

All-charm tetraquarks: a new frontier

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Dedicated to my friend and colleague Carlo Becchi (1940-2025)
for his contributions to the research reported in this talk

1. Hidden charm and beauty hadrons reveal *tetraquarks* and *pentaquarks*

M. Gell-Mann, A Schematic Model of Baryons and Mesons, PL 8, 214, 1964

Baryons can now be constructed from quarks by using the combinations (qqq) , $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest

- Heavy quark pairs are difficult to be created or destroyed by QCD forces inside hadrons. Hadrons with a $c\bar{c}$ or $b\bar{b}$ pair and ***electrically charged must contain additional light quarks***, realising the hypothesis advanced by Gell-Mann in the Sixties.

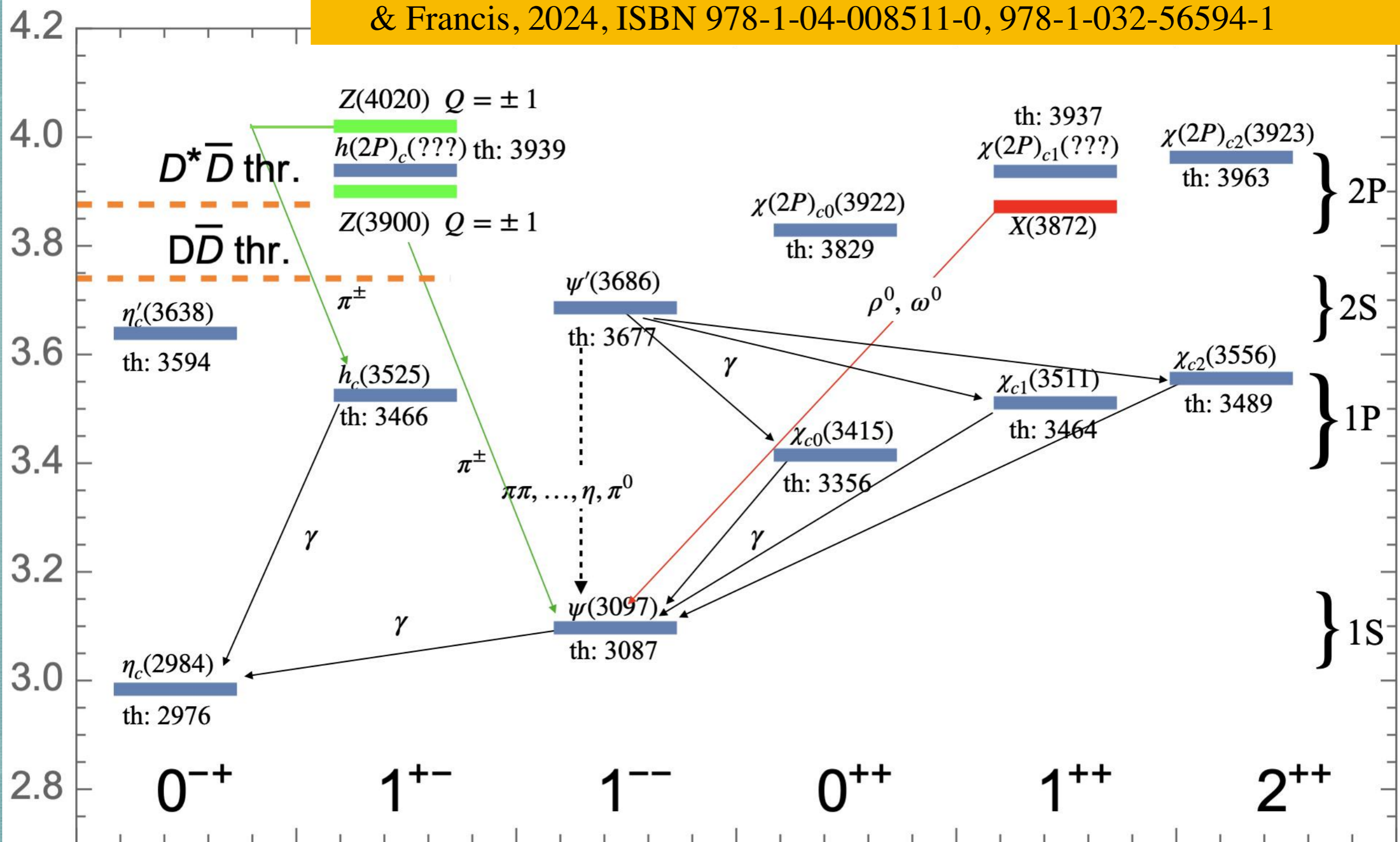
- These are the exotic X, Y, Z mesons and the pentaquarks discovered over the last decades

There are indeed new valence quark configurations !!

- First hypothesis of tetraquarks by R. Jaffe, as a model of the lightest scalar mesons
- Tetraquarks are more easy to identify at the increase of quark mass
- Hidden heavy flavors have been the first to be discovered
- The first, *unexpected charmonium* was the still controversial X(3872)
- Nearness to heavy pair threshold is to be expected, but the X(3872) is exceptionally close, we do not know yet if it is above or below the $D^0\bar{D}^{*0}$ threshold, within some 80 keV.

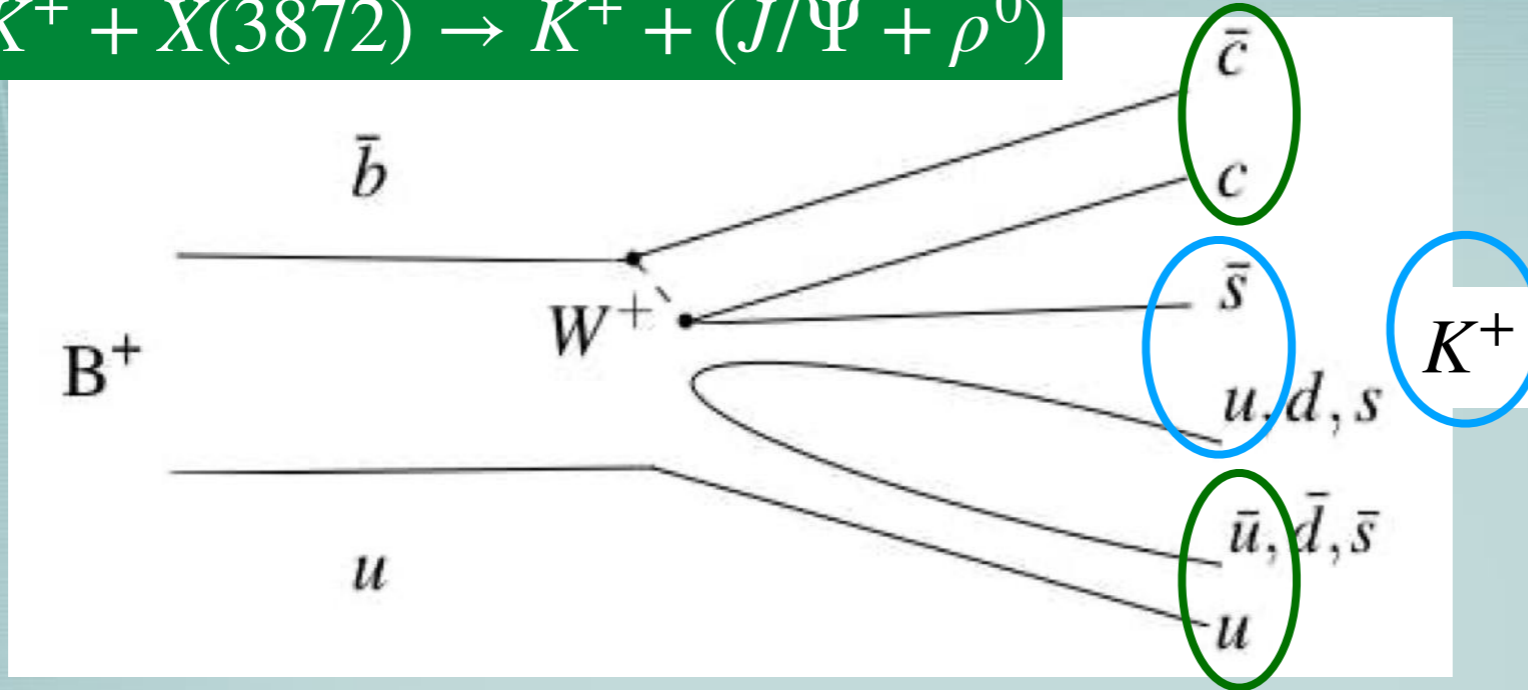
Expected and Unexpected Charmonia

see, e.g. L. Maiani and O. Benhar, *Relativistic Quantum Mechanics*, Taylor & Francis, 2024, ISBN 978-1-04-008511-0, 978-1-032-56594-1

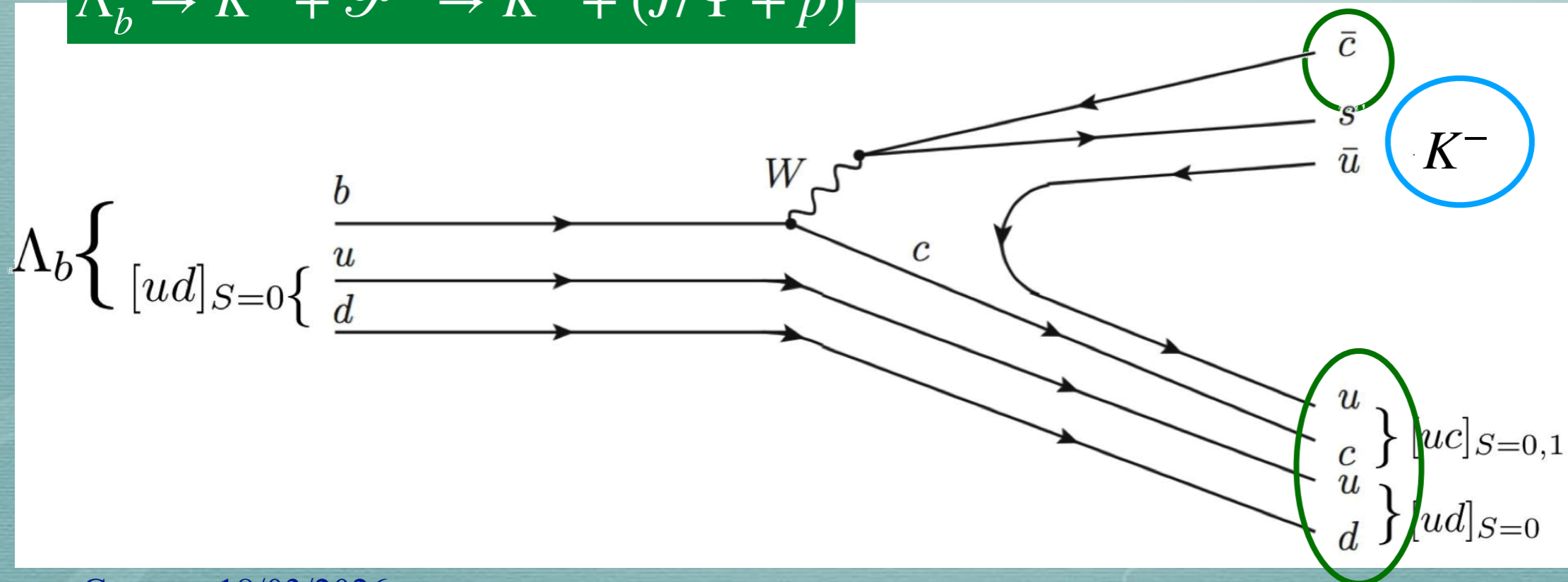


Many Exotic Hadrons have been found in the weak decays of Beauty Mesons and Baryons

$$B^+ \rightarrow K^+ + X(3872) \rightarrow K^+ + (J/\Psi + \rho^0)$$



$$\Lambda_b^0 \rightarrow K^- + \mathcal{P}^+ \rightarrow K^- + (J/\Psi + p)$$



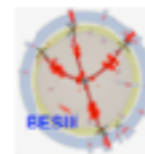
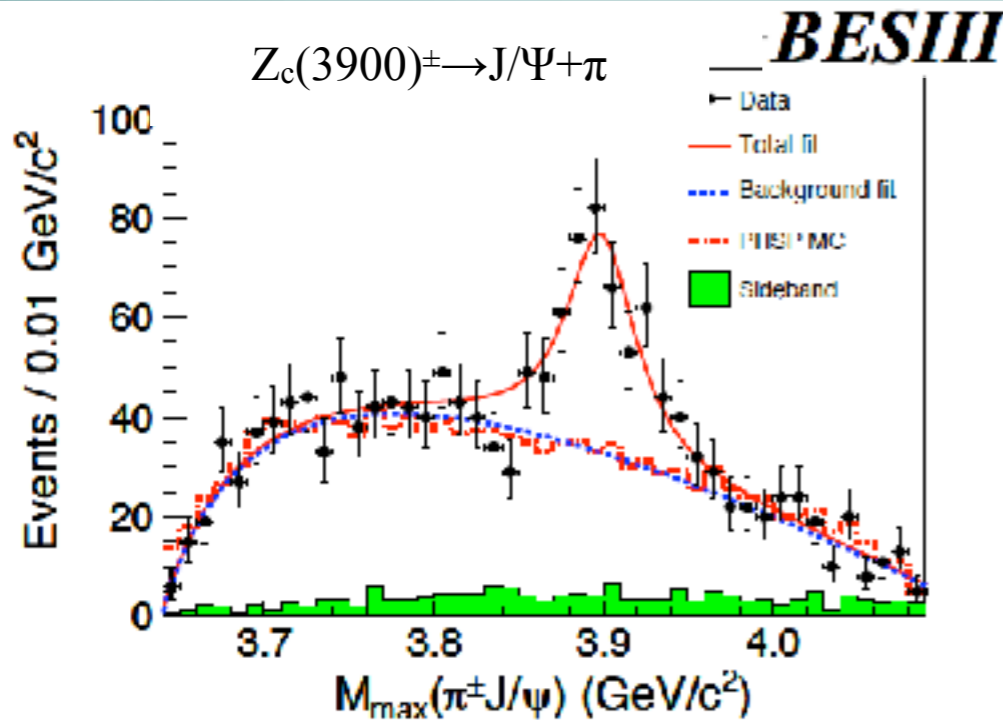
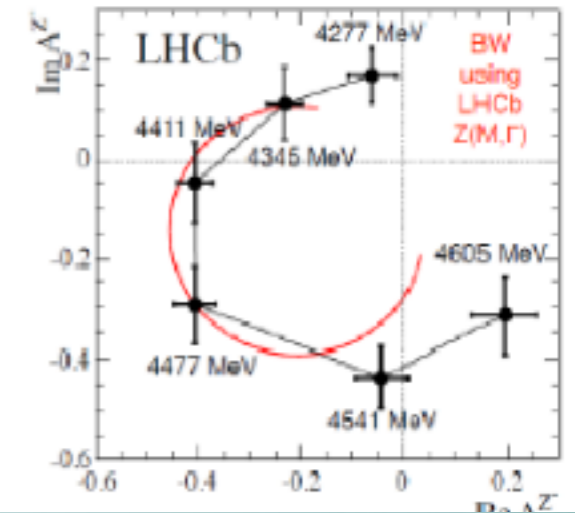
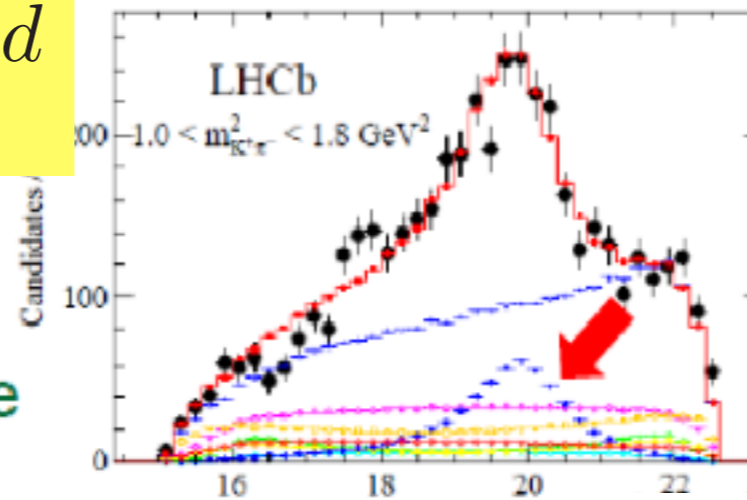
Explicit Tetraquarks

$$Z_c(4430)^\pm \rightarrow J/\Psi + \pi$$

valence quark composition: $c\bar{c}u\bar{d}$

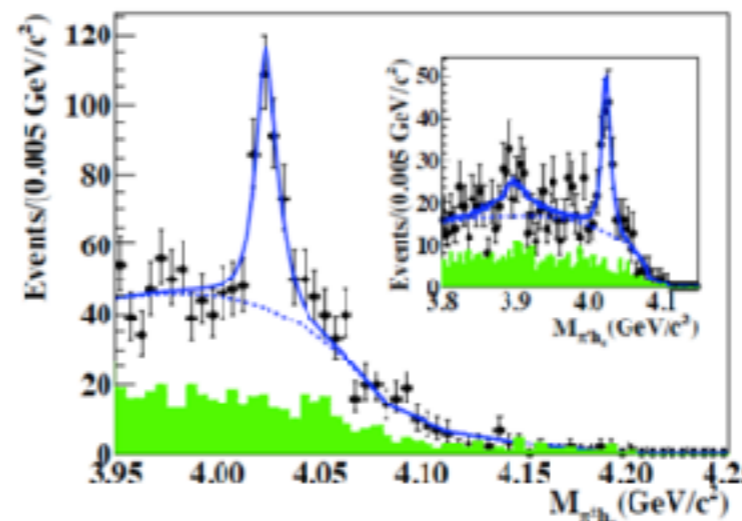
1. Confirm Belle's observation of 'bump'
2. Can NOT be built from standard states
3. Textbook phase variation of a resonance

[PRL 112 (2014) 222002]



BESIII

$$Z_c(4020)^\pm \rightarrow h_c + \pi$$



BESIII: PRL111, 242001

Simultaneous fit to
4.23/4.26/4.36 GeV data,
16 η_c decay modes. 8.9σ

$$M(Z_c(4020)) = 4022.9 \pm 0.8 \pm 2.7 \text{ MeV};$$

$$\Gamma(Z_c(4020)) = 7.9 \pm 2.7 \pm 2.6 \text{ MeV}$$

More, exotic valence quark configurations

$$\Lambda_b \rightarrow K^- + J/\psi + P$$



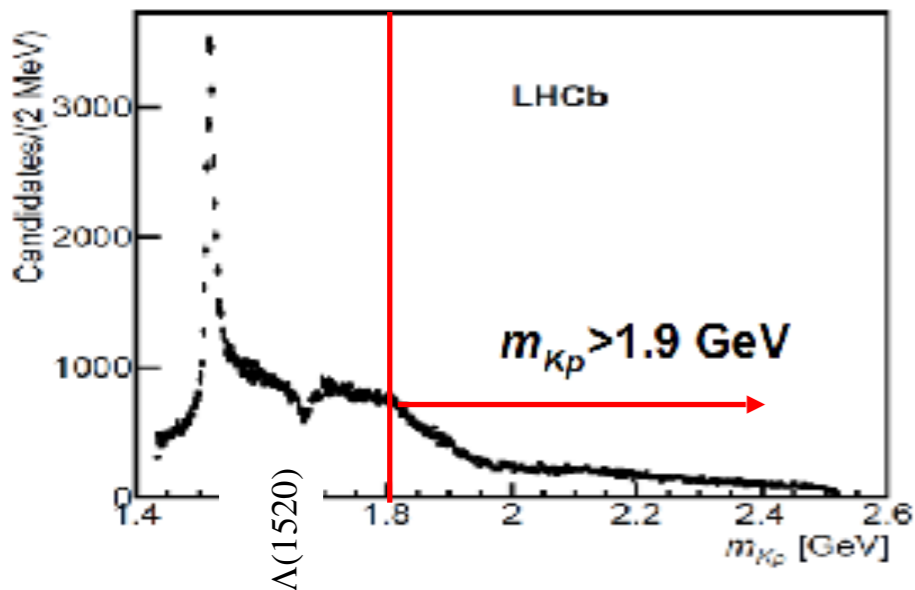
Exotic Hadrons, TD Lee Inst., Shanghai June 25, 2019 Tomasz Skwarnicki

6

Narrow $P_c^+ \rightarrow J/\psi p$ peaks with Λ^* suppression

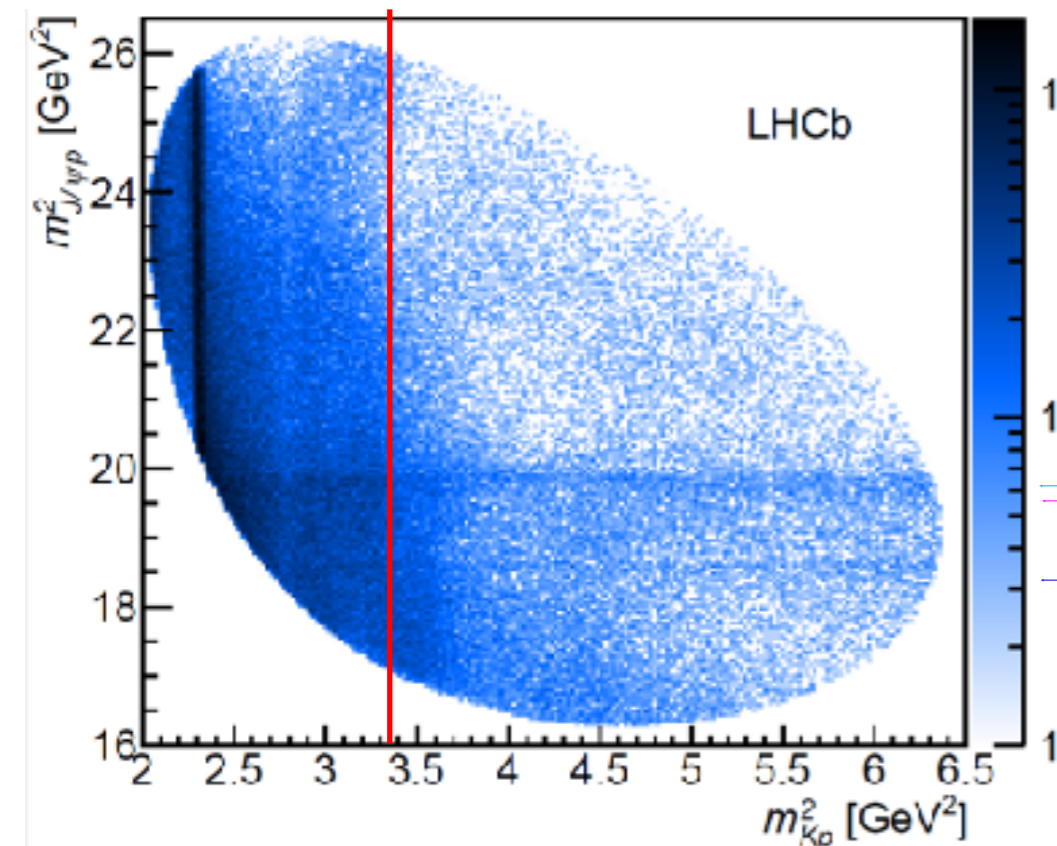
Mass resolution $\sigma=2.3-2.7$ (FWHM 5.4-6.4) MeV

Eliminates 80% of backgrounds

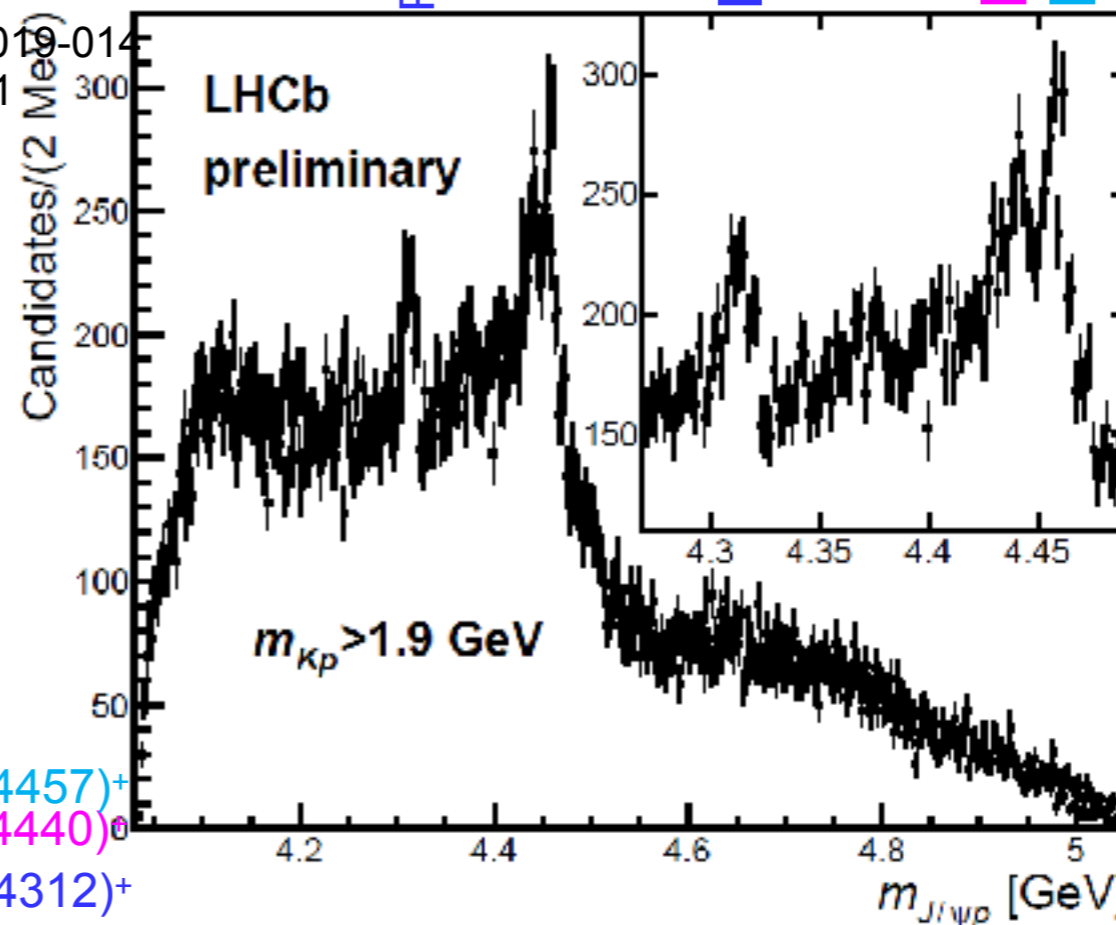


LHCb-PAPER-2019-014
PRL 122, 222001

$P_c(4312)^+$
 $P_c(4440)^+$
 $P_c(4457)^+$
 $P_c(4312)^+$
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 $P_c(4457)^+$



$P_c(4457)^+$
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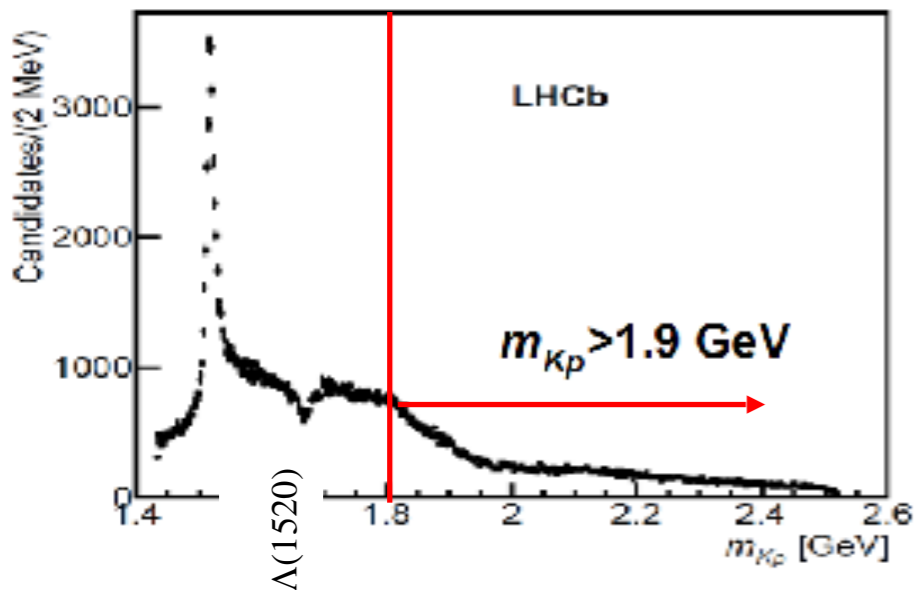
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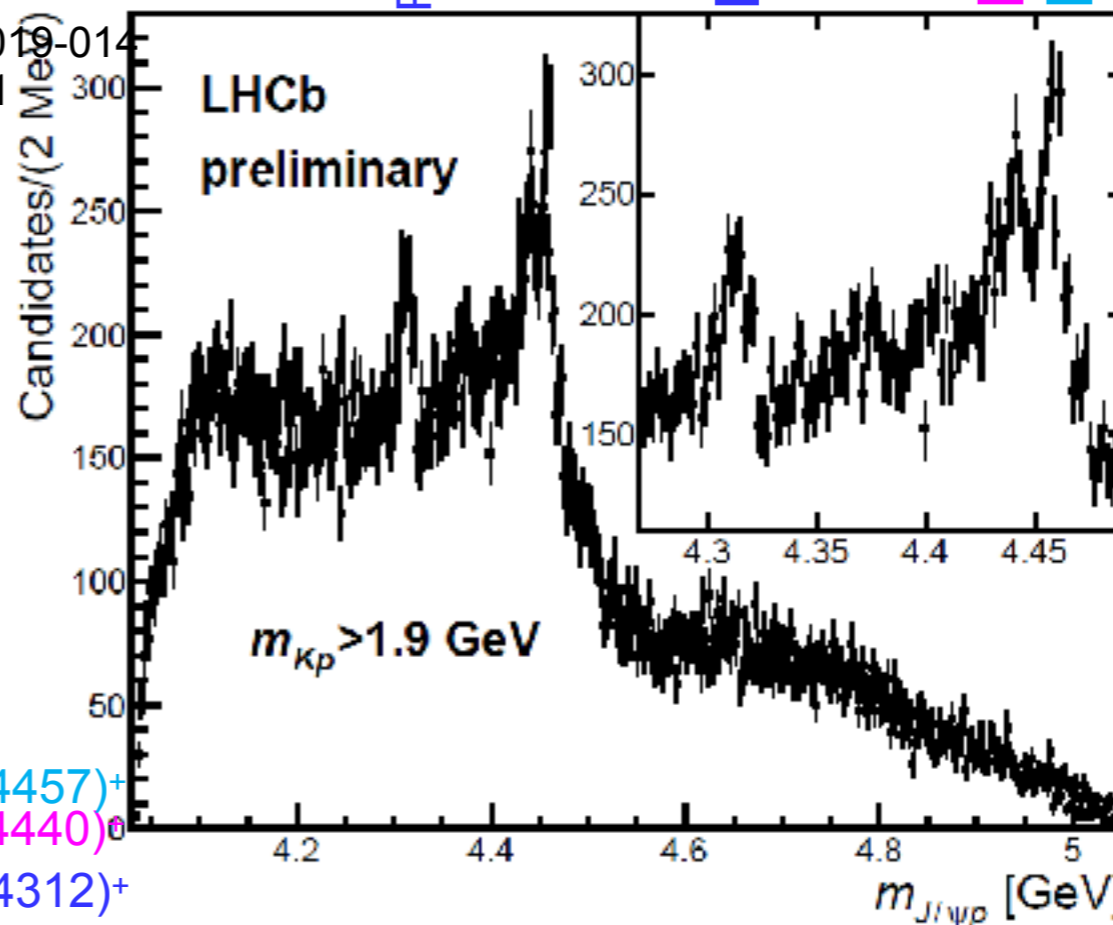
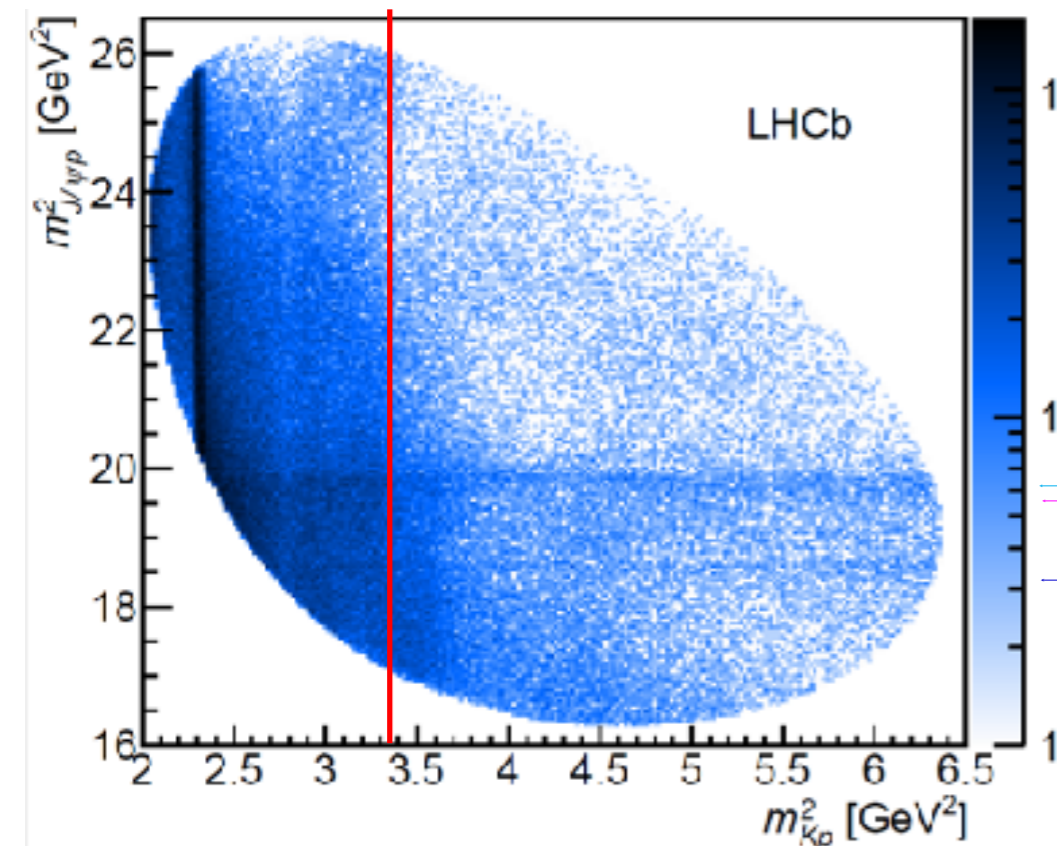
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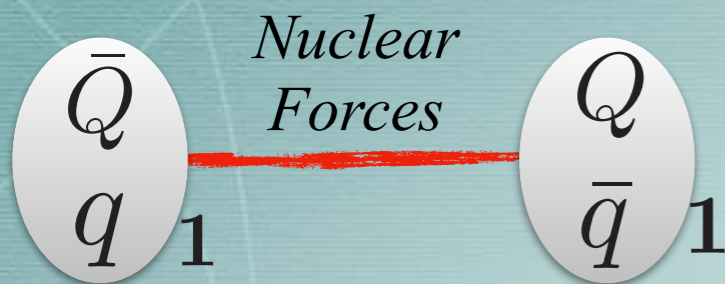


$P_c(4457)^+$
 $P_c(4440)^+$
 $P_c(4312)^+$

Pentaquarks:

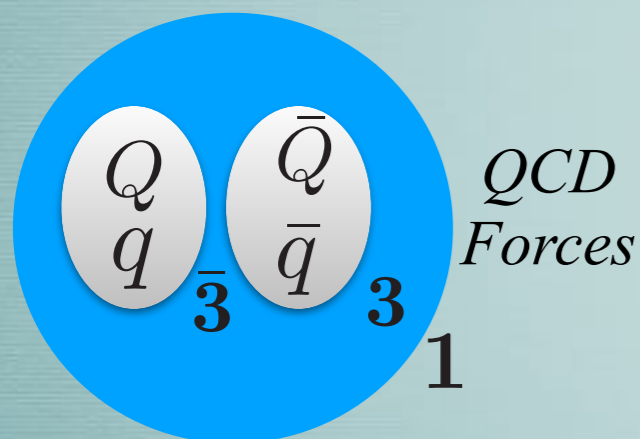
$$\mathcal{P}^+ = c\bar{c}uud$$

Twenty years after X(3872) discovery, there is no consensus yet on the internal structure of Exotic Hidden Charm hadrons



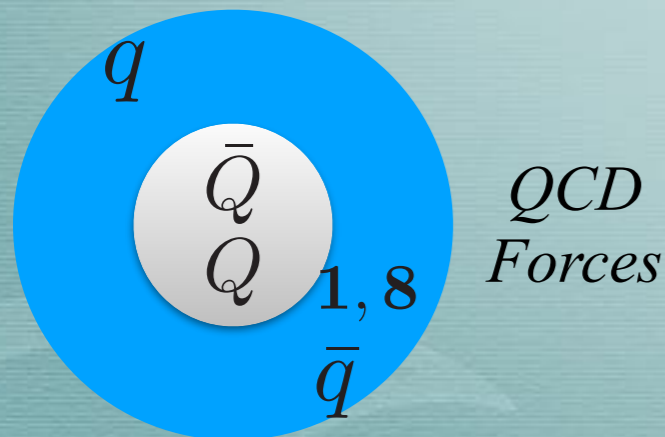
Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)



Compact Diquark-Antidiquark

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005) 014028; D 89 (2014) 114010.



HadroCharmonium (1)
Quarkonium Adjoint Meson (8)

S. Dubynskiy, S. and M. B. Voloshin, Phys. Lett. B 666, (2008) 344.

E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D 90 (2014) 01404

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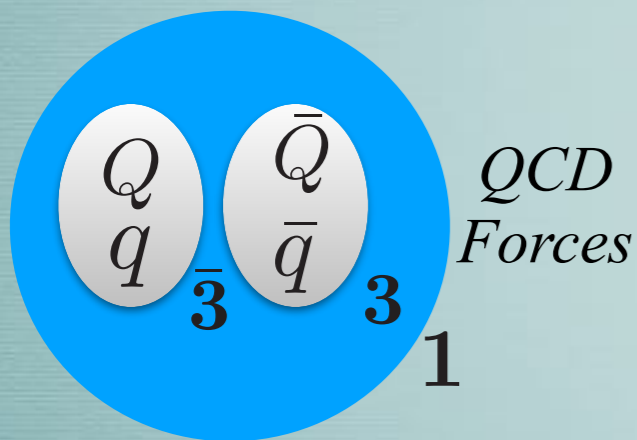


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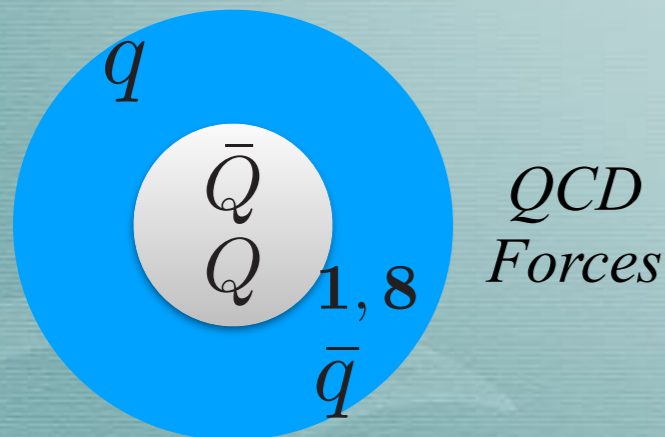
Contact meson interactions in chiral Effective Field Theory

Z. H. Zhang *et al.*, arXiv:2404.11215 [hep-ph]



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E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D 90 (2014) 01404

2. Four-Heavy Quarks Mesons

- Proposed as early as 1985

L. Heller and J. A. Tjon, *On Bound States of Heavy $Q^2\bar{Q}^2$ Systems*, Phys. Rev. **D 32** (1985) 755;

A. V. Berezhnoy, A. V. Luchinsky and A. A. Novoselov, *Tetraquarks Composed of 4 Heavy Quarks*, Phys. Rev. **D 86**, 034004 (2012).

- Widely considered after the observation of doubly heavy baryons and doubly heavy tetraquarks

W.Chen, H.X.Chen, X.Liu, T.G.Steele and S.L.Zhu, Phys. Lett. **B 773**, 247 (2017);

Y.Bai, S.Lu and J.Osborne, arXiv:1612.00012 [hep-ph];

Z.G.Wang, Eur. Phys. J. **C 77**, 432 (2017);

M.Karliner, S.Nussinov and J.L.Rosner, Phys. Rev. **D 95**, 034011 (2017);

J.M.Richard, A.Valcarce and J.Vijande, Phys. Rev. **D 95**, 054019 (2017); J.Wu, Y.R.Liu,

K.Chen, X.Liu and S.L.Zhu, Phys. Rev. **D 97**, 094015 (2018);

M.N.Anwar, J.Ferretti, F.K.Guo, E.Santopinto and B.S.Zou, Eur. Phys. J. **C 78**, 647 (2018);

A.Esposito and A.D.Polosa, Eur. Phys. J. **C 78**, 782 (2018);

M.A.Bedolla, J.Ferretti, C.D.Roberts and E.Santopinto, arXiv:1911.00960 [hep-ph].

Four-Heavy Quarks Mesons (cont'd)

- Claims of a 4μ on peak in $2Y$ spectrum circulated in 2018-2019 : not confirmed.
- In view of the luminosity update of LHCb, as a Genova-Roma collaboration, we set up to compute lifetime & branching ratios for *fully bottom tetraquarks*.

C.Becchi, A.Giachino, L.Maiani and E.Santopinto, Phys. Lett. **B 806**, 135495 (2020).

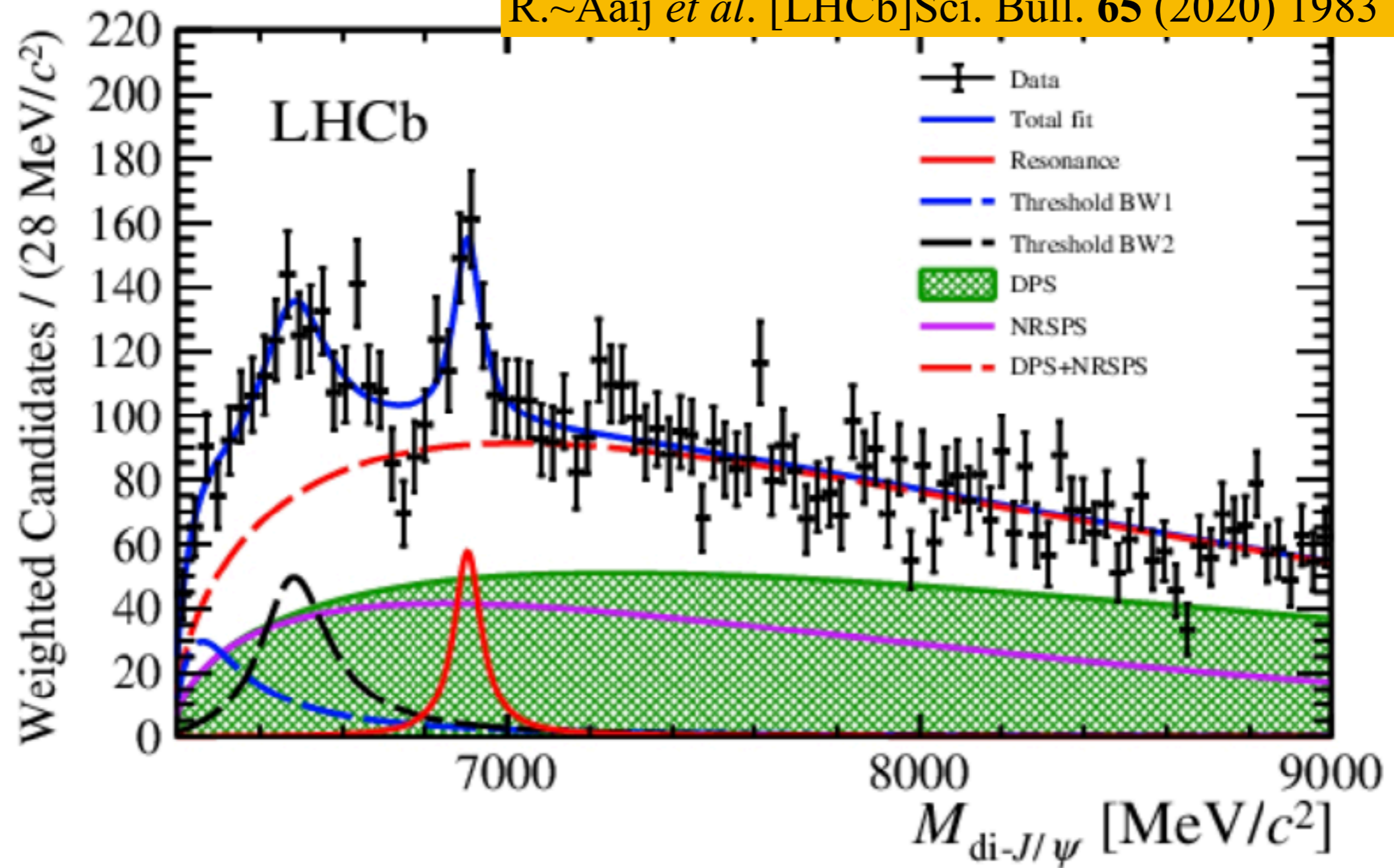
- We observed that 2^{++} states, implied by the quark structure, should be more visible than 0^{++} , with a production cross-section a factor 5 larger and therefore a larger 4μ BF !
- In March 2020, we realised that fully charmed tetraquarks would be more close to be detected and produced a new study for all-charm tetraquarks.

C.Becchi, J.Ferretti, A.Giachino, L.Maiani and E.Santopinto, Phys. Lett. **B 811** (2020) 135952

- Discussions with Sheldon Stone and Liupan An have been very useful.

Nov. 10, 2020: First observation of 4-muon, all-charm, resonances

R.~Aaij *et al.* [LHCb]Sci. Bull. **65** (2020) 1983



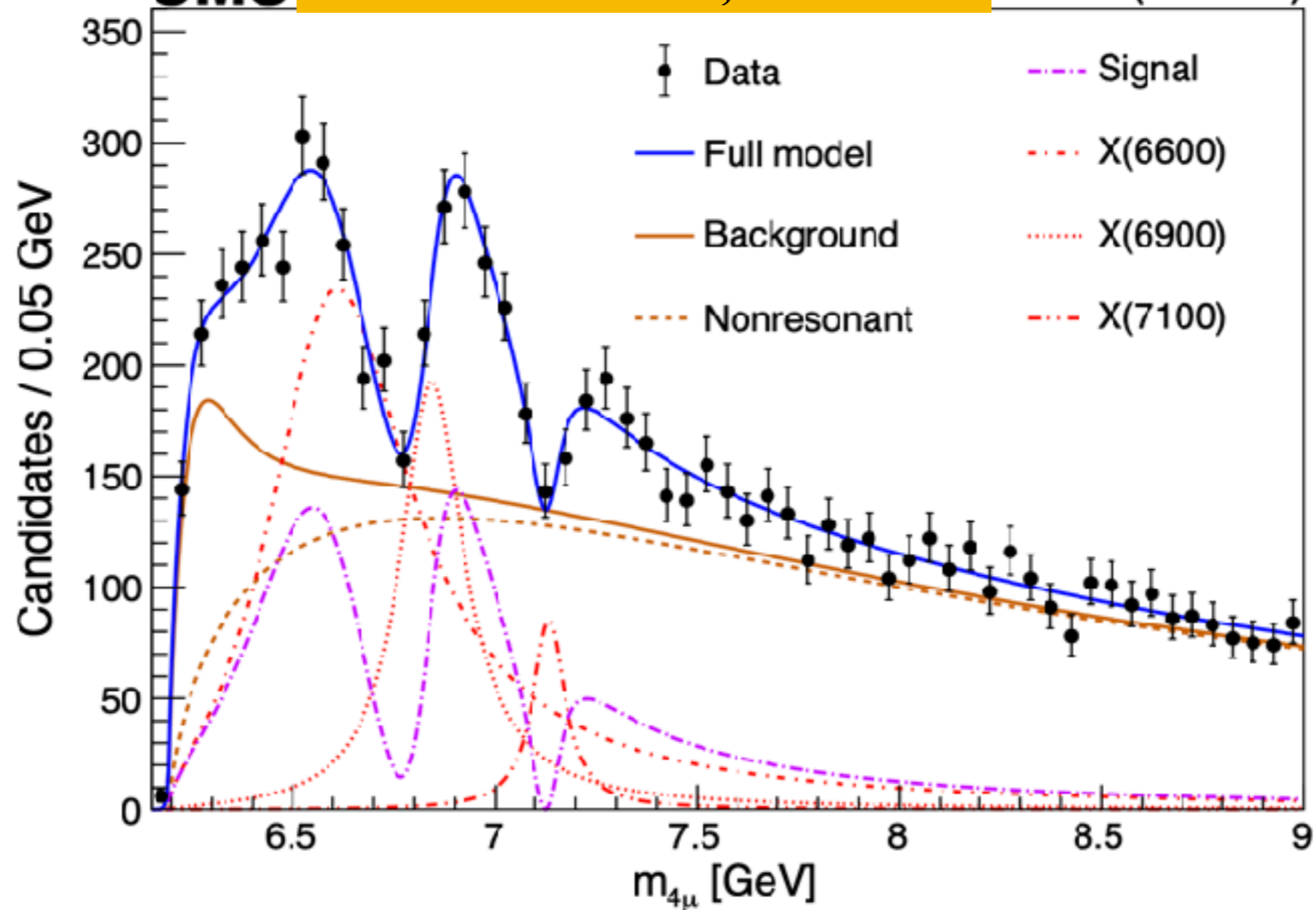


Figure 1: **Candidates for all-charm tetraquarks.** The $J/\psi/\psi \rightarrow \mu^+\mu^-\mu^+\mu^-$ invariant mass $m_{4\mu}$ spectrum shows the three exotic states, $X(6600)$, $X(6900)$, and $X(7100)$. Parameterizations of these states are displayed both individually and as a combined signal that includes quantum-mechanical interference (denoted by “Signal”).

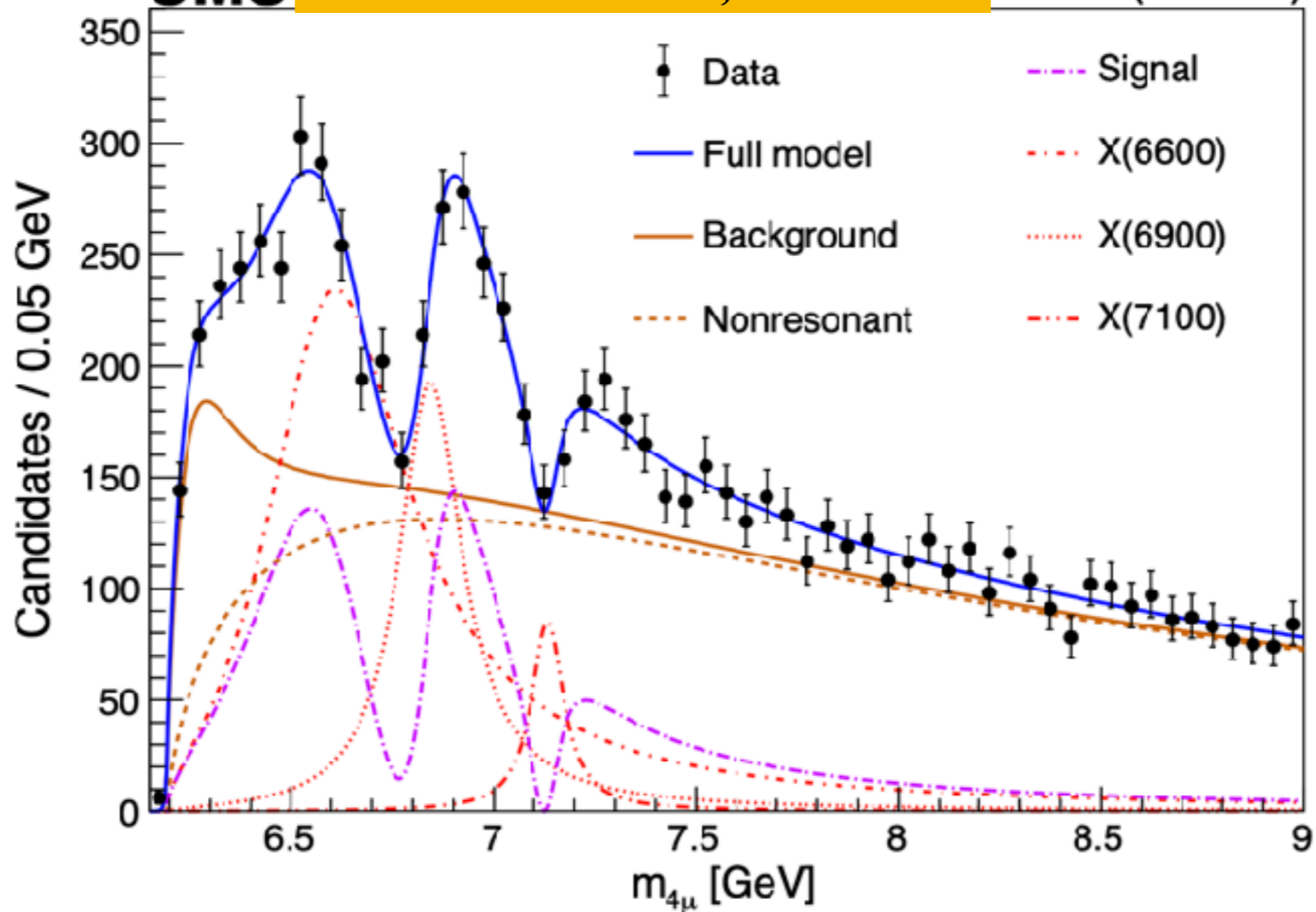


Figure 1: **Candidates for all-charm tetraquarks.** The $J/\psi/\psi \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ invariant mass $m_{4\mu}$ spectrum shows the three exotic states, $X(6600)$, $X(6900)$, and $X(7100)$. Parameterizations of these states are displayed both individually and as a combined signal that includes quantum-mechanical interference (denoted by “Signal”).

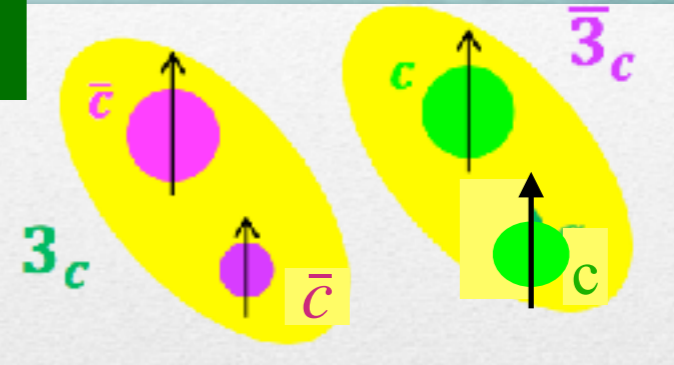
Angular analysis techniques, developed for the Higgs Boson, have been applied to the new states and show *that they have all: $J^{PC} = 2^{++}$.*

?? Radial excitations: N=1, 2, 3 or 2, 3, 4 ??

3. Tetraquark constituent picture of 2 J/Ψ resonances

S-wave, fully charm tetraquarks

$$[cc]_{S=1}^{\bar{3}} \quad [\bar{c}\bar{c}]_{S=1}^3$$



- The diquark and the antidiquark are bound by gluon exchange;
- $[cc]$ in color $\bar{3}$ (QCD interaction is attractive, in 6 is repulsive), $[\bar{c}\bar{c}]$ in color 3 ;
- color antisymmetry and Fermi statistics requires spin to be symmetric under quark exchange, i.e. $S_{cc} = S_{\bar{c}\bar{c}} = 1$:
- S-wave, $C=+1$ states: $J^{PC} = 0^{++}, 2^{++}$, may decay in 2 J/Ψ
- S-wave, $C=-1$ states: $J^{PC} = 1^{+-}$, may decay in $\eta_c + J/\Psi$, no contribution to 4μ decay.

Decays and branching fractions

C.Becchi, A.Giachino, L.Maiani, E.Santopinto, Phys. Lett. **B 806**, 135495 (2020). C.Becchi, J.Ferretti, A.Giachino, L.Maiani E.Santopinto, Phys. Lett. B 811 (2020) 135952 (4 Oct. 2020)

- Decays take place via $c\bar{c}$ annihilation. The starting point is to bring the $c\bar{c}$ pairs together.

$$\begin{aligned} \mathcal{T}(J = 0^{++}) &= \left| (cc)_{\frac{3}{3}}^1 (\bar{c}\bar{c})_{\frac{3}{3}}^1 \right\rangle_1^0 = -\frac{1}{2} \left(\sqrt{\frac{1}{3}} \left| (c\bar{c})_1^1 (c\bar{c})_1^1 \right\rangle_1^0 - \sqrt{\frac{2}{3}} \left| (c\bar{c})_8^1 (c\bar{c})_8^1 \right\rangle_1^0 \right) + \\ &\quad + \frac{\sqrt{3}}{2} \left(\sqrt{\frac{1}{3}} \left| (c\bar{c})_1^0 (c\bar{c})_1^0 \right\rangle_1^0 - \sqrt{\frac{2}{3}} \left| (c\bar{c})_8^0 (c\bar{c})_8^0 \right\rangle_1^0 \right) \\ \mathcal{T}(J = 2^{++}) &= \left| (cc)_{\frac{3}{3}}^1 (\bar{c}\bar{c})_{\frac{3}{3}}^1 \right\rangle_1^2 = \left(\sqrt{\frac{1}{3}} \left| (c\bar{c})_1^1 (c\bar{c})_1^1 \right\rangle_1^2 - \sqrt{\frac{2}{3}} \left| (c\bar{c})_8^1 (c\bar{c})_8^1 \right\rangle_1^2 \right). \end{aligned}$$

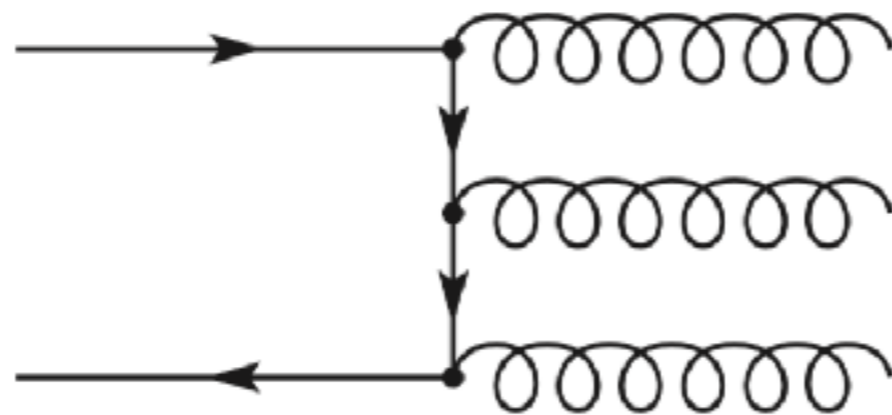
- Normalized tetraquark operators x Spectroscopic Coefficients
- The total T decay rate is the sum of individual decay rates, obtained from the simple formula

$$\Gamma((c\bar{c})_c^s) = |\Psi(0)_{\mathcal{T}}|^2 v \sigma((c\bar{c})_c^s \rightarrow f)$$

$|\Psi(0)_{\mathcal{T}}|^2$ *a new parameter* is the overlap probability of the annihilating pair, v the relative velocity, σ the spin-averaged annihilation cross section in the final state f , suffixes s and c denote spin and color.

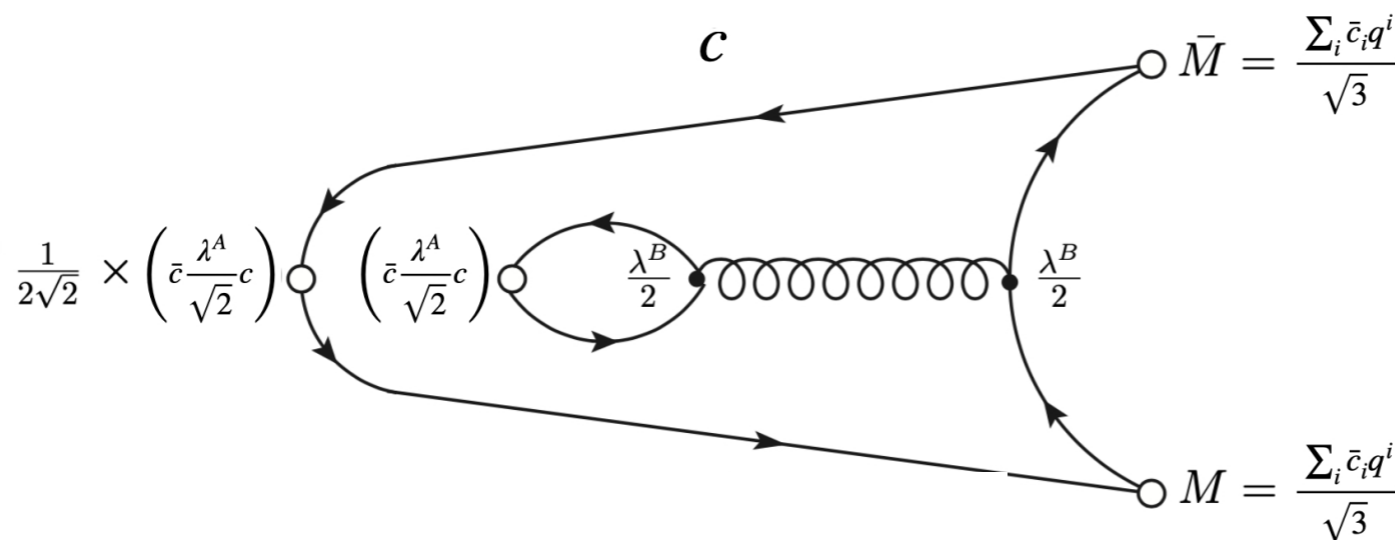
Four possible annihilations: the fate of $c\bar{c}$

Color Singlet
J=1



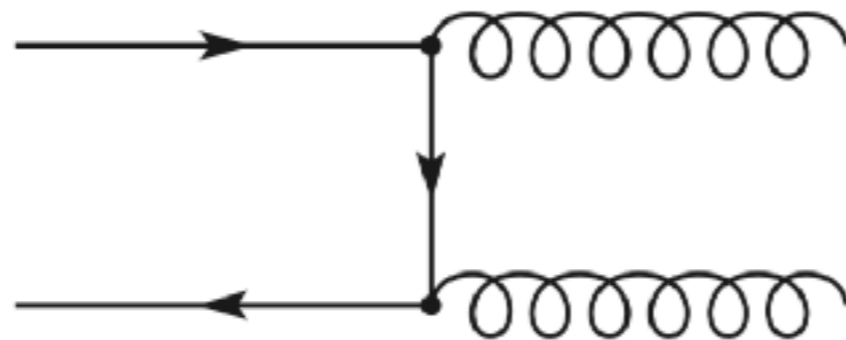
J/Ψ +light hadrons, rate,
including $2J/\Psi$ rate: α_S^3
rate of 4μ final state: α^4

Color Octet
J=1



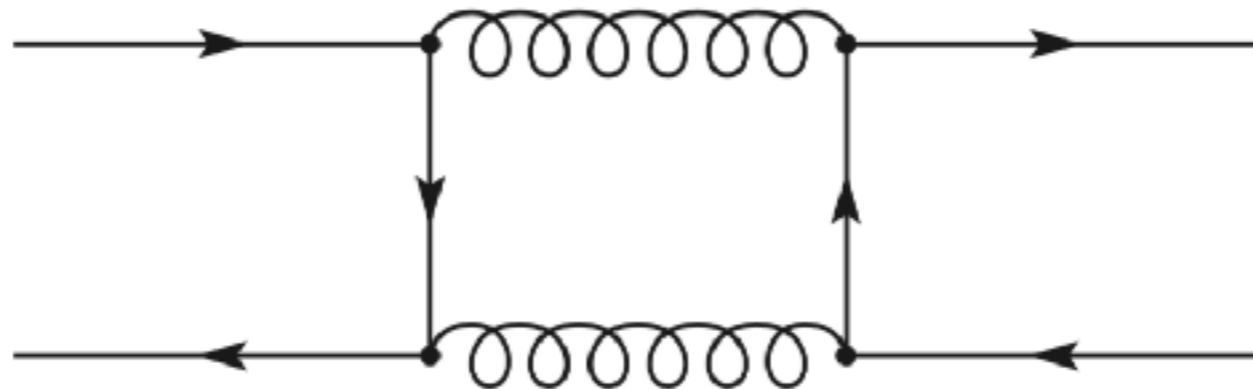
$D^{(*)} \bar{D}^{(*)}$, rate: α_S^2

Color Singlet
J=0



η_c +light hadrons, rate: α_S^2

Color Octet
J=0



Color octet light quarks
to neutralize $(c\bar{c})_8^0$,
rate: α_S^4 (*neglected*)

Visibility

- Visibility of 2^{++} vs. 0^{++} in 4μ event :

$$V = \frac{\sigma(p + p \rightarrow 2^{++})}{\sigma(p + p \rightarrow 0^{++})} \cdot \frac{BF(2^{++} \rightarrow 2 J/\Psi)}{BF(0^{++} \rightarrow 2 J/\Psi)} =$$
$$= 5 \cdot \frac{BF(2^{++} \rightarrow 2 J/\Psi)}{BF(0^{++} \rightarrow 2 J/\Psi)}$$

- we find the BF ratio ~ 6 , so $V \simeq 30$,
- justifying that CMS sees only 2^{++} states
- a different diquark-antidiquark dynamics led to the conclusion of spin $J^{PC} = 0^{++}$ for the 6.900 GeV peak.

M. Karliner, J. Rosner, Phys. Rev. D 102, 114039 (2020).

4. All-charm Tetraquark masses: the Cornell way

- One may try the same strategy of Charmonium, treating diquarks and antidiquarks as single particles with a *mass* $M_{cc} \sim 2M_c$ *left arbitrary*
- *the same Cornell potential as for Charmonia (required for color $\bar{3},3$ diquarks)*

$$V = -\frac{4}{3} \frac{\alpha_s}{r} + kr + 2M_{cc}, \quad \alpha_s = 0.33, \quad k = 0.18$$

- In this way one could try to describe radially as well as orbital excited states.

• $M_c =$	1.447	2S : 6.536;	3S : 6.895(input);	4S : 7.200
• Expt:		6.600;	6.900	7.100

We may not be so far

P-wave diquarkonia quantum numbers:

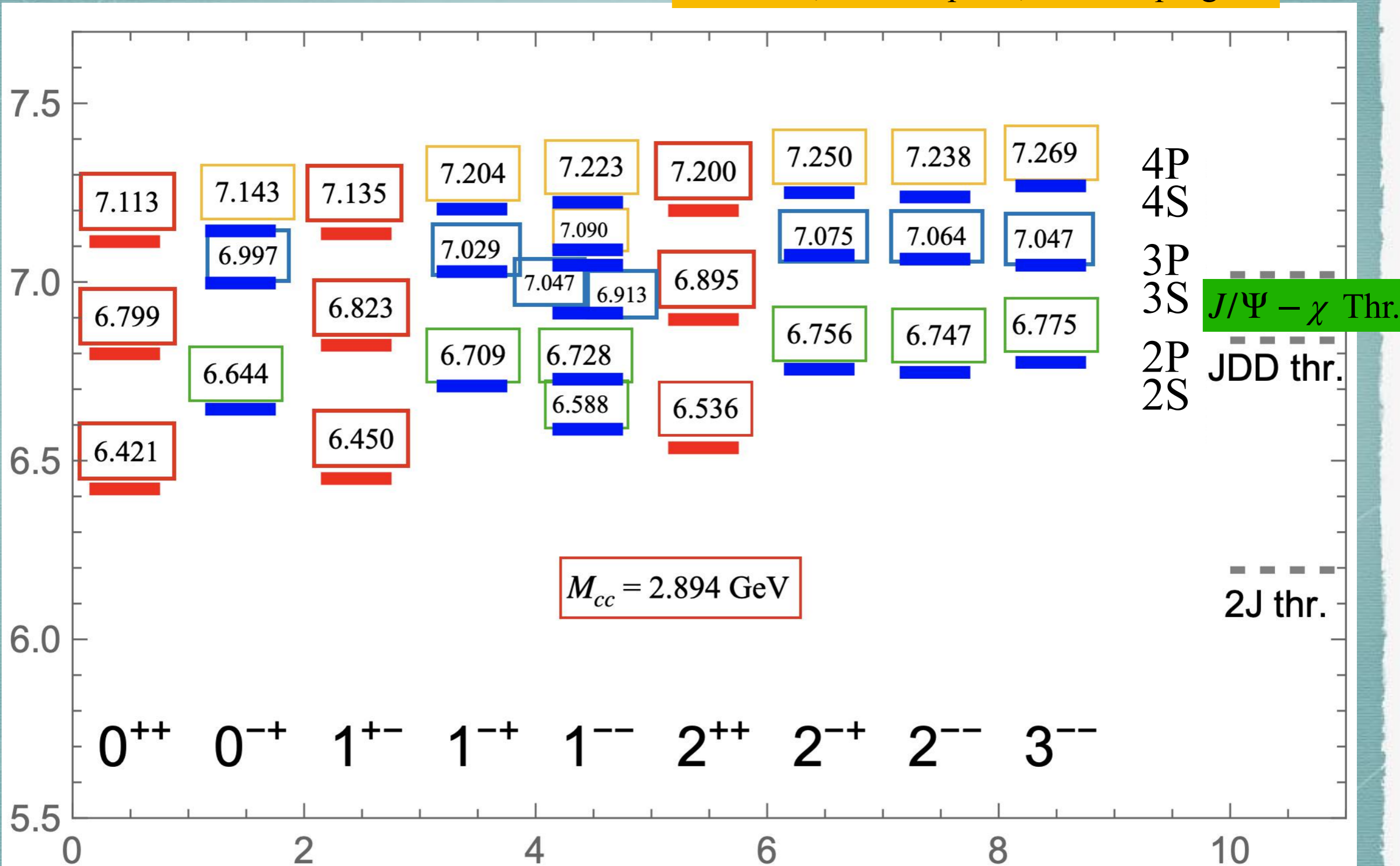
$$L = 1, S = 2, J^{PC} = 3^{--}, 2^{--}, 1^{--}$$

$$L = 1, S = 1, J^{PC} = 2^{-+}, 1^{-+}, 0^{-+}$$

$$L = 1, S = 0, J^{PC} = 1^{--}$$

The spectrum of dicharmonia

L. Maiani, E. Santopinto, Work in progress



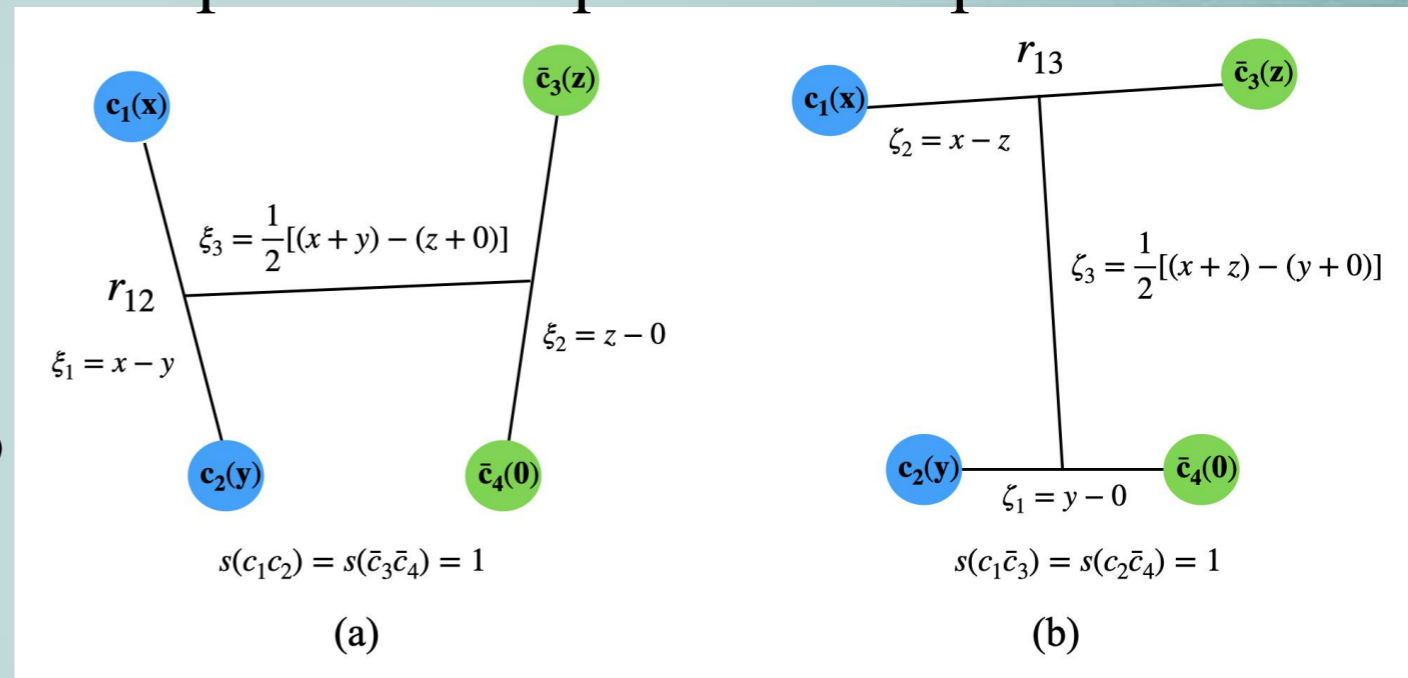
5. P-wave Tetraquark decays

M. N. Anwar, J. Ferretti, F. K. Guo, E. Santopinto and B. S. Zou, Eur. Phys. J. C 78 (2018) 647;
 G. J. Wang, L. Meng, M. Oka and S. L. Zhu, Phys. Rev. D 104 (2021) no.3, 036016;

L. Maiani, E. Santopinto, Work in progress.

• Decay happens via annihilation of a $(\bar{c}c)$ pair, which depends from the overlap parameter $|\Psi_{\mathcal{T},c\bar{c}}(0)|^2 = 0$ in the pointlike diquark-antidiquark approximation.

- we have to allow for the relative motion of the individual quark and antiquark components, see figure.
- a Fierz rearrangement brings (a) into (b) and we obtain similar structures of the previous, S-wave calculation



• Decay modes of the heaviest P-Wave tetraquark $4P, J^{PC} = 3^{--}$:

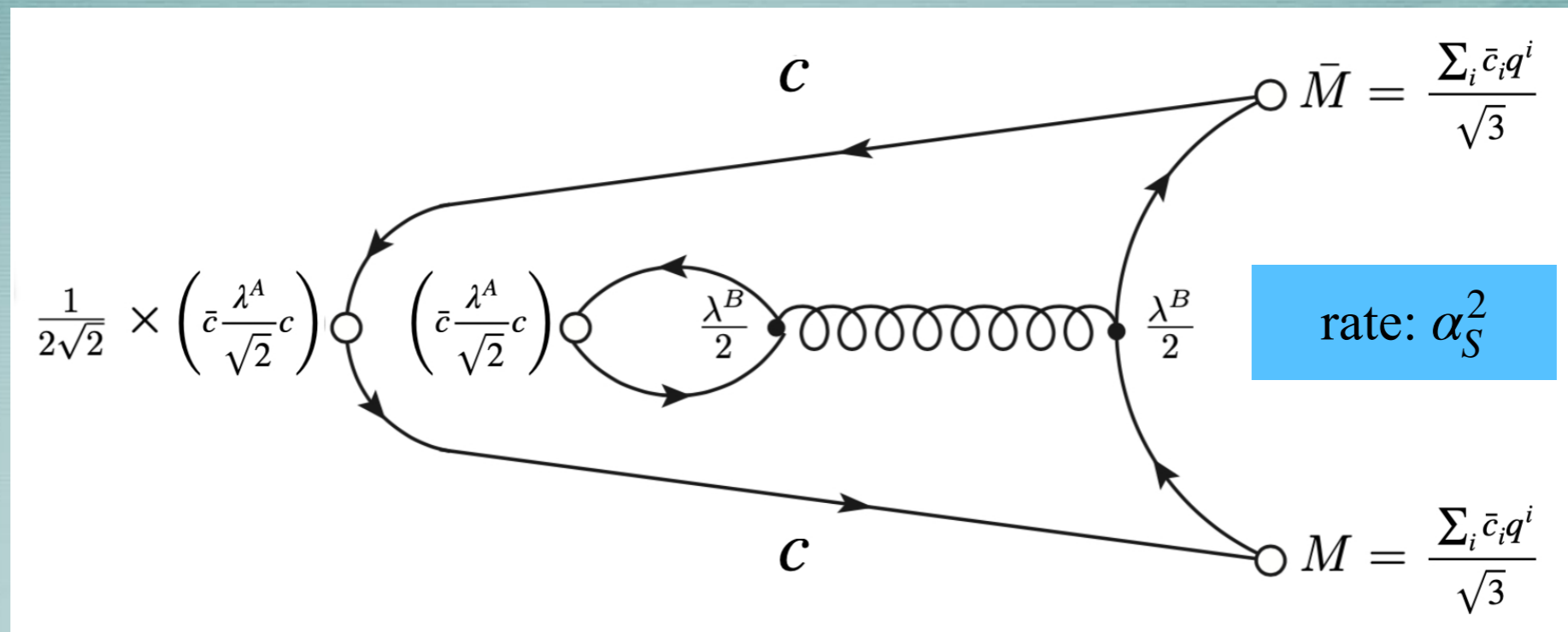
$$\left| \left[[(cc)_3^1, (\bar{c}\bar{c})_3^1]^{2 \times P \text{ wave}} \right]_1^3 \right\rangle = -\sqrt{\frac{1}{3}} \left| \left[[(\bar{c}c)_1^1, (\bar{c}c)_1^1]^{1 \times P \text{ wave}} \right]_1^3 \right\rangle + \sqrt{\frac{2}{3}} \left| \left[[(\bar{c}c)_8^1, (\bar{c}c)_8^1]^{1 \times P \text{ wave}} \right]_1^3 \right\rangle$$

$$\chi_{cc3}(4P) \rightarrow J/\Psi + \chi_{c2}(2P)(3930), \quad D^*(2007)\bar{D}_2^*(2460) + c.c.$$

Both channels can be explored to discover $\chi_{cc3}(4P)$

More explicitly...

Color Octet
J=1



$$\left| \left[[(cc)_{\frac{1}{3}}, (\bar{c}\bar{c})_{\frac{1}{3}}]^{2 \times P \text{ wave}} \right]_{\frac{3}{1}}^3 \right\rangle = -\sqrt{\frac{1}{3}} \left| \left[[(\bar{c}c)_{\frac{1}{1}}, (\bar{c}c)_{\frac{1}{1}}]^{1 \times P \text{ wave}} \right]_{\frac{3}{1}}^3 \right\rangle + \sqrt{\frac{2}{3}} \left| \left[[(\bar{c}c)_{\frac{1}{8}}, (\bar{c}c)_{\frac{1}{8}}]^{1 \times P \text{ wave}} \right]_{\frac{3}{1}}^3 \right\rangle$$

$$\left| \left[[(\bar{c}c)_{\frac{1}{8}}, (\bar{c}c)_{\frac{1}{8}}]^{1 \times P \text{ wave}} \right]_{\frac{3}{1}}^3 \right\rangle \rightarrow \left| \left[[(\bar{q}q)_{\frac{1}{8}}, (\bar{c}c)_{\frac{1}{8}}]^{1 \times P \text{ wave}} \right]_{\frac{3}{1}}^3 \right\rangle \quad (q = u, d, s, c) \rightarrow$$

$$\rightarrow \left| \left[[(\bar{q}c)_{\frac{1}{1}}, (\bar{c}q)_{\frac{1}{1}}]^{1 \times P \text{ wave}} \right]_{\frac{3}{1}}^3 \right\rangle \rightarrow [D^*(2007) \bar{D}_2^*(2460)] + [J/\Psi + \chi_{c2}(2P)]$$

q=u,d,s 1⁻⁻ 2⁺⁺
and their radial excitations

for q=c

6. An interesting challenge

- In a similar way, we expect to see the decay of the 2^{--} state according to

$$\chi_{cc2}(4P) \rightarrow J/\Psi + \chi_{c1}(2P)_{P\text{-wave}}$$

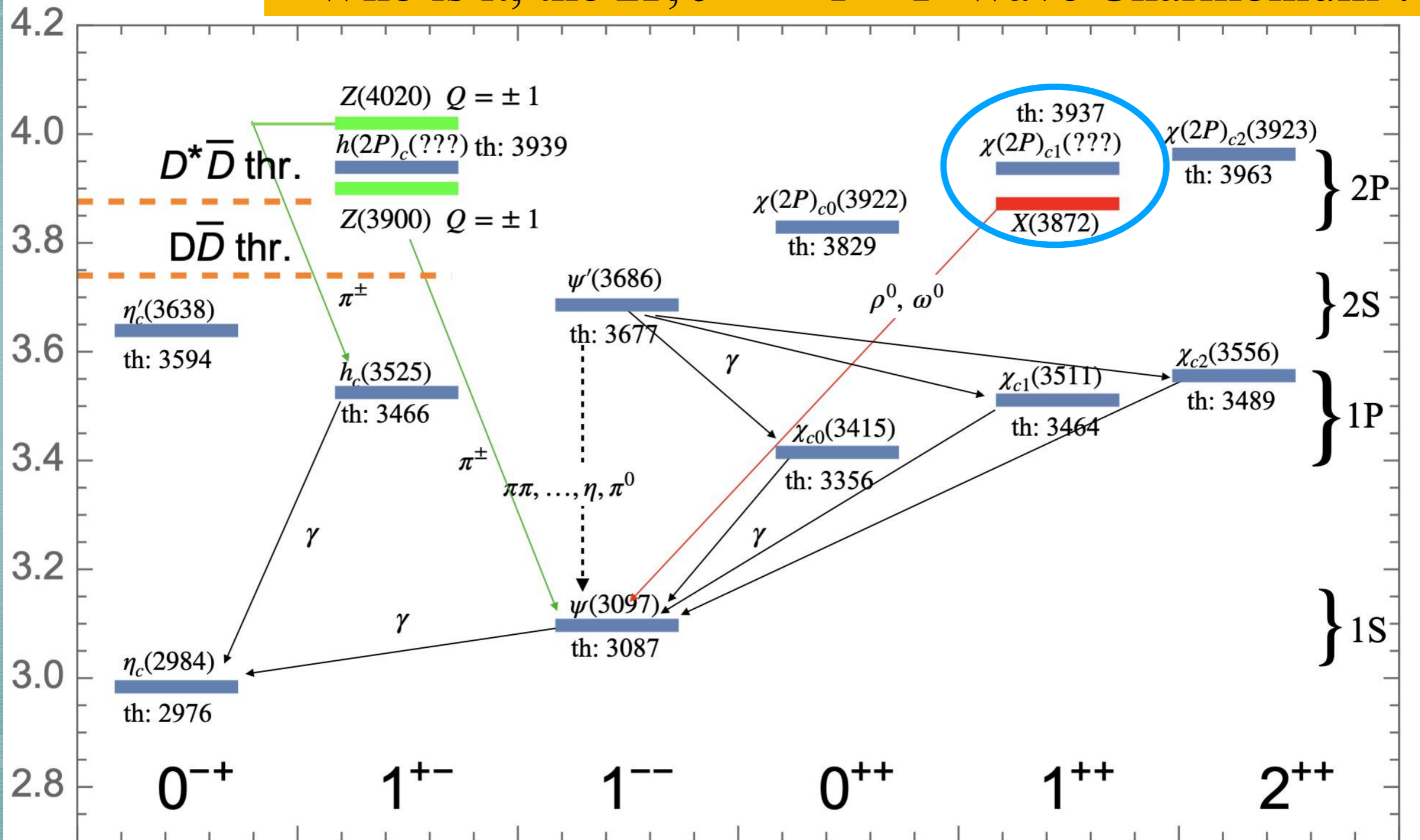
- In PdG, the charmonium state reported as $\chi_{c1}(2P)$ is the former X(3872), which was the first discovered "exotic hadron".
- X(3872) is most likely a tetraquark, judging from its consistent $J/\Psi + \rho^0$ decay. In addition, the prediction for the 2P, spin 1 charmonium, computed with the Cornell potential is rather $M = 3.940 - 3.950$ GeV.

The study of the decay of $\chi_{cc2}(4P)$ may lead to *the identification of the true $\chi_{c1}(2P)$* , some 50-70 MeV above the X(3872).

A possible candidate: the $D^*\bar{D}$ peak with mass 3940 MeV, observed by Belle in $e^+e^- \rightarrow J/\Psi + \text{hadrons}$, classified in PdG as X(3940), without specification of quantum numbers.

Charmonia, again

Who is it, the 2P, $J^{PC} = 1^{++}$ P-Wave Charmonium ??



7. QCD tetraquarks vs the latest Hadron Molecules model

- For molecular tetraquarks, in the limit of very massive charm quark, the light quark total spin is a separately conserved quantity (this is the *light quark spin symmetry in the static quark approximation* introduced by Isgur and Wise)
- For hidden charm molecules $(\bar{c}q)(\bar{q}'c)$, flavour symmetry, e.g. Isospin, is also an independent (commuting) conserved quantity. The possible combinations of light and heavy spin generate six states with definite Isospin, total angular momentum and charge conjugation: [Z. H. Zhang *et al.*, arXiv:2404.11215 \[hep-ph\]](#)

$$J_I^{PC} = 0_I^{++}, 1_I^{+-}, 1_I^{\prime+-}, 1_I^{++}, 0_I^{\prime++}, 2_I^{++}$$

- These are *the same* J_I^{PC} states predicted for diquark-antidiquark tetraquarks of the form $[cq]_S^{\bar{3}}[\bar{c}\bar{q}']_{\bar{S}}^{\bar{3}}$, with spin S and \bar{S} , $J = S + \bar{S}$.
[L. Maiani *et al.*, Phys. Rev. D 89 \(2014\), 114010; Phys. Rev. D 94 \(2016\), 054026\].](#)
- Notably, the molecular list includes the $I=1$ partner of $X(3872)$, i.e. X^+ .

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May this be an indication that new molecular paradigm and the tetraquark QCD description are two aspects of the same physics...

....as it happened in the 70's for Regge poles (t-channel physics) and quark bound states (s-channel physics)

!!??!!

A tale from the years 1970's:
Leonard Susskind in *Quark Confinement*
Cambridge University Press: 03 February 2010

By the end of the 1960s our empirical knowledge of hadrons consisted of a vast mountain of data about their spectrum, their low- and high-energy interactions, and their electromagnetic and weak properties.

To some extent the story of the eventual interpretation in terms of QCD was like digging a tunnel through the mountain with crews of diggers starting independently at the two ends.

At one end was the short-distance behavior of local currents and its interpretation in terms of freely moving quark-parton constituents.

At the other end was the low-momentum-transfer Regge structure including a spectrum of highly excited rotational states.... but no free quarks.

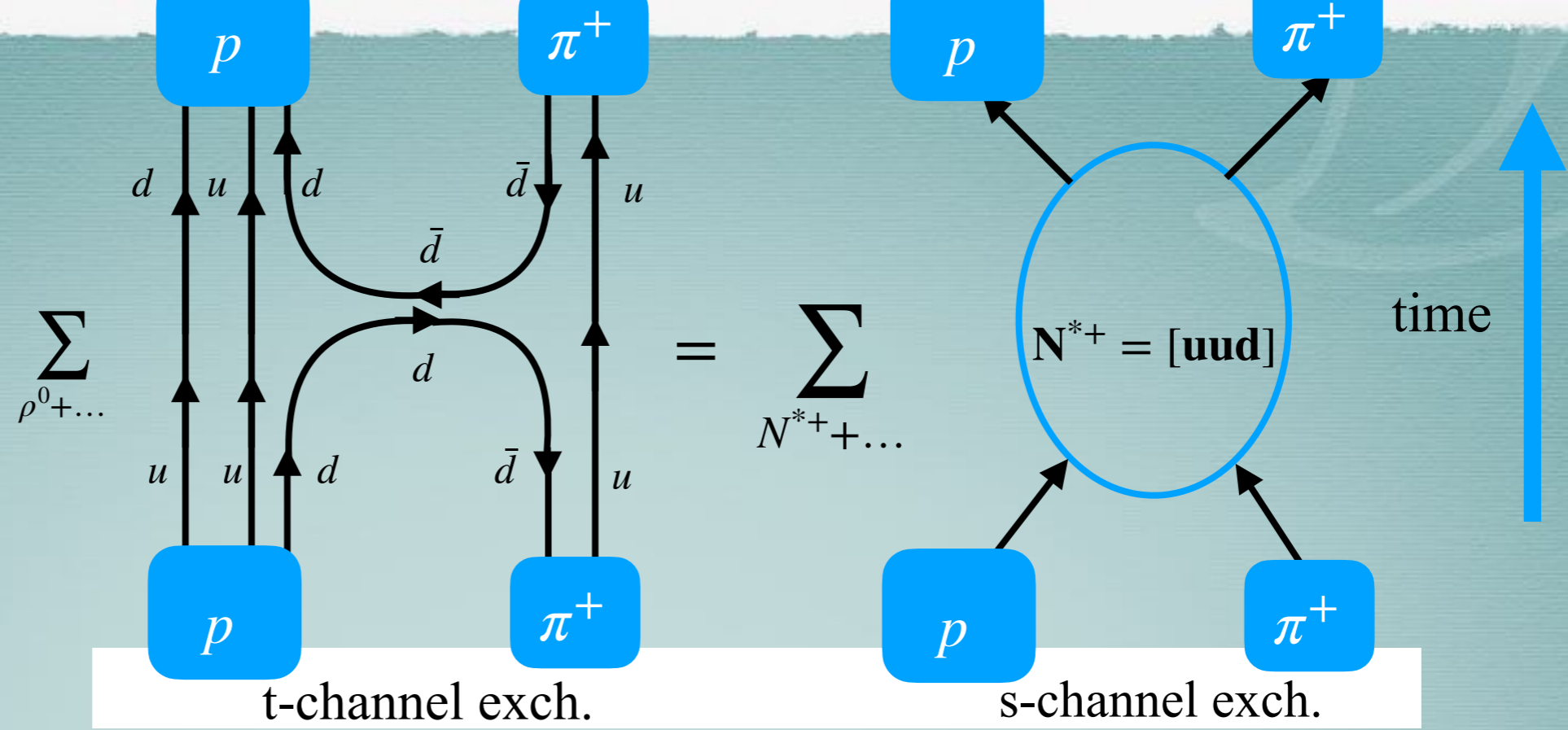
Sometime in 1973 the two tunnel crews discovered that they had met and a complete picture of the strong interactions existed. Of course the two crews were not entirely unaware of each other.

The Regge workers were beginning to organize the trajectories by quantum numbers suggested by the quark model.

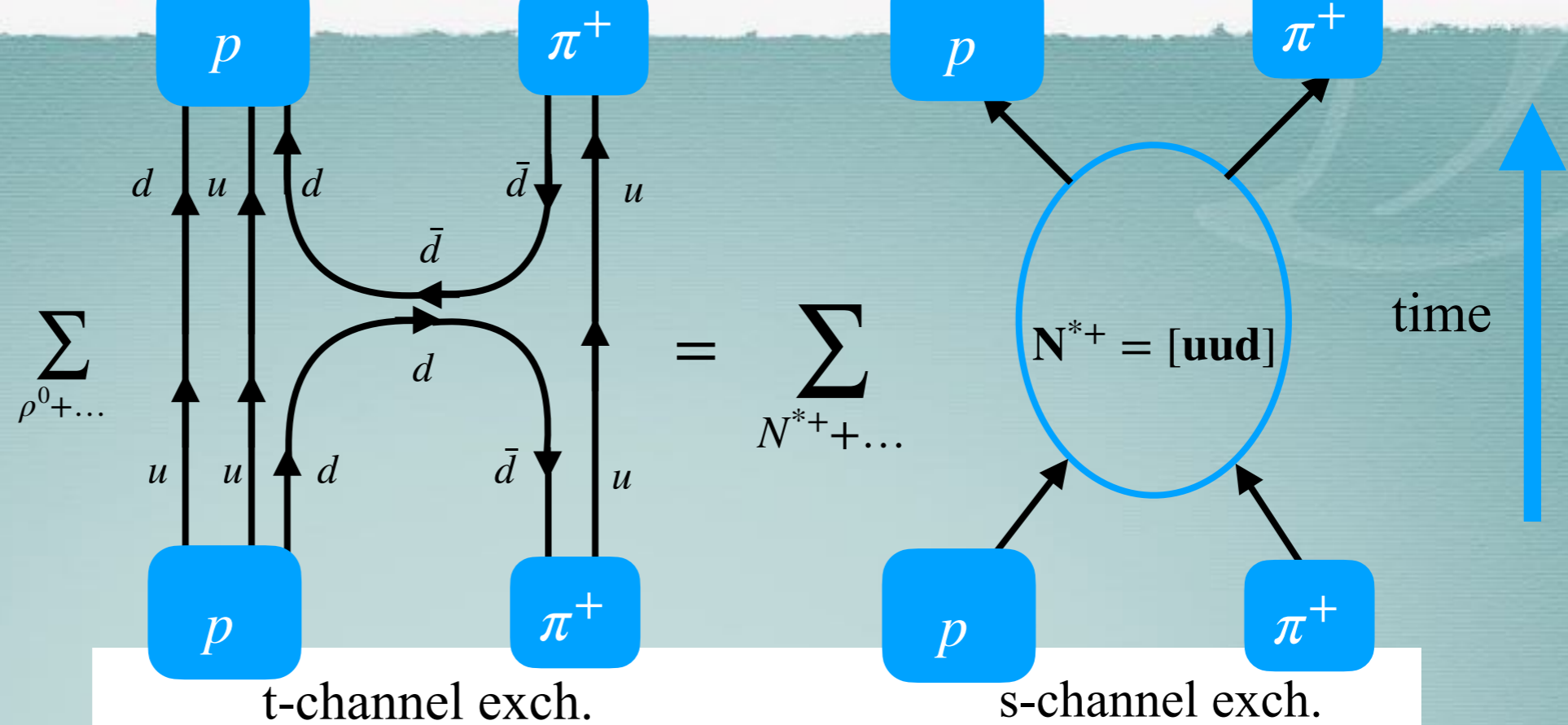
Eventually, the Regge picture culminated with a set of scattering amplitudes based on the *duality principle of R. Dolen, D. Horn, and C. Schmidt (1968):*

Sum t-exchange amplitudes = Sum s-channel resonance amplitudes

Dolen-Horn-Schmidt duality

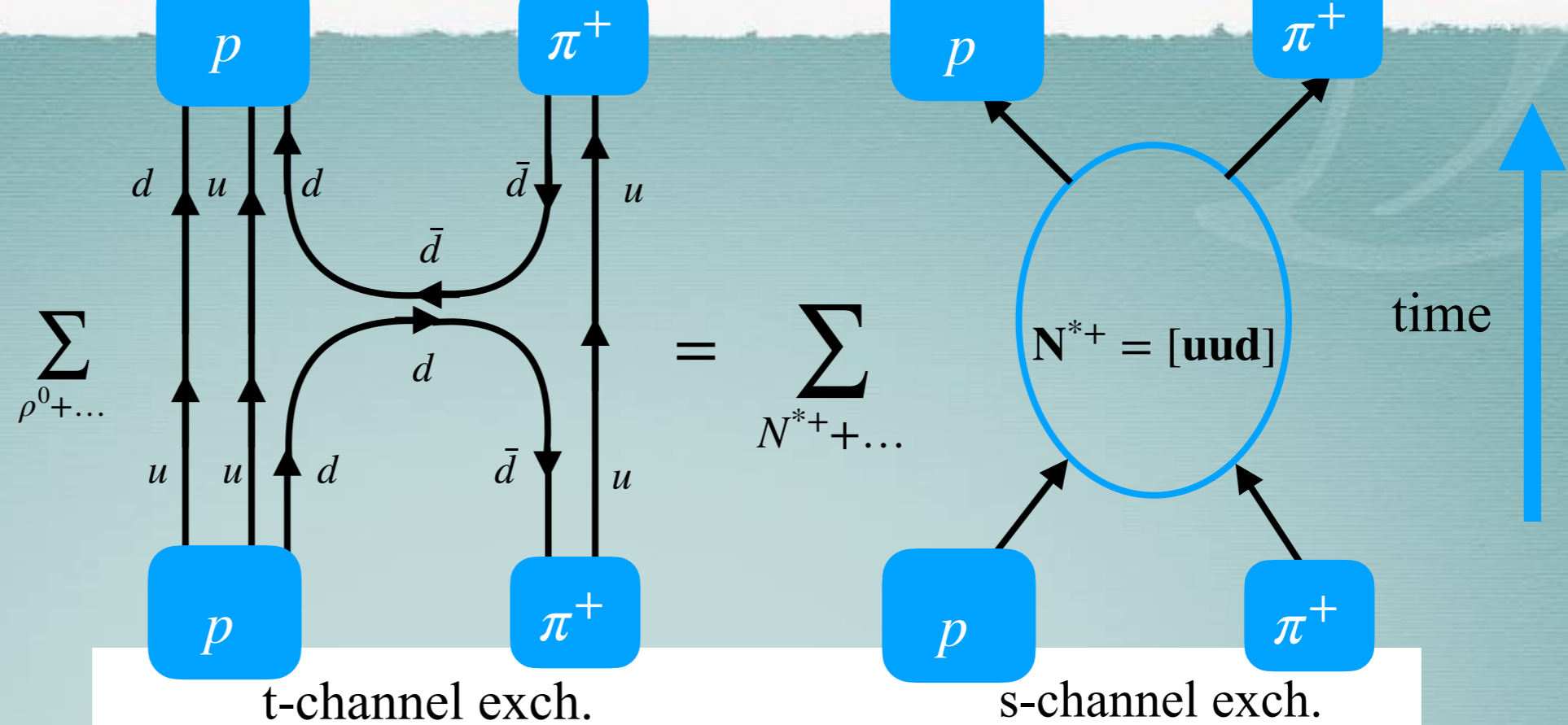


Dolen-Horn-Schmidt duality



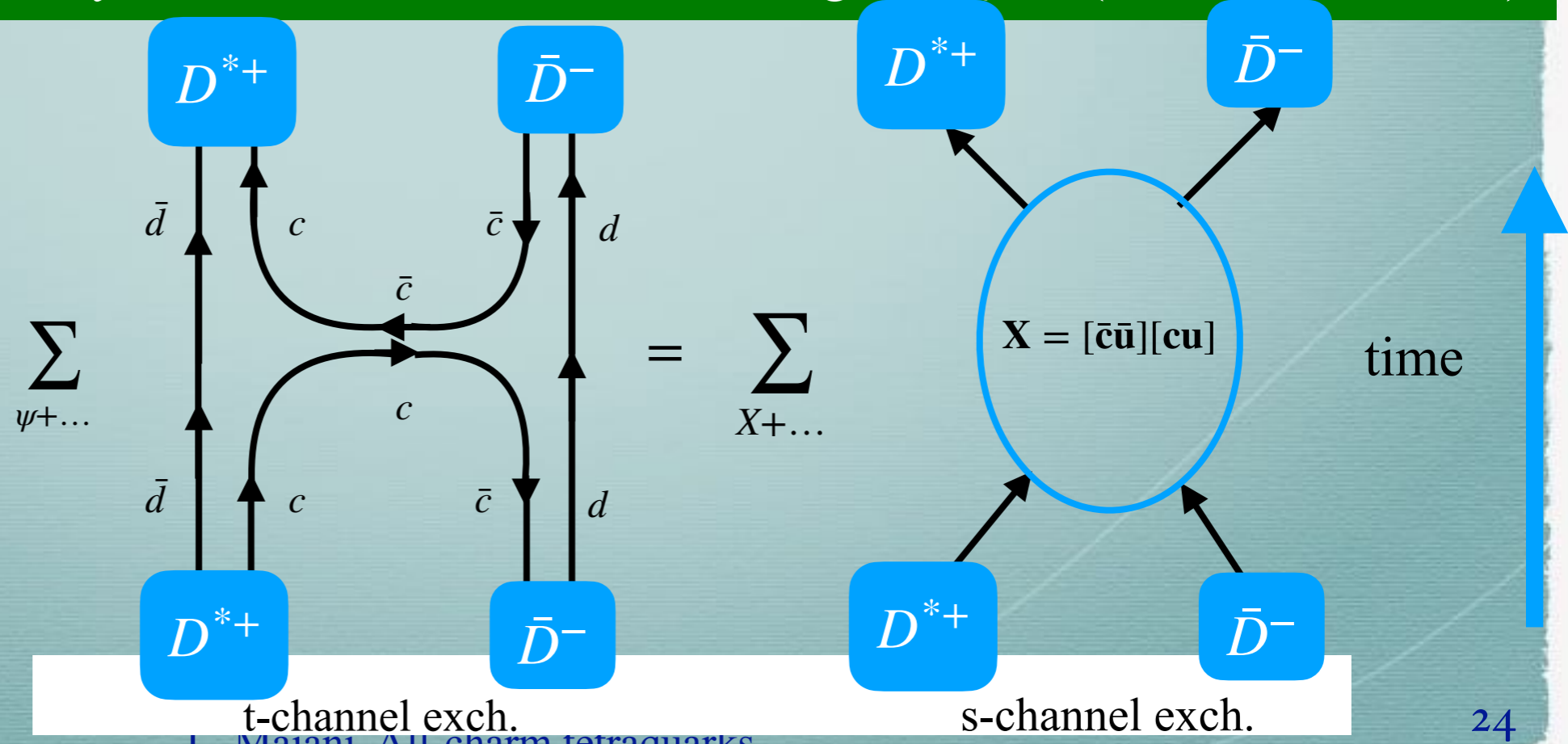
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 Duality is a property shared by the Veneziano Model and Large N QCD (G. 't Hooft, 1973)

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Extended to Exotics ? (Tetraquarks)



Summing up

- The observation of the 4 muons peaks opens several exciting possibilities and it may be a real *game changer*.
- The observed peaks above the 2 J/Ψ threshold fit well with 2^{++} [$cc\bar{c}\bar{c}$] resonances, for production rate and total width;
- a 0^{++} peak is predicted with mass ~ 6300 MeV, similar width, large decay rate in η_c pairs or $D\bar{D}$ pairs;
- P-waves:
- Challenges:
 1. Find the 0^{++} and its radial excitations
 2. Find radial and P-wave all-c tetraquarks
 3. Find all-b tetraquarks
 4. Find a convincing theoretical picture, comparable to that of Charmonia, to compute masses, production cross sections and decay rate of the new family.

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Can we end up with a convincing QCD theory of the Exotic Hadrons ?