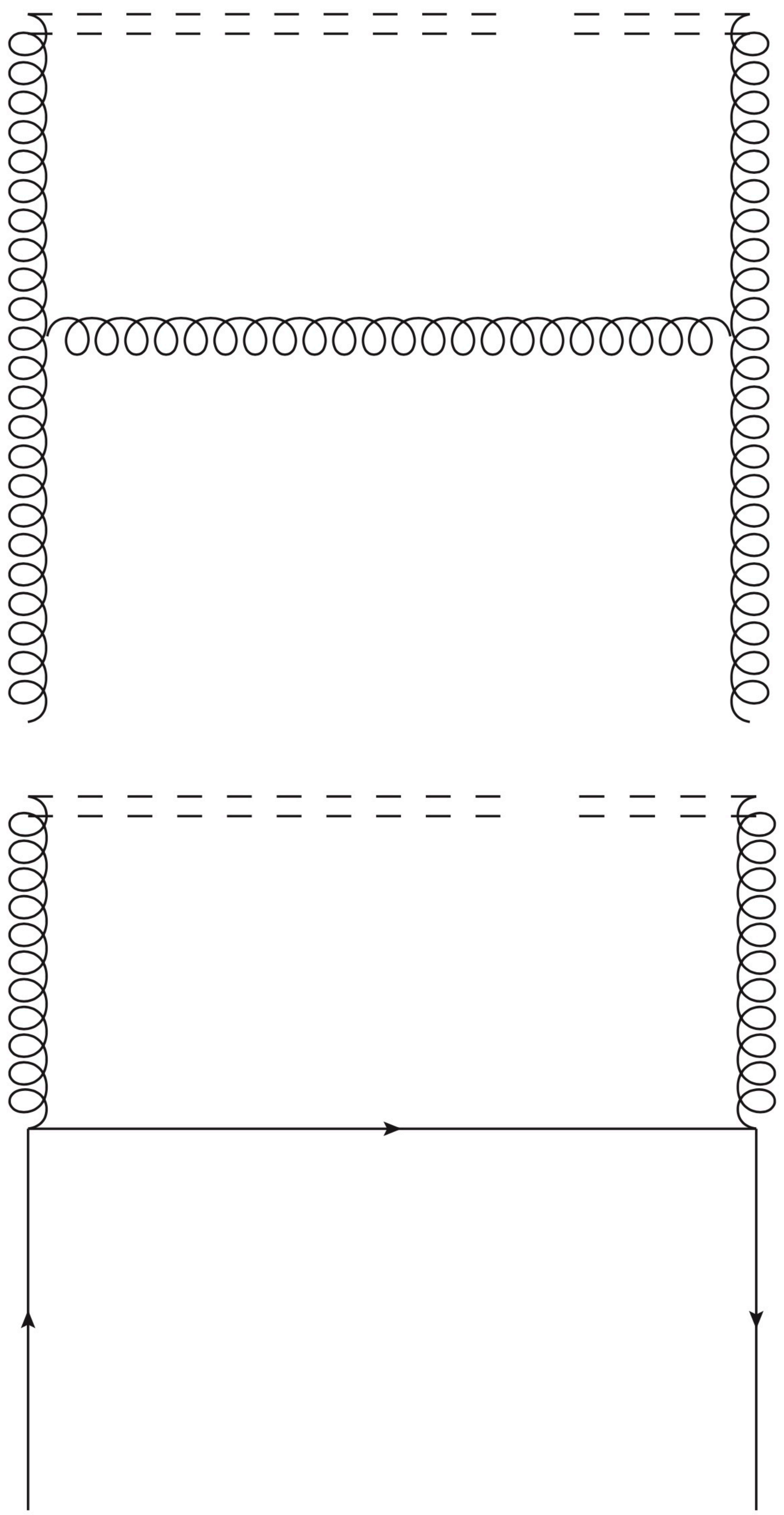
NLO: $e^{i(bp_T)} \to e^{iu(bp_T)}$

# Gluon results

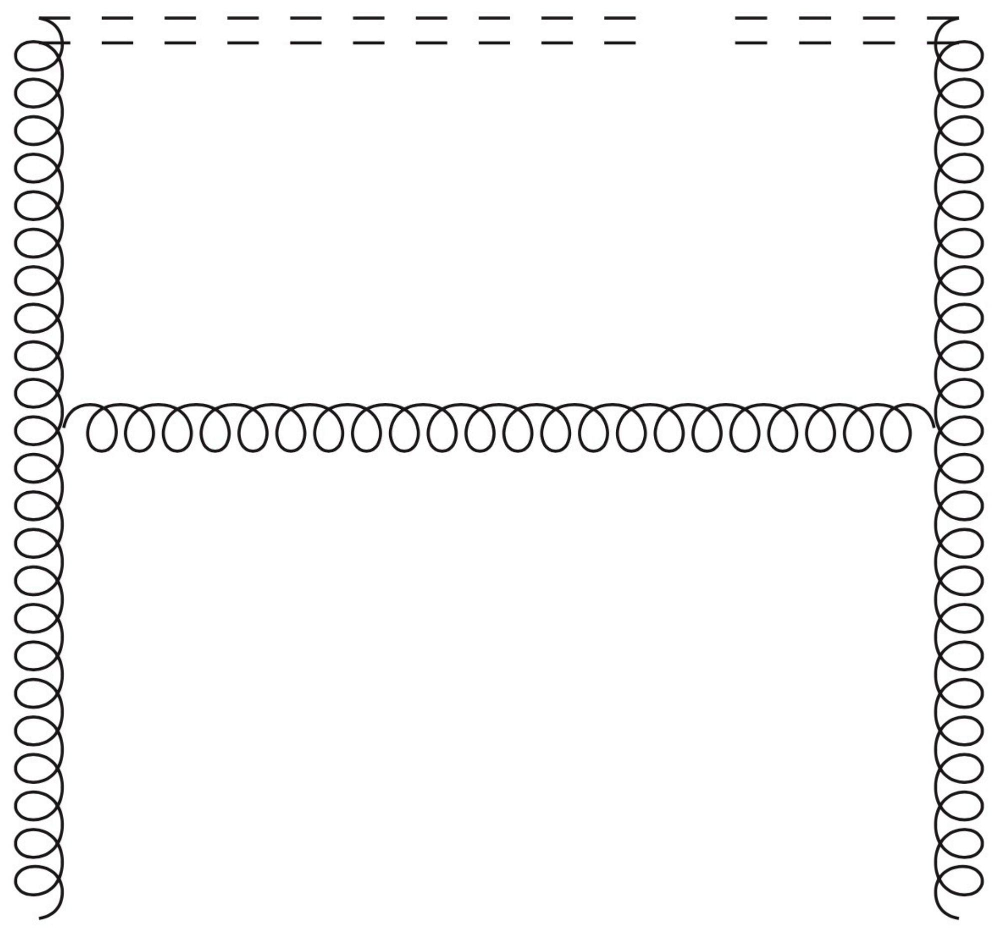
Light-cone gauge

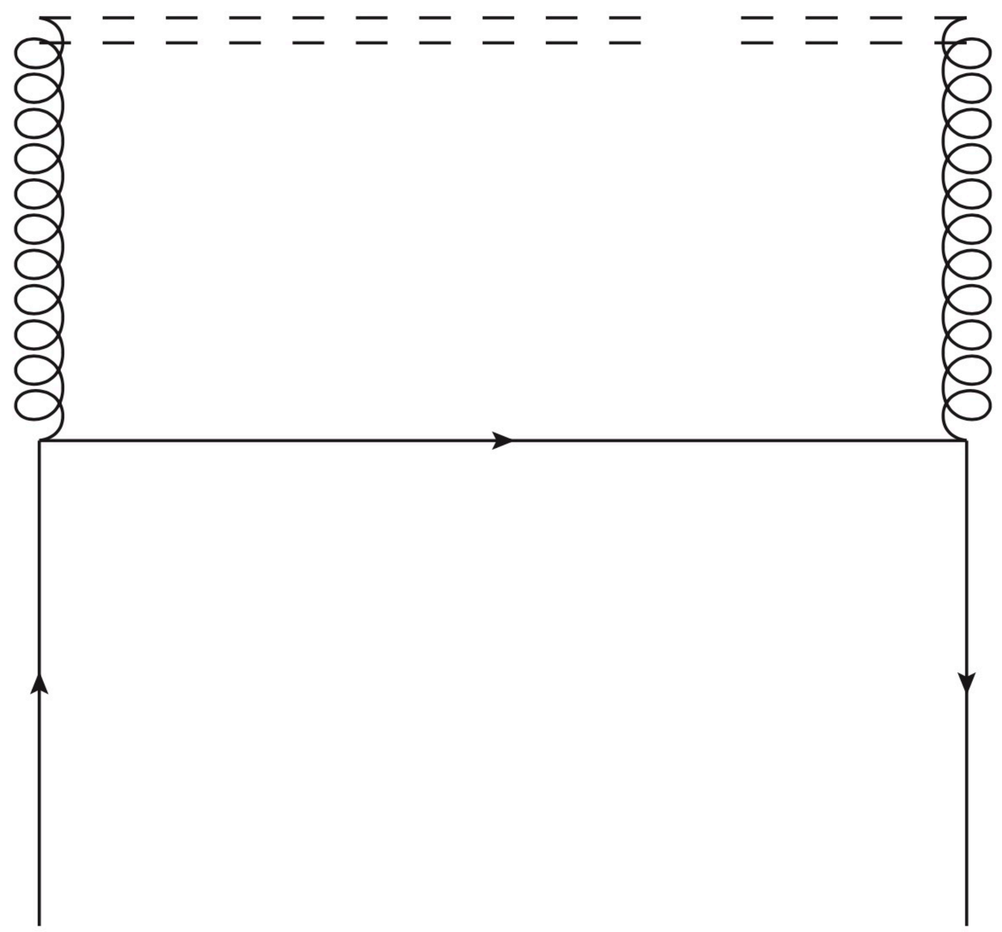


# Gluon results

Light-cone gauge

Recover known results





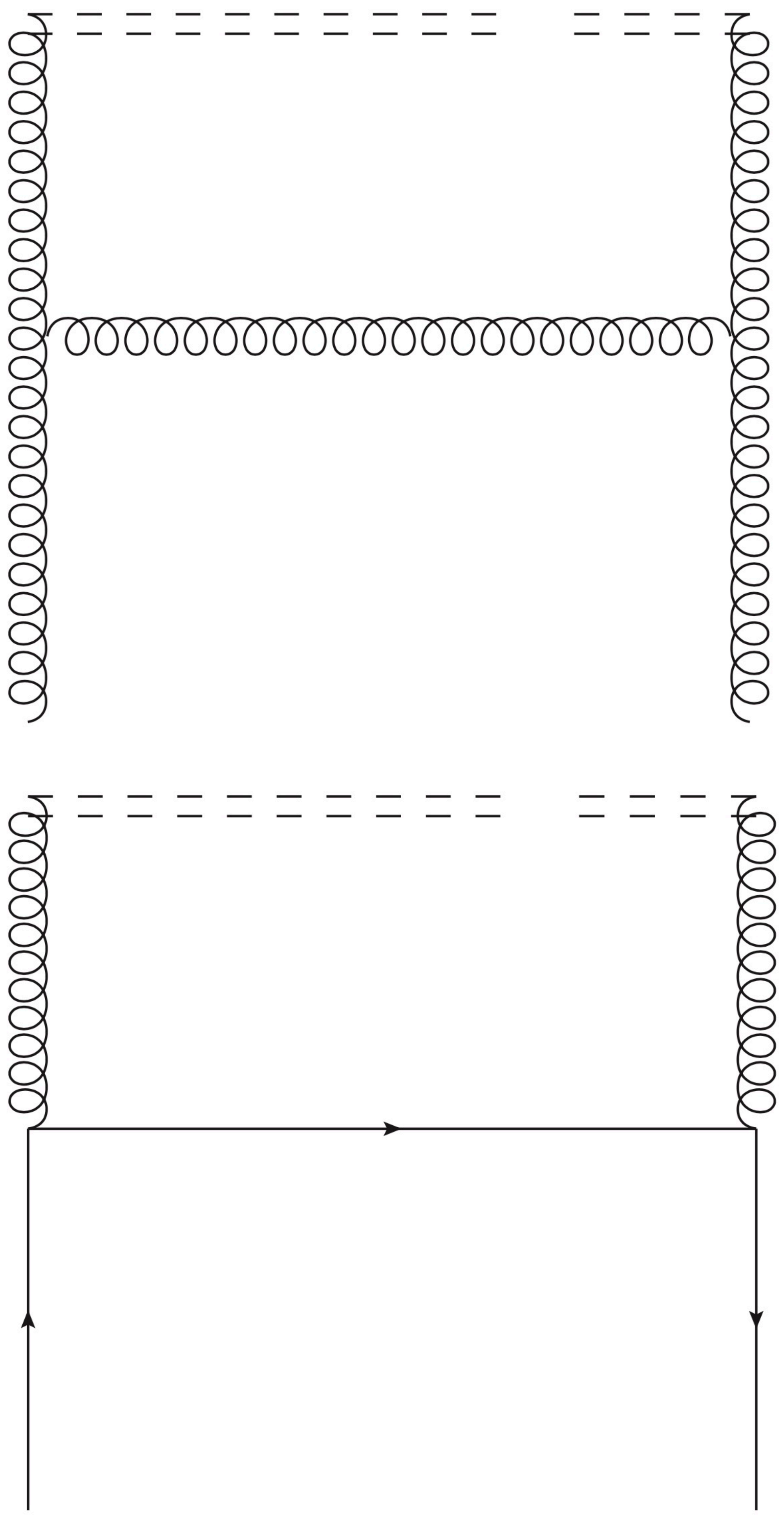
# Gluon results

Light-cone gauge

Recover known results

Wandzura-Wilczek for the $g_{1T}^{\perp,g}$:

$$g_{1T}^{g}(x,b) = \int_0^1 du \int dy\, \delta(x-uy)$$
$$\times \Big\{ 4\, C_{gg}(\mu^2, b^2, \epsilon)\, u \left( \Delta p_{gg} + 2\epsilon(1-u) \right) \left( -y g_T^g(y) \right)$$
$$+2\, C_{gq}(\mu^2, b^2, \epsilon)\, u \left( \Delta p_{gq} + 2\epsilon(1-u) \right) \left( y g_T^q(y) \right) \Big\}$$



# Mass Corrections

# ‘Universality’ of mass corrections

Exact correspondence of the mass series between gluon and quark TMDs

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$f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}$: operators have similar mathematical structure

Quark $h_{1T}, h_{1L}^{\perp}$: twist 2 PDF; gluon $h_{1T}, h_{1L}^{\perp}$: twist 3 PDF
$\Longrightarrow$ operators with different mathematical structures

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$\Longrightarrow$ operators with different mathematical structures

Underlying symmetry?

# Resummation of the series: unpolarized

$$f_1^g(x,b) = f_g(x) + \sum_{k=1}^{\infty} \frac{1}{k!(k-1)!} \left( \frac{x^2 M^2 b^2}{4} \right)^k \int_0^1 du \int dy\, \delta(x - uy) \left( \frac{\bar{u}}{u} \right)^{k-1} f_g(y)$$

$$b^2 < 0 \rightarrow |b| = \sqrt{-b^2}, \bar{u} = 1 - u$$

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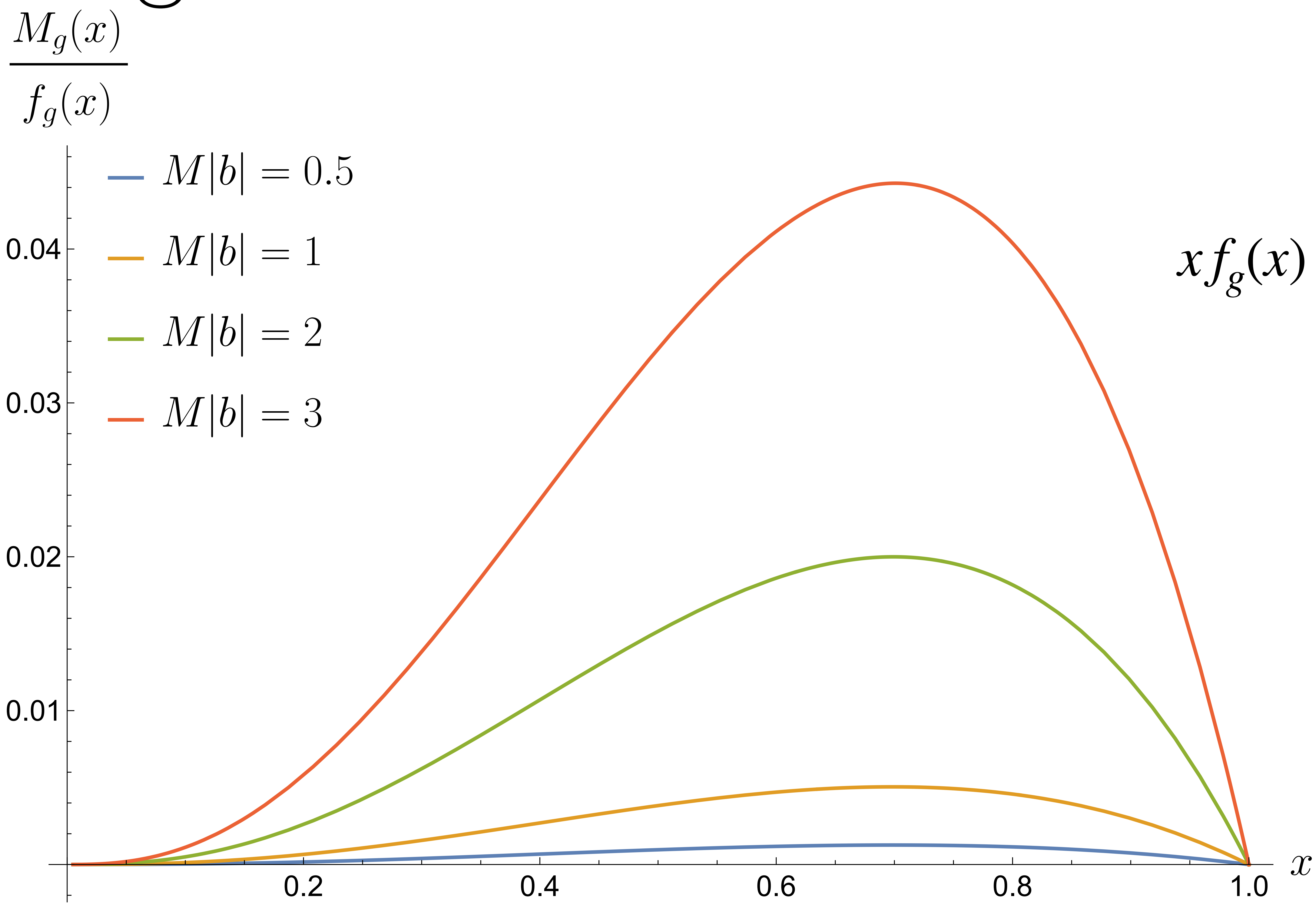
$$b^2 < 0 \rightarrow |b| = \sqrt{-b^2}\,, \bar{u} = 1 - u$$

$$f_1^g(x,b) = f_g(x) - \frac{xM|b|}{2} \int_0^1 du \int dy\, \delta(x-uy) \sqrt{\frac{u}{\bar{u}}} J_1 \left( xM|b| \sqrt{\frac{\bar{u}}{u}} \right) f_g(y)$$

# Magnitude of mass corrections: unpolarized

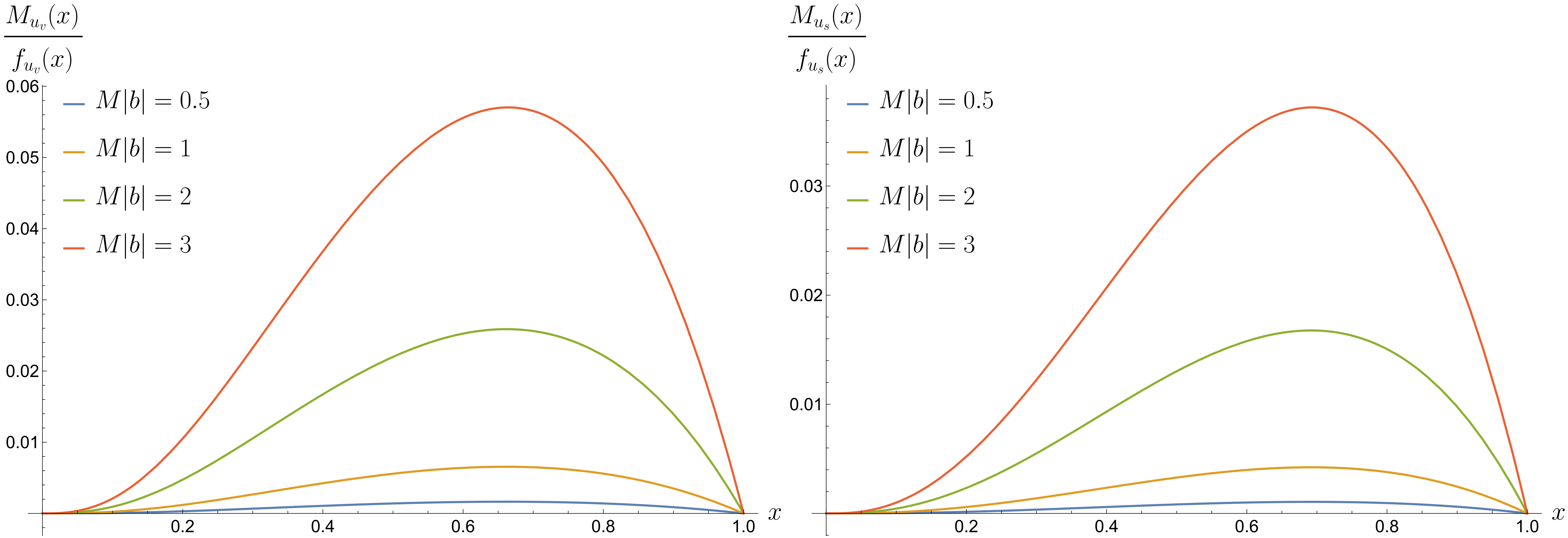



$$x f_i(x) = A_i\, x^{\alpha_i} (1-x)^{\beta_i} P_i(x)$$

$$x f_g(x) = A_g\, x^{\alpha_g} (1-x)^{\beta_g} - A_g'\, x^{\alpha_g'} (1-x)^{\beta_g'}$$

Central values of HeraPDF2.0
*Eur.Phys.J.C* 75 (2015) 12, 580

# Magnitude of mass corrections: unpolarized

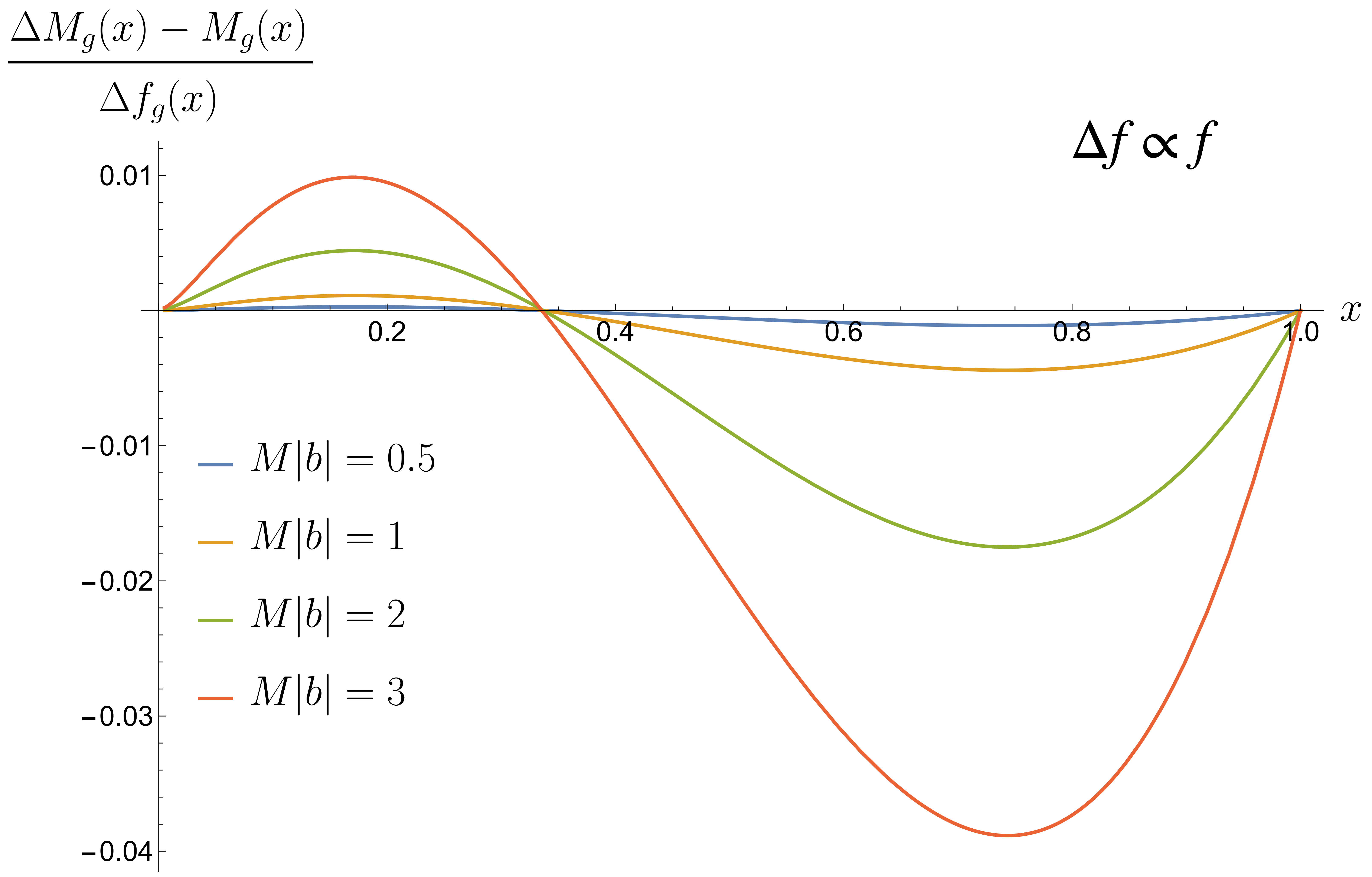


# Resummation of the series: helicity

$$\sum_{k=1}^{\infty} \frac{1}{k!(k-1)!} \left( \frac{x^2 M^2 b^2}{4} \right)^k \left( \frac{\bar{u}}{u} \right)^{k-1} \left(1 - 2\bar{u}\, {}_2F_1(1,1,k+1;\bar{u})\right) \longrightarrow$$

$$\longrightarrow -\frac{xM|b|}{2} \sqrt{\frac{u}{\bar{u}}} J_1\left( xM|b| \sqrt{\frac{\bar{u}}{u}} \right) + \frac{x^2 M^2 |b|^2}{2} \int_0^1 dt \frac{\bar{u}}{u + \bar{u}t} J_0\left( xM|b| \sqrt{\frac{\bar{u}}{u}} \sqrt{t} \right)$$

# Magnitude of mass corrections: helicity



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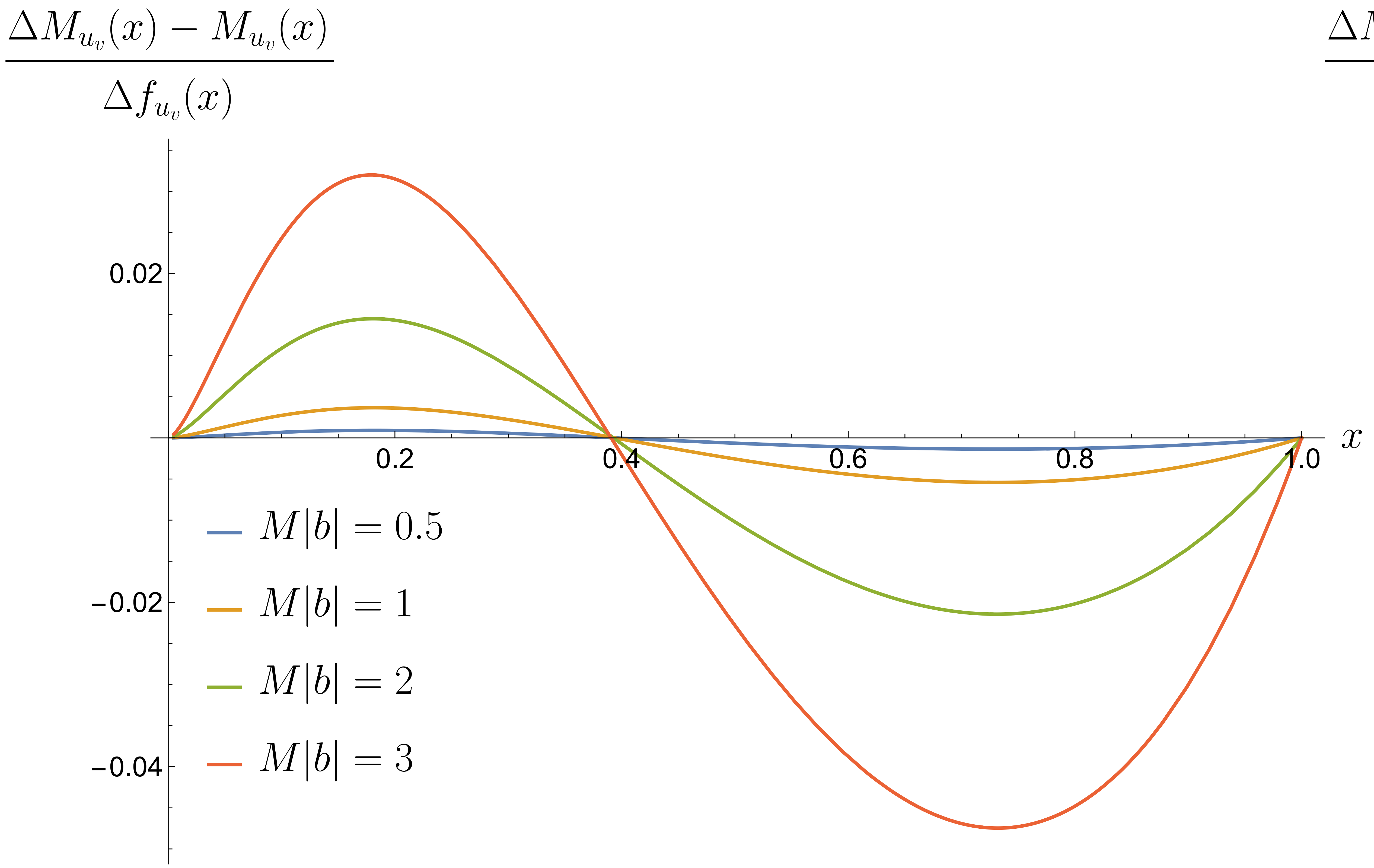


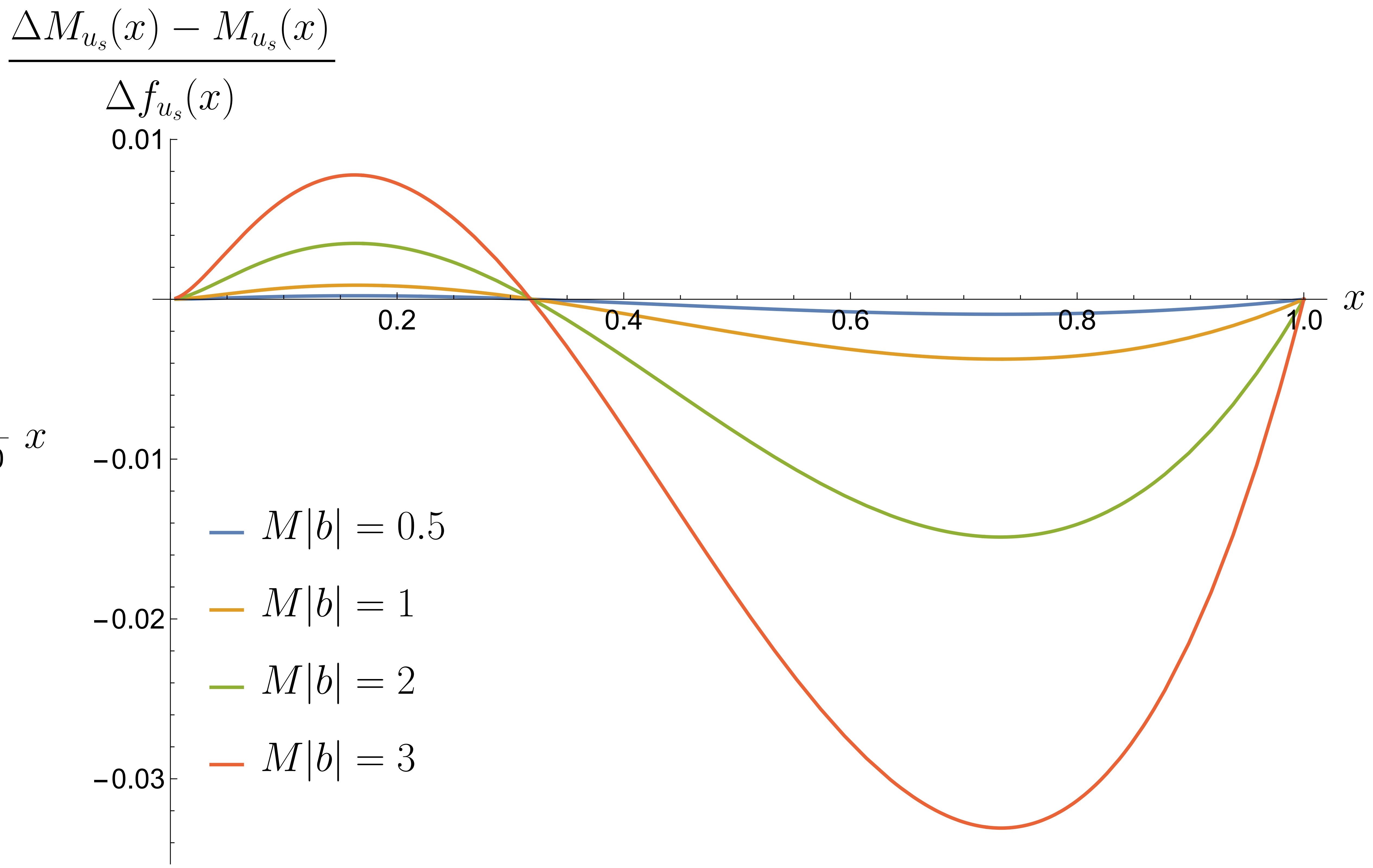

# Mass corrections at one-loop

$$\Phi_i(x,b) \overset{NLO}{=} a_s \int_0^1 du \int dy\, \delta(x-uy) \sum_j C_{ij}(u) \left( e^{ub\cdot\partial_T(0)}\, \Phi_j(y,b) \right)$$

$$f_1^g(x,b) \overset{NLO}{=}$$
$$\int_0^1 du \int_0^1 dv \int dy \sum_{k=1}^{\infty} \frac{\delta(x-uvy)}{k!(k-1)!} \left( \frac{x^2M^2b^2}{4} \right)^k \left( \frac{\bar{v}}{v} \right)^{k-1} \left( C_{gg}^{U/U}(u,b) f_g(y) + C_{gq}^{U/U}(u,b) f_1(y) \right)$$

# Mass corrections beyond one-loop

Under the hypothesis that $e^{ub\cdot\partial_T(0)} \to e^{b\cdot\partial_T(0)\prod_{l=1}^n u_l}$

$$\Phi_i^{(n)}(x,b) = a_s^n \left(\prod_{l=1}^n \int_0^1 du_l\right) \int dy\, \delta\left(x - y\prod_{l=1}^n u_l\right) \sum_j C_{ij}^{(n)}(\boldsymbol{u}) \left(e^{b\cdot\partial_T(0)\prod_{l=1}^n u_l} \Phi_j(y,b)\right)$$

$$F_i^{(n)}(x,b) = \sum_{k=1}^{\infty} \frac{a_s^n}{k!k!} \left(\frac{x^2M^2b^2}{4}\right)^k \left(\prod_{l=1}^n \int_0^1 du_l\right) \int_0^1 dv \int dy\, \delta\left(x - vy\prod_{l=1}^n u_l\right) \sum_j G_j(k,v) C_{ij}^{(n)}(\boldsymbol{u}) f_j(y)$$

# Conclusions

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| $g_{1L}^g$ | $\Delta f_g\,,g_1$ | - | $\mathrm{N^3LO}$ |
| $g_{1T}^g$ | $\Delta f_g\,,g_1$ | $\mathcal{G}_g\,,\mathcal{G}_q$ | NLO/LO |
| $f_{1T}^{\perp g}$ | - | $2F_2^+ + F_4^+$ | LO |
| $h_{1T}^g$ | - | $2F_2^+ - 2F_4^+$ | LO |
| $h_{1L}^{\perp g}$ | - | $2F_2^+ - 2F_4^+$ | LO |
| $h_{1T}^{\perp g}$ | - | - | LO |

# Outlook

Inclusion of higher-twist PDFs

Inclusion of T-odd effects

Extension to higher-twist TMDs

Extension to GTMDs