

UV asymptotics of the generating functional of correlators of $\mathcal{N} = 1$ superfield twist-2 operators in SYM theory

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Plan of the talk

- ▶ The large- N expansion
- ▶ $\mathcal{N} = 1$ SYM theory
- ▶ Superfield twist-2 operators
- ▶ UV-asymptotic generating functional
- ▶ Applications
- ▶ Epilogue

PART I

The large- N expansion

The Euclidean bare action of Quantum Chromodynamics (QCD) with N_f massless quarks and gauge group $SU(N)$ is

$$S_{\text{QCD}} = \int d^4x \left[\frac{1}{2g^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \sum_{l=1}^{N_f} \bar{\psi}_l \not{D} \psi_l \right]$$

- **Confinement:** the physical spectrum consists of *hadrons* i.e. color singlets created by gauge-invariant local operators

$$\text{mesons:} \quad \bar{\psi}_I \psi^J$$

$$\text{glueballs:} \quad \text{Tr } F^2$$

$$\text{baryons:} \quad \epsilon_{i_1 \dots i_N} \psi^{i_1 A_1} \dots \psi^{i_N A_N}$$

- **Asymptotic freedom:** the renormalized coupling $g(\mu)$ becomes small at high energies. The beta function up to the first coefficient is

$$\beta(g) = \mu \frac{dg}{d\mu} = -\frac{1}{(4\pi)^2} \left(\frac{11}{3} - \frac{2}{3} \frac{N_f}{N} \right) N^2 g^3 + \dots$$

QCD

The RG-invariant mass scale

In QCD all physical masses are proportional to the RG-invariant mass scale

$$\Lambda_{\text{QCD}} = \Lambda e^{c_0} e^{-\frac{1}{2\beta_0 g^2}} (\beta_0 g^2)^{-\frac{\beta_1}{2\beta_0^2}} \left[1 + \sum_{n=1}^{\infty} c_n g^{2n} \right]$$

where Λ is the UV cutoff.

This quantity

- ▶ Is non-analytical in the bare coupling
- ▶ Vanishes when $g \rightarrow 0$ with Λ fixed

QCD

The coupling g

Since

$$g(\mu) \xrightarrow{\mu \rightarrow \infty} 0$$

RG-improved perturbation theory becomes reliable at high energies.

QCD

What is the true coupling?

It is possible to expand quantities in the renormalized coupling $g(\mu)$ and get reliable predictions when μ is large. In this sense, $g(\mu)$ is a legitimate coupling.

On the other hand, at $g = 0$ QCD is a theory of non-interacting hadrons! Instead, it becomes a theory of free quarks and gluons.

What we need is a more physical expansion parameter that really parametrizes the strength of the interactions between hadrons.

The large- N expansion

This expansion parameter was first identified by 't Hooft¹. Redefining $g^2 \rightarrow g^2/N$ we change the normalization of the action to

$$S_{\text{QCD}} = \int d^4x \left[\frac{N}{2g^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \sum_{A=1}^{N_f} \bar{\psi}_A \not{D} \psi^A \right]$$

't Hooft's idea was to expand every observable quantity \mathcal{O} in powers of $1/N$ as

$$\mathcal{O}(g^2, N) = \sum_{n=0} \frac{1}{N^{k_n}} \mathcal{O}_n(g^2)$$

and provided a recipe to compute the exponents k_n and the coefficients \mathcal{O}_n systematically.

¹Gerard 't Hooft. "A Planar Diagram Theory for Strong Interactions". In: *Nucl. Phys. B* (1974). A cura di J. C. Taylor.

The large- N expansion

The RG-invariant mass scale revisited

Now the beta function is finite in the limit $N \rightarrow \infty$

$$\beta(g) = -\frac{1}{(4\pi)^2} \left(\frac{11}{3} - \frac{2 N_f}{3 N} \right) g^3 + \dots \xrightarrow{N \rightarrow \infty} -\frac{1}{(4\pi)^2} \frac{11}{3} g^3 + \dots$$

It is easy to check that now

$$\lim_{N \rightarrow \infty} \Lambda_{\text{QCD}} = \Lambda_{\text{QCD}}^{(N=\infty)} > 0$$

and that Λ_{QCD} is analytic in $1/N$

$$\Lambda_{\text{QCD}} = \Lambda_{\text{QCD}}^{(N=\infty)} \left[1 - \frac{N_f}{N} \frac{2}{11} \log \frac{\Lambda}{\Lambda_{\text{QCD}}^{(N=\infty)}} + \dots \right]$$

The large- N expansion

The double-line notation

The systematic procedure devised by 't Hooft to expand correlators in powers of $\frac{1}{N}$ is the double-line notation.

Key idea: draw a line everytime two color indices are contracted.

The large- N expansion

The double-line notation

The free gluon propagator

$$\langle (A_\mu)^i_j(x) (A_\nu)^k_\ell(y) \rangle = \left(\delta^i_\ell \delta^k_j - \frac{1}{N} \delta^i_j \delta^k_\ell \right) D_{\mu\nu}(x-y)$$

is represented as

The diagram shows the expansion of the free gluon propagator. On the left, a wavy line with indices i, j on the left and k, ℓ on the right is equated to the sum of two terms. The first term consists of two parallel horizontal lines: the top line has an arrow pointing left from index k to j , and the bottom line has an arrow pointing right from index i to ℓ . The second term is a fraction $\frac{1}{N}$ multiplied by two circular loops: the first loop has an arrow pointing right from index i to j , and the second loop has an arrow pointing left from index k to ℓ .

The large- N expansion

The double-line notation

The free quark propagator

$$\langle \psi^{if}(x) \bar{\psi}_{jf'}(y) \rangle = \delta^f_{f'} \delta^i_j S(x-y)$$

is represented as



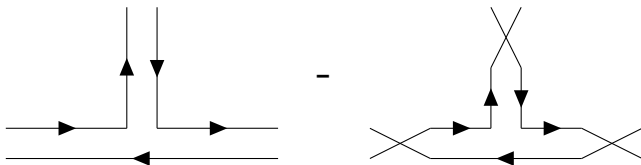
The large- N expansion

The double-line notation

The three-point vertex

$$\text{Tr}[\partial_\mu A_\nu A_\mu A_\nu] - \text{Tr}[\partial_\mu A_\nu A_\nu A_\mu]$$

is represented as



The large- N expansion

Planar vs nonplanar

Every Feynman diagram represented in the double-line notation can be drawn on a $2d$ Riemann surface with g handles and h boundaries.

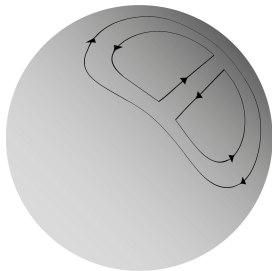
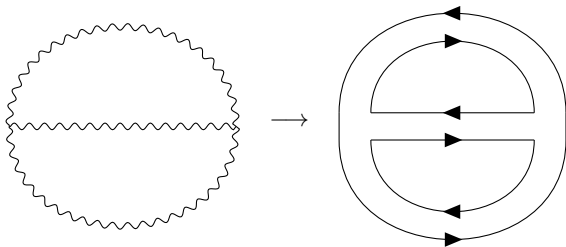
Every diagram is proportional to

$$N^{2-2g-h}$$

- ▶ **Planar diagrams:** diagrams that can be drawn on a sphere ($g = h = 0$) or a disk ($g = 0, h = 1$).
- ▶ **Nonplanar diagrams:** all the other diagrams

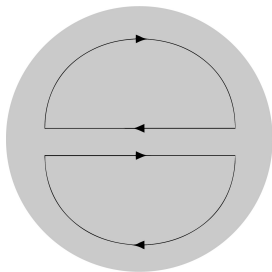
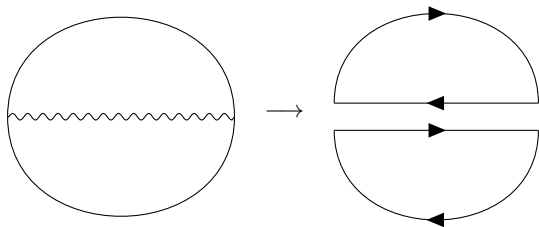
The large- N expansion

Planar diagram with spherical topology



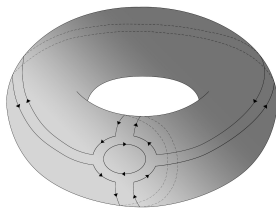
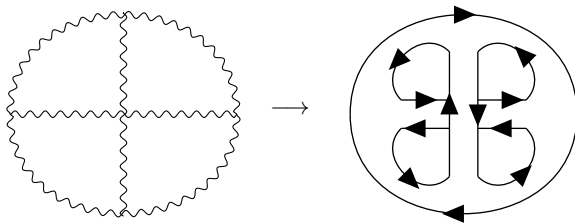
The large- N expansion

Planar diagram with disk topology



The large- N expansion

Nonplanar diagram with toroidal topology



The large- N expansion

Correlators

Consider some properly normalized² glueball operators \mathcal{G} and some meson operators \mathcal{M} . The double-line notation allows to prove that the leading contribution to their connected correlators is

$$\langle \mathcal{G}_1 \dots \mathcal{G}_n \rangle_c \sim N^{2-n}$$

$$\langle \mathcal{M}_1 \dots \mathcal{M}_k \rangle_c \sim N^{1-k/2}$$

$$\langle \mathcal{G}_1 \dots \mathcal{G}_n \mathcal{M}_1 \dots \mathcal{M}_k \rangle_c \sim N^{1-n-k/2}$$

²i.e. with 2 pt. correlators of order N^0

The large- N expansion

Some notable consequences

- ▶ The S -matrix at leading order is always trivial. The maximally disconnected part always dominates

$$\langle \mathcal{G}_1 \mathcal{G}_2 \rangle_c \cdots \langle \mathcal{G}_{n-1} \mathcal{G}_n \rangle_c \sim N^0$$

- ▶ Planar connected correlators contain only poles but not cuts: their leading large- N order describes a theory of hadrons at tree level

$$\lim_{N \rightarrow \infty} \int d^4x \langle \mathcal{G}(x) \mathcal{G}^\dagger(0) \rangle_c e^{-ipx} = \sum_n \frac{R_n}{p^2 + m_n^2}$$

The large- N expansion

The key observation

The leading non-planar part of a connected correlator is interpreted as arising from one-loop diagrams of mesons and glueballs^a.

^aG. Veneziano. "Some Aspects of a Unified Approach to Gauge, Dual and Gribov Theories". In: *Nucl. Phys. B* (1976).

PART II

$\mathcal{N} = 1$ SYM theory

$\mathcal{N} = 1$ SYM theory

Action

The matter content of supersymmetric Yang-Mills (SYM) theory consists of a

- ▶ Gluon A_μ
- ▶ Gluino λ_α (Majorana spinor)

Both transform in the adjoint representation of the gauge group.

$\mathcal{N} = 1$ SYM theory

Action

The action is

$$S_{\text{SYM}} = \frac{N}{g^2} \int \text{Tr} \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + 2i \bar{\lambda}_{\dot{\alpha}} \mathcal{D}^{\dot{\alpha}\alpha} \lambda_{\alpha} \right]$$

Supersymmetry transformations are

$$\begin{aligned} \delta A_{\mu} &= -i \bar{\epsilon}_{\dot{\alpha}} (\bar{\sigma}_{\mu})^{\dot{\alpha}\alpha} \lambda_{\alpha} - i \epsilon^{\alpha} (\sigma_{\mu})_{\alpha\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} \\ \delta \lambda_{\alpha} &= \frac{i}{2} F_{\mu\nu} (\sigma^{\mu\nu})_{\alpha}{}^{\beta} \epsilon_{\beta} + \frac{i}{2} F_{\mu\nu} (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{\epsilon}^{\dot{\beta}} \end{aligned}$$

Consequences of supersymmetry

- ▶ There are two supersymmetric charges $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ that pair bosonic and fermionic states of the same mass
- ▶ There is a $U(1)_R$ symmetry with generator R
- ▶ Non-renormalization theorems
- ▶ Exact (NSVZ) beta function³

$$\beta(g) = -\frac{3}{(4\pi)^2} \frac{g^3}{1 - \frac{g^2}{8\pi^2}}$$

- ▶ Exact gluino condensate

$$\langle \text{Tr } \lambda^2 \rangle = 16\pi^2 N \Lambda_{\text{SYM}}^3$$

³We are neglecting the rescaling anomaly, that is identically zero in dimensional regularization

Superspace

Coordinates

Superspace allows to express the action in a manifestly supersymmetric way.

We introduce a pair of Grassmann coordinates θ^α , $\bar{\theta}^{\dot{\alpha}}$ and define the SUSY-covariant derivatives

$$D^\alpha = \frac{\partial}{\partial \theta_\alpha} - i\bar{\theta}_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \partial_\mu \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}} \partial_\mu$$

and the corresponding gauge-covariant derivatives $\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}$

$\mathcal{N} = 1$ SYM theory in superspace

We pack all elementary fields into a unique *superfield*

$$W_\alpha = i \left(\lambda_\alpha + i\theta_\alpha D + i(\sigma^{\mu\nu})_\alpha{}^\beta \theta_\beta F_{\mu\nu} - i\theta^2 (\sigma^\mu)_{\alpha\dot{\alpha}} \mathcal{D}_\mu \bar{\lambda}^{\dot{\alpha}} \right)$$

where D is an auxiliary field. The fields that appear in front of the powers of θ^α and $\bar{\theta}^{\dot{\alpha}}$ are called *components* of the superfield.

$\mathcal{N} = 1$ SYM theory in superspace

The action becomes

$$S_{\text{SYM}} = \frac{N}{2g^2} \int d^4x d^2\theta \text{Tr}(W_\alpha W^\alpha) + \text{h.c.}$$

where $\int d^2\theta$ is a Berezin integration.

Some properties

- ▶ The theory is confining, asymptotically free, and has a mass gap
- ▶ In the 't Hooft's large- N limit the spectrum of the theory consists of *glueballs* (bosonic) and *gluinoballs* (fermionic)
- ▶ Glueballs and gluinoballs are created by single-trace operators made by W_α , $\bar{W}_{\dot{\alpha}}$ and their SUSY gauge-covariant derivatives e.g.

$$\text{Tr}(W_\alpha W^\alpha)$$

glueball operator

$$\text{Tr}(W_\alpha \bar{\nabla}_{\dot{\beta}} W^\alpha)$$

gluinoball operator

The large- N limit

In the large- N limit $\mathcal{N} = 1$ SYM becomes a theory of free glueballs and gluinoballs.

Recall:

- ▶ The coupling that parametrizes the interaction strength between hadrons is $\frac{1}{N}$
- ▶ Planar connected correlators are interpreted as arising from tree glueball/gluinoball diagrams
- ▶ Leading nonplanar connected correlators are interpreted as arising from one-loop glueball/gluinoball diagrams

The large- N limit

We call O_s and M_s the glueball and gluinoball operators respectively. The generating functional of their connected correlators is

$$\mathcal{W}^E[J_O, J_M] = \log \left(\frac{1}{\mathcal{Z}^E} \int e^{-S_{SYM} + \sum_s \int J_{O_s} O_s + J_{M_s} M_s} \right)$$

whose large- N expansion is

$$\mathcal{W}^E[J_O, J_M] = \mathcal{W}_{\text{sphere}}^E[J_O, J_M] + \mathcal{W}_{\text{torus}}^E[J_O, J_M] + \dots$$

$\mathcal{W}_{\text{sphere}}^E$ includes all tree glueball/gluinoball diagrams, while $\mathcal{W}_{\text{torus}}^E$ includes all one-loop glueball/gluinoball diagrams.

Glueball/gluinoball effective action

The effective action for glueballs and gluinoballs is⁴

$$S_{\text{glueball/gluinoball}}(\Phi, \Psi) = \frac{1}{2} \int \left[\Phi *_2 (-\Delta + M^2)\Phi + \Psi *_2' (-\Delta + M^2)\Psi \right. \\ \left. + \frac{1}{3N} \Phi *_3 \Phi *_3 \Phi + \frac{1}{N} \Psi *_3' \Phi *_3' \Psi + \dots \right]$$

where Φ, Ψ are the glueball/gluinoball superfields.

⁴Marco Bochicchio, Mauro Papinutto e Francesco Scardino. “On the structure of the large-N expansion in SU(N) Yang-Mills theory”. In: *Nucl. Phys. B* (2025), Marco Bochicchio, Mauro Papinutto e Francesco Scardino. “Generating functional of correlators of twist-2 operators in $\mathcal{N} = 1$ SUSY Yang-Mills theory, I”. In: *To appear in Eur. Phys. J. C* (2025).

Glueball/gluinoball effective action

One-loop

A simple functional integration shows that its one-loop generating functional is

$$\mathcal{W}_{\text{glueball/gluinoball, 1-loop}}^E[J_\Phi, J_\Psi] = \frac{1}{2} \log \text{sdet} \begin{pmatrix} *'_2(-\Delta + M^2) + \frac{1}{N} *'_3 \Phi_J *'_3 & & \\ & \frac{1}{N} *'_3 *'_3 \Psi_J & \\ & & *'_2(-\Delta + M^2) + \frac{1}{N} *'_3 \Phi_J *'_3 \end{pmatrix}$$

where Φ_J and Ψ_J are implicitly defined by

$$\left. \frac{\delta \mathcal{S}_{\text{glueball/gluinoball}}}{\delta \Phi} \right|_{\Phi_J} = *'_1 J_\Phi, \quad \left. \frac{\delta \mathcal{S}_{\text{glueball/gluinoball}}}{\delta \Psi} \right|_{\Psi_J} = *'_1 J_\Psi$$

Key question

Q: Can we observe the "log sdet" structure of $\mathcal{W}_{\text{glueball}/\text{gluinoball}, 1\text{-loop}}^E[J_\Phi, J_\Psi]$ in $\mathcal{W}_{\text{torus}}^E[J_O, J_M]$?

A: Yes, in the short-distance limit and with a suitable choice of the interpolating operators O_s and M_s . The occurrence of the "log sdet" structure was verified asymptotically in $\mathcal{N} = 1$ SU(N) SYM theory in the *component formalism*⁵.

⁵Marco Bochicchio, Mauro Papinutto e Francesco Scardino. "Generating functional of correlators of twist-2 operators in $\mathcal{N} = 1$ SUSY Yang-Mills theory, I". In: *To appear in Eur. Phys. J. C* (2025), M. Bochicchio, M. Papinutto e F. Scardino. "Generating functional of correlators of twist-2 operators in $\mathcal{N} = 1$ SUSY Yang-Mills theory, II". In: *To appear in Eur. Phys. J. C* (mag. 2025).

Aim of this work

The aim of this work is to obtain the "log sdet" structure in the *superspace formalism*.

Aim of this work

Q: Why is the superspace approach so important to deserve a separate work?

A: Superspace formalism has several advantages

- ▶ Manifest supersymmetry
- ▶ Shorter expressions
- ▶ Clearer renormalization constraints
- ▶ Clearer physical interpretation in the glueball/gluinoball theory
- ▶ Easier to generalize to other supersymmetric theories

PART III

Superfield twist-2 operators

Why twist-2?

- ▶ The generating functional of their connected correlators exhibits the "log sdet" structure at order g^0 .
- ▶ At order g^0 they transform under irreducible representations of the collinear superconformal group. This makes the calculation of their one-loop anomalous dimension especially simple.
- ▶ Let $\gamma(g)$ be their anomalous dimension matrix. It is possible to prove that there is a operator basis in which the matrix $\gamma(g)/\beta(g)$ is diagonal and one-loop exact.
- ▶ As a consequence of the above the "log sdet" structure is preserved asymptotically by the RG-improvement.
- ▶ They are related to the light-cone parton distribution functions.

Collinear superconformal algebra

Recall that:

- ▶ Gauge theories are *conformal* up to order g^2 in perturbation theory⁶
- ▶ Conformal symmetry + SUSY \Rightarrow Superconformal symmetry

The full superconformal algebra consists of

- ▶ The generators of the Poincaré group: translations P_μ and Lorentz transformations $M_{\mu\nu}$
- ▶ Dilatations D and special conformal transformations K_μ
- ▶ The SUSY generators: the spinorial charges Q_α , $\bar{Q}_{\dot{\alpha}}$ and the R -symmetry generator R
- ▶ To close the algebra we need to add the spinorial special conformal transformations S_α , $\bar{S}_{\dot{\alpha}}$

⁶V. M. Braun, G. P. Korchemsky e Dieter Müller. "The Uses of conformal symmetry in QCD". In: *Prog. Part. Nucl. Phys.* (2003).

Collinear superconformal algebra

The collinear superconformal algebra is the superalgebra consisting of the generators

$$\begin{aligned}L_+ &= -iP_+ & L_- &= \frac{i}{2}K_- & L &= \frac{i}{2}(D + M_{-+}) \\B &= -\frac{3}{4}R + \frac{1}{2}M_{12} & E &= \frac{i}{2}(D - M_{-+}) & V_+ &= i2^{-3/4}Q_1 \\V_- &= 2^{-5/4}\bar{S}_2 & W_+ &= -2^{-3/4}\bar{Q}_1 & W_- &= -2^{-5/4}S_2\end{aligned}$$

where the generator E is the *twist*. The algebra is isomorphic to $\mathfrak{sl}(2|1)$.

Superfield twist-2 operators

Recipe constructing superfield twist-2 operators:

- ▶ Consider the free ($g = 0$) theory.
- ▶ Identify primary elementary superfields and their charges.
- ▶ Construct single-trace superfield operators of twist 2 out of them and their derivatives.
- ▶ Require the operator to be a primary.
- ▶ Dress the operator to ensure gauge-invariance at $g \neq 0$.

Superfield twist-2 operators

Irreducible representations and elementary superfields

Each irreducible representation is characterized by three quantum numbers

- ▶ The collinear superconformal spin j
- ▶ The collinear twist τ
- ▶ The b -charge

The building blocks that we used to construct twist-2 superfield operators are

	j	b	τ
W_1	1	$-\frac{3}{2}$	1
\bar{W}_1	1	$+\frac{3}{2}$	1
∇_1	$\frac{1}{2}$	+1	0
$\bar{\nabla}_1$	$\frac{1}{2}$	-1	0

Superfield twist-2 operators

Twist-2

Twist-2 gauge-invariant single-trace superfield operators must have the form⁷

$$\mathbb{W}_n = \text{Tr} \left[(e^{-V} \bar{W}_1 e^V) \mathcal{P}_n(\overleftarrow{\nabla}_1, \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1, \overrightarrow{\nabla}_i) W_1 \right] \quad (\text{balanced})$$

$$\mathbb{W}_n^+ = \text{Tr} \left[W_1 \mathcal{P}_n^+(\overleftarrow{\nabla}_1, \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1, \overrightarrow{\nabla}_i) W_1 \right] \quad (\text{unbalanced})$$

$$\mathbb{W}_n^- = \text{Tr} \left[\bar{W}_1 \mathcal{P}_n^-(\overleftarrow{\nabla}_1, \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1, \overrightarrow{\nabla}_i) \bar{W}_1 \right] \quad (\text{unbalanced})$$

⁷ V is the superfield such that $W_\alpha = \frac{1}{8} \bar{D}^2 (e^{-V} D_\alpha e^V)$

Superfield twist-2 operators

Primaries

The polynomials \mathcal{P}_n are determined by the condition that \mathbb{W}_n are primaries

$$L_- \mathbb{W}_n = V_- \mathbb{W}_n = W_- \mathbb{W}_n = 0$$

Same for the \mathcal{P}_n^\pm for the \mathbb{W}_n^\pm . Descendants can be constructed by differentiation.

Superfield twist-2 operators

Primaries

The calculation is very long and technical and cannot be shown here. Just some observations:

- ▶ A similar calculations are for the non-supersymmetric case are available in the literature⁸
- ▶ Previous results on this topic in $\mathcal{N} = 1$ SYM are not manifestly supersymmetric/gauge-invariant and heavily relied on the light-cone gauge⁹
- ▶ This task is equivalent to find the Clebsch-Gordan coefficients¹⁰ for $\mathfrak{sl}(2|1)$. To our knowledge, this is the first time this task has been performed in full generality.

⁸V. M. Braun, G. P. Korchemsky e Dieter Müller. "The Uses of conformal symmetry in QCD". In: *Prog. Part. Nucl. Phys.* (2003).

⁹A. V. Belitsky et al. "Superconformal operators in Yang-Mills theories on the light-cone". In: *Nucl. Phys. B* (2005).

¹⁰Gerhard Gotz, Thomas Quella e Volker Schomerus. "Representation theory of $\mathfrak{sl}(2|1)$ ". In: *J. Algebra* (2007).

Superfield twist-2 operators

Solution

The result is

$$\mathcal{P}_n(\overleftarrow{\nabla}_1, \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1, \overrightarrow{\nabla}_i) \propto \mathbb{C}_{2n}^{1,1}(\overleftarrow{\nabla}_1 + i\overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1 + i\overrightarrow{\nabla}_i)$$
$$\mathcal{P}_n^\pm(\overleftarrow{\nabla}_1, \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1, \overrightarrow{\nabla}_i) \propto \mathbb{C}_{4n+1}^{1,1}(\overleftarrow{\nabla}_1 + i\overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1 + i\overrightarrow{\nabla}_i)$$

The polynomial

$$\mathbb{C}_n^{j_1, j_2}(\alpha, \beta) = \sum_{k_1+k_2=n} \frac{(-1)^{\lfloor \frac{k_1}{2} \rfloor} \alpha^{k_1} \beta^{k_2}}{\Gamma(1 + \lfloor \frac{k_1}{2} \rfloor) \Gamma(1 + \lfloor \frac{k_2}{2} \rfloor) \Gamma(2j_1 + \lfloor \frac{k_1+1}{2} \rfloor) \Gamma(2j_2 + \lfloor \frac{k_2+1}{2} \rfloor)}$$

is the SUSY counterpart of a Jacobi polynomial.

Superfield twist-2 operators

Normalization

The normalization is chosen to enforce the hermiticity condition

$$(\mathbb{W}_n)^\dagger = \mathbb{W}_n \qquad (\mathbb{W}_n^\pm)^\dagger = \mathbb{W}_n^\mp$$

and the correct large- N scalings

$$\mathbb{W}_n = -\frac{2\sqrt{2}\pi^2 i^n}{g^2} \text{Tr} \left[(e^{-V} \bar{W}_i e^V) \mathbb{C}_{2n}^{1,1} (\overleftarrow{\nabla}_1, \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1, \overrightarrow{\nabla}_i) W_1 \right]$$
$$\mathbb{W}_n^+ = -\frac{2\sqrt{2}\pi^2 i^{2n}}{g^2} \text{Tr} \left[W_1 \mathbb{C}_{4n+1}^{1,1} (\overleftarrow{\nabla}_1, \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1, \overrightarrow{\nabla}_i) W_1 \right]$$

Superfield twist-2 operators

Component expansion

Superfields contain fields belonging to the same supermultiplet¹¹. In the light-cone gauge:

$$\begin{aligned}\mathbb{W}_n \propto & \left(\bar{\lambda}^a C_n^{3/2} \lambda^a + \frac{6}{n+3} \partial_+ A^a C_{n-1}^{5/2} \partial_+ \bar{A}^a \right) \\ & - 2^{-\frac{3}{4}} \theta^1 \left(\frac{n+2}{2} \right) \bar{\lambda}^a P_n^{(1,2)} \partial_+ \bar{A}^a \\ & - 2^{-\frac{3}{4}} \bar{\theta}^1 \left(\frac{n+2}{2} \right) \partial_+ A^a P_n^{(2,1)} \lambda^a \\ & - \sqrt{2} \theta^1 \bar{\theta}^1 \frac{n+1}{n+3} \left(\bar{\lambda}^a C_{n+1}^{3/2} \lambda^a - \frac{6}{n+1} \partial_+ A^a C_n^{5/2} \partial_+ \bar{A}^a \right) \\ & + \text{off-the-light-cone terms}\end{aligned}$$

The C_n^α and $P_n^{(\alpha,\beta)}$ are Gegenbauer and Jacobi polynomials.

¹¹We use a condensed notation $\bar{\lambda}^a C_n^{3/2} \lambda^a \equiv \bar{\lambda}^a (i \overleftarrow{\partial}_+ + i \overrightarrow{\partial}_+)^n C_n^{3/2} (\overleftarrow{\partial}_+ - \overrightarrow{\partial}_+ / \overleftarrow{\partial}_+ + \overrightarrow{\partial}_+) \lambda^a$ etc.

PART IV

UV-asymptotic generating functional

The generating functional at order g^0

Basic fact: in a free theory the generating functional of the connected correlators of a bilinear operator always has a "log sdet" structure.

Take an operator $\mathcal{O} = \sum_k a_k \partial_+^k \phi \partial_+^{n-k} \phi$ in a free scalar theory

$$\begin{aligned}\mathcal{W}[J] &= \log \left(\int [d\phi] e^{-\frac{1}{2} \phi(-\square)\phi + J \cdot \mathcal{O}} \right) - \log \left(\int [d\phi] e^{-\frac{1}{2} \phi(-\square)\phi} \right) \\ &= \log \det \left(1 - \sum_k a_k \frac{1}{-\square} \overleftarrow{\partial}^k J \overrightarrow{\partial}^{n-k} \right) \\ &= \log \det \left[\delta_{k_1 k_2} \delta(x_1 - x_2) - a_{k_1} \mathcal{D}_{k_1 k_2}^{-1}(x_1 - x_2) J(x_2) \right]\end{aligned}$$

with $\mathcal{D}_{k_1 k_2}^{-1} = (-1)^{k_2} \partial_+^{n_1 - k_1 + k_2} (-\square)^{-1}$.

The generating functional at order g^0

The generating functional

From now on we will concentrate on the balanced operators \mathbb{W}_n alone.

In Minkowski superspace

$$\begin{aligned} & \mathcal{W}_{\text{sconf}}[J_{\mathbb{W}}, 0, 0] \\ &= (N^2 - 1) \log \det \left[\delta_{n_1 k_1, n_2 k_2} \delta^{(8)}(Z_1, Z_2) - \Delta_{n_1 k_1, n_2 k_2}^{-1}(Z_1, Z_2) i^{\lfloor \frac{n_2}{2} \rfloor} \frac{J_{\mathbb{W} n_2}(Z_2)}{N} \right] \end{aligned}$$

The generating functional at order g^0

The kernel

The kernel is

$$\Delta_{nk, n'k'}^{-1}(Z_1, Z_2) = \frac{1}{\Gamma(1 + \lfloor \frac{n-k}{2} \rfloor) \Gamma(2 + \lfloor \frac{n-k+1}{2} \rfloor)} \frac{1}{\Gamma(1 + \lfloor \frac{k'}{2} \rfloor) \Gamma(2 + \lfloor \frac{k'+1}{2} \rfloor)}$$
$$(-1)^{\lfloor \frac{k'+1}{2} \rfloor} (D_1^{(1)} + i\bar{D}_1^{(1)})^{n-k} (D_1^{(2)} + i\bar{D}_1^{(2)})^{k'} \frac{(x_{1\bar{2}})_+}{(x_{1\bar{2}})^2}$$

with

$$Z = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$$

and

$$x_{1\bar{2}}^\mu = x_1^\mu - x_2^\mu - i\theta_1 \sigma^\mu \bar{\theta}_1 - i\theta_2 \sigma^\mu \bar{\theta}_2 + 2i\theta_1 \sigma^\mu \bar{\theta}_2$$

being the super-Poincaré-invariant interval.

Short-distance limit

Short distances in superspace

Take a Euclidean correlator

$$G_{I_1 \dots I_n}(x_i, \theta_i, \bar{\theta}_i; g(\mu), \mu) \equiv \langle \mathcal{O}_{I_1}(x_1, \theta_1, \bar{\theta}_1) \cdots \mathcal{O}_{I_n}(x_n, \theta_n, \bar{\theta}_n) \rangle$$

rescale all coordinates by a factor $\lambda > 0$

$$G_{I_1 \dots I_n}(x_i, \theta_i, \bar{\theta}_i; g(\mu), \mu) \xrightarrow{\text{rescaling}} G_{I_1 \dots I_n}(\lambda x_i, \lambda^{1/2} \theta_i, \lambda^{1/2} \bar{\theta}_i; g(\mu), \mu)$$

then take the limit

$$\lambda \rightarrow 0$$

Short-distance limit

Limit of correlators

The Callan-Symanzik equation implies

$$\begin{aligned} & G_{J_1 \dots J_n}(\lambda x_i, \lambda^{1/2} \theta_i, \lambda^{1/2} \bar{\theta}_i; g(\mu), \mu) \\ &= \sum_{J_1, \dots, J_n} \lambda^{-\sum_{k=1}^n D_{I_k}} \mathcal{Z}_{I_1 J_1}(\lambda) \dots \mathcal{Z}_{I_n J_n}(\lambda) G_{J_1 \dots J_n}(x_i, \theta_i, \bar{\theta}_i; g(\mu/\lambda), \mu) \end{aligned}$$

where

$$\begin{aligned} \mathcal{Z}(\lambda) &= \text{P exp} \left(\int_{g(\mu)}^{g(\mu/\lambda)} \frac{\gamma(g')}{\beta(g')} dg' \right) \\ g^2(\mu/\lambda) &\stackrel{\lambda \rightarrow 0}{\sim} \frac{1}{\beta_0 \log(\frac{1}{\lambda^2})} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log \log(\frac{1}{\lambda^2})}{\log^2(\frac{1}{\lambda^2})} \right) \end{aligned}$$

with D_I the canonical dimensions of the operators.

Non-resonant diagonal basis

The problem

The renormalization factors $\mathcal{Z}_{IJ}(\lambda)$ are infinite matrices that take contributions from all orders of perturbation theory.

They look impossible to compute, apparently.

Good news!

These renormalization factors can be computed *exactly!* The key move is to pass to an operator basis called *non-resonant diagonal basis*. The existence of this basis is granted by two theorems.

Non-resonant diagonal basis

Theorem I: statement

This is the SUSY counterpart of the main result in¹². We extended it to a SUSY setting by means of the superspace formalism.

Theorem I

Consider a massless asymptotically free theory that is superconformal at order g^0 , and let \mathcal{O}_s be gauge-invariant twist-2 superfields with dimensions $D_s = 2 + s$ that at order g^0 are superconformal primaries.

⇒ In the MS scheme each \mathcal{O}_s is multiplicatively renormalizable at one-loop.

¹²N. S. Craigie, V. K. Dobrev e I. T. Todorov. "Conformally Covariant Composite Operators in Quantum Chromodynamics". In: *Annals Phys.* 159 (1985), pp. 411–444.

Non-resonant diagonal basis

Theorem I: consequences

At one-loop

$$\mathbb{W}_n = \left(\mathbf{1} + \frac{\gamma_0^{(\mathbb{W}_n)} \mathbf{g}^2}{\varepsilon} \right) \mathbb{W}_n^{(\text{bare})} + \dots$$

$$\mathbb{W}_n^\pm = \left(\mathbf{1} + \frac{\gamma_0^{(\mathbb{W}_n^\pm)} \mathbf{g}^2}{\varepsilon} \right) \mathbb{W}_n^{\pm(\text{bare})} + \dots$$

and do not mix with their descendants.

Non-resonant diagonal basis

Theorem I: observations

- ▶ The key idea of our proof is that *superfields mix only with other superfields (and their derivatives)*

$$\Phi_n(x, \theta, \bar{\theta}) = \sum_m \mathcal{L}_{nm} \Phi_m^{(\text{bare})}(x, \theta, \bar{\theta})$$

- ▶ Using the component formalism and imposing the SUSY Ward identities by hand leads to extremely cumbersome calculations¹³

¹³Andrei V. Belitsky, Dieter Mueller e A. Schafer. "Implications of N=1 supersymmetry for QCD conformal operators". In: *Phys. Lett. B* (1999).

Non-resonant diagonal basis

Theorem II: statement

The second theorem was proven in¹⁴.

Theorem II

Let $\gamma(g)$ be the anomalous dimension matrix of a set of gauge-invariant operators that mix under renormalization in a massless, asymptotically free, QCD-like theory with beta function $\beta(g)$.

Suppose that the matrix $\frac{\gamma_0}{\beta_0}$ is

- ▶ Diagonal
- ▶ Non-resonant: the sequence of its eigenvalues in nonincreasing order $\lambda_1, \lambda_2, \dots$ satisfies

$$\lambda_i - \lambda_j \neq 2k, \quad i > j$$

for any nonvanishing integer k .

⇒ There exists a basis of renormalized operators \mathcal{O}_s in which the matrix $\frac{\gamma(g)}{\beta(g)}$ is diagonal and one-loop exact to all orders of perturbation theory.

¹⁴Marco Bochicchio. "On the geometry of operator mixing in massless QCD-like theories". In: *Eur. Phys. J. C* 81.8 (2021), p. 749, Matteo Becchetti e Marco Bochicchio. "Operator mixing in massless QCD-like theories and Poincarè–Dulac theorem". In: *Eur. Phys. J. C* 82.10 (2022), p. 866.

Non-resonant diagonal basis

Theorem II: consequences

In the non-resonant diagonal scheme the solution of the Callan-Symanzik equation simplifies

$$\begin{aligned} & G_{l_1 \dots l_n}(\lambda x_i, \lambda^{1/2} \theta_i, \lambda^{1/2} \bar{\theta}_i; g(\mu), \mu) \\ &= \lambda^{-\sum_{k=1}^n D_{l_k}} \mathcal{Z}_{l_1}(\lambda) \dots \mathcal{Z}_{l_n}(\lambda) G_{l_1 \dots l_n}(x_i, \theta_i, \bar{\theta}_i; g(\mu/\lambda), \mu) \end{aligned}$$

where now

$$\mathcal{Z}_l(\lambda) \stackrel{\lambda \rightarrow 0}{\sim} \left(\frac{g^2(\mu)}{g^2(\mu/\lambda)} \right)^{\frac{\gamma_0^{(l)}}{\beta_0}}$$

Non-resonant diagonal basis

Theorem II: consequences

What happens in the limit $\lambda \rightarrow 0$?

We can expand the correlator in the rhs in powers of $g^2(\mu/\lambda)$

$$G_{l_1 \dots l_n}(x_i, \theta_i, \bar{\theta}_i; g(\mu/\lambda), \mu) = G_{l_1 \dots l_n}^{(\text{sconf})}(x_i, \theta_i, \bar{\theta}_i) + G_{l_1 \dots l_n}^{(1)}(x_i, \theta_i, \bar{\theta}_i; \mu) g^2(\mu/\lambda) + \dots$$

The zeroth order must correspond to the correlator at order g^0 i.e. superconformal correlator. AF implies

$$G_{l_1 \dots l_n}(x_i, \theta_i, \bar{\theta}_i; g(\mu/\lambda), \mu) \stackrel{\lambda \rightarrow 0}{\sim} G_{l_1 \dots l_n}^{(\text{sconf})}(x_i, \theta_i, \bar{\theta}_i)$$

Non-resonant diagonal basis

Theorem II: consequences

Putting everything together

$$G_{I_1 \dots I_n}(\lambda x_i, \lambda^{1/2} \theta_i, \lambda^{1/2} \bar{\theta}_i; g(\mu), \mu) \stackrel{\lambda \rightarrow 0}{\sim} \lambda^{-\sum_{k=1}^n D_{I_k}} \mathcal{Z}_{I_1}(\lambda) \dots \mathcal{Z}_{I_n}(\lambda) G_{I_1 \dots I_n}^{(\text{sconf})}(x_i, \theta_i, \bar{\theta}_i)$$

In the non-resonant diagonal basis the short-distance limit of a correlator is just the "dressing" of the superconformal correlator with the factors $\lambda^{-D} \mathcal{Z}(\lambda)$

RG-improved generating functional

Anomalous dimension

The one-loop anomalous dimensions of \mathbb{W}_n are

$$\gamma_0^{(\mathbb{W}_n)} = \frac{1}{4\pi^2} \left(\psi(n+4) + \psi(n+1) - 2\psi(1) - \frac{2(-1)^n}{(n+1)(n+2)(n+3)} - \frac{3}{2} \right)$$

where $\psi(i)$ is the polylogarithmic function.

It has been proven¹⁵ that they satisfy the assumptions of Theorem II.

¹⁵Francesco Scardino. "Nonresonant renormalization scheme for twist-2 operators in $\mathcal{N} = 1$ SUSY $SU(N)$ Yang-Mills theory". In: (2025).

RG-improved generating functional

Asymptotic generating functional

The generating functional of Euclidean connected correlators of the balanced twist-2 operators in the short-distance limit is

$$\mathcal{W}_{\text{asympt}}^E[J_{\mathbb{W}}, 0, 0; \lambda]$$
$$= (N^2 - 1) \log \text{sdet} \left[\delta_{n_1 k_1, n_2 k_2} \delta^{(8)}(Z_1^E, Z_2^E) - \frac{\mathcal{Z}_{n_2}(\lambda)}{\lambda^{n_2+2}} \Delta_{n_1 k_1, n_2 k_2}^{-1}(Z_1^E, Z_2^E) i^{\lfloor \frac{n_2}{2} \rfloor} \frac{J_{\mathbb{W} n_2}(Z_2^E)}{N} \right]$$

RG-improved generating functional

Asymptotic generating functional: leading nonplanar part

Its leading non-planar part is

$$\mathcal{W}_{\text{asympt,torus}}^E[J_{\mathbb{W}}, 0, 0; \lambda]$$
$$= -\log \text{sdet} \left[\delta_{n_1 k_1, n_2 k_2} \delta^{(8)}(Z_1^E, Z_2^E) - \frac{Z_{n_2}(\lambda)}{\lambda^{n_2+2}} \Delta_{n_1 k_1, n_2 k_2}^{-1}(Z_1^E, Z_2^E) i^{\lfloor \frac{n_2}{2} \rfloor} \frac{J_{\mathbb{W} n_2}(Z_2^E)}{N} \right]$$

as expected from first principles.

PART V
Applications

Applications (in progress)

- ▶ Construction of the twist-2 superfield operators in $\mathcal{N} = 1$ supersymmetric QCD (SQCD)
- ▶ Computation of their anomalous dimensions
- ▶ Construction of the supersymmetric parton distribution functions

$\mathcal{N} = 1$ SQCD

Action

To obtain $\mathcal{N} = 1$ SQCD add to $\mathcal{N} = 1$ SYM theory

- ▶ N_f fundamental (antifundamental) quarks ψ ($\tilde{\psi}$)
- ▶ N_f fundamental (antifundamental) squarks q (\tilde{q})

The action is¹⁶

$$S_{SQCD} = \int d^4x \left\{ \frac{N}{g^2} \text{Tr} \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + 2i\lambda \not{D} \bar{\lambda} + D^2 \right] + |\mathcal{D}q|^2 + |\mathcal{D}\tilde{q}|^2 \right. \\ \left. + i\bar{\psi} \not{D} \psi + i\tilde{\psi} \not{D} \tilde{\psi} - i\sqrt{2}\tilde{\psi} \lambda \tilde{q} + i\sqrt{2}\tilde{q} \bar{\lambda} \tilde{\psi} + i\sqrt{2}\tilde{q} \lambda \psi - i\sqrt{2}\bar{\psi} \bar{\lambda} q + (\bar{q} D q - \tilde{q} D \tilde{q}) \right\}$$

¹⁶We omitted all color, flavor and spinor indices. With a slight abuse of notation we denoted $\not{D} \equiv \mathcal{D}_{\alpha\dot{\alpha}}$.

$\mathcal{N} = 1$ SQCD

Superfields

We can pack quarks and squarks in two superfields

$$Q = q + \sqrt{2}\theta\psi + \theta^2 F \qquad \tilde{Q} = \tilde{q} + \sqrt{2}\theta\tilde{\psi} + \theta^2 \tilde{F}$$

at the price of introducing two more auxiliary fields F, \tilde{F} .

The superspace action is

$$S_{SQCD} = \int d^4x \left[\left(\frac{N}{2g^2} \int d^2\theta \operatorname{Tr} W^2 + \text{h.c.} \right) + \int d^4\theta \bar{Q} e^V Q + \int d^4\theta \tilde{Q} e^{-V} \bar{\tilde{Q}} \right]$$

Twist-2 operators in $\mathcal{N} = 1$ SQCD

Twist-2 superfield operators in $\mathcal{N} = 1$ SQCD are those of $\mathcal{N} = 1$ SYM plus those in the matter sector

$$Q_n^\alpha = \frac{4\pi^2}{\sqrt{N}} i^{n+|\mathbf{t}^\alpha|-1} (t^\alpha)^I{}_J \bar{Q}_I e^V C_{2n}^{\frac{1}{2}, \frac{1}{2}} \left(\overleftarrow{\nabla}_1 + i \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1 + i \overrightarrow{\nabla}_i \right) Q^J$$

$$\tilde{Q}_n^\alpha = \frac{4\pi^2}{\sqrt{N}} i^{n+|\mathbf{t}^\alpha|-1} (t^\alpha)^I{}_J \tilde{Q}_I e^{-V} C_{2n}^{\frac{1}{2}, \frac{1}{2}} \left(\overleftarrow{\nabla}_1 + i \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1 + i \overrightarrow{\nabla}_i \right) \tilde{Q}^J$$

$$Q_n^{\alpha+} = \frac{4\pi^2}{\sqrt{N}} i^{n+|\mathbf{t}^\alpha|-1} (t^\alpha)^I{}_J \tilde{Q}_I C_{2n+1}^{\frac{1}{2}, \frac{1}{2}} \left(\overleftarrow{\nabla}_1 + i \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1 + i \overrightarrow{\nabla}_i \right) Q^J$$

$$Q_n^{\alpha-} = \frac{4\pi^2}{\sqrt{N}} i^{n+|\mathbf{t}^\alpha|-1} (t^\alpha)^I{}_J \bar{Q}_I C_{2n+1}^{\frac{1}{2}, \frac{1}{2}} \left(\overleftarrow{\nabla}_1 + i \overleftarrow{\nabla}_i, \overrightarrow{\nabla}_1 + i \overrightarrow{\nabla}_i \right) \tilde{Q}^J$$

The flavor matrices t^α are chosen so that $(t^\alpha)^t = (-1)^{|\mathbf{t}^\alpha|} t^\alpha$

Parton distribution functions

Non-SUSY case

In QCD, nucleon scattering amplitudes in the forward limit are controlled by parton distribution functions¹⁷

$$\int \frac{dx^+}{2\pi} e^{ip_+x^+} \langle \mathbf{p}, \sigma | \bar{\psi}_I(-\frac{x^+}{2}) \mathcal{W}(-\frac{x^+}{2}, \frac{x^+}{2}) \bar{\sigma}_+ \psi^J(\frac{x^+}{2}) | \mathbf{p}, \sigma \rangle$$

where

$$\mathcal{W}(-\frac{x^+}{2}, \frac{x^+}{2}) = \text{P exp} \left(-i \int_{-\frac{1}{2}}^{\frac{1}{2}} A_+(\tau x^+) d\tau \right)$$

is a straight Wilson line along the light-cone.

¹⁷Xiang-Dong Ji. "Deeply virtual Compton scattering". In: *Phys. Rev. D* (1997).

Parton distribution functions

Non-SUSY case: relation to twist-2 operators

Expanding the correlator inside the expectation value in powers of x^+ results in a sum over twist-2 operators

$$\langle \mathbf{p}, \sigma | \bar{\psi}_I(-\frac{x^+}{2}) \mathcal{W}(-\frac{x^+}{2}, \frac{x^+}{2}) \bar{\sigma}_+ \psi^J(\frac{x^+}{2}) | \mathbf{p}, \sigma \rangle = \sum_{\text{twist-2}} \frac{(x^+)^n}{n!} \langle \mathbf{p}, \sigma | \mathcal{O}(0) | \mathbf{p}, \sigma \rangle$$

Parton distribution functions

SUSY case

In $\mathcal{N} = 1$ SQCD several more parton distribution functions appear

$$\int \frac{dx^+}{2\pi} e^{ip_+ x^+} \langle \mathbf{p}, \sigma | \bar{q}_I(-\frac{x^+}{2}) \mathcal{W}(-\frac{x^+}{2}, \frac{x^+}{2}) q^J(\frac{x^+}{2}) | \mathbf{p}, \sigma \rangle$$
$$\int \frac{dx^+}{2\pi} e^{ip_+ x^+} \langle \mathbf{p}, \sigma | \text{Tr} \left[\bar{\lambda}(-\frac{x^+}{2}) \mathcal{W}(-\frac{x^+}{2}, \frac{x^+}{2}) \bar{\sigma}_+ \lambda(\frac{x^+}{2}) \mathcal{W}(\frac{x^+}{2}, -\frac{x^+}{2}) \right] | \mathbf{p}, \sigma \rangle$$

...

Since they are related by SUSY transformations they can be packed into a unique superspace parton distribution function.

Parton distribution functions

SUSY case

As an example consider the (s)quark parton distribution function:

$$\int \frac{dx^+}{2\pi} e^{ip_+ x^+} \langle \mathbf{p}, \sigma | \bar{Q}_I(-\frac{x^+}{2}, \theta, \bar{\theta}) \mathcal{W}(-\frac{x^+}{2}, \frac{x^+}{2}; \theta, \bar{\theta}) Q^J(\frac{x^+}{2}, \theta, \bar{\theta}) | \mathbf{p}, \sigma \rangle$$

where now \mathcal{W} is now a superspace Wilson line¹⁸

$$\mathcal{W}(-\frac{x^+}{2}, \frac{x^+}{2}; \theta, \bar{\theta}) = \text{P exp} \left(-i \int_{-\frac{1}{2}}^{\frac{1}{2}} \Gamma_+(\tau x^+, \theta, \bar{\theta}) d\tau \right)$$

where $\Gamma_\mu = \frac{1}{4}(\bar{\sigma}_\mu)^{\dot{\alpha}\alpha} \bar{D}_{\dot{\alpha}} (e^{-V} D_\alpha e^V)$ is the supersymmetric vectorial connection.

We worked out their expansion in terms of their components and their relation to twist-2 superfield operators.

¹⁸T. Hagiwara, So-Young Pi e H. S. Tsao. "Point Splitting Regularization in a Supersymmetric Gauge Field Theory". In: *Phys. Lett. B* (1980).

Anomalous dimensions

Superfields allow to compute anomalous dimensions of quark superfield twist-2 operators and their mixing with their gluonic counterparts in a relatively clean manner.

The computation was performed by the master student Dario Valerio D'Antonio in his thesis *Anomalous Dimensions of Twist-2 Operators in Large- N Planar SUSY QCD* (Supervisors: Prof. Mauro Papinutto, Dr. Giacomo Santoni)

Anomalous dimensions

Methodology

The method that we employed is the SUSY version of¹⁹:

- ▶ Recall that twist-2 superfield operators are multiplicatively renormalizable at one-loop (Theorem I)
- ▶ Write the twist-2 superfield operators in nonlocal form via a point-splitting regularization
- ▶ Expand in non-local components and choose the one you like most
- ▶ Compute their mixing i.e. the *Balitsky-Braun kernel*
- ▶ Expand the divergent parts in powers of the separation between elementary superfields to get the anomalous dimensions

¹⁹I. I. Balitsky e Vladimir M. Braun. “Evolution Equations for QCD String Operators”. In: *Nucl. Phys. B* (1989).

Epilogue

Summary of the main results

- ▶ A complete classification of the irreducible representation of the superalgebra $\mathfrak{sl}(2|1)$ and their Clebsch-Gordan coefficients.
- ▶ A manifestly gauge-invariant and SUSY-covariant expression for the twist-2 superfield operators in $\mathcal{N} = 1$ superconformal field theories and SUSY gauge theories.
- ▶ A new derivation of the renormalization properties of the twist-2 superfield operators in $\mathcal{N} = 1$ SYM theory.
- ▶ We verified that the short-distance asymptotic generating functional of the connected correlators of the twist-2 superfield operators in $\mathcal{N} = 1$ SYM theory has indeed the form of the logarithm of a functional superdeterminant.

What's next?

- ▶ Generalization of our construction of the superfield twist-2 operators to higher twists and higher supersymmetries.
- ▶ Find the UV-asymptotic generating functional of the connected correlators of twist-2 superfield operators in $\mathcal{N} = 1$ SQCD
- ▶ Find the constraints on the structure of the glueball/gluinoball non-perturbative effective action²⁰.

²⁰ Marco Bochicchio, Mauro Papinutto e Francesco Scardino. "On the structure of the large-N expansion in SU(N) Yang-Mills theory". In: *Nucl. Phys. B* (2025), Marco Bochicchio, Mauro Papinutto e Francesco Scardino. "Topology of the large-N expansion in SU(N) Yang-Mills theory and spin-statistics theorem". In: *EPJ Web Conf.* 314 (2024).

Thank you for your attention!