High amplitude oscillations in dense matter

Mark Alford
Washington University in St. Louis

M. Alford, S. Mahmoodifar, K. Schwenzer,
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M. Alford, S. Reddy, K. Schwenzer, `arXiv:1110.6213`
(accepted by Phys. Rev. Lett.)

Outline

Overview: new phenomena at high amplitude

R-modes and signatures of dense matter

Bulk viscosity: generalities

Bulk viscosity: subthermal and suprathermal

Gap-bridging in a superfluid/superconductor

Overview

- Some oscillations of neutron stars can reach high amplitude. E.g. violent local accretion impacts, or unstable "r-modes".
- Response of different phases of dense matter to compression may provide signatures of their presence in neutron stars.
- At high enough amplitude, processes that depend on flavor equilibration have amplitude-dependent ("suprathermal") enhancements.
- Suprathermal enhancement may be strong enough to overcome suppression due to
  - slowness of flavor equilibration
  - Cooper pairing of relevant fermions

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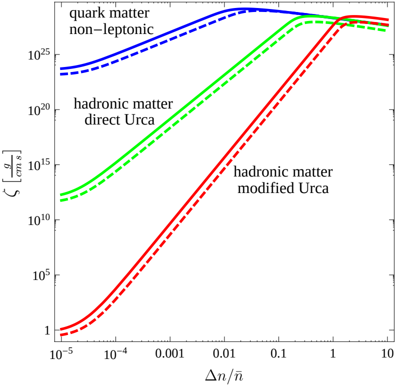
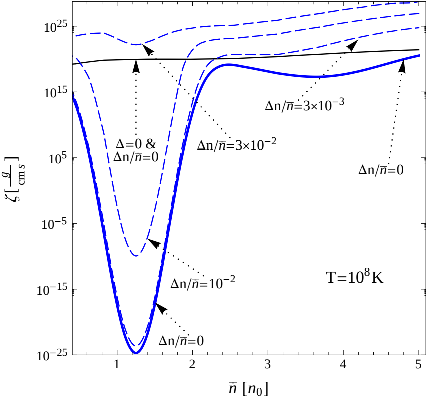
- Suprathermal enhancement may be strong enough to overcome suppression due to
  - slowness of flavor equilibration
  - Cooper pairing of relevant fermions
- Consequences for high amplitude oscillations:
  - Enhancement of heating and neutrino emission
  - Unsuppressed bulk viscosity and neutrino emission in superfluid phases
  - Enhanced bulk viscosity is capable of stopping r-mode growth

Suprathermal enhancement of bulk viscosity

Unpaired matter, $T = 10^6\,\mathrm{K}$

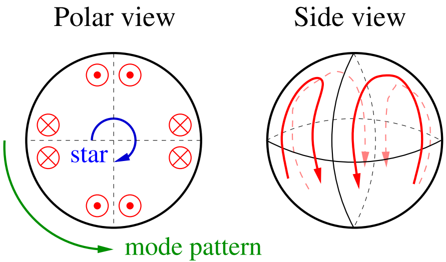


Nuclear superfluid, $T = 10^8\,\mathrm{K}$



Signature: r-mode-induced spindown

An r-mode is a mainly quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.



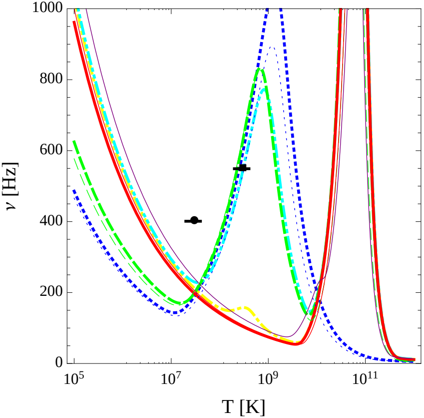
The unstable *r*-mode can spin the star down very quickly, exactly how fast depends on the amplitude at which it saturates.

(Andersson `gr-qc/9706075`; Friedman and Morsink `gr-qc/9706073`; Lindblom `astro-ph/0101136`).

So if we see a star spinning quickly, we can infer that the interior viscosity must be high enough to damp the *r*-modes.

Constraints from r-modes: current results

Regions above curves are forbidden $\Leftarrow$ viscosity is too low to damp r-modes.



Neutron star, nuclear matter

Hybrid star, medium quark matter core

Hybrid star, large quark matter core

Quark star

LMXB data: Aql X-1 (square), SAX J1808.4-3658 (circle).

Damping by crust is not included.

Alford, Mahmoodifar, Schwenzer `arXiv:1012.4883`

What is bulk viscosity?

Energy consumed in a compression cycle:

$$\begin{aligned} V(t) &= \bar{V} + \delta V \sin(\omega t) \\ p(t) &= \bar{p} + \delta p \sin(\omega t + \phi) \end{aligned}$$

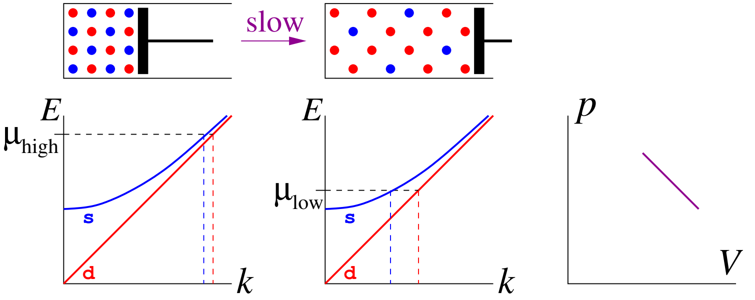
$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\zeta}{\tau}\int_0^\tau (\mathrm{div}\,\vec{v})^2 dt = \frac{\zeta}{2}\omega^2 \frac{\delta V^2}{\bar{V}^2} = -\frac{1}{\tau \bar{V}}\int_0^\tau p(t)\frac{dV}{dt}dt$$

- Bulk viscosity arises from re-equilibration processes.
- If some quantity goes out of equilibrium on compression, and re-equilibrates on a timescale comparable to τ, then pressure gets out of phase with volume.
- The driving force then does net work in each cycle.
- There is an exact analogy with V and Q in an R-C circuit.

Bulk viscosity and pressure phase lag

$d \leftrightarrow s$ stays in equilibrium

$\mu_d = \mu_s$



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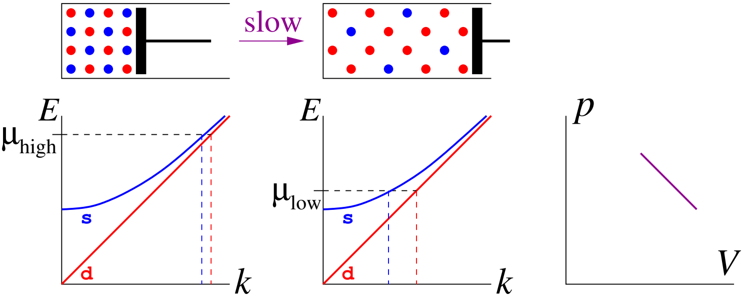
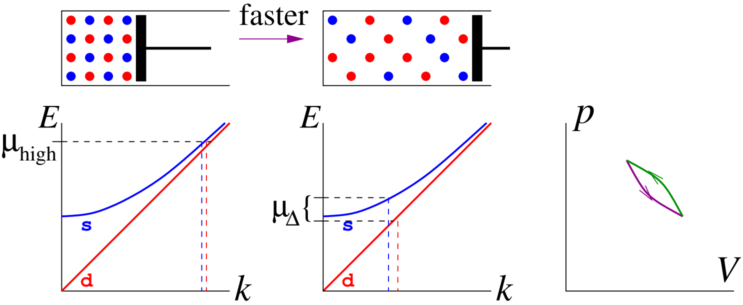


$d \leftrightarrow s$ goes out of equil.

$\mu_\Delta = \mu_d - \mu_s$



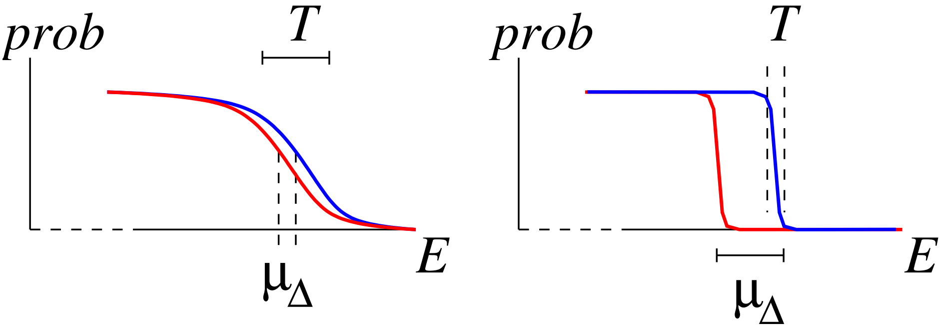
Subthermal vs Suprathermal

Subthermal

$\mu_\Delta \ll T \ll \mu$

Suprathermal

$T \ll \mu_\Delta \ll \mu$



Madsen, Phys. Rev. D46,3290 (1992); Reisenegger, Bonacic, astro-ph/0303454

Calculating bulk viscosity

- Compression at frequency ω. Density of conserved charge oscillates as $n(t) = \bar{n} + \delta n \sin(\omega t)$
- One quantity "Δ" goes out of equilibrium (eg $S - D$ in quark matter). In equilibrium, $\mu_\Delta = 0$.
- EoS is characterized by susceptibilities B, C.

$$\zeta = -\frac{1}{\pi} \frac{\bar{n}}{\delta n} \frac{C}{B} \int_0^\tau \mu_\Delta(t) \cos(\omega t) dt$$

Bulk visc arises from component of μ_Δ that lags behind the forcing oscillation by a phase of 90°; $\mu_\Delta(t)$ is given by

$$\frac{d\mu_\Delta}{dt} = \underbrace{C\omega \frac{\delta n}{\bar{n}} \cos(\omega t)}_{\text{forcing osc.}} - \underbrace{\Gamma(\mu_\Delta, T)}_{\text{equilibration}}$$

Re-express this in dimensionless variables:

Computing departure from equilibrium

- Define dimensionless time (i.e. phase) $\varphi = \omega t$
- Define dimensionless departure from equilibrium $\bar{\mu}_\Delta = \mu_\Delta / T$
- Driving coeff $d = \dfrac{C}{T}\dfrac{\delta n}{\bar{n}}$
- Equilibration rate: $\Gamma(\mu_\Delta, T) = \tilde{\Gamma}\, T^{2N} \gamma(\mu_\Delta / T)$.

  Equilibration coeff $f = \dfrac{B}{\omega}\tilde{\Gamma}\, T^{2N}$.

$$\boxed{\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\,\gamma(\bar{\mu}_\Delta)}$$

Dependence on density, EoS, driving amplitude, and temperature is contained in d and f.

Dependence of equilibration rate on $\bar{\mu}_\Delta$ for *unpaired* fermions:

$\gamma(\bar{\mu}_\Delta) = \bar{\mu}_\Delta + \chi_1 \bar{\mu}_\Delta{}^3 + \cdots + \chi_N \bar{\mu}_\Delta{}^{2N}$

Final term gives T-independent equilibration.

Suprathermal and subthermal bulk viscosity

Subthermal: assume $\bar{\mu}_\Delta \ll 1$ (i.e. $\mu_\Delta \ll T$), so $\gamma(\bar{\mu}_\Delta) = \bar{\mu}_\Delta$,

$$\boxed{\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\,\bar{\mu}_\Delta}$$

$$\bar{\mu}_\Delta(\varphi) = -\frac{f\,d}{1+f^2}\cos\varphi + \frac{d}{1+f^2}\sin\varphi$$

$$\zeta_{\rm sub} = \frac{C^2}{B\omega}\frac{f}{1+f^2} = \frac{C^2}{B}\frac{\gamma_{\rm eff}}{\omega^2+{\gamma_{\rm eff}}^2} \qquad (\gamma_{\rm eff} \equiv B\tilde{\Gamma}T^{2N})$$

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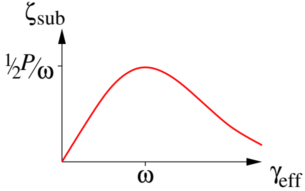
Suprathermal: allow $\bar{\mu}_\Delta \gtrsim 1$ (always assuming $\delta n \ll \bar{n}$),

$$\boxed{\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\bar{\mu}_\Delta\Big(1+\chi_1\bar{\mu}_\Delta{}^2+\cdots+\chi_N\bar{\mu}_\Delta{}^{2N}\Big)}$$

Now there are nonlinear effects; $\bar{\mu}_\Delta(\varphi)$ may not be harmonic.

The subthermal bulk viscosity

$$\zeta_{\rm sub}(\omega, T) = P \frac{\gamma_{\rm eff}(T)}{\gamma_{\rm eff}(T)^2 + \omega^2}$$



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- Prefactor $P = C^2/B$ is a combination of susceptibilities.
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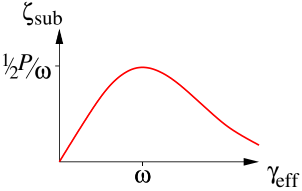

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- As $\gamma_{\rm eff} \to 0$, $\zeta \to 0$. No equilibration.
- As $\gamma_{\rm eff} \to \infty$, $\zeta \to 0$. Infinitely fast equilibration.

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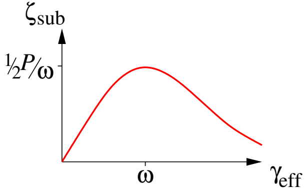

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- In phases where Fermi surface modes dominate equilibration (nuclear, unpaired quark matter, 2SC) P is constant for $T \ll \mu_q$, and subthermal bulk viscosity peaks when $\gamma_{\rm eff}(T) = \omega$.
- In phases where bosons dominate equilibration (CFL, CFL-K0), $P(T)$ washes out the peak.

The general bulk viscosity

To include the suprathermal regime, we have to solve the diffeq for $\bar{\mu}_\Delta(\varphi)$ numerically.

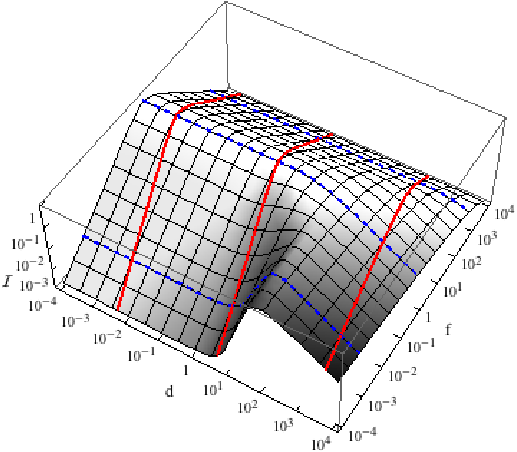
For a given form of matter, we can summarize dependence on driving amplitude and temperature in dimensionless function $\mathcal{I}(d, f)$,

$$\zeta = \frac{C^2}{2\omega B}\mathcal{I}(d, f)$$

$f = \gamma_{\mathrm{eff}}/\omega \quad d = (C/T)\,\delta n/\bar{n}$

This could then be used to calculate damping time of r-modes.

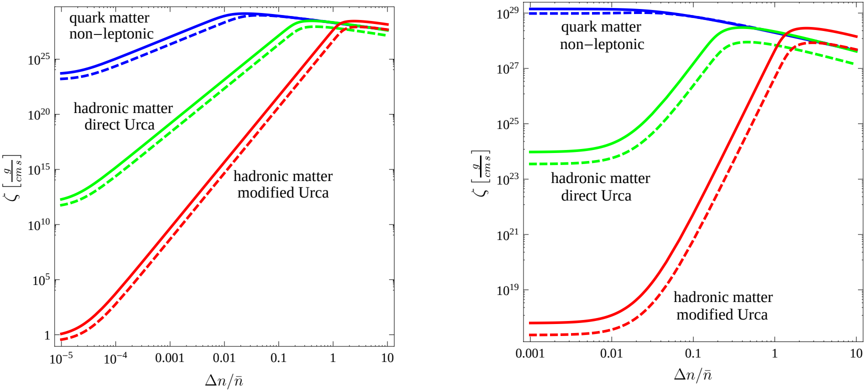
Nuclear matter, modified Urca



Suprathermal enhancement of bulk viscosity

(ms pulsar)

$T = 10^6$ K $\qquad$ $T = 10^9$ K



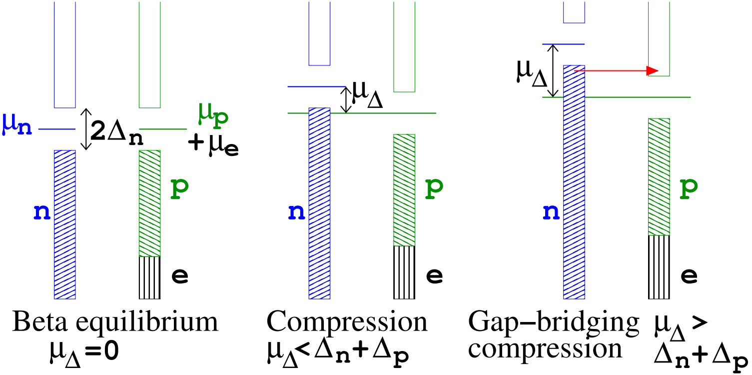
- Bulk visc rises very steeply in suprathermal regime
- Max reached at $\delta n/n \sim 0.1$; max value indp of temperature
- Suprathermal enhancement is greater at low T and for matter where ζ goes as higher power of μ_Δ.

Consequences of suprathermal enhancement

- Superthermal bulk viscosity can stop r-mode growth, but only at very high amplitude $\alpha \sim 1$ ($\delta n/\bar{n} \sim 0.03$).
  Other mechanisms may saturate r-mode first, e.g. mode-coupling at $\alpha \sim 10^{-4}$ (Bondarescu, Teukolsky, Wasserman, `arXiv:0809.3448`)
- Superthermal bulk viscosity and neutrino emission affect heating and cooling of stars undergoing r-mode spindown
- Response of stars to other high-amplitude compressions will also be affected.

Suprathermal enhancement in a superfluid ("gap-bridging")

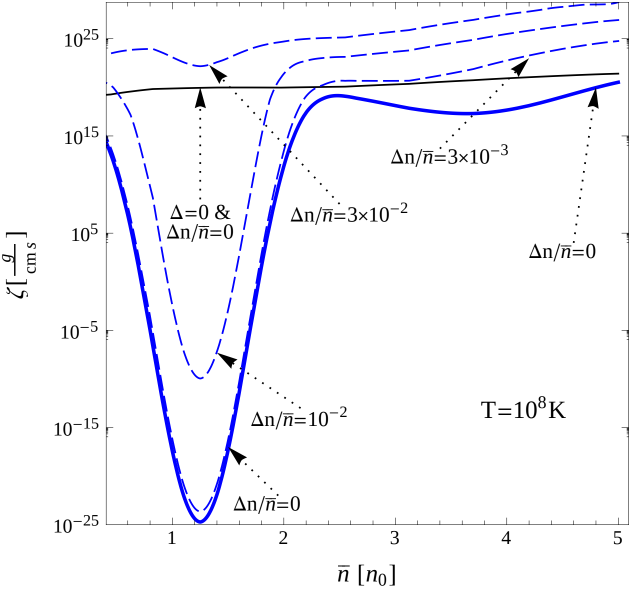
If density amplitude is high enough, μ_Δ can be large enough to open up phase space above the gap, overcoming $\exp(-\Delta/T)$ suppression.



$\mu_\Delta = C\frac{\delta n}{\bar{n}}$ For APR, $C \sim 20$ to 150 MeV

Oscillation with $\delta n/\bar{n} \sim 0.01$ can overcome $\Delta \sim 1$ MeV.

Gap-bridging enhancement of bulk viscosity



Illustrative example:

direct Urca allowed

s-wave pairing for *p* and *n*

Δ_p peak of 1 MeV at $n = 1.3n_0$

Δ_n peak of 0.12 MeV at $n = 3.7n_0$ [Cas A]

There is similar enhancement for neutrino emissivity.

Future directions

Transport:

- Study suprathermal enhancement in other phases, e.g. Hyperonic nuclear matter, neutron star crust
- Gap-bridging: apply to realistic case, modified Urca, $^3P_2$ neutron pairing, etc.
- Investigate effect of multiple equilibrating quantities

Astrophysics:

- Evolution of r-mode spindown, trajectory in (T, Ω) space
  (requires assumed r-mode saturation amplitude and a cooling model)
- Complications with r-modes: layered stars, role of crust, etc
- Apply to other modes, e.g. pulsations, f-modes (which emit grav waves), violent accretion events