QCD axion at the QCD scale ?

Axion Theory and Lattice QCD
8 April 2026 - INFN Frascati

Luca Di Luzio

INFN PADOVA

What is this talk about ?

[https://arxiv.org/abs/2601.02465]

[Submitted on 5 Jan 2026 (v1), last revised 23 Jan 2026 (this version, v2)]

GeV-scale QCD Axion

Hitoshi Murayama

In order to solve the strong CP problem, we study the possibility that the Peccei--Quinn symmetry is broken {\it below}\/ the QCD scale. We find that a QCD axion can be above GeV, and may be among the observed η resonances. It is immune to quantum gravity corrections. The only fermion that has a $U(1)$ Peccei--Quinn charge is the right-handed up quark. Flavor-changing neutral currents are surprisingly small. All accelerator and astrophysical limits can be evaded. The most significant constraint is the mass splitting between $\pi^\pm$ and π^0. In a UV completed model, LHC can look for a heavy quark pair $\mathcal{D}\overline{\mathcal{D}}$ followed by the decay $\mathcal{D} \to W + u$ or a single production $qu \to q\,\mathcal{U}$ followed by $\mathcal{U} \to uZ, uh$. There can be an $O(1)$ contribution $h \to u\bar{u}\varphi$ in the measurement of $h \to gg$ or permille effects on the hadronic Z width at a Higgs factory.

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→ *We found this to be a novel and very interesting idea, deserving further exploration*

What is this talk about ?

[https://arxiv.org/abs/2602.10057]

[Submitted on 10 Feb 2026 (v1), last revised 24 Feb 2026 (this version, v2)]

The Too Visible QCD Axion

Luca Di Luzio, Michele Redi, Alessandro Strumia, Andrea Tesi, Arsenii V. Titov

Murayama proposed a GeV-scale axion theory where the up-quark mass term is generated dynamically by the QCD chiral condensate, spontaneously breaking a Peccei-Quinn symmetry. It predicts a too large mass splitting between neutral and charged pions. Trying to solve this problem we explore extensions. Despite some partial improvements, we identify a structural obstruction: the new Peccei-Quinn spurion breaks the accidental isospin symmetry of the chiral Lagrangian, leading to an enhanced higher-order operator. As a consequence, pion scatterings too are distorted. We also examine the limit in which the axion becomes light, finding that it is excluded by fifth-force constraints.

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What is this talk about ?

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What is this talk about ?

final word might require lattice simulation
of new, deformed QCD theory

We found this to be a novel and very interesting idea, deserving further exploration

Strong CP ($m_u = 0$)

- Most elegant solution would be massless up-quark

  - accidental PQ symmetry acting on $u_R \to e^{i\alpha} u_R$

  - excluded @ 27σ by the lattice $\quad \overline{m}_u = 2.20(8)\,\text{MeV},\ \overline{m}_d = 4.69(5)\,\text{MeV},\ r \equiv \dfrac{\overline{m}_u}{\overline{m}_d} = 0.477(19)$ [PDG]

Strong CP ($m_u = 0$)

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- How chiral Lagrangian "reacts" to $m_u = 0$ — proxy for η' mass

  - quark mass matrix spurion would have a zero eigenvalue — [Witten NPB 156 (1979), Veneziano NPB 159 (1979)]

$$U \approx 1_3 + \sqrt{2} i \Pi / f_\pi + \cdots \qquad \mathcal{L} \supset \Lambda^3 \mathrm{tr}[M_{m_u=0} U^* + h.c] + \frac{\Lambda^4}{N_c} (i \log \frac{\det U}{\det U^*} - \theta)^2$$

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  - PQ rotation leaves the action invariant, up to a shift in theta

$$U \to L U R^\dagger = U e^{-i\alpha T_{\mathrm{PQ}}} \qquad \theta \to \theta - 2\alpha \mathrm{tr}[T_{\mathrm{PQ}}] \qquad T_{\mathrm{PQ}} = \mathrm{diag}\,(1,0,0)$$

→ in the massless up-quark limit η' acts as an axion

Kaplan-Manohar ambiguity

- For some time (from 1986 to ~ 2000) $m_u = 0$ was not obviously ruled out

  - mass ratios m_u/m_d and m_s/m_d can be extracted from fit to pseudo-scalar masses in χPT

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  - at LO [Weinberg (1977)]

$$
\begin{aligned}
m_{\pi^0}^2 &= B\,(m_u + m_d)\,, \\
m_{\pi^\pm}^2 &= B\,(m_u + m_d) + \Delta_{\rm em}, \\
m_{K^0}^2 &= m_{\overline{K}^0}^2 = B\,(m_d + m_s)\,, \\
m_{K^\pm}^2 &= B\,(m_u + m_s) + \Delta_{\rm em}, \\
m_\eta^2 &= \frac{1}{3} B\,(m_u + m_d + 4m_s)\,,
\end{aligned}
$$

$$
\begin{aligned}
\frac{m_u}{m_d} &= \frac{2m_{\pi^0}^2 - m_{\pi^+}^2 + m_{K^+}^2 - m_{K^0}^2}{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2} = 0.56, \\
\frac{m_s}{m_d} &= \frac{m_{K^0}^2 + m_{K^+}^2 - m_{\pi^+}^2}{m_{K^0}^2 + m_{\pi^+}^2 - m_{K^+}^2} = 20.2,
\end{aligned}
$$

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  - potential ambiguity at NLO [Kaplan Manohar PRL 56 (1986)]

$M_q\,,\ (M_q^{-1})^\dagger \det(M_q^\dagger)$ → transform in the same way under the chiral symmetry

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- correspond to a linear combination of $\mathcal{O}(p^4)$ terms (L_6, L_7, L_8)

$$\mathcal{O}_{\rm KM} \equiv -\frac{1}{2}\left[\mathrm{tr}(MU^\dagger MU^\dagger) - \mathrm{tr}(MU^\dagger)^2 + \mathrm{h.c.}\right]$$

$$\Delta\mathscr{L} = \frac{B_0 f_\pi^2}{2} x_{\rm KM}\mathcal{O}_{\rm KM}$$

↓

$\mathcal{O}\left(1/(4\pi f_\pi)\right)$

- KM operator "dresses" LO spurion

$$M_q \to M_q|_{\rm KM} = M_q + x_{\rm KM}(M_q^{-1})^\dagger \det(M_q^\dagger)$$

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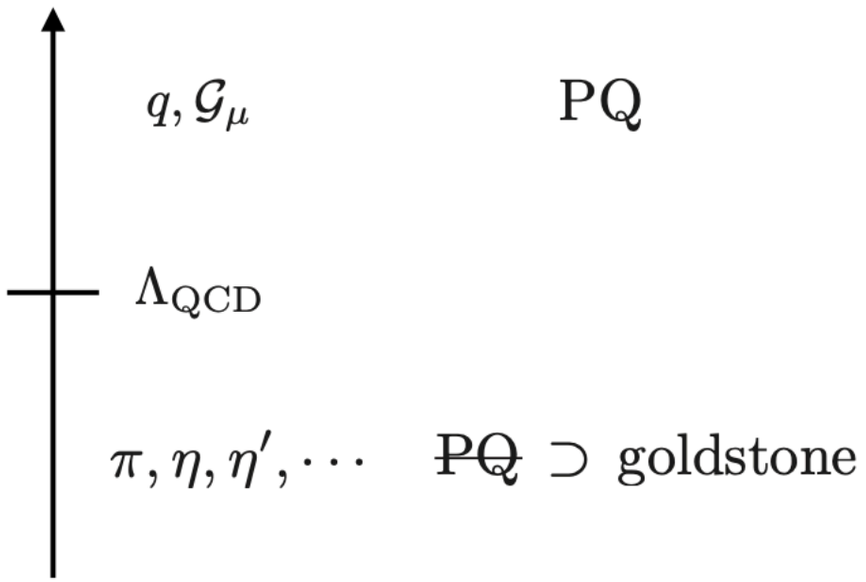
- KM operator "dresses" LO spurion

$$M_q \to M_q|_{\rm KM} = M_q + x_{\rm KM}(M_q^{-1})^\dagger \det(M_q^\dagger) \quad \longrightarrow \quad m_u|_{\rm KM} = m_u + x_{\rm KM} m_s m_d$$

- $m_u = 0$ *could be made consistent in χPT fits thanks to NLO effects*

Murayama's proposal

- Breaking a new PQ symmetry spontaneously, thanks to chiral symmetry breaking



How to realise this in practice ?

PQ needs anomaly under color

New light colored fermions <u>not</u> allowed

Murayama's proposal

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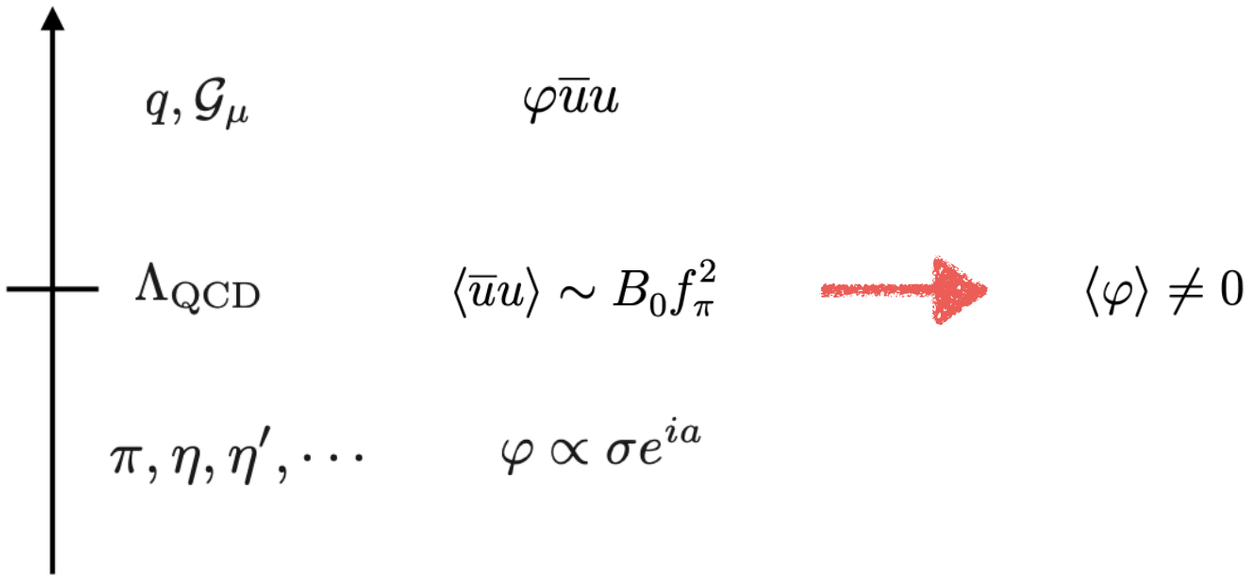

[akin to bosonic technicolor with elementary Higgs; Samuel NPB 347 (1990)]

- The only way to have PQ broken at the QCD scale:

  - is to deform the massless up-quark solution …

  - … rendering the up-quark massless prior to chiral SSB

  - to mitigate the impact of lattice results, effective quark masses induced by a PQ scalar

→ *very different from usual QCD axion, where PQ is broken at high energies !*

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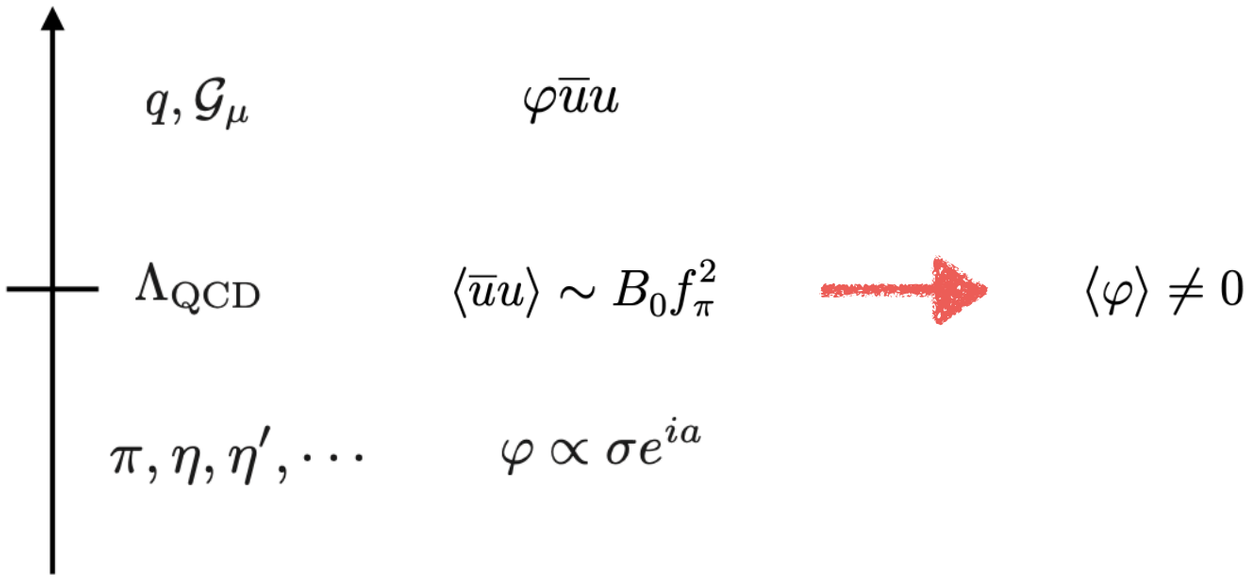

- Interesting (but critical) consequences:

  - new scalar/pseudo-scalar around GeV scale (hidden among QCD resonances)

  - no astrophysical limits, lab constraints (flavor & collider) seem OK

  - solves PQ quality problem, but no dark matter since unstable

  - sizeable coupling to light quarks, alters χPT predictions

Model and generalizations

- Renormalizable Lagrangian below the electroweak scale*

$$\mathscr{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} + i \sum_{q=u,d,s} \bar{q} \not{D} q$$

$$+ f_\varphi^2 \left[|\partial_\mu \varphi|^2 - m_\varphi^2 |\varphi|^2 - \lambda f_\varphi^2 |\varphi|^4 \right] - \sum_{q=\{u,d,s\}} m_q (\sqrt{2}\varphi)^{n_q} \bar{q}_L q_R + \text{h.c.} \qquad m_q = \kappa_q \frac{f_\varphi}{\sqrt{2}}$$

  - PQ complex field φ (dimensionless), no quadratic instability $m_\varphi^2 > 0$

  - if you prefer, work with $\varphi_{\text{can}} \equiv f_\varphi \, \varphi$

**UV completion at the EW scale discussed by Murayama (see backup slides)*

Model and generalizations

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- Light quarks can have PQ charges:

$$\frac{E}{N} \equiv \frac{2}{3}\frac{(4n_u + n_d + n_s)}{\sum_q n_q}, \quad N_{\rm DW} = \left|\sum_q n_q\right|$$

$\{n_u, n_d, n_s\} = \{1,0,0\}$ [massless up] **[Murayama]**

$\{n_u, n_d, n_s\} = \{0,1,0\}$ [massless down]

$\{n_u, n_d, n_s\} = \{1,1,0\}$ [massless up-down]

$\{n_u, n_d, n_s\} = \{0,0,1\}$ [massless strange :)]

$$T_{\rm PQ} = {\rm diag}[n_u, n_d, n_s]$$

PQ symmetry generator,
needs to have non-zero trace

Matching to chiral Lagrangian

- PQ scalar φ as an external field, keep track in the mass matrix spurion $M_q(\varphi)$

$$\mathcal{L}_{\pi\varphi} = f_\varphi^2\left[|\partial_\mu\varphi|^2 - m_\varphi^2|\varphi|^2 - \lambda f_\varphi^2|\varphi|^4\right]$$
$$+ \frac{f_\pi^2}{4}\mathrm{tr}[D_\mu U D^\mu U^\dagger] + \frac{f_\pi^2 B_0}{2}\mathrm{tr}[M_q U^\dagger + M_q^\dagger U] - \frac{m_1^2 f_\pi^2}{12}\left[\left(-\frac{i}{2}\ln\det U + h.c.\right) - \theta\right]^2$$

- $U = \exp(i\sqrt{2}\Pi/f_\pi)$ transforms as $U \to LUR^\dagger$ under $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$

$$\frac{\Pi}{f_\pi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} \end{pmatrix}$$

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- quark mass matrix depends on PQ scalar

$$M_q = \mathrm{diag}\left[m_u(\sqrt{2}\varphi)^{n_u},\, m_d(\sqrt{2}\varphi)^{n_d},\, m_s(\sqrt{2}\varphi)^{n_s}\right]$$

- SSB of PQ controlled by chiral condensate, e.g. for $\{n_u, n_d, n_s\} = \{1,0,0\}$ model

$$\frac{f_\pi^2 B_0}{2}\mathrm{tr}[M_q + h.c.] = \sqrt{2} f_\pi^2 B_0 m_u \varphi + \cdots$$

- VEV from tadpole condition

$$f_\varphi^2 = \frac{f_\pi^2 B_0 m_u}{m_\varphi^2}$$

Spectrum

- Polar decomposition $\varphi = \frac{1+\sigma}{\sqrt{2}} e^{ia}$

  - tadpole condition from minimum in $\sigma = 0$ and $a = 0$ $\longrightarrow$ $\langle \varphi \rangle = \frac{1}{\sqrt{2}}$

$$m_\varphi^2 = \frac{B_0 f_\pi^2}{f_\varphi^2} \sum_q m_q n_q - \lambda f_\varphi^2$$

  - $m_\varphi^2 > 0$ for $f_\varphi \lesssim \mathrm{GeV} \left(\frac{5 \times 10^{-5}}{\lambda} \right)^{1/4}$ → *upper bound on quartic for fixed* f_φ

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$$m_\varphi^2 = \frac{B_0 f_\pi^2}{f_\varphi^2} \sum_q m_q n_q - \lambda f_\varphi^2 \approx \frac{B_0 f_\pi^2}{\sqrt{2} f_\varphi} \sum_q \kappa_q n_q = \frac{B_0 f_\pi^2}{2} \sum_q \frac{n_q \kappa_q^2}{m_q}$$

neglect quartic + $m_q = \frac{\kappa_q}{\sqrt{2}} f_\varphi$

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  - e.g. in Murayama's model $\{n_u, n_d, n_s\} = \{1,0,0\}$

$$m_\varphi \approx 2.3 \kappa_u \mathrm{GeV} \qquad\qquad f_\varphi \approx 3 \kappa_u^{-1} \mathrm{MeV}$$

- CP-even radial mode: $m_\sigma^2 = m_\varphi^2 + 3\lambda f_\varphi^2$
- CP-odd Goldstone mode mixes with π, η, η' (next slides): $m_a^2 \approx m_\varphi^2 - 2\lambda f_\varphi^2$

→ *does not look like the usual axion since* $f_\varphi \ll f_\pi \approx 92\,\mathrm{MeV}$

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  - actual numbers depend on model (which quark is massless prior to chiral SSB)

Where is φ ?

- New state coupled to QCD, hidden among heavy iso-singlet resonances

State	J^{PC}	Mass [MeV]	Width [MeV]
$\eta(547)$	0^{-+}	547.9 ± 0.0	$(1.31 \pm 0.05) \times 10^{-3}$
$\eta'(958)$	0^{-+}	957.8 ± 0.1	0.198 ± 0.009
$\eta(1295)$	0^{-+}	1294 ± 4	55 ± 5
$\eta(1405)$	0^{-+}	1409 ± 2	51 ± 7
$\eta(1475)$	0^{-+}	1476 ± 4	85 ± 9
$\eta(1760)$	0^{-+}	1756 ± 9	96 ± 70
$\eta(2225)$	0^{-+}	2221 ± 8	185 ± 40
$f_0(500)$	0^{++}	400–550	400–700
$f_0(980)$	0^{++}	990 ± 20	40–100
$f_0(1370)$	0^{++}	1200–1500	200–500
$f_0(1500)$	0^{++}	1505 ± 6	109 ± 7
$f_0(1710)$	0^{++}	1723 ± 6	135 ± 8
$f_0(2020)$	0^{++}	1992 ± 16	442 ± 60
$f_0(2200)$	0^{++}	2189 ± 13	238 ± 50
$f_0(2330)$	0^{++}	2325 ± 35	200 ± 70

[PDG]

- main decay channels hadronic: $\Gamma(a \to 3\pi) \approx 0.1\kappa_u^2 m_a, \;\; \Gamma(\sigma \to \pi\pi) \approx \frac{\kappa_u^2}{8\pi} m_\sigma.$

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$\kappa_u \sim 1/2$

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[PDG]

[The η(1295) was first reported in 1979 in $\pi^- p \to \eta\pi^+\pi^- n$ at Argonne National Labs; Stanton et al., Phys. Rev. Lett. 42, 346 (1979)]

[Evidence for $f_0(1370)$ first emerged in $p\bar{p}$ annihilation data in 1994-1995, although the resonance remained controversial for several years; C. Amsler et al. (Crystal Barrel Coll. at CERN), Phys. Lett B355, 425 (1995)]

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Main issue

- New state a mixes with pseudo-scalar mesons: (a, π^0, η, η')

$$\mathcal{M}^2 = B_0 \begin{pmatrix} \frac{f_\pi^2}{f_\varphi^2}\sum_q m_q n_q^2 & \frac{f_\pi}{f_\varphi}(m_d n_d - m_u n_u) & \frac{-f_\pi}{\sqrt{3}\,f_\varphi}(m_u n_u + m_d n_d - 2 m_s n_s) & \frac{-\sqrt{2} f_\pi}{\sqrt{3}\,f_\varphi}\sum_q m_q n_q \\ - & m_u + m_d & \frac{(m_u - m_d)}{\sqrt{3}} & \sqrt{\frac{2}{3}}(m_u - m_d) \\ - & - & \frac{1}{3}(m_d + m_u + 4 m_s) & \frac{\sqrt{2}}{3}(m_d + m_u - 2 m_s) \\ - & - & - & m_1^2 + \frac{2}{3}(m_u + m_d + m_s) \end{pmatrix}$$

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- π^0 gets negative shift at tree-level (while $\pi^\pm$ unaffected)

$$\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi} \approx -\frac{1}{2}\frac{(m_u n_u - m_d n_d)^2}{(m_d n_d^2 + m_u n_u^2)(m_u + m_d)}$$

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$$\mathcal{M}^2 = B_0 \begin{pmatrix} \frac{f_\pi^2}{f_\varphi^2}\sum_q m_q n_q^2 & \frac{f_\pi}{f_\varphi}(m_d n_d - m_u n_u) & \frac{-f_\pi}{\sqrt{3}\, f_\varphi}(m_u n_u + m_d n_d - 2 m_s n_s) & \frac{-\sqrt{2} f_\pi}{\sqrt{3}\, f_\varphi}\sum_q m_q n_q \\ - & m_u + m_d & \frac{(m_u - m_d)}{\sqrt{3}} & \sqrt{\frac{2}{3}}(m_u - m_d) \\ - & - & \frac{1}{3}(m_d + m_u + 4 m_s) & \frac{\sqrt{2}}{3}(m_d + m_u - 2 m_s) \\ - & - & - & m_1^2 + \frac{2}{3}(m_u + m_d + m_s) \end{pmatrix}$$

- π^0 gets negative shift at tree-level (while $\pi^\pm$ unaffected)

$$\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi} \approx -\frac{1}{2}\frac{(m_u n_u - m_d n_d)^2}{(m_d n_d^2 + m_u n_u^2)(m_u + m_d)} \approx -0.15$$

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$$\left.\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi}\right|_{\text{exp}} \approx -0.03 \approx \left.\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi}\right|_{\text{EM}}$$

[$(m_{\pi^\pm} - m_{\pi^0})_{\text{EM}}$ = 4.622(95) MeV in good agreement (2%) with $(m_{\pi^\pm} - m_{\pi^0})_{\text{exp}}$ = 4.5936(5) MeV from Frezzotti, Gagliardi, Lubicz, Martinelli, Sanfilippo, Simula, Phys. Rev. D 106 (2022)]

KM solution

- Murayama suggested KM ambiguity can help mitigating the tension

$$M_q|_{\rm KM} = {\rm diag}\left(\begin{array}{c} m_u\sqrt{2}\varphi + x_{\rm KM} m_s m_d e^{i\sqrt{6}\eta'} \\ m_d + x_{\rm KM} m_u m_s \sqrt{2}\varphi^* e^{i\sqrt{6}\eta'} \\ m_s + x_{\rm KM} m_u m_d \sqrt{2}\varphi^* e^{i\sqrt{6}\eta'} \end{array} \right)$$

- KM does not modify tadpole condition, but allows for a reduced m_u (enters pion mixing)

- Define: $m_u = \dfrac{m_u|_{\rm KM}}{R}$ (where $m_u|_{\rm KM}$ identified with lattice value)

$$\frac{\Delta m_{\pi^0}}{m_{\pi^0}} \approx -\frac{0.15}{R}, \qquad m_\varphi \approx 2.3\,\kappa_u\,\sqrt{R}\,{\rm GeV}$$

- remember tadpole condition:

$$m_\varphi^2 = \frac{B_0 f_\pi^2}{2} \sum_q \frac{n_q \kappa_q^2}{m_q}$$

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- $R \sim 5$ to match exp. value

[Murayama's point of view: "we conservatively require the mass shift to be at most the observed value"]

- $R \sim 200$ to match lattice error of EM contribution

- $R = \infty$ corresponds to massless up-quark solution

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- $R = \infty$ corresponds to massless up-quark solution

→ *caveat: lattice results assume SM (decoupling regime for heavy φ = QCD)*

We tried to improve…

- Model $\{n_u, n_d, n_s\} = \{1,1,0\}$

$$\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi} \approx -\frac{1}{2}\frac{(m_u n_u - m_d n_d)^2}{(m_d n_d^2 + m_u n_u^2)(m_u + m_d)} \approx -0.07$$

- splitting down to 7 % (need $< 3\,\%$ to match $(\Delta m_\pi / m_\pi)_{\rm exp}$)

- KM does not help here, since φ enters also KM correction

$$M_q|_{\rm KM} = {\rm diag}\begin{pmatrix} m_u\sqrt{2}\varphi + x_{\rm KM} m_s m_d \sqrt{2}\varphi^* e^{i\sqrt{6}\eta'} \\ m_d\sqrt{2}\varphi + x_{\rm KM} m_u m_s \sqrt{2}\varphi^* e^{i\sqrt{6}\eta'} \\ m_s + x_{\rm KM} m_u m_d (\sqrt{2}\varphi^*)^2 e^{i\sqrt{6}\eta'} \end{pmatrix}$$

We tried to improve…

- Model $\{n_u, n_d, n_s\} = \{2,1,0\}$

$$\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi} \approx -\frac{1}{2}\frac{(m_u n_u - m_d n_d)^2}{(m_d n_d^2 + m_u n_u^2)(m_u + m_d)} \approx -0.0005$$

  - splitting down to $0.05\,\%$ (need $< 0.07\,\%$ to match lattice error on $(\Delta m_\pi / m_\pi)_{\rm EM}$)

  - accident, due to $m_d/m_u \sim 2$

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- unfortunately, the cut-off of this effective theory is around m_u

$$\mathcal{L} \supset -\kappa_u \frac{\varphi^2}{\Lambda}\bar{u}_L u_R - \kappa_d \varphi \bar{d}_L d_R + \text{h.c.} \qquad \langle \overline{q_L} q_R \rangle = -\frac{B f_\pi^2}{2} \equiv -\mu^3$$

$$\langle\varphi\rangle = \frac{\kappa_d \mu^3}{m_\varphi^2 - 2\kappa_u \mu^3/\Lambda}$$

$$m_u = \kappa_u \frac{\langle\varphi\rangle^2}{\Lambda}$$

$$m_d = \kappa_d \langle\varphi\rangle$$

$$m_\varphi = \sqrt{\frac{\kappa_d^2 \mu^3}{m_d}\left(2\frac{m_u}{m_d} + 1\right)} = 2.1\ \text{GeV}\ \kappa_d$$

$$\Lambda = \frac{\kappa_u}{\kappa_d^2}\frac{m_d^2}{m_u} = 10.2\ \text{MeV}\ \frac{\kappa_u}{\kappa_d^2}$$

We tried to improve…

- Model $\{n_u, n_d, n_s\} = \{0,0,1\}$

  - preserves SU(2) symmetry of u-d sector → pion mass splitting OK

  - new problems arise:

    1) too light mass parameter (CP-odd component even lighter upon diagonalization)

$$m_\varphi = \left(\frac{B_0 f_\pi^2}{2m_s}\right)^{1/2} \kappa_s \approx 0.36\ \text{GeV}\, \kappa_s$$

2) pion mass issue effectively shifted to eta sector, i.e. $\Delta m_\eta / m_\eta \approx +15\%$

We tried to improve…

- "Fallo di confusione"

  - models with multiple PQ scalars

  - add a PQ-neutral scalar just to make the π^0 heavier

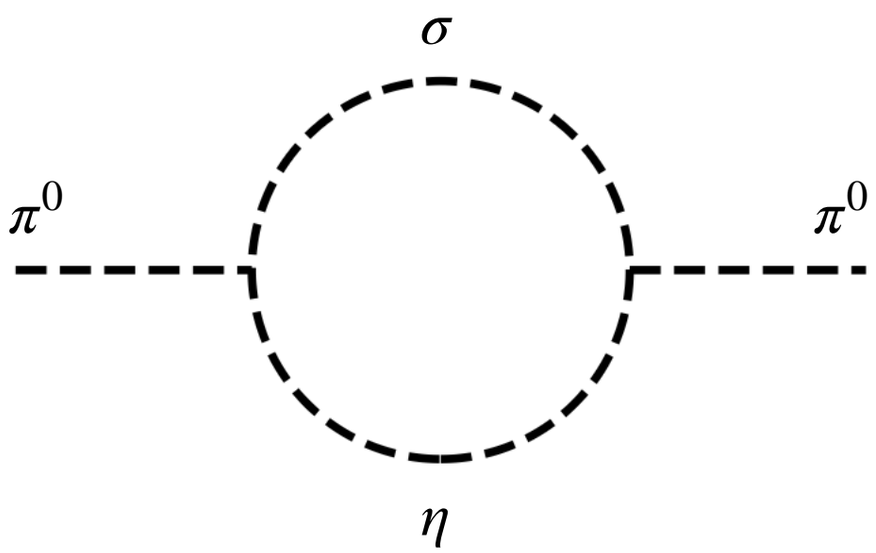
→ *extensive scans of parameter space show that configurations which can fix the pion mass problem require new light states, which cannot be hidden among QCD resonances (consistent with pattern emerging for* $\{n_u, n_d\} = \{2,1\}$ *and* $\{n_s\} = \{1\}$ *models)*

We tried to improve…

- Loops

  - κ is sizeable, so one should worry about loop corrections

  - e.g. $\pi^0\sigma\eta$ vertex $\propto B_0(\kappa_u - \kappa_d)$ affect only π^0 mass



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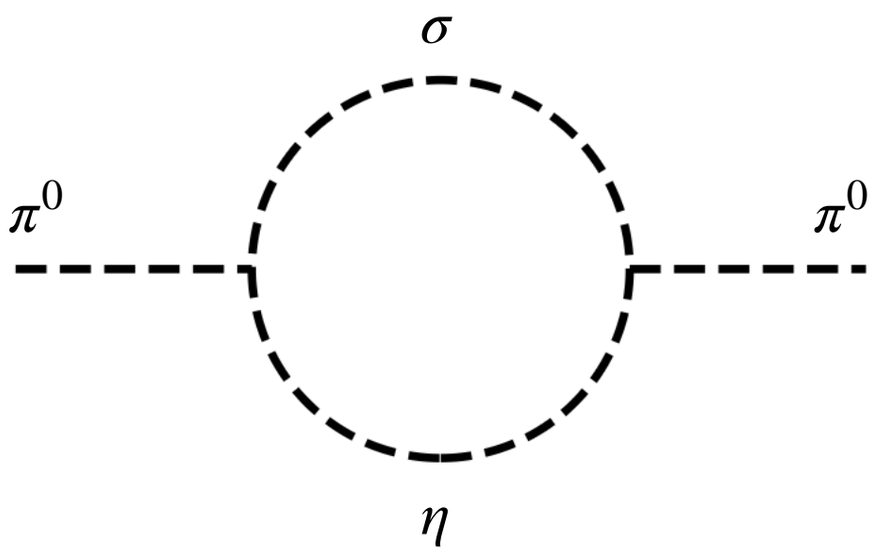
  - in general:

$$\Delta M_{ij}^2 = -(\partial_\varphi M_{ik}^2)\left[\int \frac{d^4Q}{(2\pi)^4}\frac{1}{Q^2+m_\varphi^2}\frac{1}{Q^2+m^2}\right]_{kl}(\partial_\varphi M_{lj}^2),$$

trilinear couplings from derivative of φ-dependent meson mass matrix



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  - attempt an estimate by taking a cut-off $Q \lesssim \Lambda = 1$ GeV and expand for $m_\varphi \gtrsim \Lambda$

$$\Delta M_{ij}^2 \approx -\frac{(\partial_\varphi M_{ik}^2)}{16\pi^2 m_\varphi^2}\left[\Lambda^2 - M_{kk}^2 \ln\frac{\Lambda^2+M_{kk}^2}{M_{kk}^2}\right](\partial_\varphi M_{kj}^2)$$

  - e.g. for $n_u = 1$ model

$$\Delta(m_{\pi^0}^2 - m_{\pi^\pm}^2) \approx -\frac{\kappa^2 B_0^2 \Lambda^2}{(4\pi m_\varphi)^2}$$

confirmed also by a more serious calculation with QCD correlators (see backup slides)

→ *unlikely a cancellation with tree-level (cf. size and sign)*

Why is it difficult to make it work ?

- Some general lessons:

  - playing little with KM, otherwise one ends up into the SM massless up "solution"

  - find models where isospin is accidentally restored (loop has no reason to cancel tree)

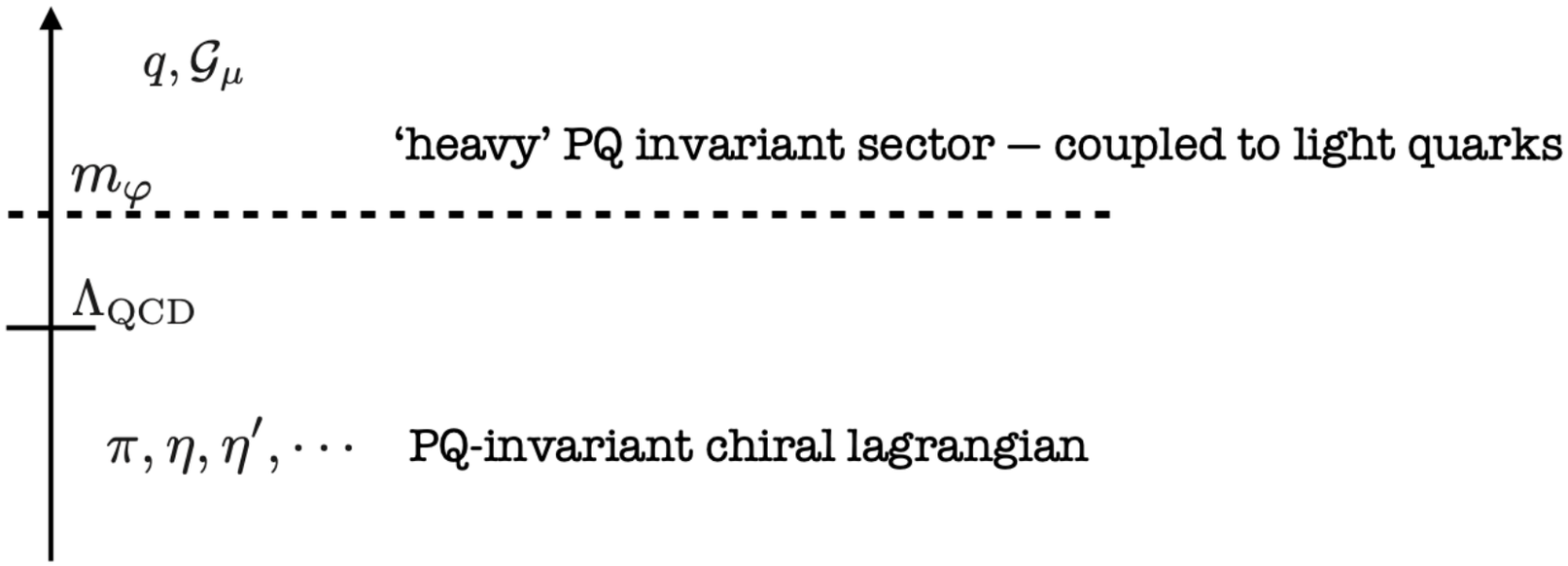
  - invoke a new lattice simulation (QCD + new scalar) to see if spectrum is OK

  - $m_\varphi \gtrsim$ GeV, so one can attempt an EFT expansion to gain further insight

$$\partial/m_\varphi \ll 1$$

EFT of heavy PQ sector

- Chiral expansion with PQ symmetry



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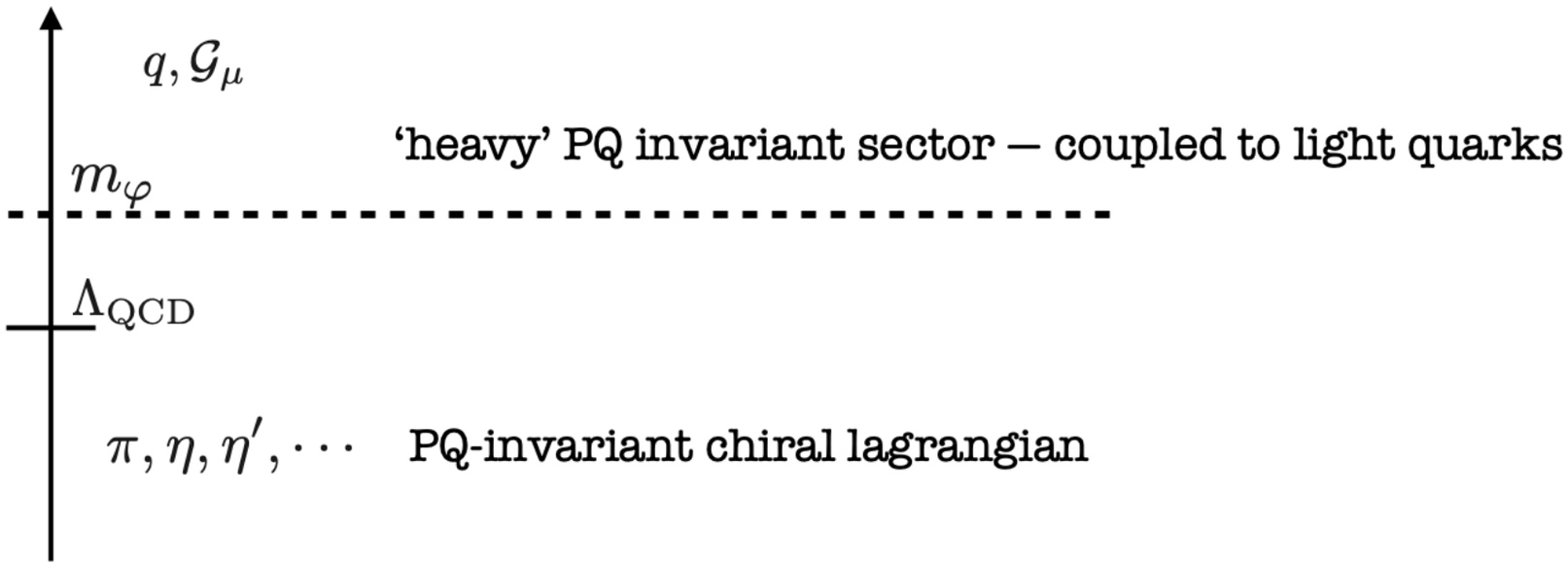

- Two sets of spurions:

  - mass matrix with some zero eigenvalues (here $n_s = 0$)

$$\tilde{M}_q = \mathrm{diag}(m_u(1-n_u), m_d(1-n_d), m_s)$$

  - couplings to light quark of the "heavy" PQ invariant sector

$$I_{\rm PQ} = \mathrm{diag}(\kappa_u n_u, \kappa_d n_d, 0)$$

EFT of heavy PQ sector

- Chiral expansion with PQ symmetry

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD}(\tilde{M}_q) + \sum_i C_i \mathcal{O}_i^{\rm PQ}(I_{\rm PQ})$$

- first term: usual chiral expansion, with the new mass matrix (with zeroes)

- second term: tower of operators generated by the PQ sector at m_φ

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- second term: tower of operators generated by the PQ sector at m_φ

- first non-trivial operator arises at $\mathcal{O}(p^4)$ (only one, assuming a single mediator)

$$\mathcal{O}_1^{\rm PQ} = {\rm tr}[I_{\rm PQ}^\dagger U]{\rm tr}[I_{\rm PQ} U^\dagger]$$

- manifestly chiral and PQ invariant: $I_{\rm PQ} \to L I_{\rm PQ} R^\dagger$ and $I_{\rm PQ} \to I_{\rm PQ} e^{-i\alpha Q_{\rm PQ}}$

$$I_{\rm PQ} = {\rm diag}(\kappa_u n_u, \kappa_d n_d, 0)$$

A main obstruction

- What is the effect of the new operator ?

$$\mathcal{O}_1^{\rm PQ} = {\rm tr}[I_{\rm PQ}^\dagger U]{\rm tr}[I_{\rm PQ} U^\dagger]$$

- it controls the pion mass splitting

$$m_{\pi^0}^2 - m_{\pi_\pm}^2 = -2\frac{C_1}{f_\pi^2}(\kappa_u n_u - \kappa_d n_d)^2$$

- only operator that breaks iso-spin while preserving PQ, others appear only at $\mathcal{O}(p^6)$

$$\mathcal{O}_2^{\rm PQ} = {\rm tr}[I_{\rm PQ}^\dagger \partial_\mu U]\,{\rm tr}[I_{\rm PQ}\partial^\mu U^\dagger], \qquad \mathcal{O}_3^{\rm PQ} = {\rm tr}[I_{\rm PQ}^\dagger (\partial_\mu U)(\partial^\mu U)^\dagger I_{\rm PQ}]$$

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- tree-level matching gives $C_1 = \frac{1}{4}\frac{f_\pi^4 B_0^2}{m_\varphi^2}$

- $\mathcal{O}(p^6)$ terms cannot cancel $\mathcal{O}(p^4)$ term, as long as chiral expansion holds

- loop corrections cannot cancel the tree-level effect, as long as one operator dominates

- $\pi\pi$ scattering affected too (see backup slides)

A curiosity: invisible axion limit

- For fixed m_u, a large decay constant = light axion limit $\quad f_\varphi^2 = \dfrac{f_\pi^2 B_0 m_u}{m_\varphi^2}$

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$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_\varphi^2} \left(\sum_q n_q \right)^2 \qquad f_a \equiv \frac{f_\varphi}{\sum_q n_q}$$

  - requires a tiny quartic $\quad f_\varphi \lesssim \mathrm{GeV} \left(\dfrac{5 \times 10^{-5}}{\lambda} \right)^{1/4}$

  - interesting cosmology: axion exists only after QCD cross-over

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- Problem: CP-even radial mode light as well

$$m_a^2 < \frac{m_d}{m_u + m_d} m_\sigma^2 < 3 m_a^2$$

→ *ruled out by 5th force constraints in the sub-eV axion mass window (needs $f_a \sim M_{\rm Pl}$ and $\lambda \sim 10^{-80}$)*

Conclusions

- Deform "massless up" by adding an operator "that contribute to quark mass matrix"

  - Such operator controls the pion mass splitting

  - If the coefficient is set to zero: SM massless up → excluded by lattice

  - This limit corresponds to decoupling the PQ complex field

$$m_\varphi^2 = \frac{B_0 f_\pi^2}{2} \sum_q \frac{n_q \kappa_q^2}{m_q}$$

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- What will save these "massless up" deformations ?

  - Models with an accidentally small splitting, at finite "m_u"

  - Despite some partial improvement, we did not find a good model

  - For a final word we need to redo the lattice (QCD + new sector)

Backup slides

UV completion

- Need a dim = 5 operator:

$$\frac{y\kappa_u}{M_{\mathcal{Q}}}\,\bar{Q}_u\varphi H u_R + c.c. \quad \longrightarrow \quad \kappa = \frac{y\kappa_u v}{M_{\mathcal{Q}}}$$

- possible UV completion in terms of a vector-like $\mathcal{Q} = (\mathcal{U}, \mathcal{D})$ with PQ charge -1

$$\mathcal{L}_{\mathcal{Q}} = \bar{\mathcal{Q}}(i\not{D} - M_{\mathcal{Q}})\mathcal{Q} + (\kappa_i \bar{Q}_i \varphi \mathcal{Q} + y\bar{\mathcal{Q}} H u_R + c.c.)$$

- LHC limits: $M_{\mathcal{D}} > 1.53\ \mathrm{TeV}$

- With $y, \kappa_u \approx 1$ and $\mathcal{Q} \approx 1.6$ TeV $\rightarrow \kappa \approx 0.11$, enhanced to $\kappa \approx 0.25$ down to QCD scale

- Peculiar LHC phenomenology of UV completion, which does not decouple (see Murayama)

$\pi\pi$ scattering

- In usual QCD one has

$$\mathrm{tr}\left[M_q U^\dagger + \mathrm{h.c.}\right] = m_s + (m_u + m_d)\cos\pi \qquad \text{where} \qquad \pi^2 \equiv \pi^a\pi^a = (\pi^0)^2 + 2\pi^-\pi^+$$

$$\mathscr{A}(\pi_a\pi_b \to \pi_c\pi_d) = \frac{1}{f_\pi^2}\left[\delta_{ab}\delta_{cd}(s - m_\pi^2) + \delta_{ac}\delta_{bd}(t - m_\pi^2) + \delta_{ad}\delta_{bc}(u - m_\pi^2)\right]$$

- O_1^{PQ} gives a different isospin-breaking function of pions

$$\mathcal{O}_1^{\mathrm{PQ}} = (n_u\kappa_u + n_d\kappa_d)^2\cos^2\pi + (n_u\kappa_u - n_d\kappa_d)^2\frac{(\pi^0)^2}{\pi^2}\sin^2\pi$$

- tuning $n_u\kappa_u - n_d\kappa_d$ to zero allows to restore SU(2) invariance, but with a different function of π

$$\mathscr{A}(\pi^0\pi^0 \to \pi^0\pi^0) = (4 - 3X_0)\frac{m_{\pi^0}^2}{f_\pi^2}, \qquad X_0 = \begin{cases} 1 & \text{if one } n_q = 0 \\ 4 & \text{if } n_u = n_d = 1 \end{cases},$$

$$\mathscr{A}(\pi^+\pi^- \to \pi^+\pi^-) = \frac{s + t - 2X_\pm m_{\pi^\pm}^2}{f_\pi^2}, \qquad X_\pm = 1 + 3\frac{m_u n_u + m_d n_d}{m_u + m_d}$$

→ *differences w.r.t. QCD case of order one for* $X_{0,\pm} \neq 1$

Pion effective potential

- Electromagnetic pion mass splitting

  - Computed from current correlators [See e.g. TASI Lecture by Contino, arXiv:1005.4269]

  - Effective potential for pions integrating over the full effective Lagrangian for photons

- Computation can be done similarly in our case

  - Computed by "estimating" quark-antiquark scalar correlators

  - Effective potential for pions integrating over the full effective Lagrangian for φ

  - Promote $\kappa_q \varphi$ to a spurion Φ transforming as $\Phi \to L \Phi R^\dagger$

$$\mathscr{L}_{\text{eff}} = \Pi_A(p)\,\text{tr}[\Phi\Phi^\dagger] + \Pi_B(p)\,\text{tr}[\Phi U^\dagger]\,\text{tr}[\Phi^\dagger U] + \Pi_0\,\text{tr}[\Phi U^\dagger + \text{h.c.}]$$

  - Effective potential for pions:

$$V(U) = -i \int d^4p \log \frac{\delta \mathcal{L}_{\text{eff}}}{\delta\Phi(p)\delta\Phi^*(-p)}$$

Pion effective potential

- EFT matching to QCD+scalar

$$e^{i\int \mathcal{L}_{\rm eff}(\Phi,\Phi^*)} = Z = e^{iS_{\rm QCD}+i\int \Phi_{ij}\bar{q}^i_L q^j_R + h.c.}$$

  - field φ as a source for the following operators: $i\bar{q}\gamma_5 T^a q$, $i\bar{q}\gamma_5 q$, $\bar{q}q$, $\bar{q}T^a q$

- Exploiting SU(3) symmetry, four relevant form factors:

$$\Pi_{\mathbf{1},P} = \frac{f^2_{\eta'}B_0^2}{p^2-m^2_{\eta'}}+\cdots, \qquad \Pi_{\mathbf{8},P} = \frac{f^2_{\pi}B_0^2}{p^2-m^2_{\pi}}+\cdots., \qquad \Pi_{\mathbf{1},S} = \frac{f^2_{\Sigma}B_0^2}{p^2-m^2_{\Sigma}}+\cdots, \qquad \Pi_{\mathbf{8},S} = \frac{f^2_{\Sigma_8}B_0^2}{p^2-m^2_{\Sigma_8}}+\cdots$$

  - decomposed in CP even/odd, singlet/octet

- Leading correction to the mass splitting

$$m^2_{\pi^0} - m^2_{\pi^\pm} \approx 2(\kappa_u n_u - \kappa_d n_d)^2 \times \frac{1}{6f_\pi^2}\int \frac{d^4Q}{(2\pi)^4}\frac{\Pi_{\mathbf{8},S}+\Pi_{\mathbf{8},P}-\Pi_{\mathbf{1},S}-\Pi_{\mathbf{1},P}}{-Q^2-m^2_\varphi}$$

  - saturated at the mass of the CP-even scalar resonances (the QCD sigma a.k.a. f_0)