A toy model for understanding the space point resolution of silicon pixel detectors with digital readout

The work presented here is carried out with A. Kalweit and will be published soon

Marianna Mazzilli - 08/04/2026

Physics motivations

Silicon pixel detectors are widely used for tracking close to the primary interaction vertex:

- Provide excellent spatial resolution and fast readout capabilities

- **Space point resolution (σ_p) is a key performance parameter**:
  → Directly affects momentum and distance-of-closest approach (d_0) resolution
  → d_0 is crucial for identifying charm and beauty decays
  → At high momentum: $d_0 \approx \sigma_p / \sqrt{(N + 4)}$, with N = number of layers
  → Optimizing σ_p is therefore essential in detector design

- Many modern detectors use digital readout (no analogue information)
  → Limits the applicability of centre-of-gravity methods
  → Example: ALPIDE chip in ALICE ITS2

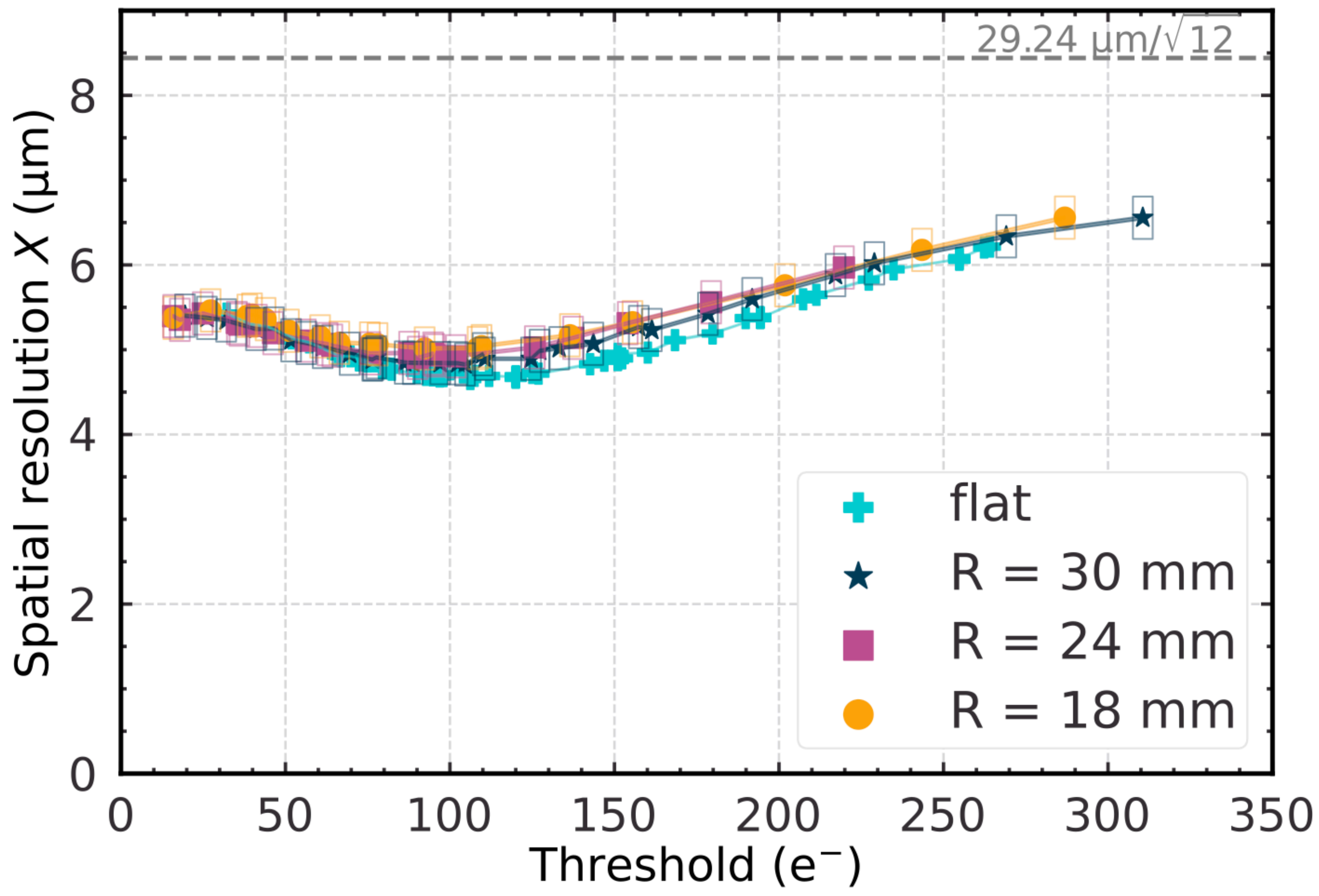
Physics motivations

- Charge sharing can improve resolution beyond the single-pixel limit
- For analogue readout: resolution improves with cluster size (more information)
- For digital readout: equal pixel weights make the improvement less intuitive

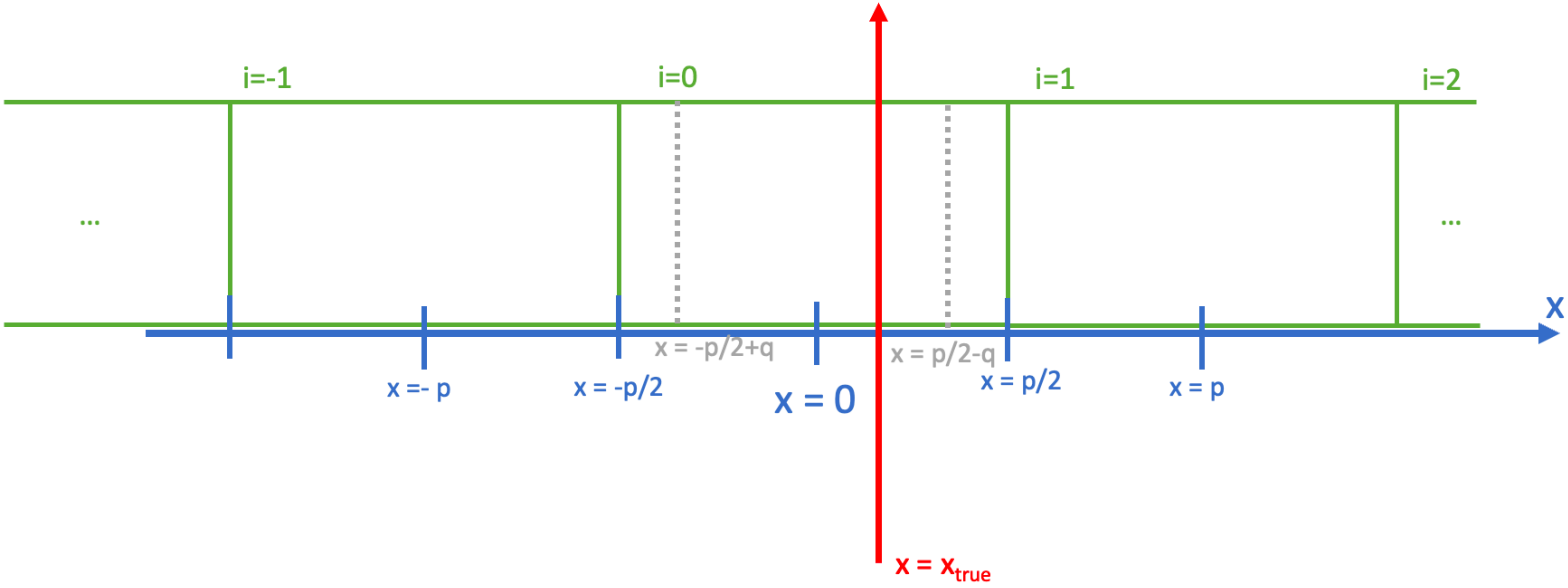
Quantitative understanding of charge sharing effects remains limited

This work introduces a simplified model to estimate the maximum achievable resolution gain (in 1D and 2D)

ALPIDE paper



One-pixel cluster



- In the simplified 1-dimensional case, the detector is segmented into pixels along the x-direction, with a given pitch *p*
- A particle crosses the detector in the position x_{true} (we assume the particle can impinge the detector only perpendicularly)

One-pixel cluster

- For symmetry reasons, all cases can be described by considering only impinging particles on the central pad (i=0)
- The probability distribution f(x_true) of the impinging particles can be described as a box distribution according to:

$$f(x_{true}) = \begin{cases} 1/p & -p/2 < x_{true} < p/2 \\ 0 & elsewhere \end{cases} .$$

- The reconstructed position x_{rec} is given by the average of the pad center *ip* weighted by the signal $S(i, x_{true})$ on pad-i for a given x_{true}:

$$x_{rec} = \frac{\sum_i S(i, x_{true}) \cdot ip}{\sum_i S(i, x_{true})} .$$

One-pixel cluster

- For one-pixel cluster: $$S(i, x_{true}) = \begin{cases} 1 & -p/2 < x_{true} < p/2, i = 0 \\ 0 & elsewhere \end{cases} .$$

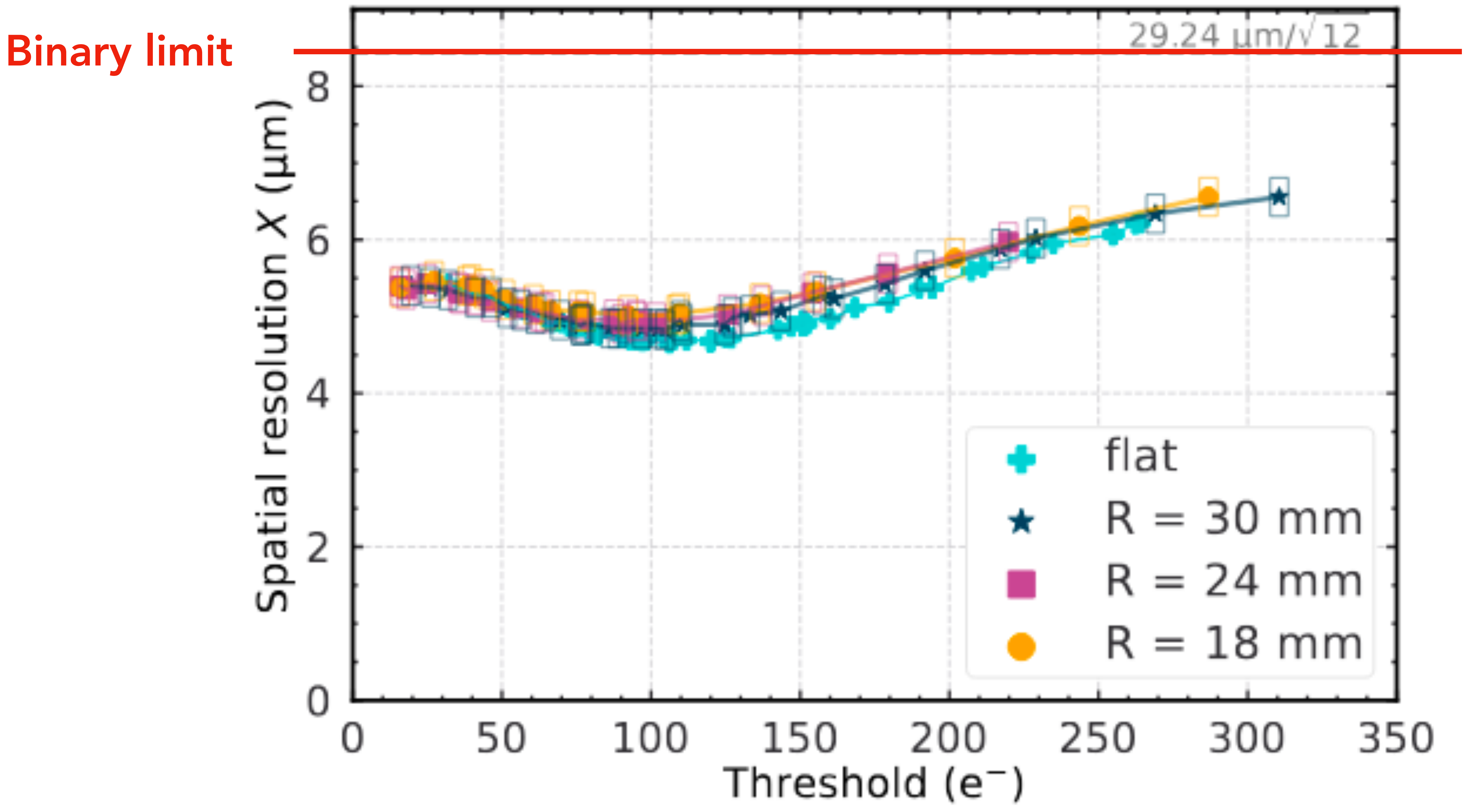
- Therefore, the reconstructed center of the hit is simply given by the center of the central pixel, i.e.

$$\begin{aligned} \sigma_p^2 &= \int_{-p/2}^{p/2} (x_{rec} - x_{true})^2 f(x_{true}) \, \mathrm{d}x_{true} = \int_{-p/2}^{p/2} x_{true}^2 \frac{1}{p} \, \mathrm{d}x_{true} \\ &= \frac{1}{p} \left[\frac{1}{3} x_{true}^3 \right]_{-p/2}^{p/2} = \frac{1}{3p} \left(\frac{p^3}{8} + \frac{p^3}{8} \right) \\ &= \frac{p^2}{12} . \end{aligned}$$

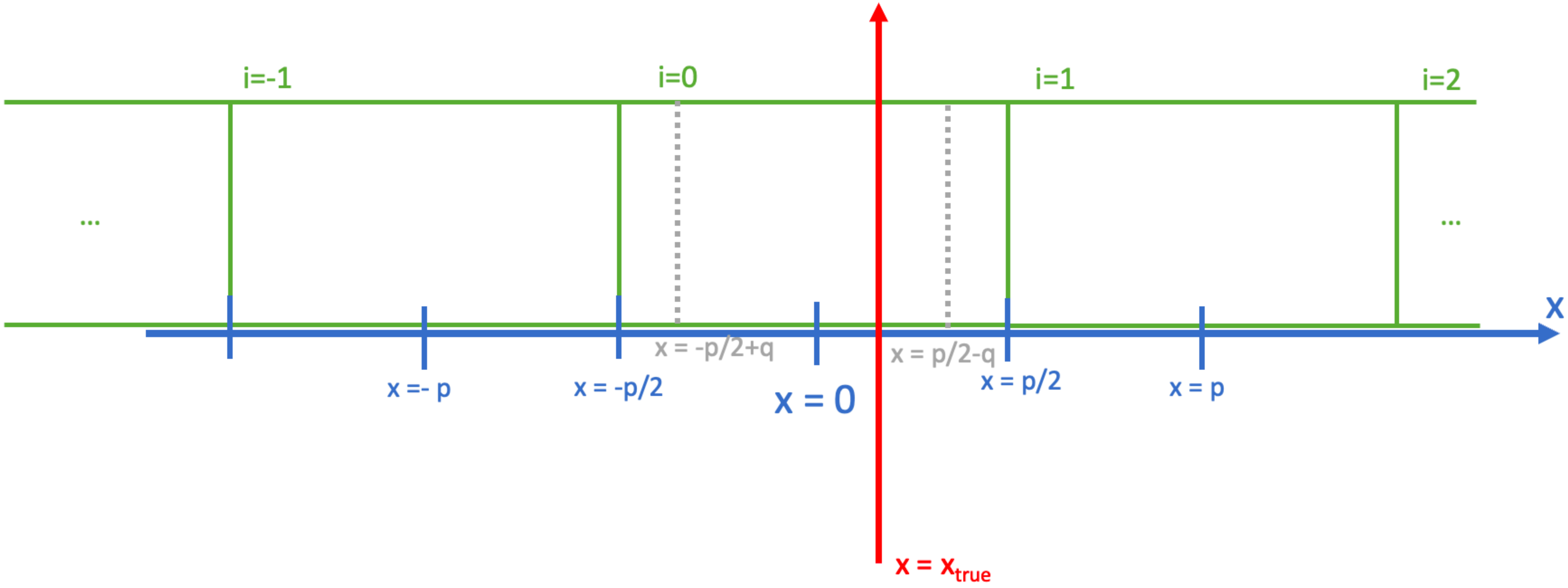
- Which is the well-known $\sigma_p = p/\sqrt{12}$ relation for the one-pixel clusters

ALPIDE resolution

- In practice, a better resolution is observed due to the charge sharing
- We will quantify this in the following



One and two pixel clusters



- Analogously to the one-pixel case, the probability distribution is given by:

$$f(x_{true}) = \begin{cases} 1/p & -p/2 < x_{true} < p/2 \\ 0 & elsewhere \end{cases} .$$

One and two pixel clusters

- The reconstructed position changes depending on the fact if the neighbouring pixels fire.
  - For $-p/2 < x_{true} < -p/2 + q$, the pixels i=-1 and i=0 fire,
  - For $-p/2 + q < x_{true} < p/2 - q$ only the central pixel i=0 fires,
  - For $p/2 - q < x_{true} < p/2$ the pixels i=0 and i=1 fire.
- We therefore obtain from the weighted average:

$$x_{rec} = \begin{cases} -p/2 & -p/2 < x_{true} < -p/2 + q \\ 0 & -p/2 + q < x_{true} < p/2 - q \\ p/2 & p/2 - q < x_{true} < p/2 \end{cases} .$$

One and two pixel clusters

- For the corresponding resolution, we then obtain:

$$\begin{aligned}
\sigma_p^2 &= \int_{-p/2}^{p/2} (x_{true} - x_{rec})^2 f(x_{true}) \, \mathrm{d}x_{true} \\
&= \int_{-p/2}^{-p/2+q} (x_{true} + p/2)^2 \frac{1}{p} \, \mathrm{d}x_{true} + \\
&+ \int_{-p/2+q}^{p/2-q} x_{true}^2 \frac{1}{p} \, \mathrm{d}x_{true} + \int_{p/2-q}^{p/2} (x_{true} - p/2)^2 \frac{1}{p} \, \mathrm{d}x_{true}
\end{aligned}$$

with the subsitutions $y = x + p/2$ and $z = x - p/2$ this reduces to

$$\begin{aligned}
\sigma_p^2 &= \int_0^q y^2 \frac{1}{p} \, \mathrm{d}y + \int_{-p/2+q}^{p/2-q} x_{true}^2 \frac{1}{p} \, \mathrm{d}x_{true} + \int_{-q}^{0} z^2 \frac{1}{p} \, \mathrm{d}z \\
&= \frac{1}{p} \left[\frac{1}{3} y^3 \right]_0^q + \frac{1}{p} \left[\frac{1}{3} x_{true}^3 \right]_{-p/2+q}^{p/2-q} + \frac{1}{p} \left[\frac{1}{3} z^3 \right]_{-q}^{0} \\
&= \frac{p^2 - 6pq + 12q^2}{12} \, .
\end{aligned}$$

One and two pixel clusters

- With $d\sigma_p^2/dq = 0$, we find that the optimal (minimal) resolution is reached for q=p/4 and amounts to $\sigma_p = \frac{1}{2\sqrt{12}} p \approx 0.144p$, i.e. half of the one-pixel case

- This result is also intuitive: if one knows that only one pixel fires for $-p/4 < x_{true} < p/4$ and two fire for $p/4 < x_{true} < p/2$, the position of the particle is effectively constrained to half the pixel pitch.

- The one and two pixel case with q=p/4 in practice corresponds to an average cluster size with N=1.5, as in half of the cases one pixel fires and half of the other cases, two fixes fire.

- The non-intuitive part of the result is that this corresponds to the **best achievable resolution in two-pixel case**
- This limit also holds for larger cluster sizes in the one-dimensional case.

N-pixel clusters

For the investigation of larger cluster sizes, the previous case can be generalized under the assumption that the charge is shared symmetrically around x_{true}.

- In this scenario, for sufficiently large charge and within the range $-\frac{p}{2} + q < x_{\text{true}} < \frac{p}{2} - q$, the central pixel (i = 0) and its two neighboring pixels (i = -1) and (i = +1) are activated, resulting in a reconstructed position $x_{\text{rec}} = 0$.

- As x_{true} increases towards larger values, the charge collected by pixel (i = -1) decreases. Consequently, within the interval $\frac{p}{2} - q < x_{\text{true}} < \frac{p}{2}$, only pixels (i = 0) and (i = +1) fire, leading to a reconstructed position $x_{\text{rec}} = \frac{p}{2}$.

- Similarly, for the range $-\frac{p}{2} < x_{\text{true}} < -\frac{p}{2} + q$, the reconstructed position becomes $x_{\text{rec}} = -\frac{p}{2}$.

In summary, this leads to the same qualitative behavior observed in the previous case.

N-pixel clusters

In order to facilitate the comparison with the toy model in the following, we express this formally as a function of the average cluster size l with the help of the Heaviside Theta-function Θ. With:

$$f(x_{\text{True}}) = \frac{1}{p}\left(\Theta\left(x_{\text{True}} + \frac{p}{2}\right) - \Theta\left(x_{\text{True}} - \frac{p}{2}\right)\right)$$

And the center of each pad given by $x_{pc} = i \cdot p$, the signal is given as

$$S(i,\, x_{\text{True}},\, l) = \Theta\left(x_{\text{True}} - \left(x_{\text{pc}}(i) - \frac{l \cdot p}{2}\right)\right) - \Theta\left(x_{\text{True}} - \left(x_{\text{pc}}(i) + \frac{l \cdot p}{2}\right)\right)$$

One can then verify that the average cluster size is given by

$$l = \frac{1}{p}\int_{-p/2}^{p/2} \sum_{i=-\infty}^{+\infty} S(i,\, x_{\text{True}},\, l)\, dx_{\text{True}}\ .$$

N-pixel clusters

- Based on the reconstructed position

$$x_{\text{Rec}}(x_{\text{True}},\, l) = \frac{\sum\limits_{i=-\infty}^{+\infty} x_{\text{pc}}(i) \cdot S(i,\, x_{\text{True}},\, l)}{\sum\limits_{i_{\text{Pad}}=-\infty}^{+\infty} S(i,\, x_{\text{True}},\, l)}$$

N-pixel clusters

- We finally obtain for the resolution:

$$
\begin{aligned}
\sigma_p^2 &= \int_{-p/2}^{p/2} \left(x_{\text{Rec}}(x_{\text{True}},\, l) - x_{\text{True}}\right)^2 \cdot f_{\text{Beam}}(x_{\text{True}})\, dx_{\text{True}} \\
&= \begin{cases}
\dfrac{p^2}{12} & l = 1,\, 2,\, 3,\, 4,\, \dots \\
\dfrac{1}{12}\left(7 - 9\,l + 3\,l^2\right) p^2 & 1 < l < 2 \\
\dfrac{1}{12}\left(19 - 15\,l + 3\,l^2\right) p^2 & 2 < l < 3 \\
\dfrac{1}{12}\left(37 - 21\,l + 3\,l^2\right) p^2 & 3 < l < 4 \\
\dots
\end{cases} .
\end{aligned}
$$

Toy model results

- 1D model with linear array of pixels (pitch (p = 1), (n = 21), centered at (x = 0))
  - Particle deposits charge with Gaussian profile (width σ_{charge})
  - Pixel charge computed via analytical integration $Q_i = \frac{1}{2}\left[\text{erf}\left(\frac{x_i - x_{\text{true}} + p/2}{\sqrt{2}\sigma_{\text{charge}}}\right) - \text{erf}\left(\frac{x_i - x_{\text{true}} - p/2}{\sqrt{2}\sigma_{\text{charge}}}\right)\right],$
  - No electronic noise included
- Fixed per-pixel threshold (Q_{thr}) applied

  → Pixels above threshold ($Q_i > Q_{thr}$) marked as active

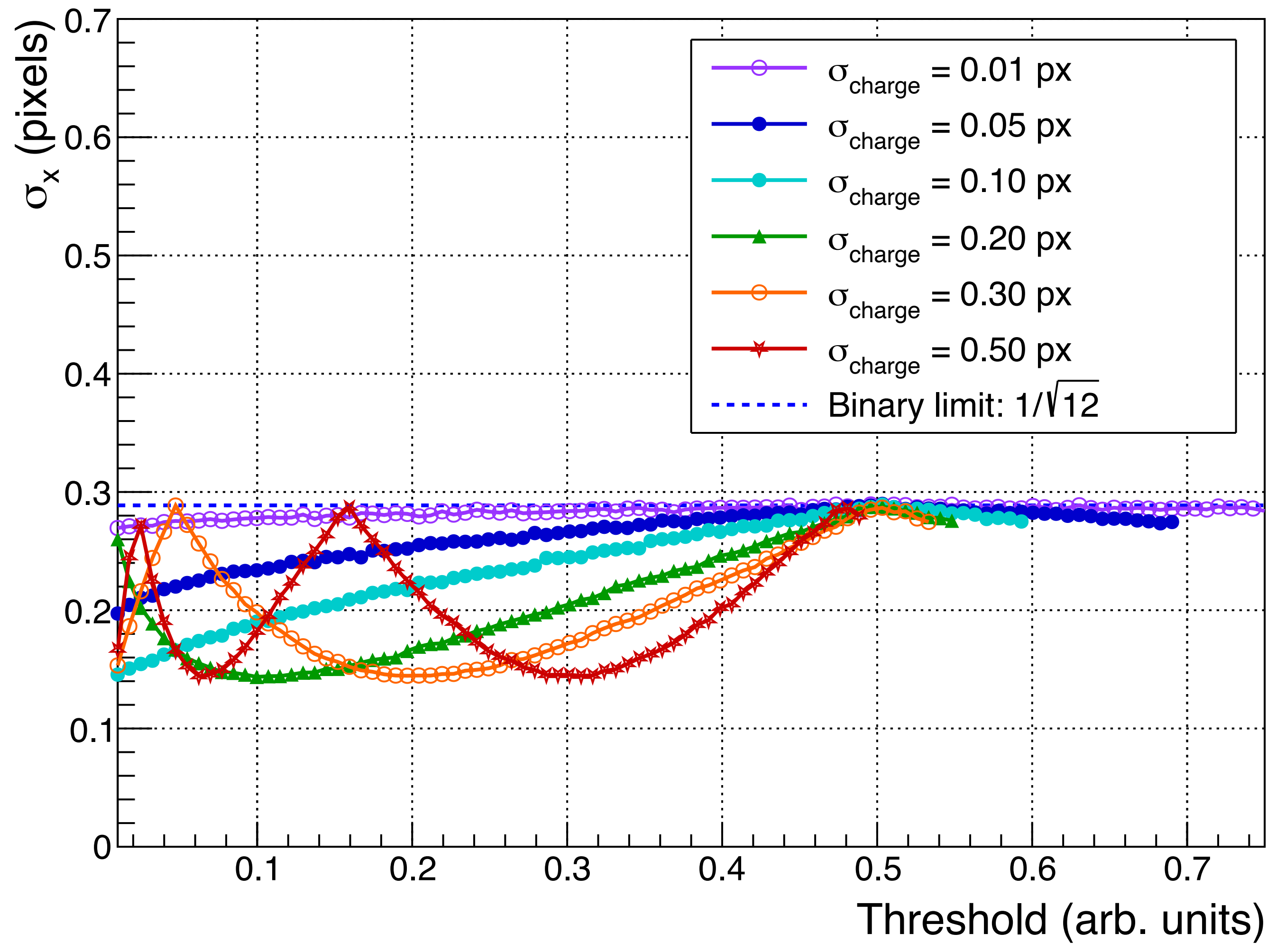
  → Clusters formed from contiguous active pixels around a seed (max charge)

  → One cluster reconstructed per event
- Position reconstructed using binary centroid

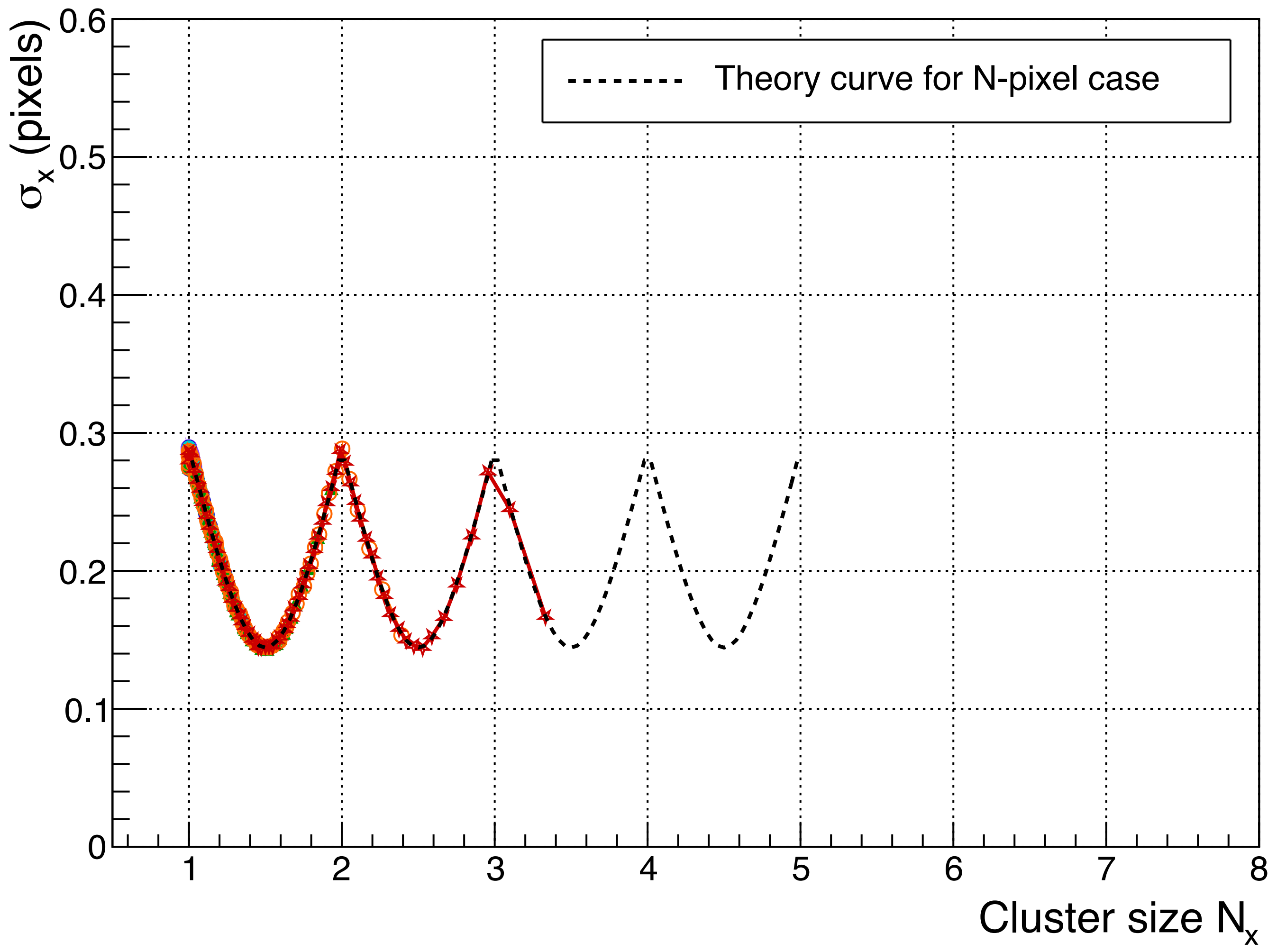
  → Average of pixel positions within the cluster
- Spatial resolution defined as RMS of residuals → ($\sigma_x = \sqrt{\langle (x_{\text{reco}} - x_{\text{true}})^2 \rangle}$)

Toy model results

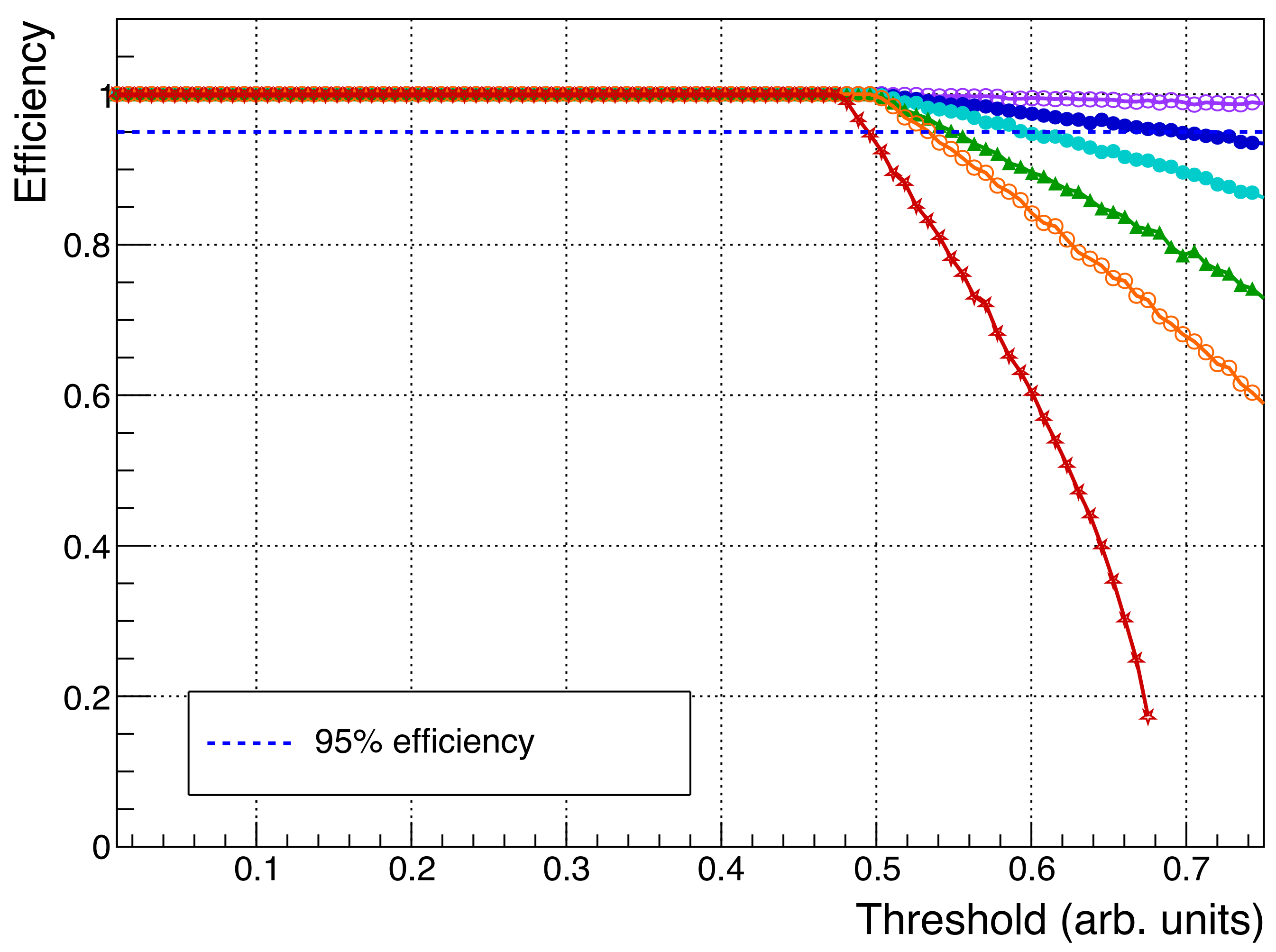
Resolution vs threshold



Resolution vs avg cluster size



Efficiency vs threshold



- Events generated uniformly within central pixel→ Reproduces binary limit: ($\sigma_x = \dfrac{1}{\sqrt{12}}$) for single-pixel clusters
- Performance studied by scanning threshold
  → Evaluate resolution, cluster size, and detection efficiency
- Key results:
  → Best resolution: ($\dfrac{p}{2\sqrt{12}}$)
  → Worst resolution (1-pixel limit): ($\dfrac{p}{\sqrt{12}}$)
  → **Excellent agreement with analytical expectations**

2D toy model

- 2D model: (21×21) matrix of square pixels (pitch (p = 1)), centered at (x, y)=(0,0)
- Particle deposits charge with a 2D Gaussian σ_{charge}
- Pixel charge computed via analytical integration over pixel area

  → Factorizes: $Q_{ij} = Q_x(i), Q_y(j)$
  → Exact description of charge sharing in both directions

- Per-pixel threshold Q_{thr} applied

  → Clusters built using 2D connected-component algorithm (4-neighbor)
  → All contiguous pixels above threshold grouped into clusters

- Position reconstructed via binary centroid in both coordinates: $x_{\text{reco}} = \frac{1}{N} \sum_{k \in \text{cluster}} x_k, \qquad y_{\text{reco}} = \frac{1}{N} \sum_{k \in \text{cluster}} y_k,$
- Spatial resolution defined as RMS of residuals in x and y: $\sigma_x = \sqrt{\left\langle (x_{\text{reco}} - x_{\text{true}})^2 \right\rangle}, \qquad \sigma_y = \sqrt{\left\langle (y_{\text{reco}} - y_{\text{true}})^2 \right\rangle}.$
- Events generated uniformly in central pixel
  → Larger clusters naturally arise due to 2D charge sharing
  → Enables realistic study of cluster topology, efficiency, and resolution

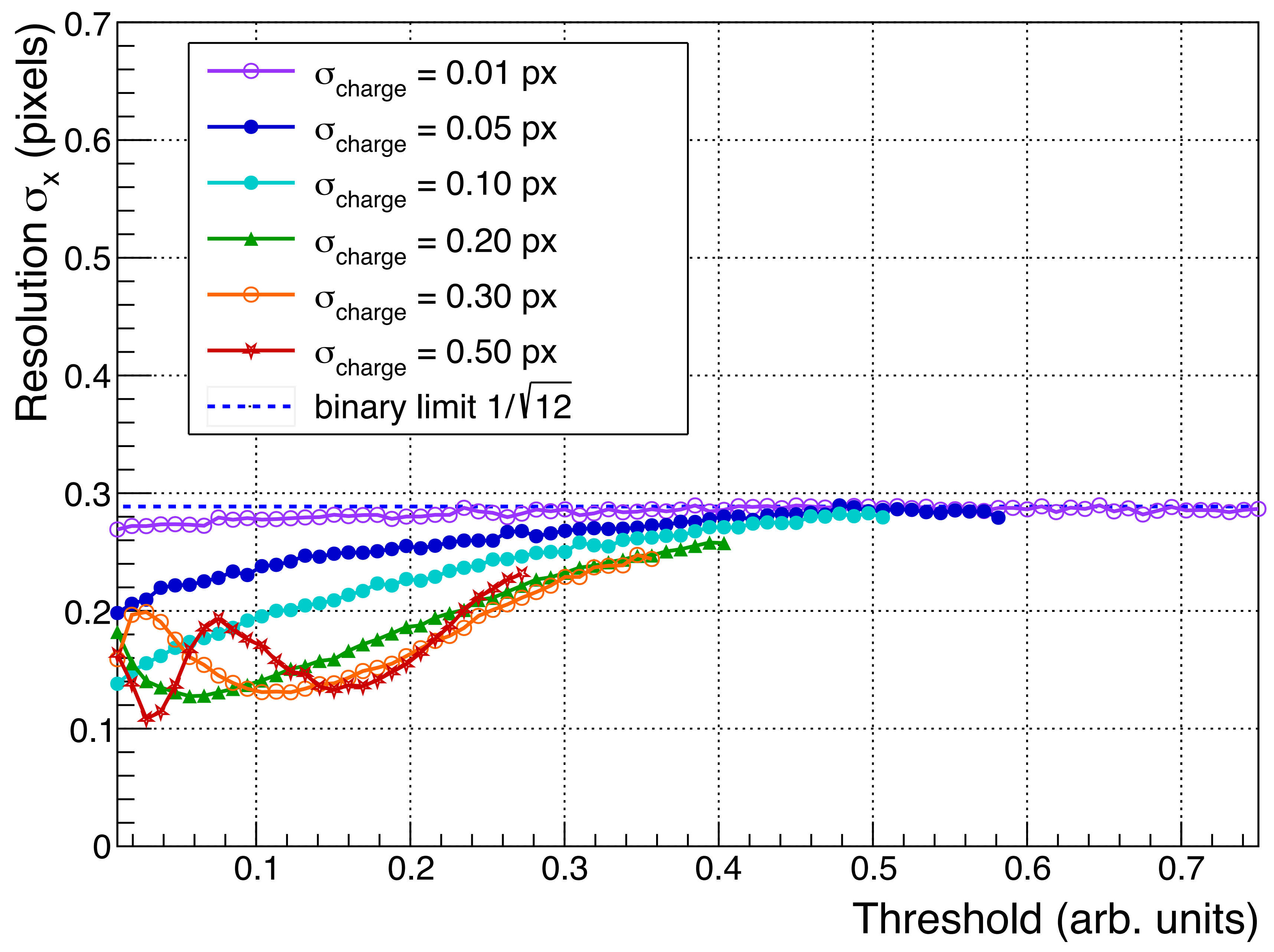
2D toy model

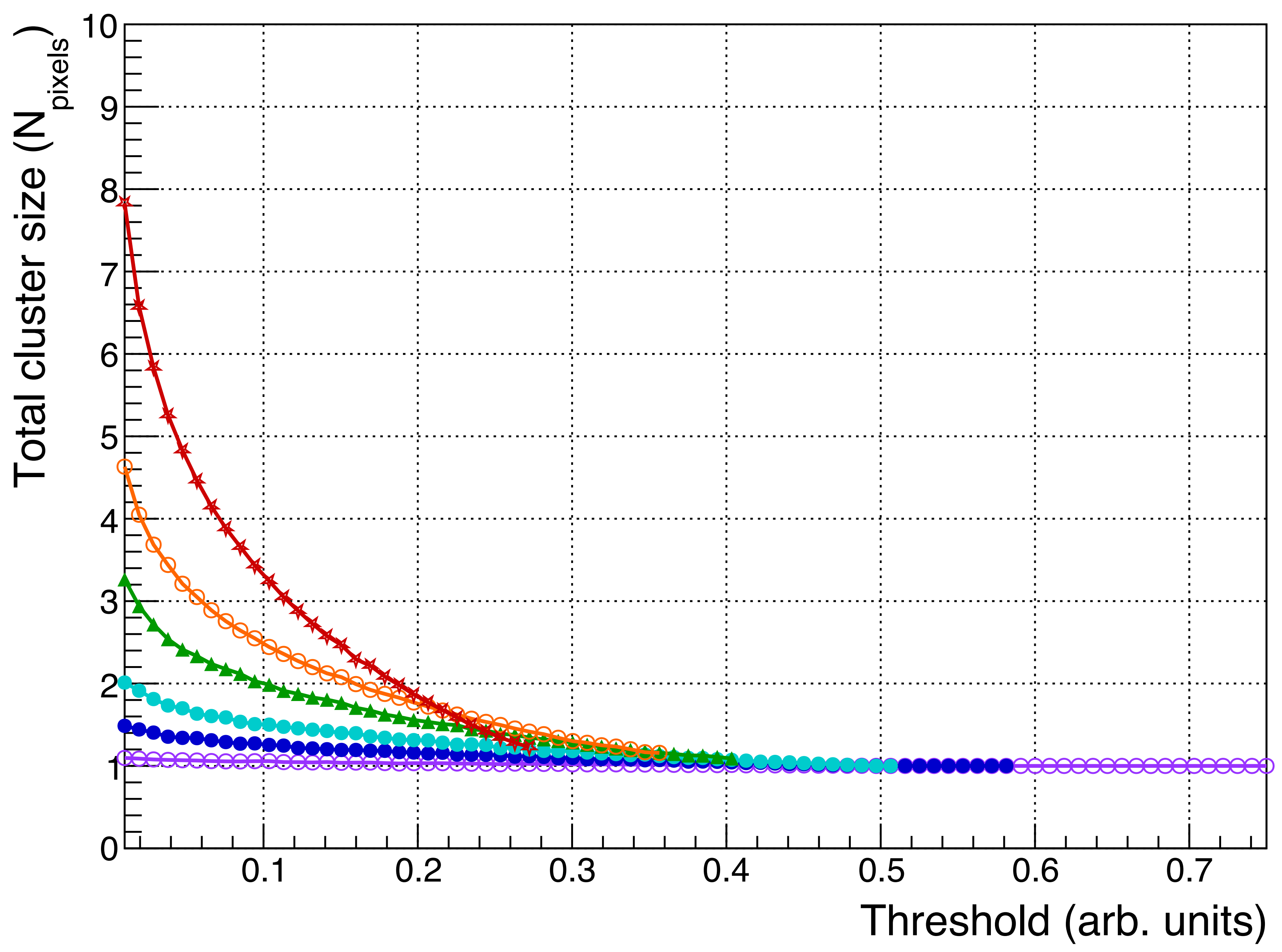
- Threshold scans performed
  - → Study resolution, cluster size, and detection efficiency
  - → Results shown for efficiency > 95%
  - → Negligible edge effects confirmed
- **Key observations:**
  - → **Average cluster size is a universal scaling variable for resolution**
  - → **Same resolution obtained for fixed cluster size, independent of charge width**
- Resolution parametrized as function of cluster size l
  - → Phenomenological 4th-order polynomial fit :

$$\sigma_p = p \cdot \left(\frac{1}{\sqrt{12}} + a_1(l-1) + a_2(l^2-1) + a_3(l^3-1) + a_4(l^4-1) \right) \ , \ 1 \leq l \leq 4.5$$

- 1D and 2D models provide complementary insight into
  - → Effects of thresholding and charge sharing

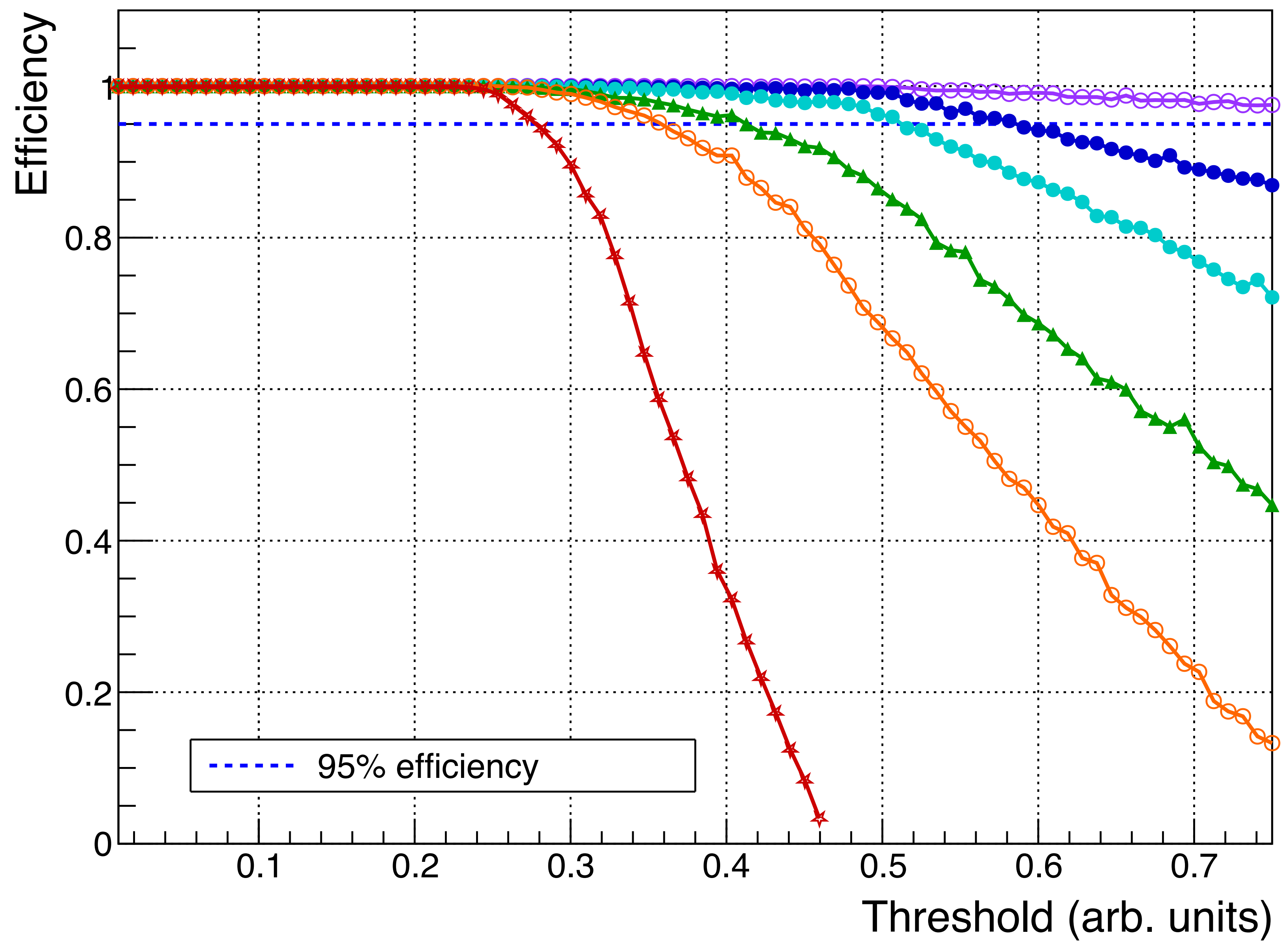
2D model results

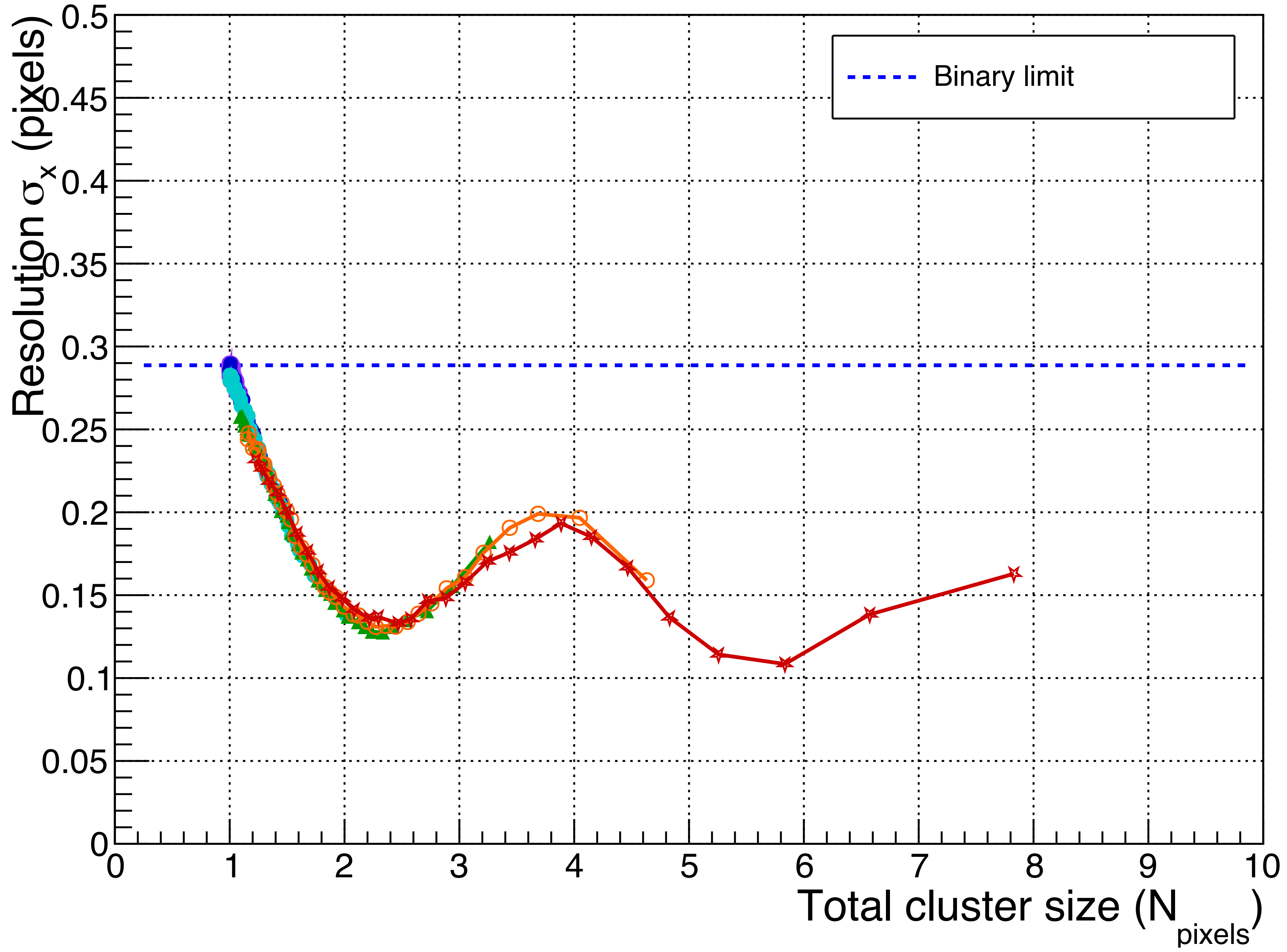




- Threshold scans performed
  - → Study resolution, cluster size, and detection efficiency
  - → Results shown for efficiency > 95%
  - → Negligible edge effects confirmed (events generated uniformly around the central pixel)

2D model results





Key observations:

→ **Average cluster size is a universal scaling variable for resolution**

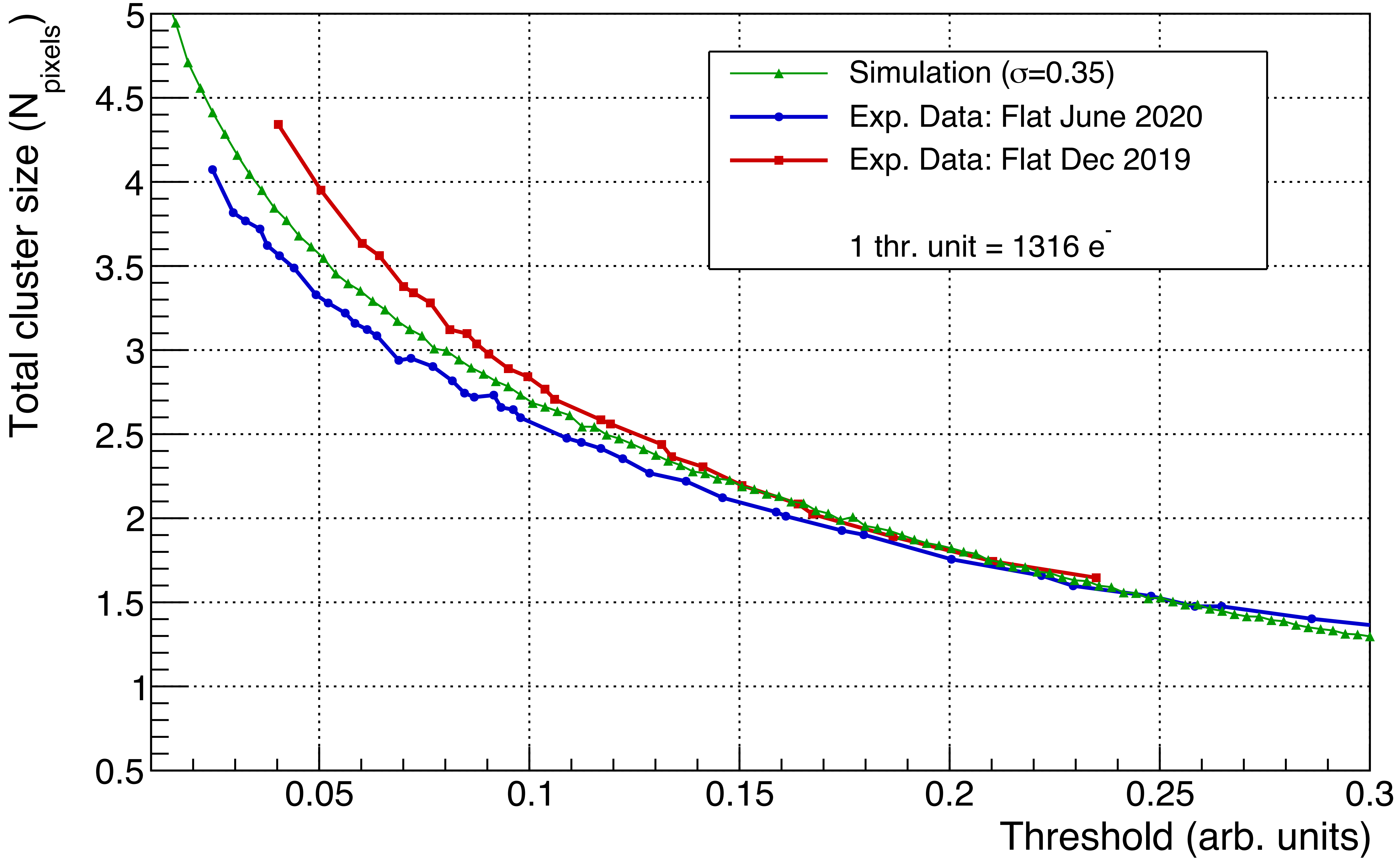
→ **Same resolution obtained for fixed cluster size, independent of charge width**

- Resolution parametrized as function of cluster size l

→ Phenomenological 4th-order polynomial fit : $\sigma_p = p \cdot \left(\frac{1}{\sqrt{12}} + a_1(l-1) + a_2(l^2-1) + a_3(l^3-1) + a_4(l^4-1) \right)$, $1 \leq l \leq 4.5$

Comparison with data

ALPIDE paper: https://arxiv.org/pdf/2502.04941



Summary & outlook

Stay tuned!

A toy model for understanding the space point resolution of silicon pixel detectors with digital readout

Marianna Mazzilli[1] and Alexander Kalweit[2]

[1]Dipartimento Interateneo di Fisica M. Merlin and Sezione INFN, Bari, Italy
[2]Experimental Physics department, CERN

Abstract

Silicon pixel detectors are widely used in high energy physics experiments as tracking detectors close to the primary interaction vertex. They provide excellent space point resolution together with fast electronic readout. Many of them do not have analogue readout which makes the applicability of centre-of-gravity algorithms less obvious. Charge sharing improves the resolution beyond the one-pixel limit, but in practice there is a lack of quantitative understanding. This work provides a simplified model with which the maximum improvement achievable with charge sharing is quantified in one and two-dimensional cases.

Keywords: keyword 1, keyword 2, keyword 3, keyword 4, keyword 5, keyword 6

1 Introduction

One of the key performance parameters of modern tracking detectors is the space point resolution σ_p. In particular, it directly enters into the momentum and distance-of-closest approach resolution of charged particles. For instance, the resolution d_0 of the distance-of-closest approach is the deceisive quantity in identifying charm and beauty decays and is at high momenta (negligible multiple scattering) given by [2]:

Physics motivations

Silicon pixel detectors are widely used for tracking close to the primary interaction vertex:

- Provide excellent spatial resolution and fast readout capabilities
- Many designs lack analogue readout, limiting the use of centre-of-gravity algorithms
- Charge sharing can enhance resolution beyond the single-pixel limit

However, there is a lack of quantitative understanding

This work introduces a simplified model to quantify the maximum resolution gain from charge sharing in 1D and 2D