Thermal Production of QCD Axions from the Early Universe

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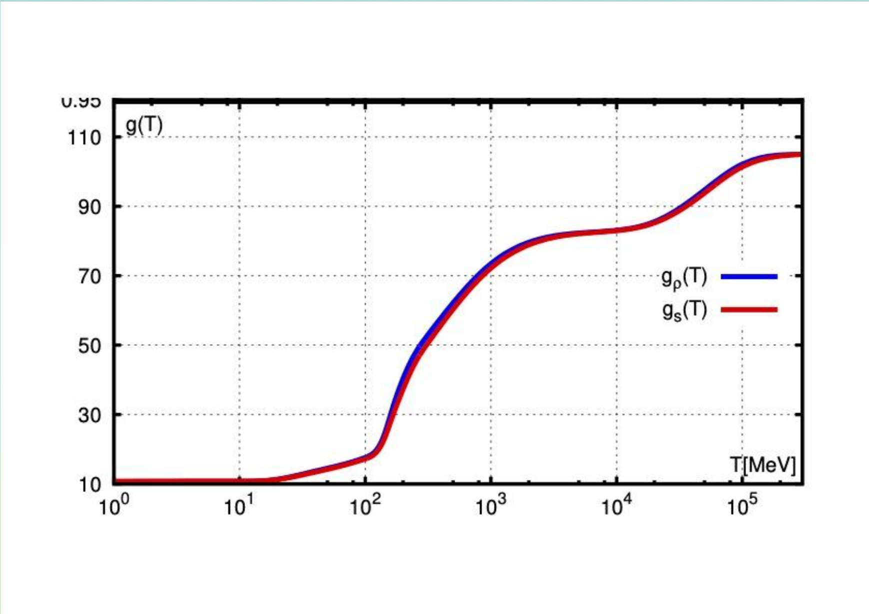
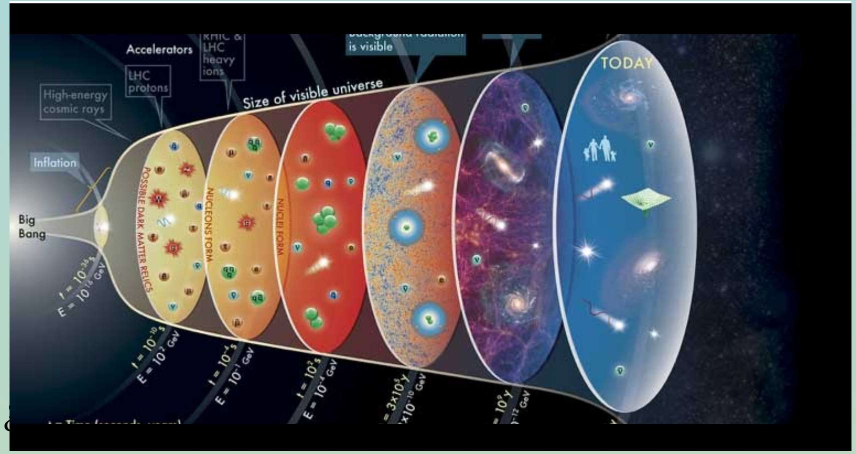
Relic light particles in Cosmology

- Primordial plasma, g_* degrees of freedom and temperature T





Total plasma energy density: $\rho_{\mathrm{TOT}} \propto g_* T^4$

$$g_* \equiv \sum_{i=\mathrm{RELATIVISTIC\ BOSONS}} g_i + \frac{7}{8} \sum_{i=\mathrm{RELATIVISTIC\ FERMIONS}} g_i$$

- Conservation of entropy: $g_*^{1/3} T \propto 1/a$
- When a species becomes non-relativistic (e.g. $e^+ - e^-$ at $T \ll m_e$) g_* decreases

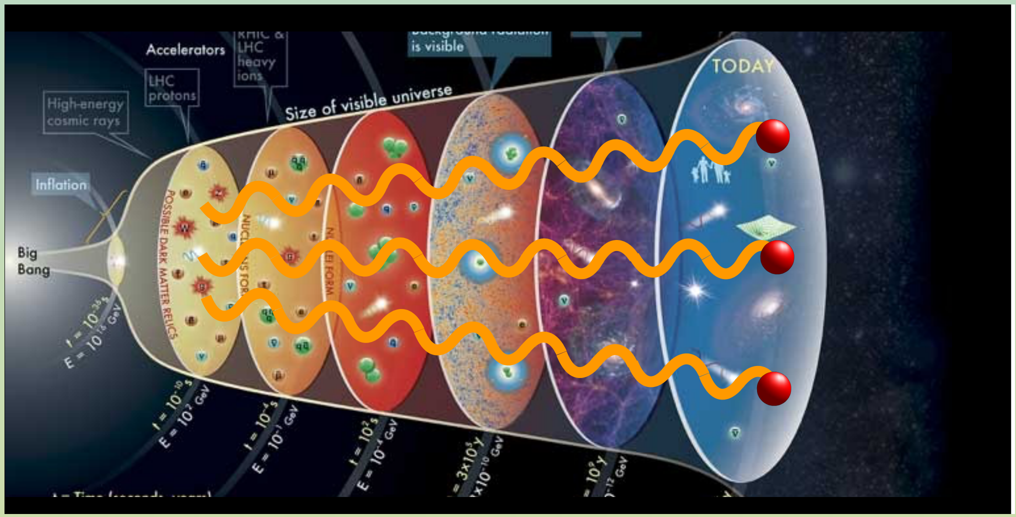
T slightly “increases” (photons get slightly “heated”)

Relic light particles in Cosmology

- Light particles with small interaction ("thermalization rate" Γ), (e.g. neutrinos, axions)

- Compare with Hubble rate H ⇨ Decoupling

- If Particle Decouples below some Temperature $T_{\rm DEC}$, its distribution freezes at its "own temperature" and freely evolves, $\rho_P \propto T_P^4$, with $T_P = T_{\rm DEC}/a$



Relic light particles in Cosmology

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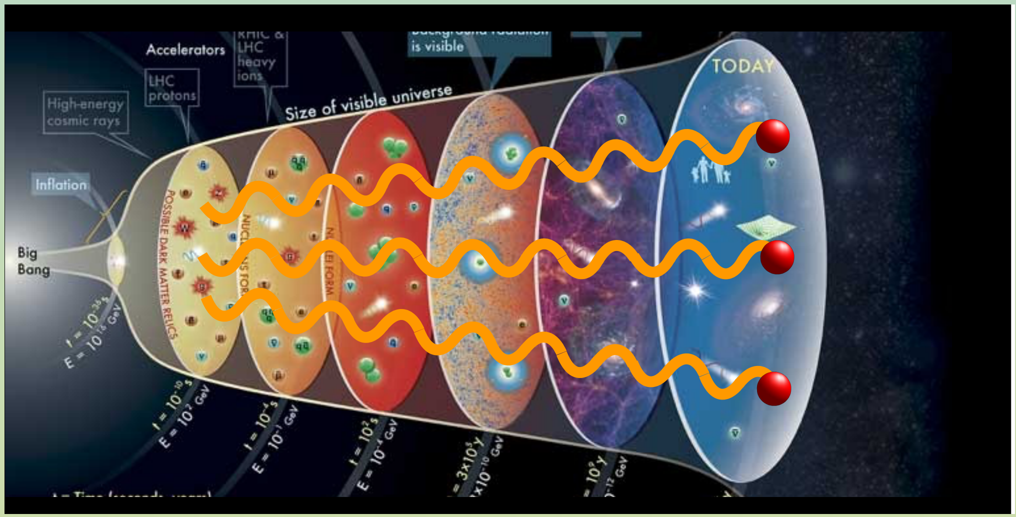
- Compare with Hubble rate ($H \equiv \dot{a}/a$)

- If Particle Decouples below some Temperature T_{DEC}, its distribution freezes at its "own temperature" and freely evolves, $\rho_P \propto T_P^4$, with $T_P = T_{\mathrm{DEC}}/a$

- Compared to photons it does not get heated after decoupling

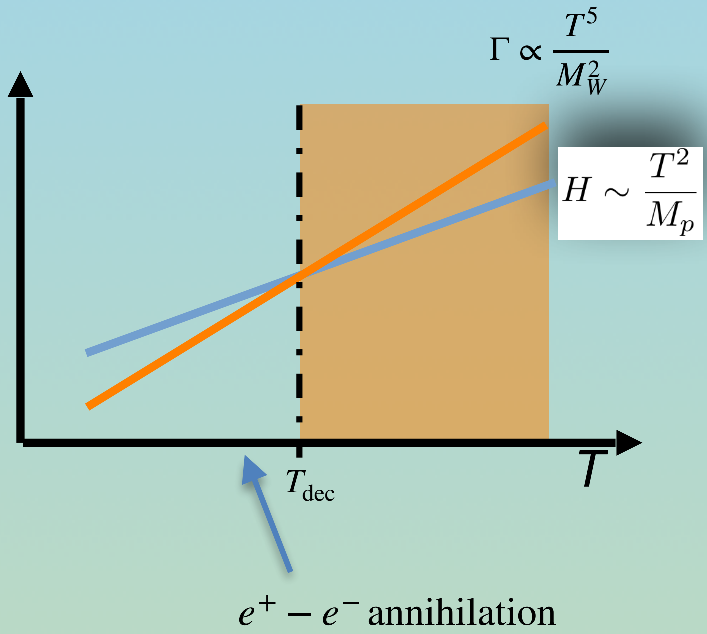
$$\rho_P/\rho_\gamma \propto T_P^4/T^4 \propto 1/g_{*DEC}^{4/3}$$

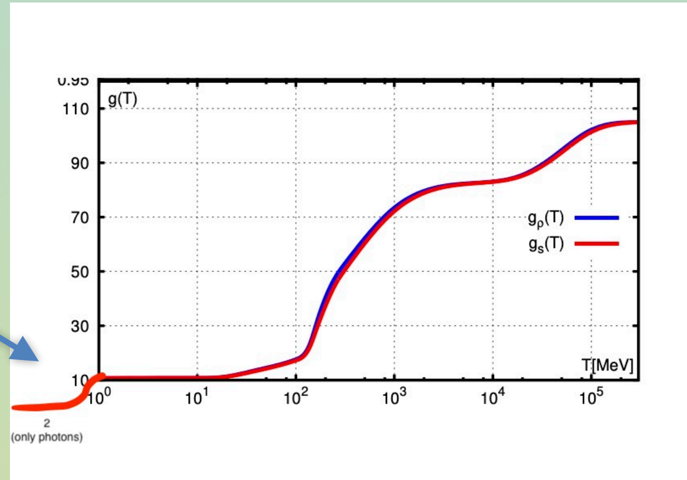


Example: Relic Neutrinos

- Neutrinos decouple at $T \approx MeV$

$$\frac{\rho_\nu}{\rho_\gamma} \propto \frac{1}{g_{*,DEC}^{4/3}} = \left(\frac{4}{11}\right)^{4/3}, \qquad T_\nu \approx 0.7\, T_\gamma \approx 1.96\,\mathrm{K}$$





Example: Cosmic Neutrino Background

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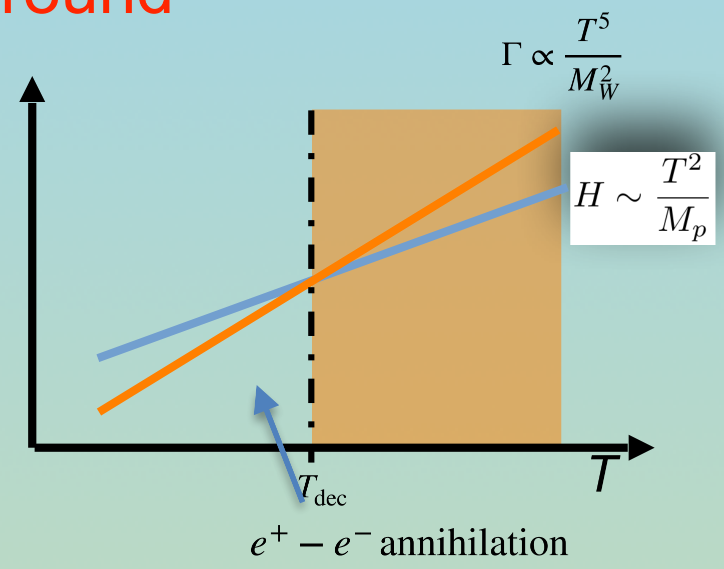


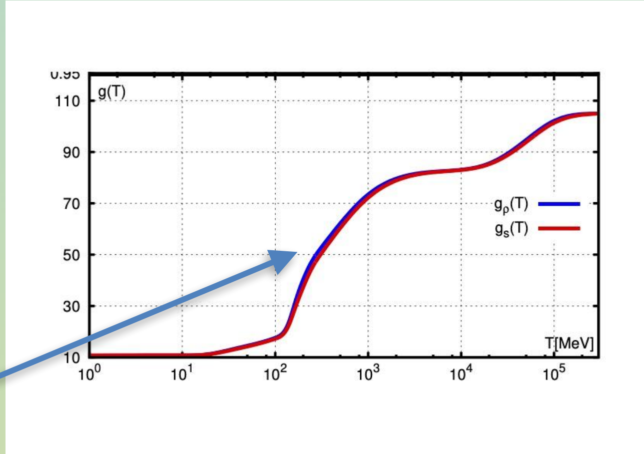
- Any light particle (axions,…) can do the same.

- Traditional parameterization as "extra neutrinos species":

$$\Delta N_{\rm eff} \equiv \left(\frac{8}{7}\right)\left(\frac{11}{4}\right)^{4/3} \frac{\rho_P}{\rho_\gamma}\Big|_{\rm CMB}$$

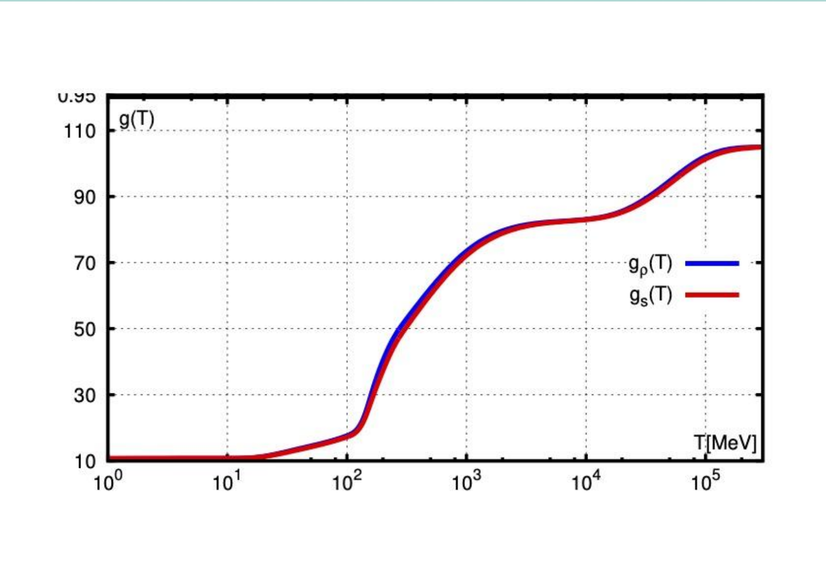
- Relic abundance suppressed as:

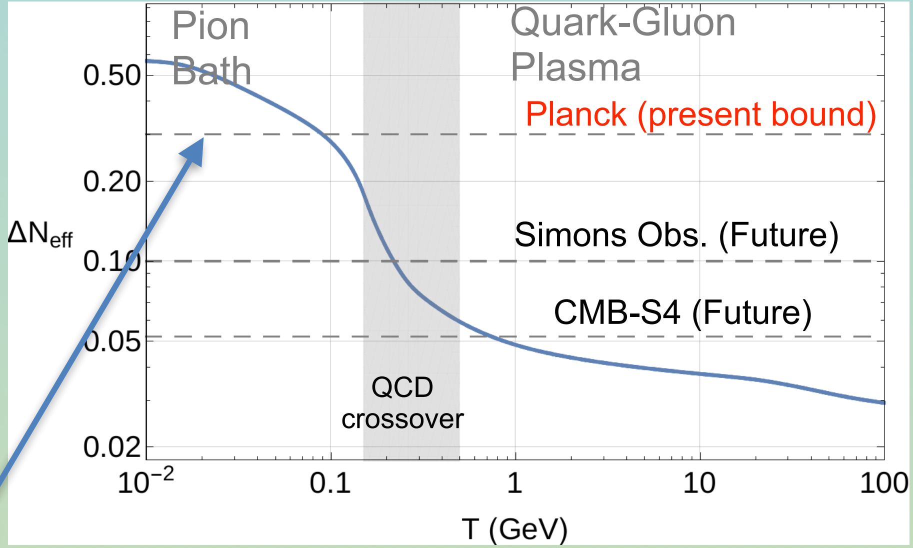
$$\Delta N_{\rm eff} \propto \frac{\rho_P}{\rho_\gamma}\Big|_{\rm CMB} \propto \frac{1}{g_{*,DEC}^{4/3}}$$



Example: Relic Scalars

- Relic abundance $\Delta N_{\rm eff} \propto \frac{\rho_P}{\rho_\gamma}|_{\rm CMB} \propto \frac{1}{g_{*,DEC}^{4/3}}$





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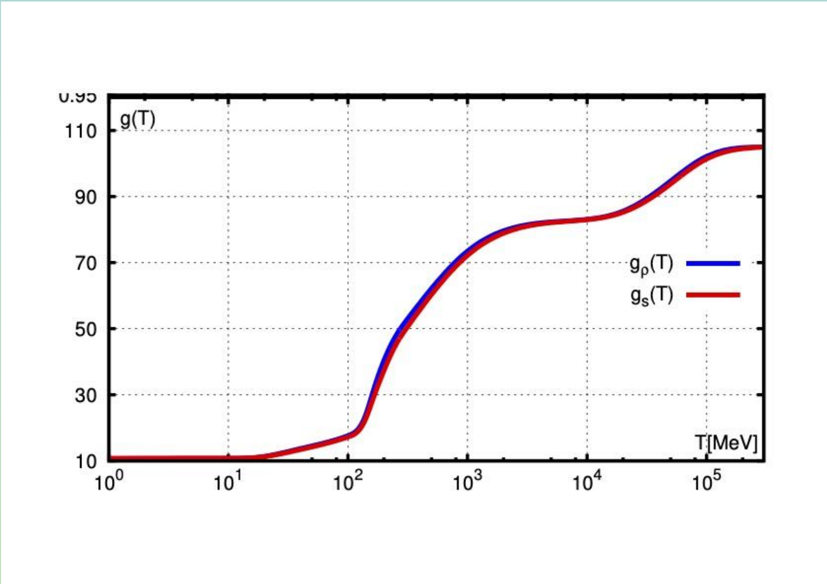

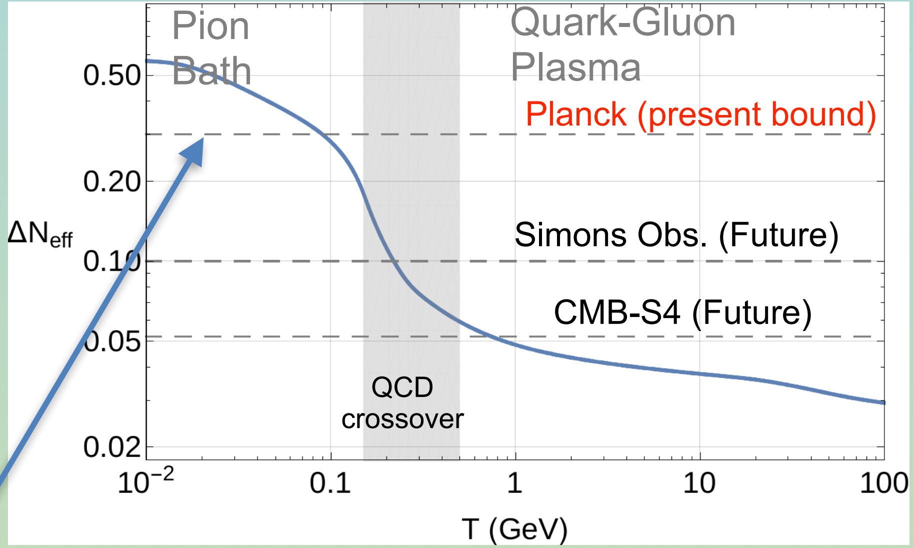

- Main effect: Extra "radiation" at CMB time ($T \approx 0.1 eV$) ⇒ $\Delta N_{\rm eff}$ affects CMB spectra
- If massive ($m \lesssim 0.1\text{eV}$) becomes non-relativistic after CMB time ⇒ adds to Dark Matter (affecting expansion rate of the universe and dark matter fluctuations)

The (Minimal) QCD Axion

$$\mathscr{L}_{\rm SM} \supset \theta_{\rm strong} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Why CP-violation in QCD is tiny $(\bar{\theta}_{\rm strong} \ll 1)$?

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- Dynamical explanation of $\theta_{\text{strong}} \ll 1$
- Light scalar particle, $m_a \approx \Lambda_{QCD}^2/f_a \approx 0.57eV\left(\frac{10^7 GeV}{f_a}\right)$

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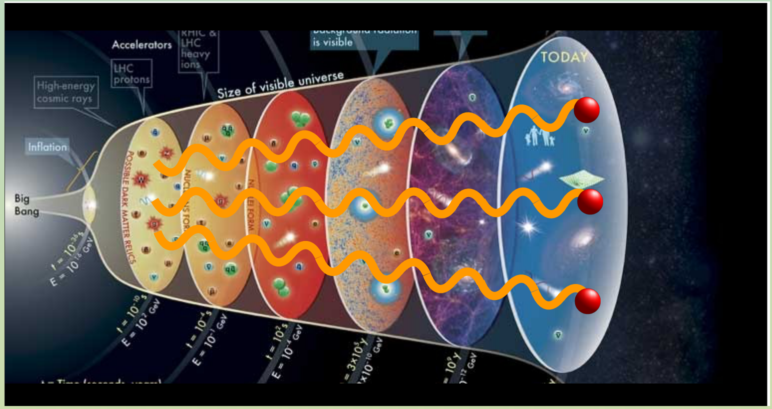
- Two populations of cosmological relic axions:
  - "Cold axions", non-relativistic, candidate for Dark matter (or part of it),
  - "Thermal axions", relativistic at production, May become non-relativistic later ⇨ small part of dark matter (like relic neutrinos)



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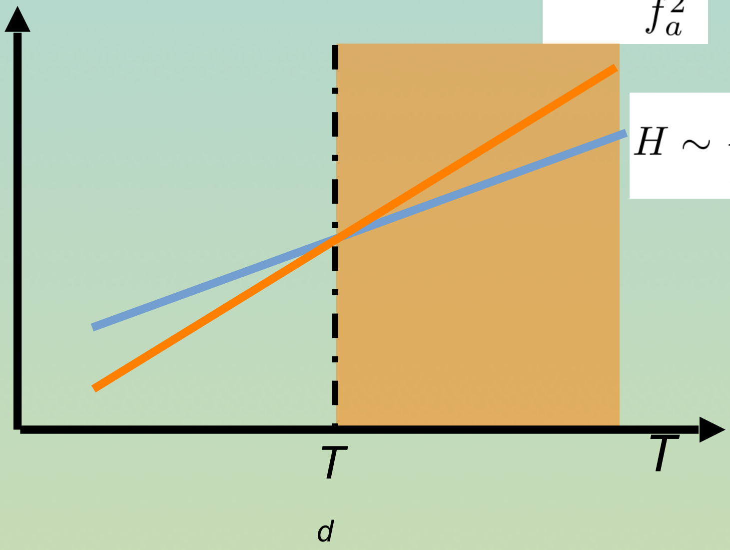
Axion-gluon scatterings ($T \gtrsim T_{QCD}$)

$$\Gamma \sim \frac{T^3}{f_a^2}$$

$$H \sim \frac{T^2}{M_p}$$



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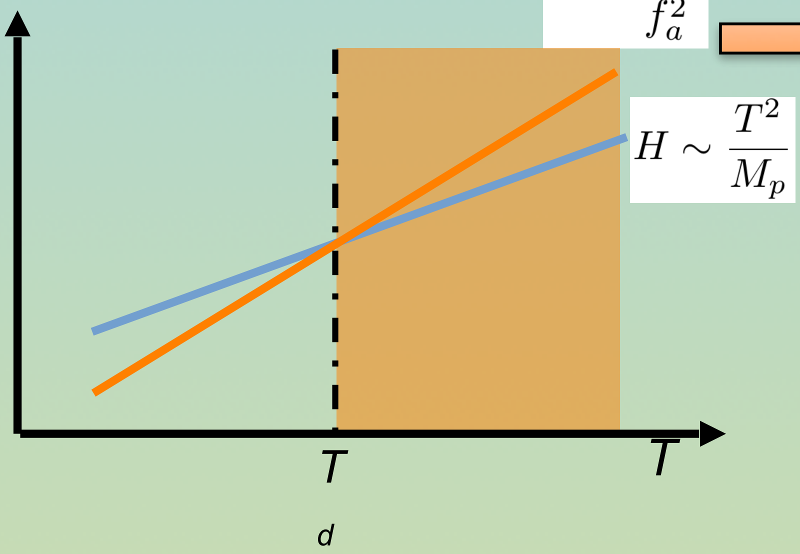

$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8\ \text{GeV}}\right)^2$$

Cosmic Axion Background

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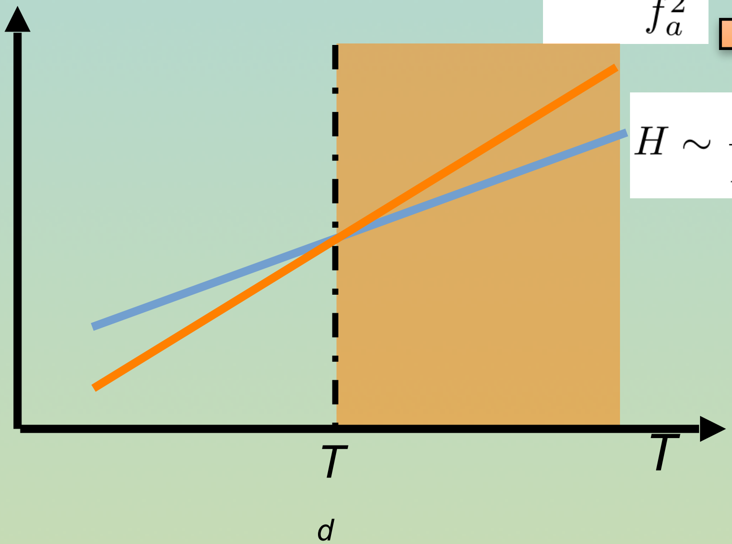

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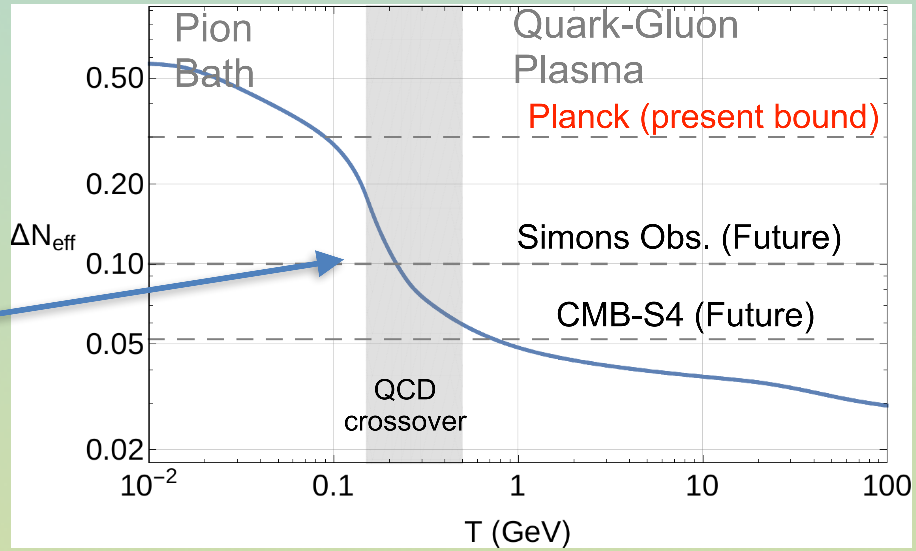
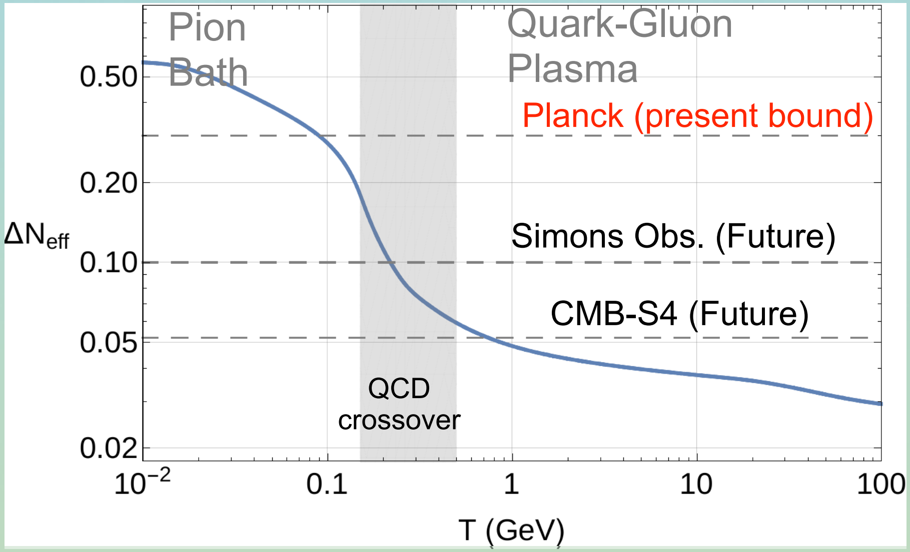


Axion $\Delta N_{\rm eff}$ has a long history:

$\pi\pi \leftrightarrow a\pi$ $\quad$ $gg \leftrightarrow ga$



Arias-Aragon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, AN, Piazza, Raffelt, Rompineve, Rota, Salvio, Sakharov, Silk, Slosar, Steffen, Strumia, Wallisch, Wong, Yun, Zsembinszki, Xue, …

“Standard” treatments:

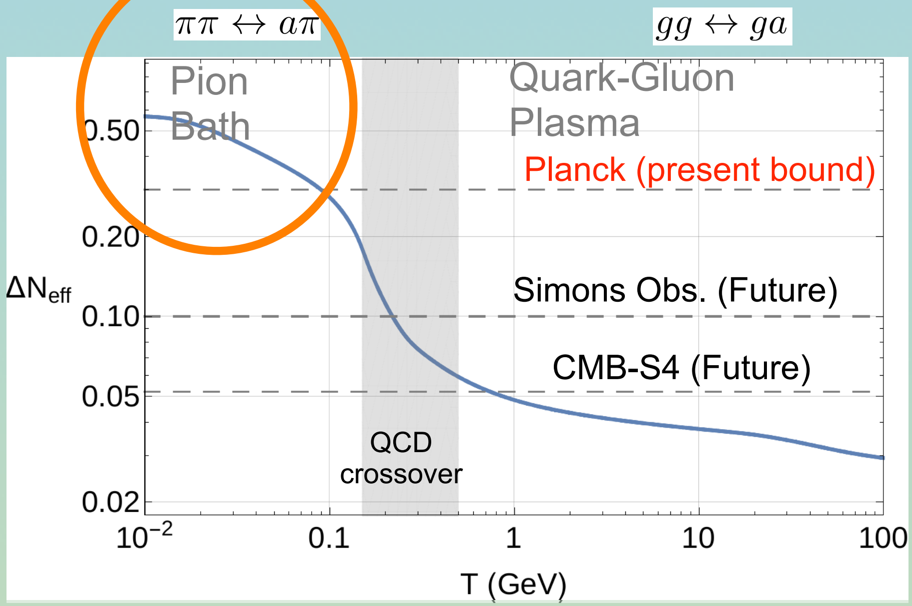
1. Instantaneous decoupling ($\Gamma = H$)
2. Single Boltzmann Eq.for abundance Y.

$$\frac{dY}{d\log x} = (Y^{\rm eq} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right) \qquad (x \equiv m/T)$$

Axion $\Delta N_{\rm eff}$ has a long history:

Improved bounds from pion scatterings

(AN, Rompineve, Villadoro, PRL '23)



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Momentum-dependent Boltzmann Equation and Thermalization Rate Γ

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

$$\Gamma^{<} = e^{-\frac{E}{T}}\Gamma^{>}$$

(Detailed balance, plasma particles in equilibrium)

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Perturbatively, due to scatterings with pions:

$$\Gamma^{<} = \frac{1}{2E}\int\left(\prod_{i=1}^{3}\frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i}\right) f_1^{\rm eq} f_2^{\rm eq}(1 + f_3^{\rm eq})(2\pi)^4\delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu)|\mathcal{M}|^2_{2\,\leftrightarrow\,2}$$

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO chiral perturbation theory rate
(Chang Choi ‘93)

NLO chiral perturbation theory rate
(Chang Choi ‘93)

(Di Luzio, Martinelli, Piazza ‘21)

→ breaks down at $T \gtrsim 60$ MeV !

$$|\mathcal{M}^{\mathrm{LO}}|^2 = \theta_{a\pi}^2 \frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4}$$

$$\theta_{a\pi} = \frac{m_u - m_d}{m_u + m_d} \frac{f_\pi}{2 f_a}$$

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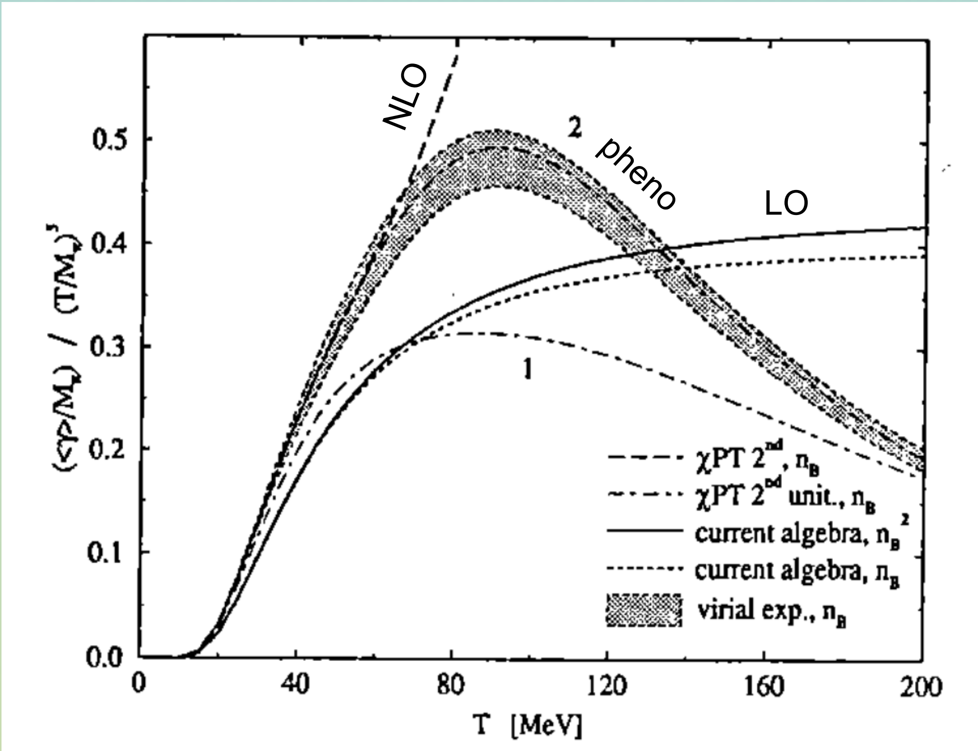

Schenk '94

1. The Thermalization Rate Γ

General form of low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right) q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}}$$

$$\frac{\partial_\mu a}{2f_a} j_A^\mu \overset{\chi\text{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\text{phys}} + \sin(\theta_{a\pi})\,a_{\text{phys}} \simeq \pi^0_{\text{phys}} + \theta_{a\pi} a_{\text{phys}}$$

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$$\mathcal{M}_{a\pi^i \to \pi^j \pi^k} = \theta_{a\pi} \cdot \mathcal{M}_{\pi^0 \pi^i \to \pi^j \pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

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@ all orders in χPT

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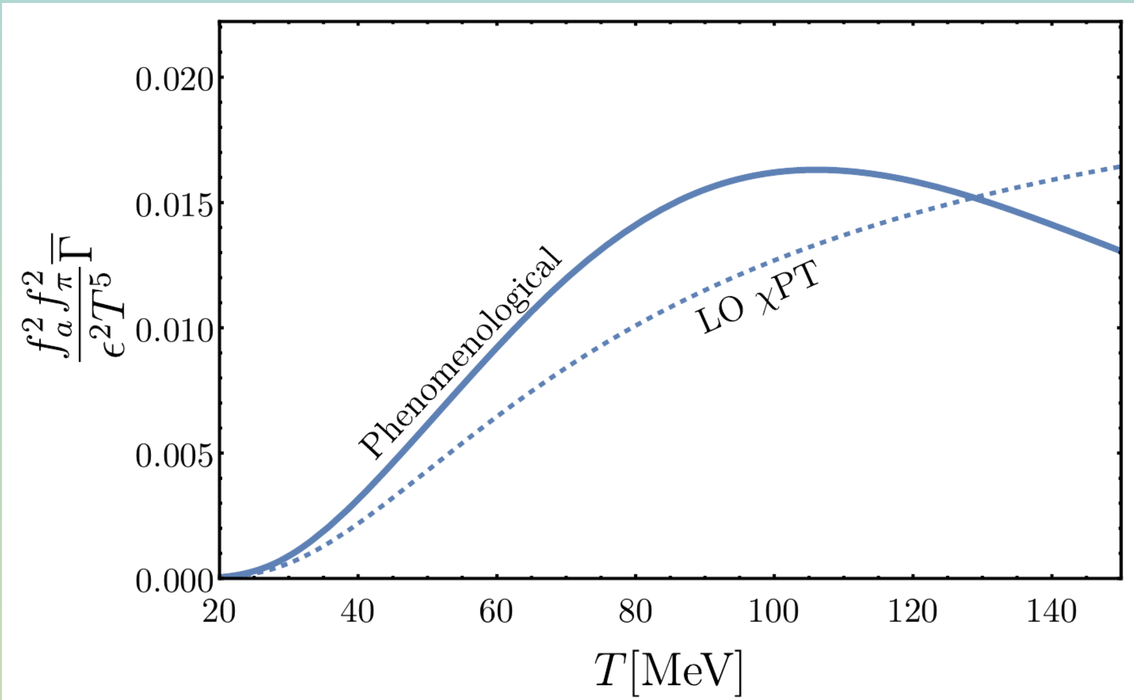
e.g. @ LO

$$|\mathcal{M}^{\mathrm{LO}}|^2 = \theta_{a\pi}^2 \frac{s^2 + t^2 + u^2 - 3m_\pi^4}{f_\pi^4}$$

$$|\mathcal{M}^{\mathrm{LO}}_{\pi\pi}|^2 = \frac{s^2 + t^2 + u^2 - 4m_\pi^4}{f_\pi^4}$$

$\lesssim 10\%$

1. The Axion Thermalization Rate Γ (from pions): our result



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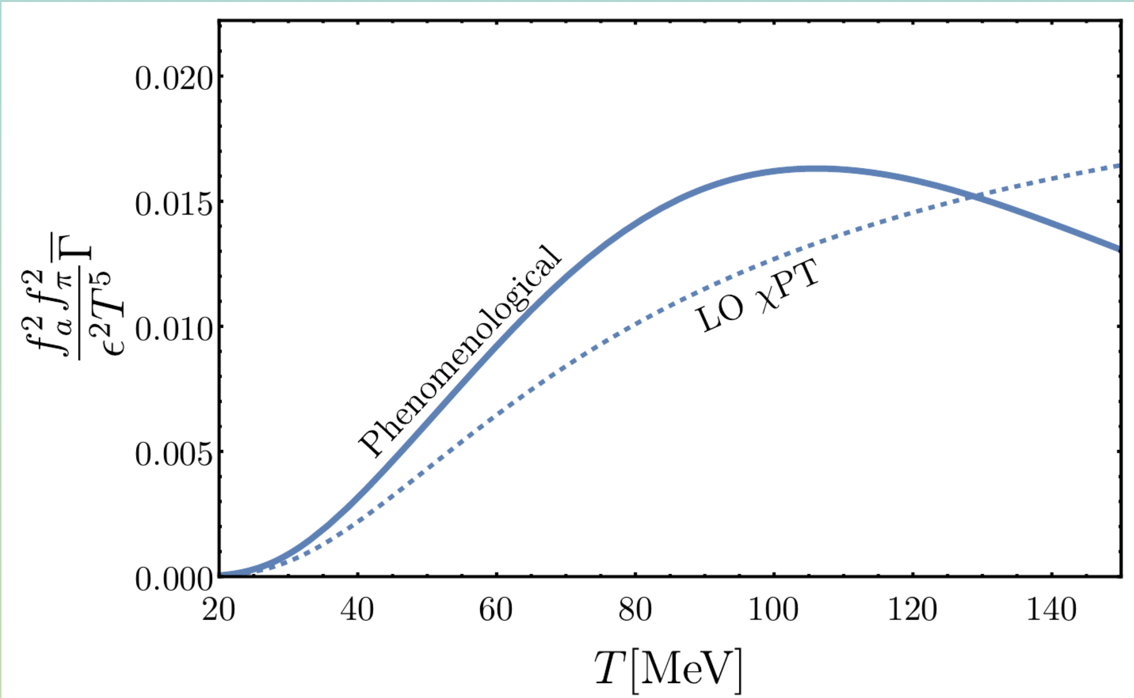

In reasonable agreement with:
Di Luzio, Camalich, Martinelli, Oller, Piazza PRD '23
(using NLO+unitarization)

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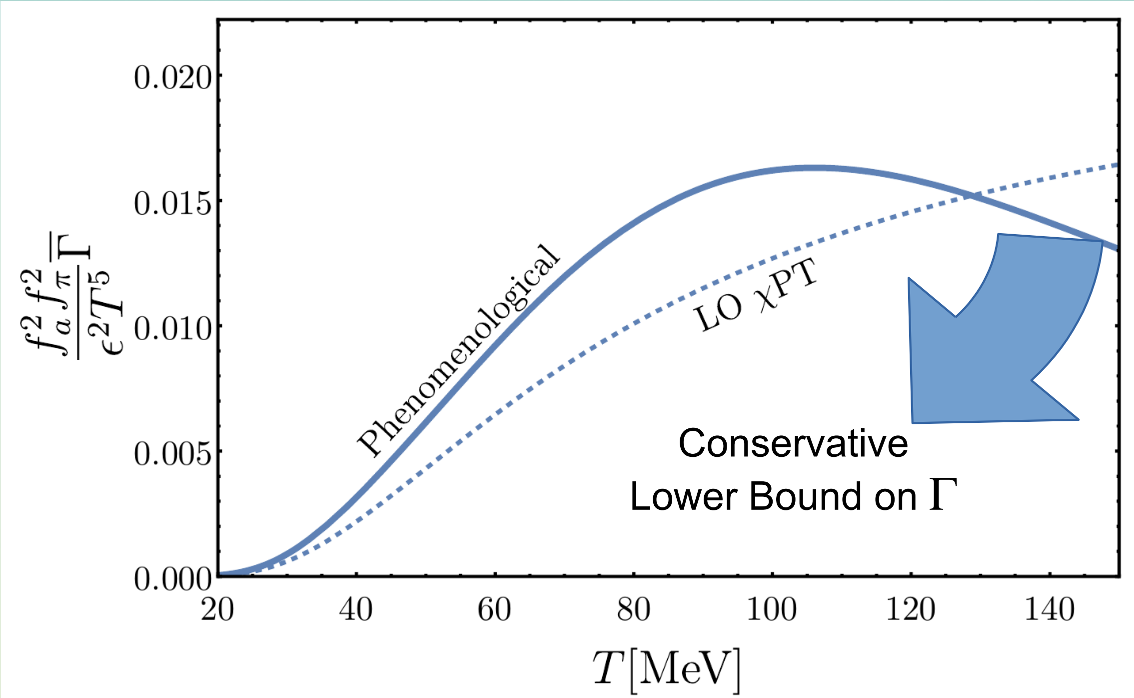

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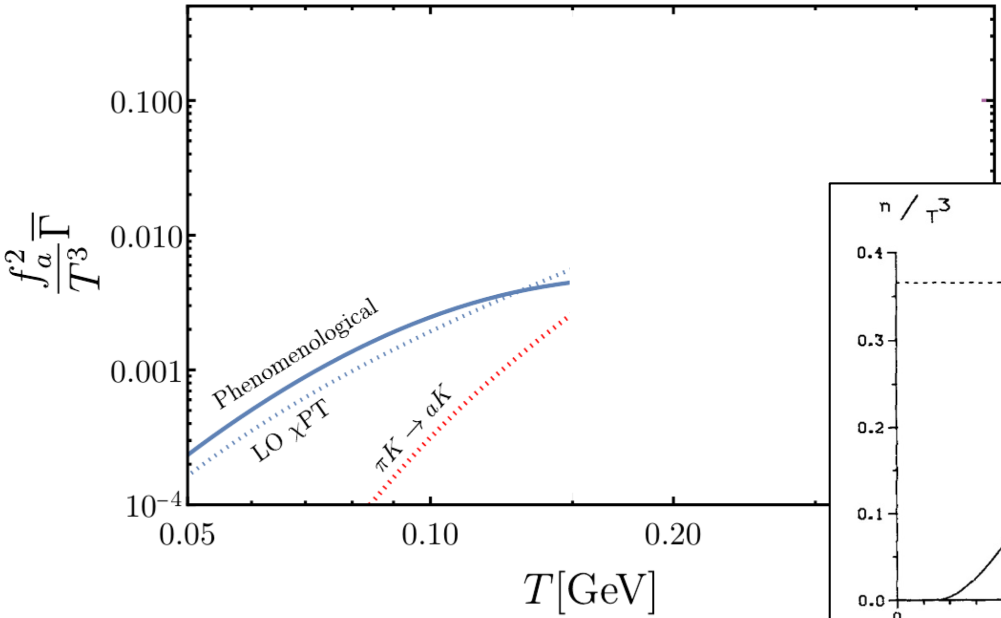
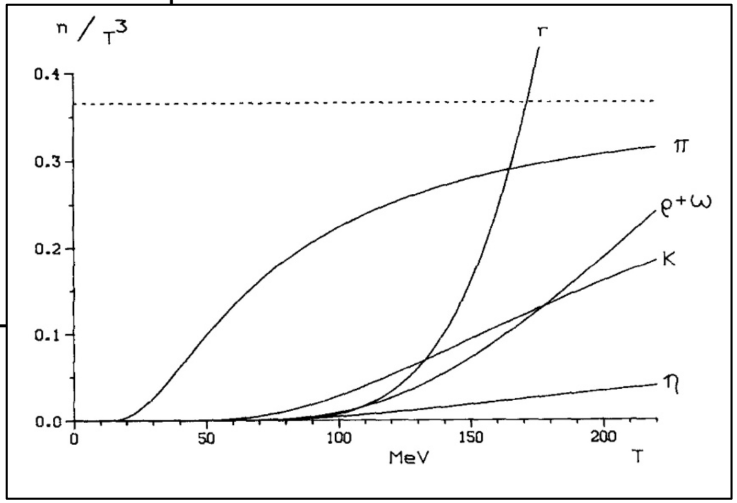
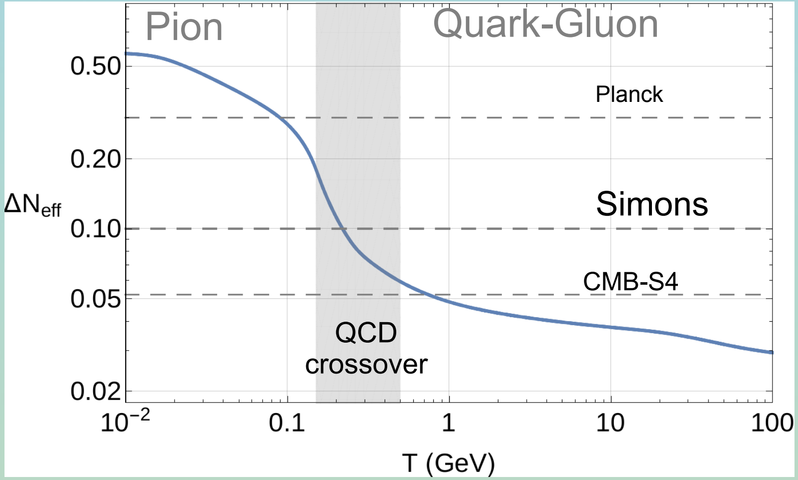
1. The Thermalization Rate Γ
(Possible other channels: Kaons,…)





Gerber Leutwyler '89

2. Momentum Dependence

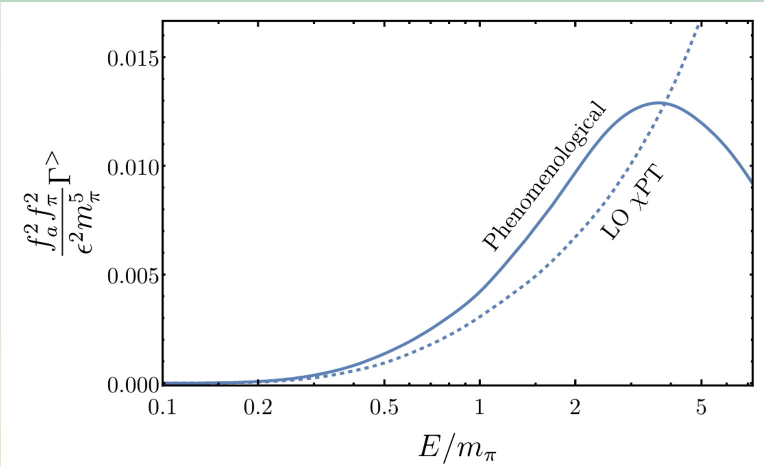


Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

High momenta k decouple later than low k

They see a lower g_* → More abundant



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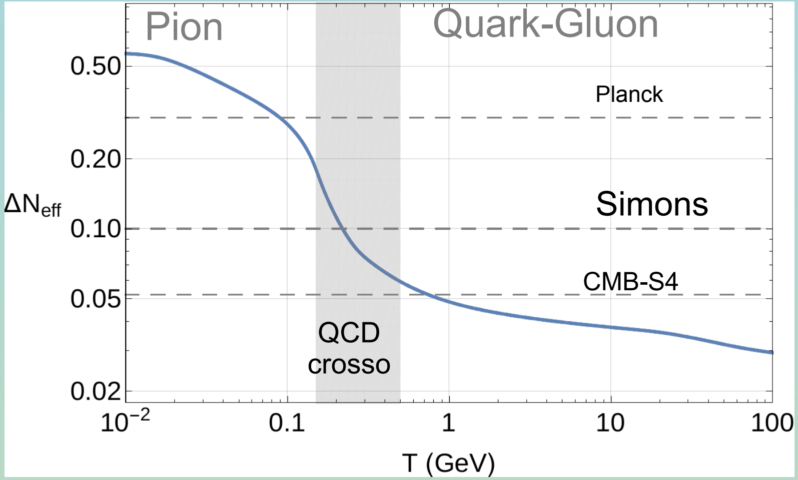

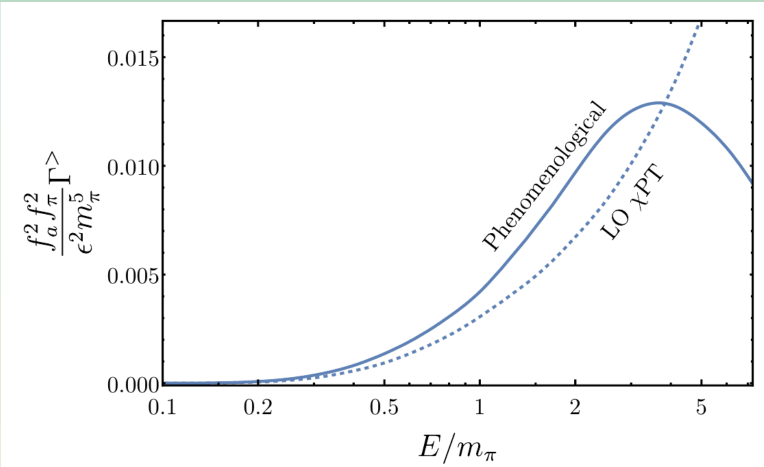

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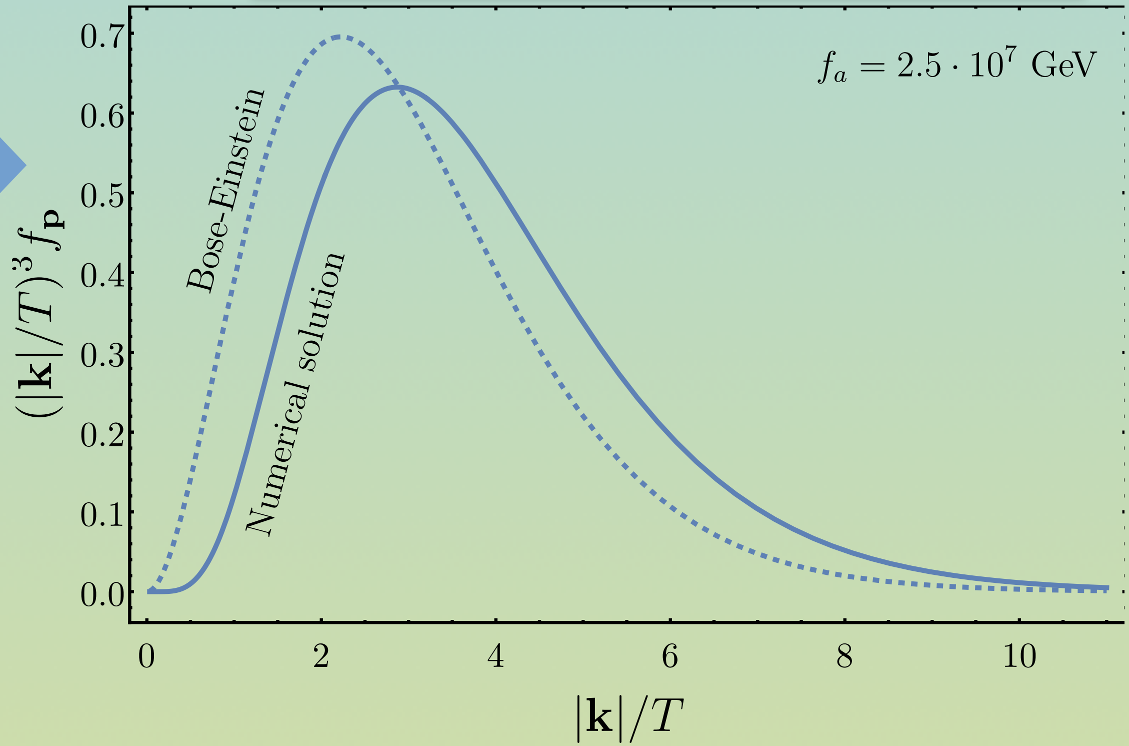
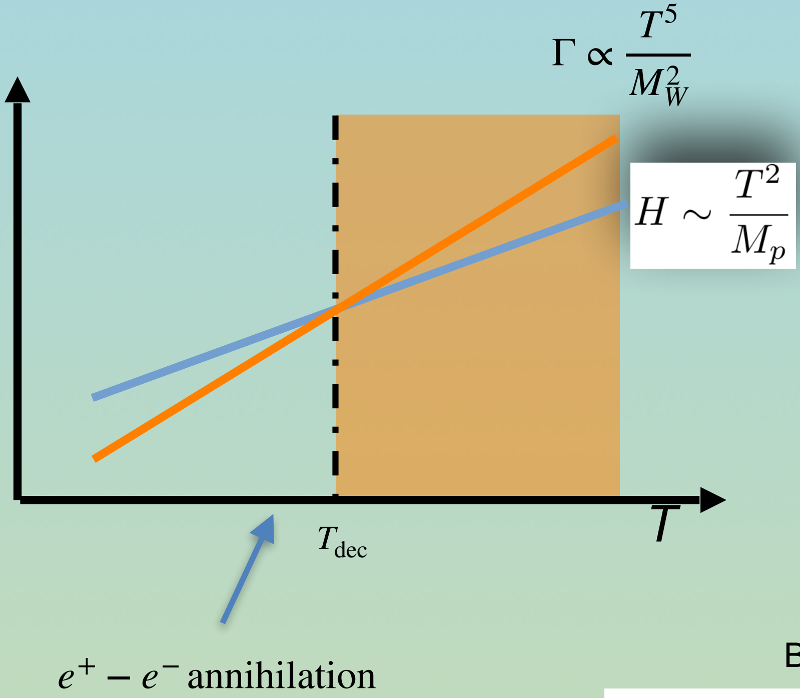
~ 40% enhanced total abundance



2. Momentum Dependence: Neutrinos



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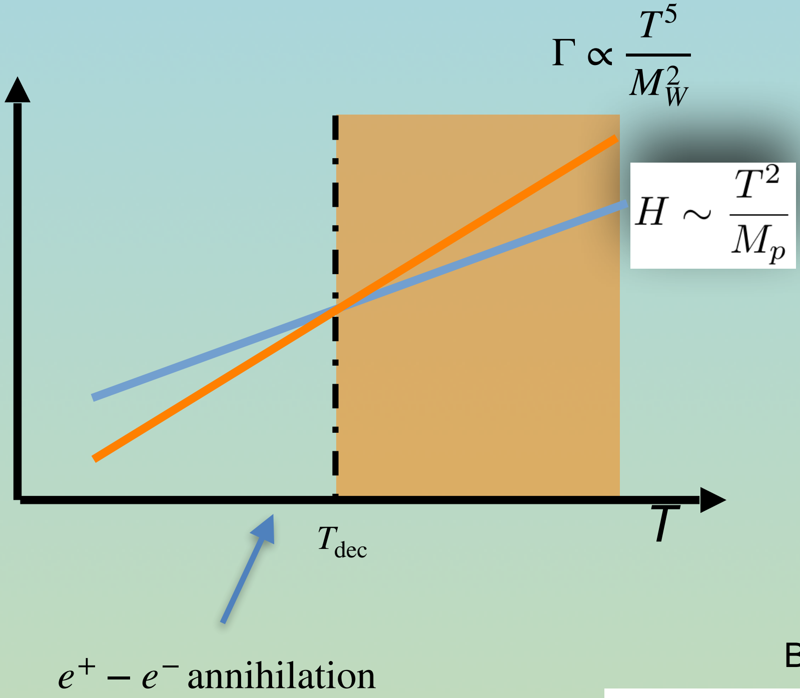

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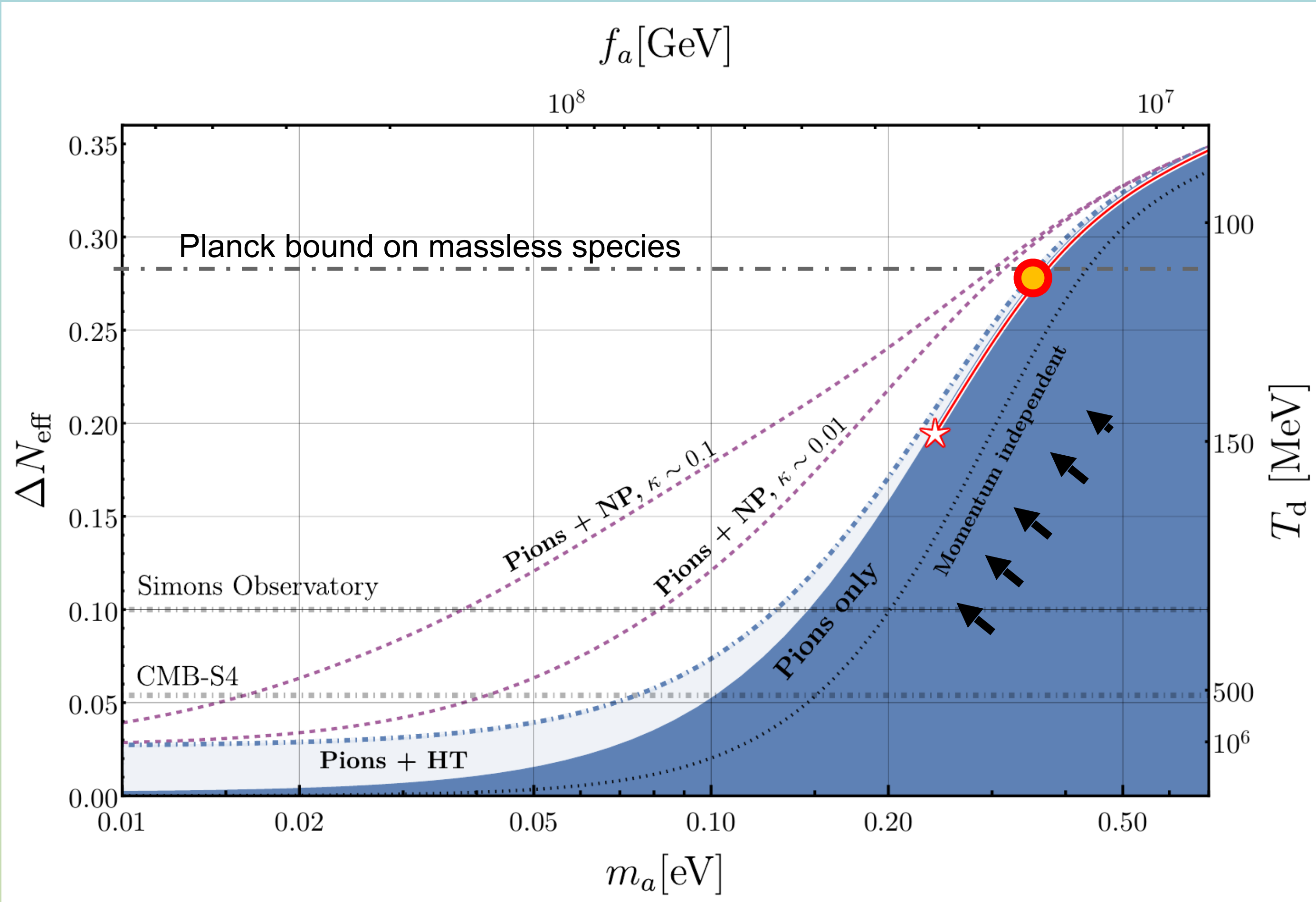
But m_e and $T_{\rm dec}$ are more separated →

Only ~ 1% enhancement

$N_{\rm eff} \approx 3.044$

Present bound+Future Reach



Dataset: Planck18+CMB lensing+BAO (BOSS)+Pantheon Supernovae

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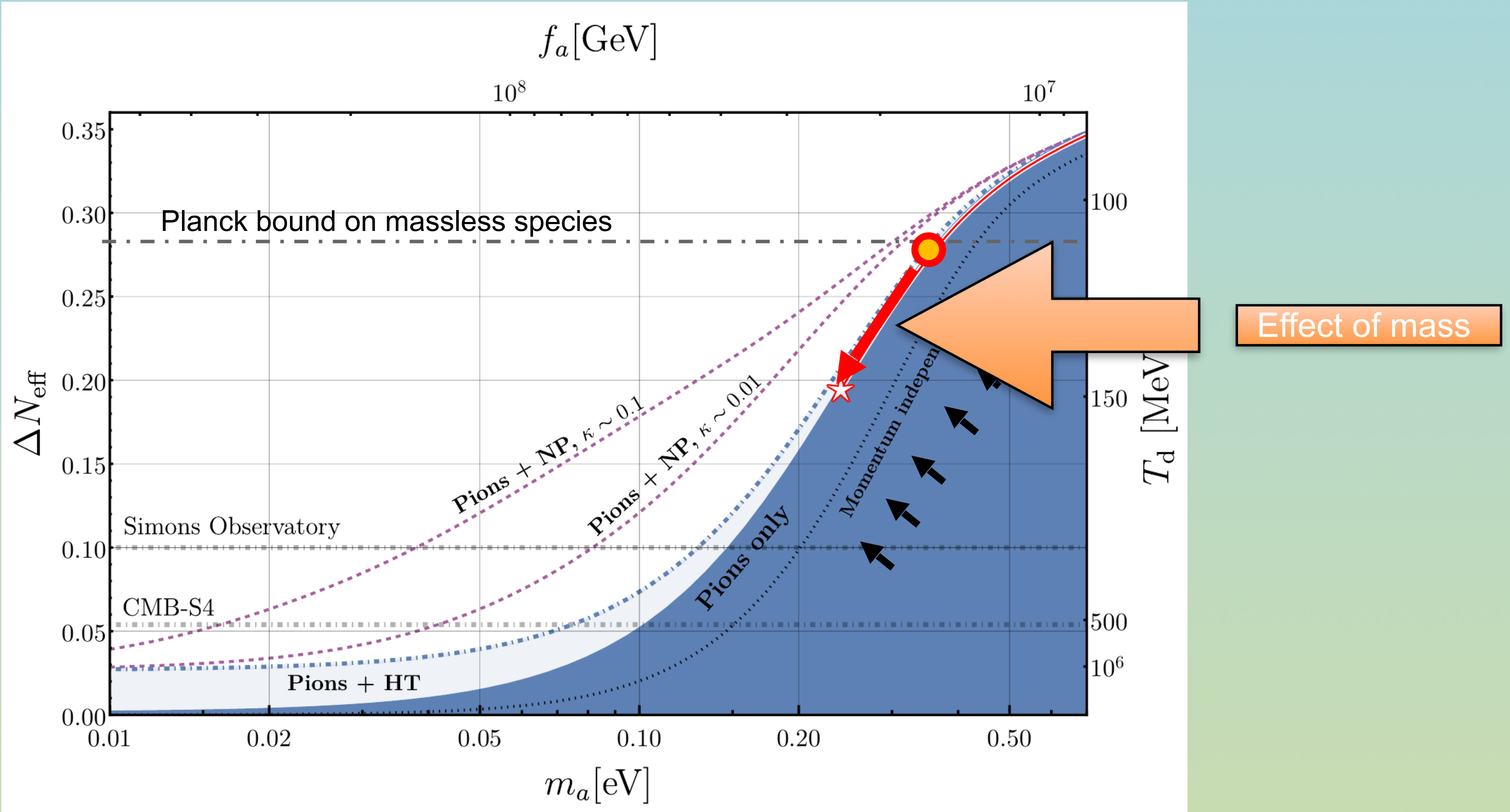

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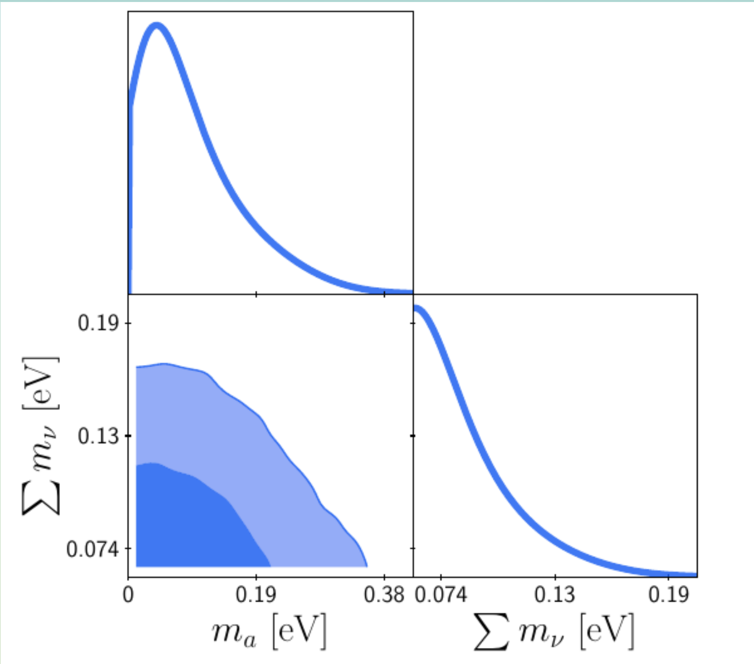
3. Combined cosmological Fit

(Λ_{CDM} + massive neutrinos + *axions*)



$$m_a \leq 0.24 \text{ eV} \qquad f_a \geqslant 2.4 \cdot 10^7 \text{ GeV}$$

$$\Leftrightarrow$$

$$\Delta N_{\text{eff}} \lesssim 0.19$$

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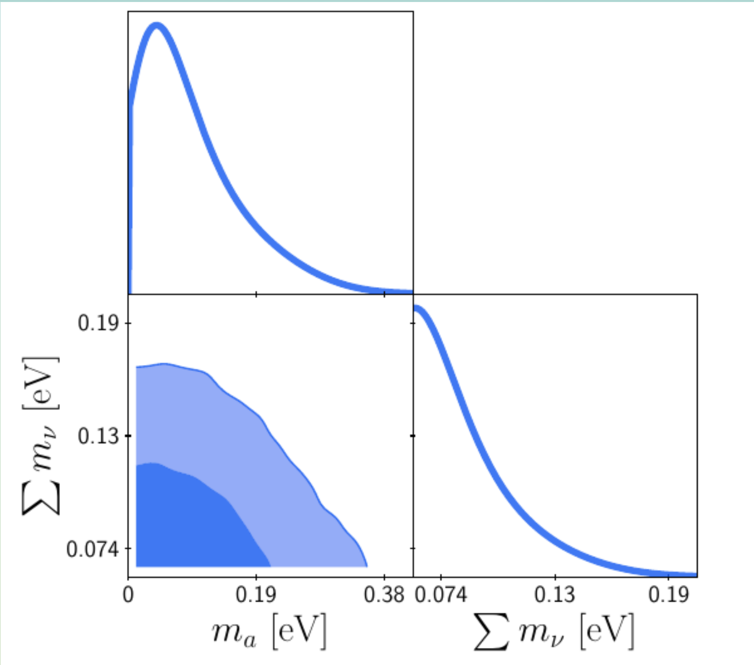

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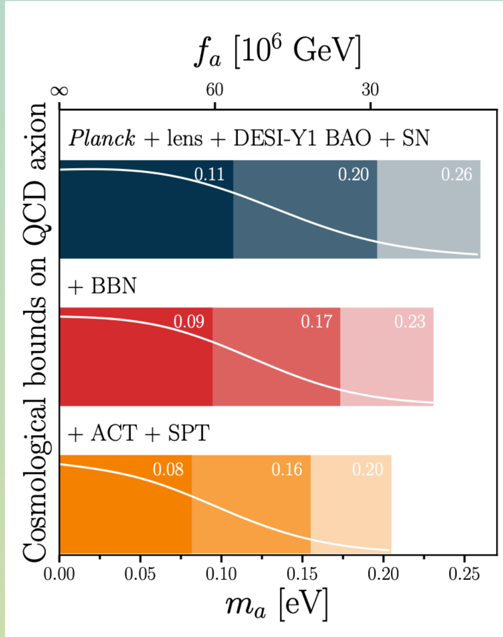
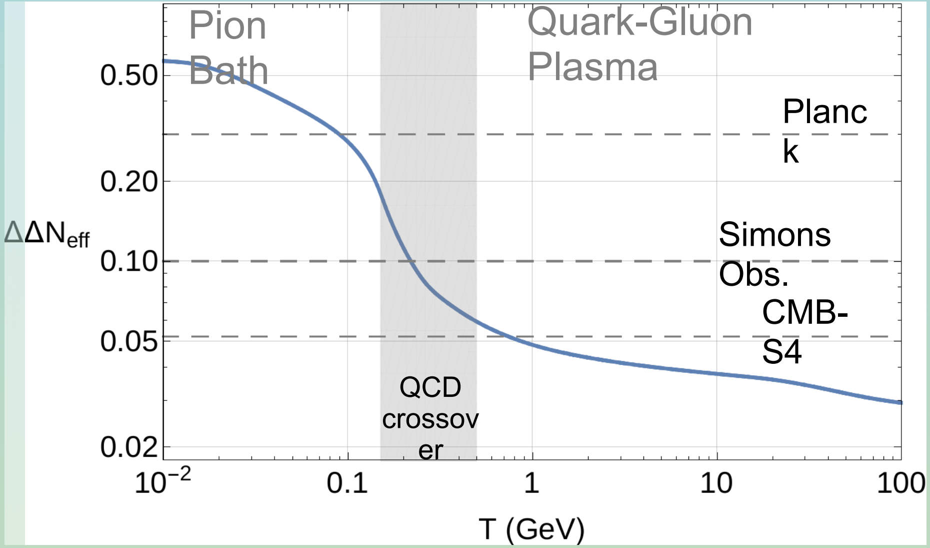
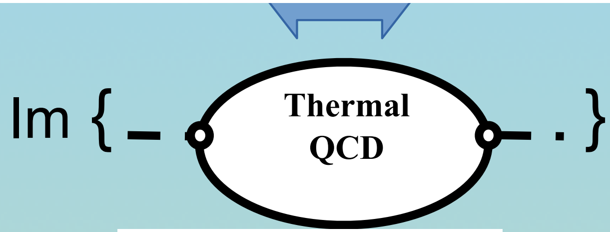
Adding BBN
(Bianchini et al. PRD'24):
$m_a \leq 0.16$ eV



Future Reach

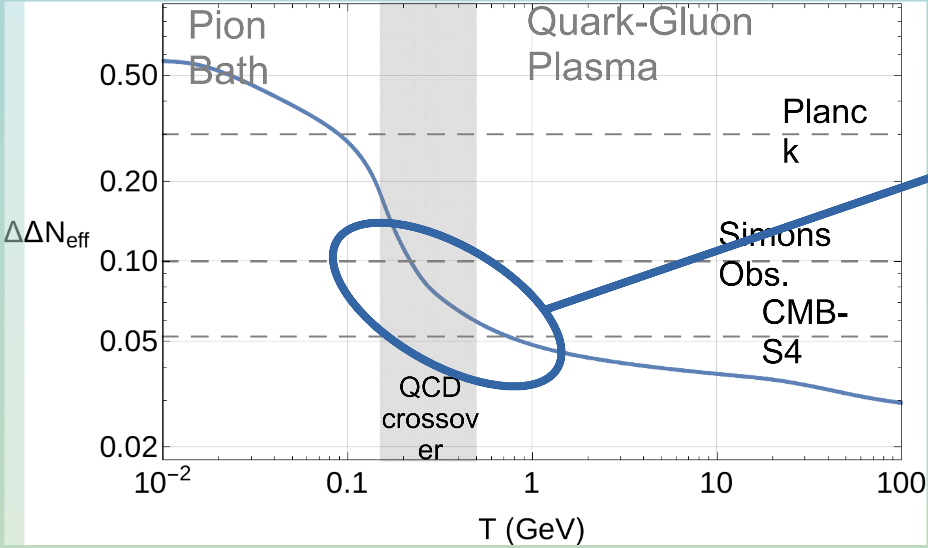


Future Reach



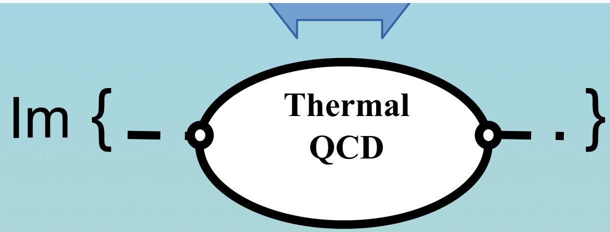
$$\Gamma^{>} = e^{\frac{E}{T}}\Gamma^{<} = \frac{\Gamma^{>}_{\rm top}}{2Ef_a^2}$$

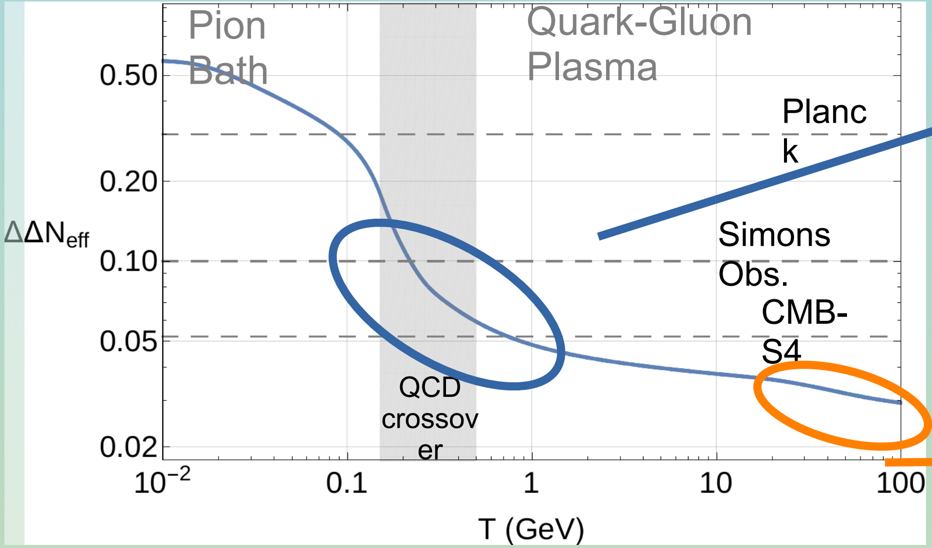
$$\Gamma^{>}_{\rm top} \equiv \int d^4x\, e^{ik^\mu x_\mu} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu)\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$



Non-Perturbative

Future Reach





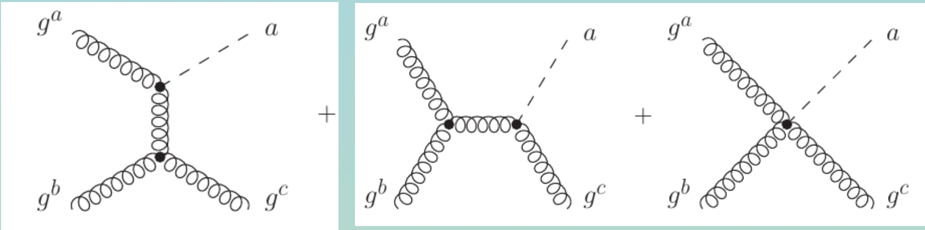
Non-Perturbative

$gg \leftrightarrow ga$

Perturbative ?

$$\overline{\Gamma}_{\text{pert}} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

High Temperatures Regime



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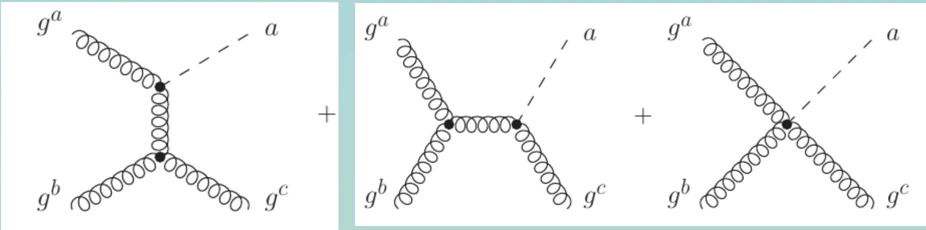

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Masso, Rota, Zsembinszki '02
Graf, Steffen '10

$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

for $g_s \ll 1$

IR divergent

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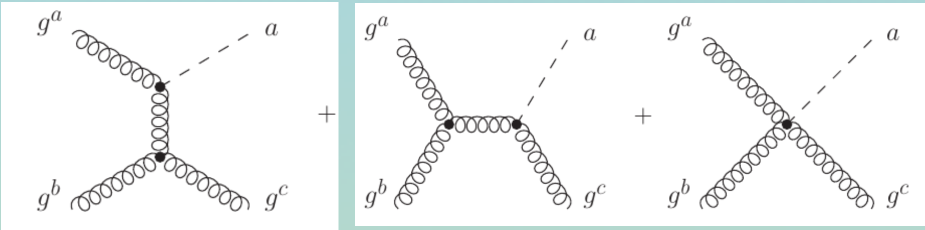

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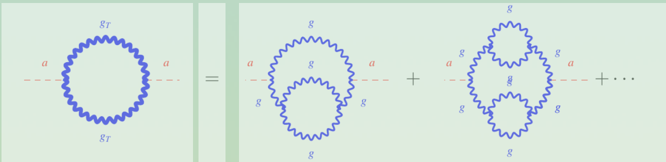

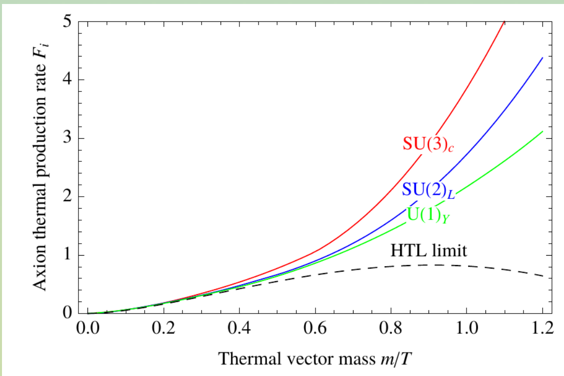

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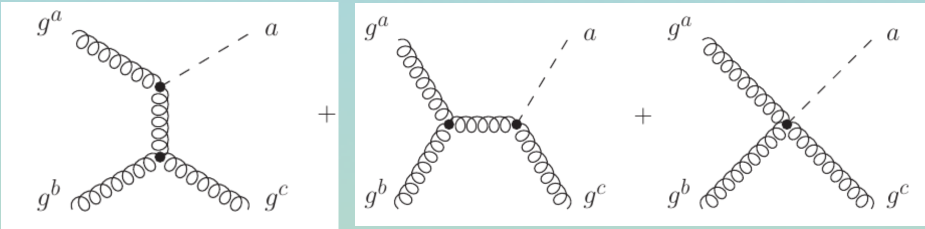

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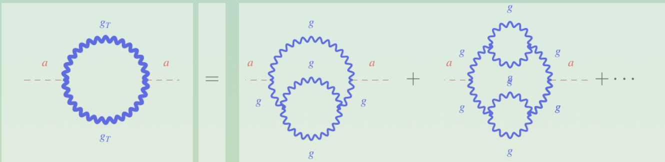
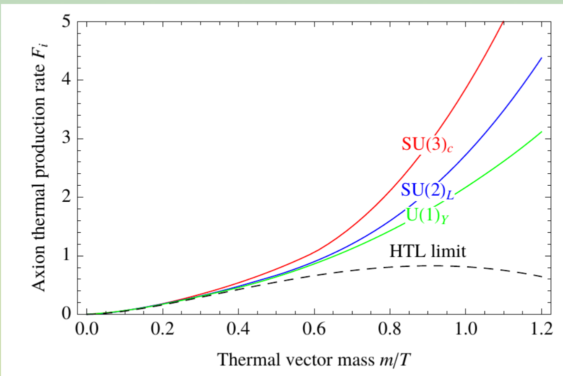
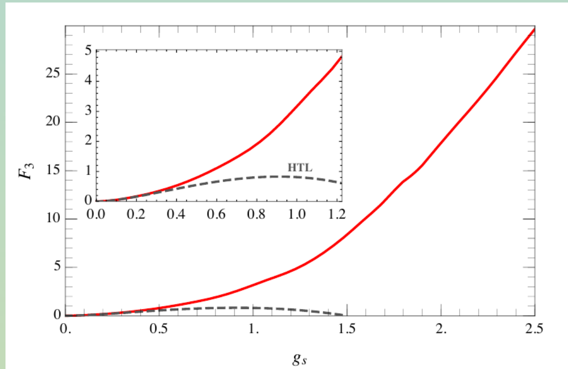


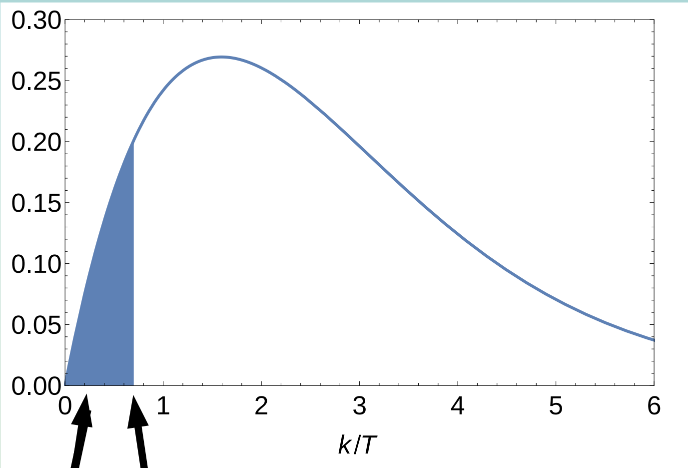


D'Eramo, Hajkarim, Yun ('21):
extrapolated F_3 from Salvio et al. to $g_s > 1$
(Beyond regime of validity?)



*Matching gluon to pions through QCD crossover?
Pion-axion: suppressed by $\theta_{a\pi} \propto \dfrac{m_u - m_d}{m_u + m_d}$, gluon is not
Pion rates not monotonic with T
Rates could have sudden jumps, as g_* does

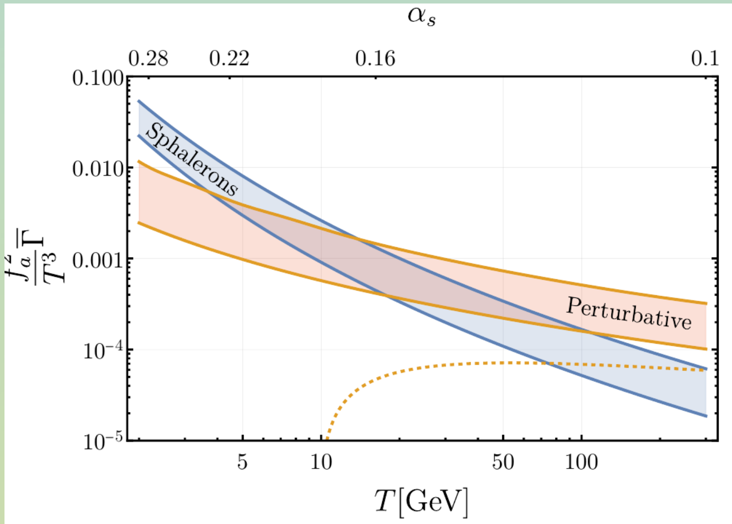
Intermediate Temperatures Regime



$k \sim m_e \sim g_s T$

$k \sim m_m \sim g_s^2 T$
$\# \sim 1 / g_s^2$

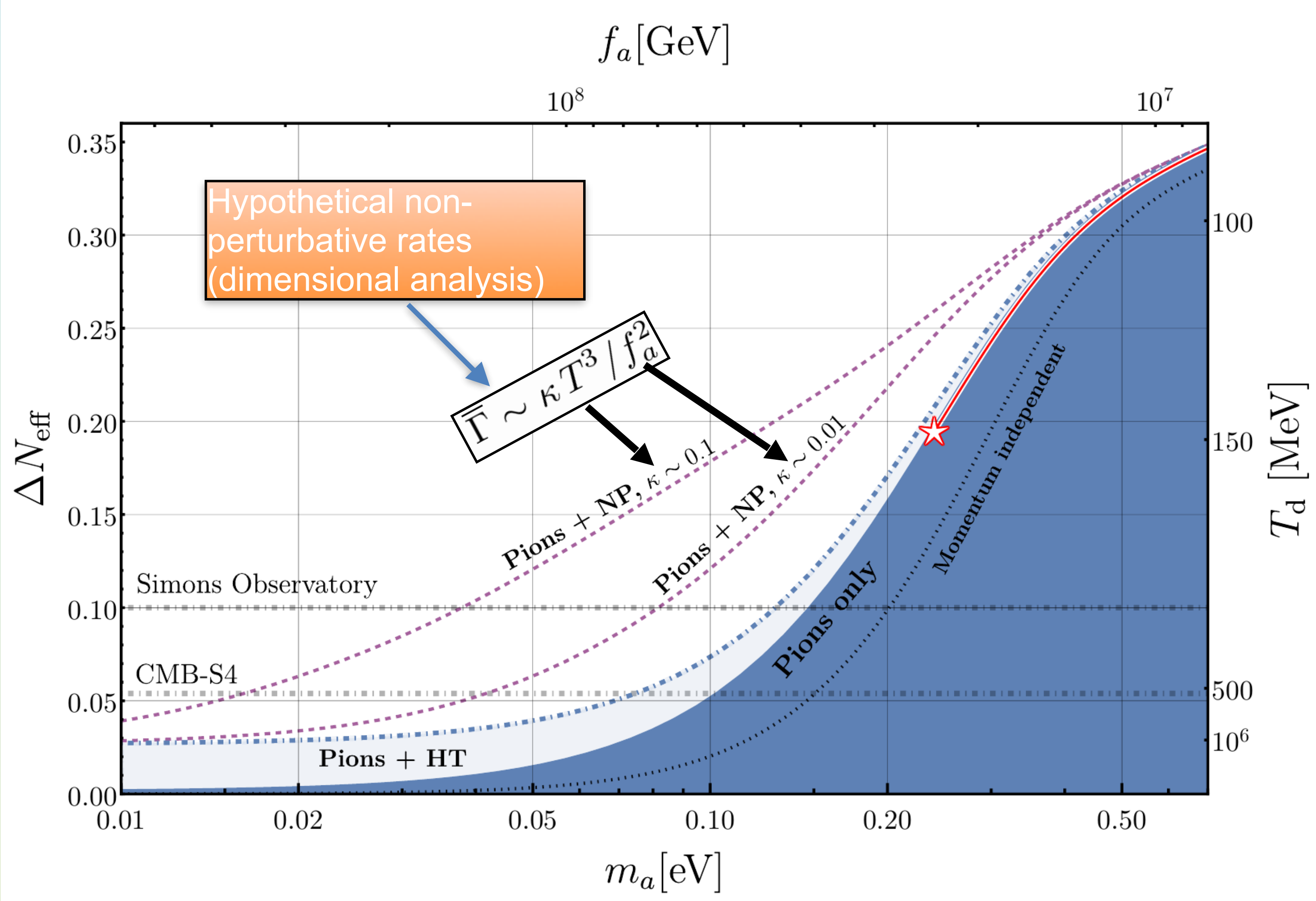
Comparison: dissipation from strong sphalerons vs perturbative rate



$$\Gamma_{\text{sphal}} \simeq \frac{(N_c \alpha_s)^5 T^3}{f_a^2}$$

(Adapted from: Moore, Tassler '10)

Future Reach ($T_{\rm DEC} \gtrsim T_c$ region)



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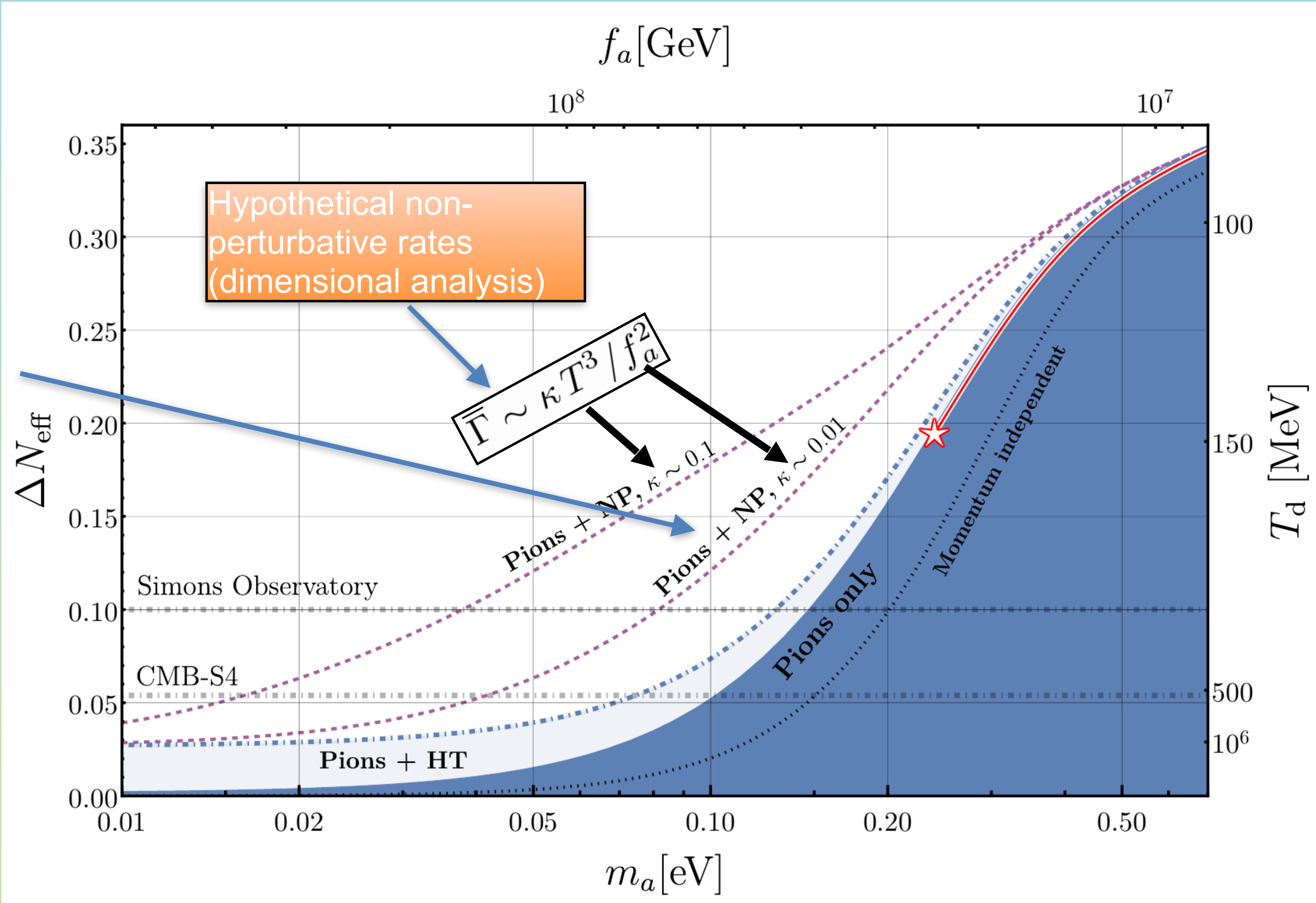
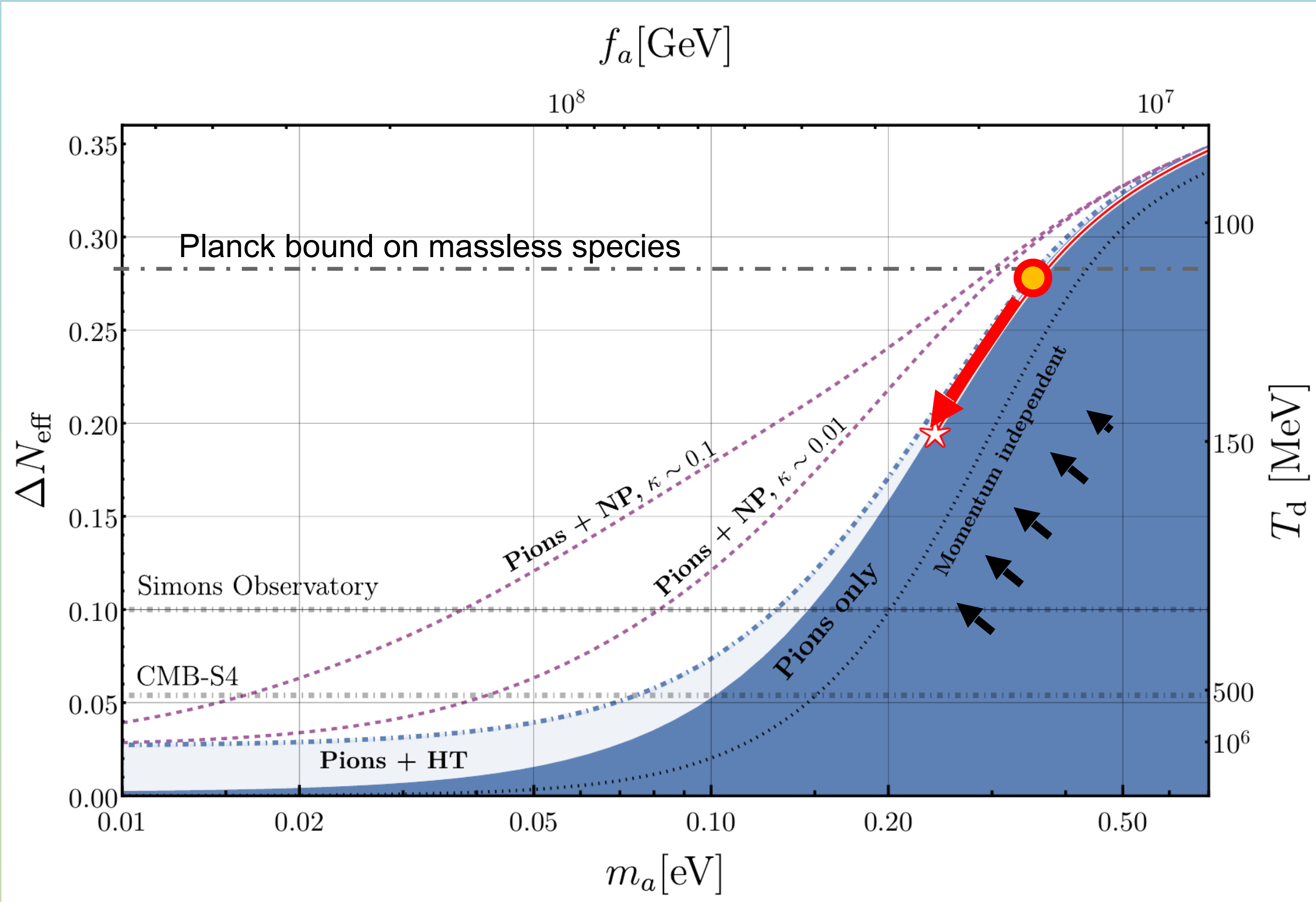


Consistent with very recent Lattice QCD simulation ([Bonanno](#) et al. PRL '24) on $\Gamma_{\rm sphal}$

Present bound+Future Reach



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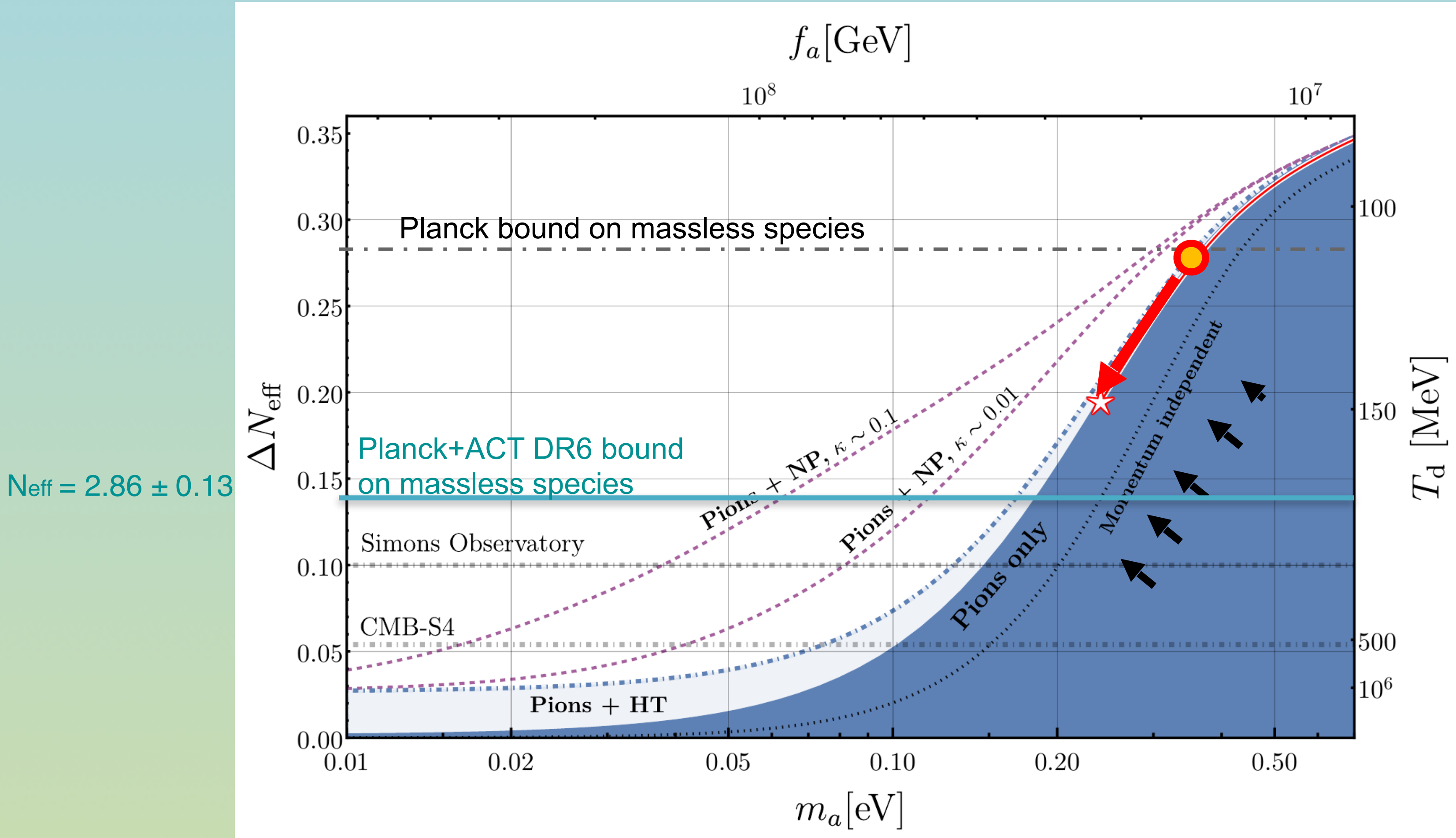
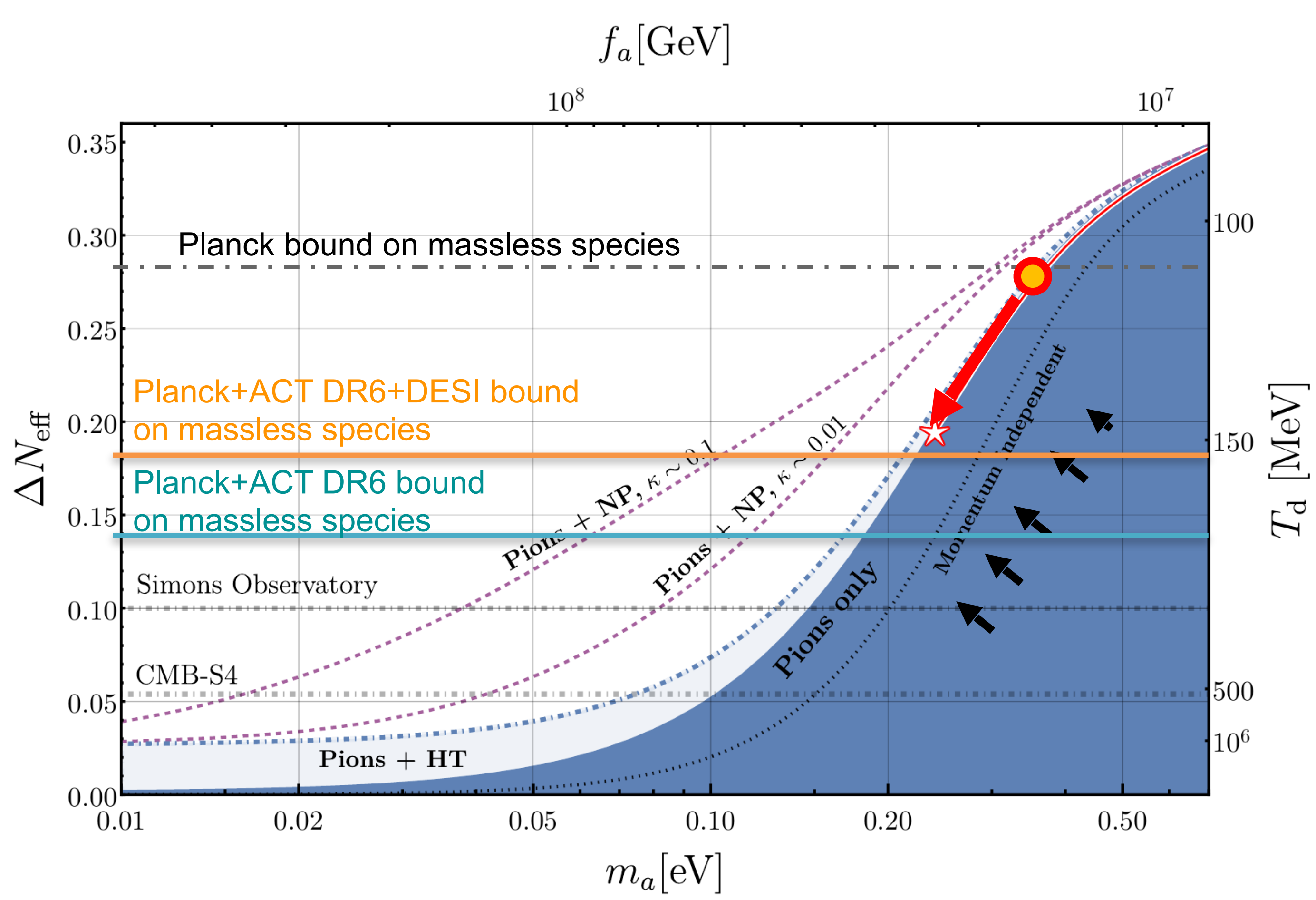
Current bound+Future Reach(95%C.L.)



Current bound+Future Reach(95%C.L.)

Neff = 2.86 ± 0.13



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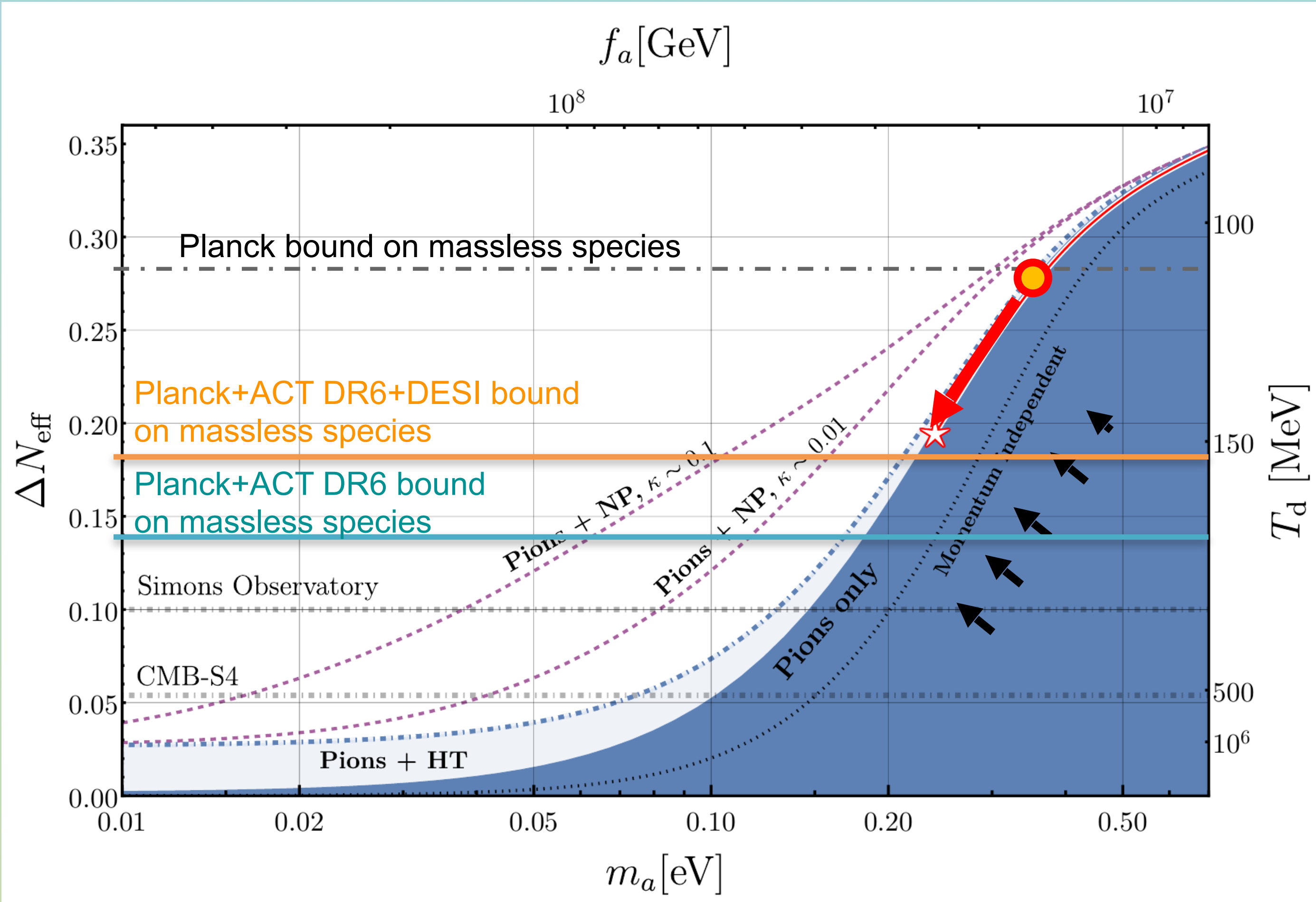
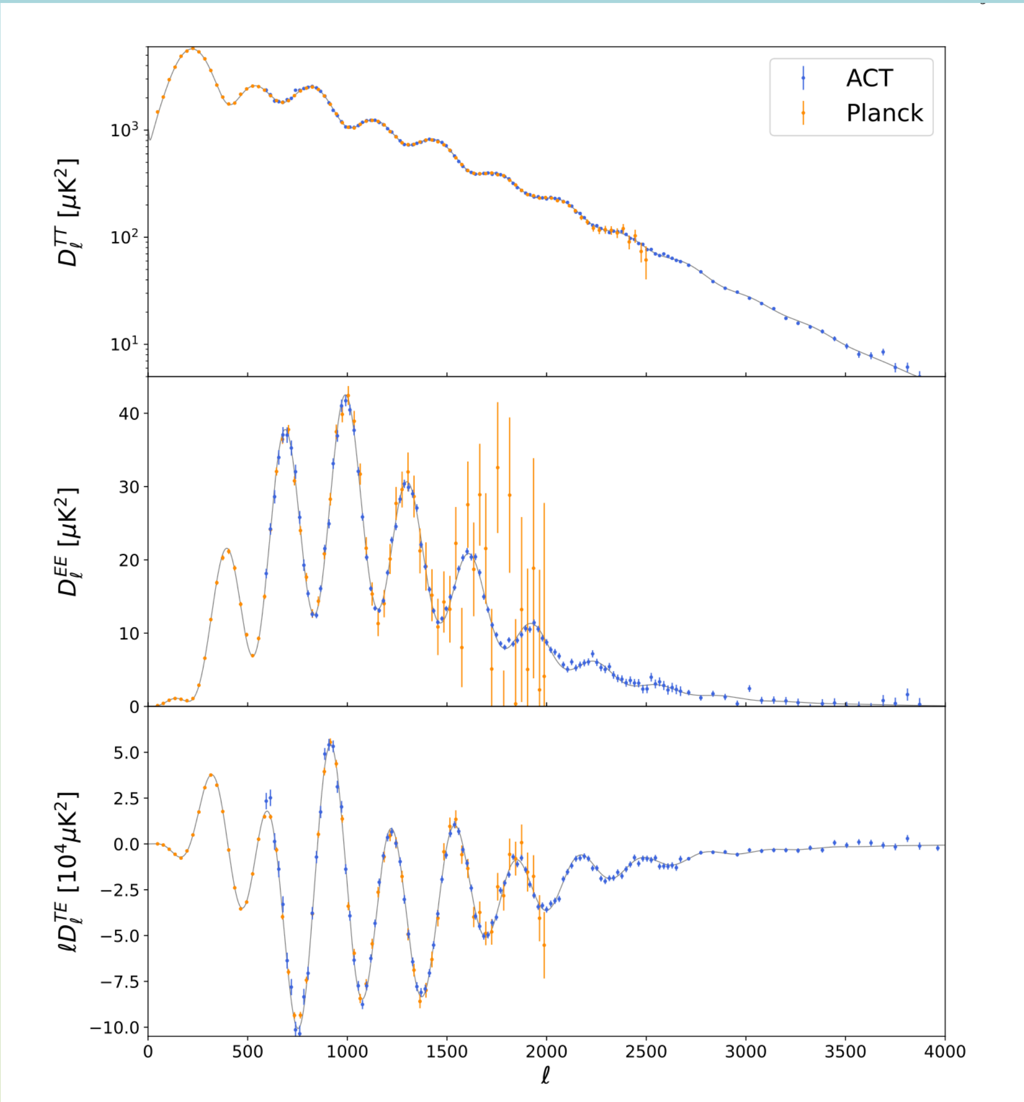


$N_{\rm eff}$ = 2.86 ± 0.13

Soon SPT-3G Ext-10k (Prabhu et al., 2024) survey will improve : $\sigma(\Delta N_{\rm eff})$ = 0.07

PLANCK+ACT DR6+DESI BAO



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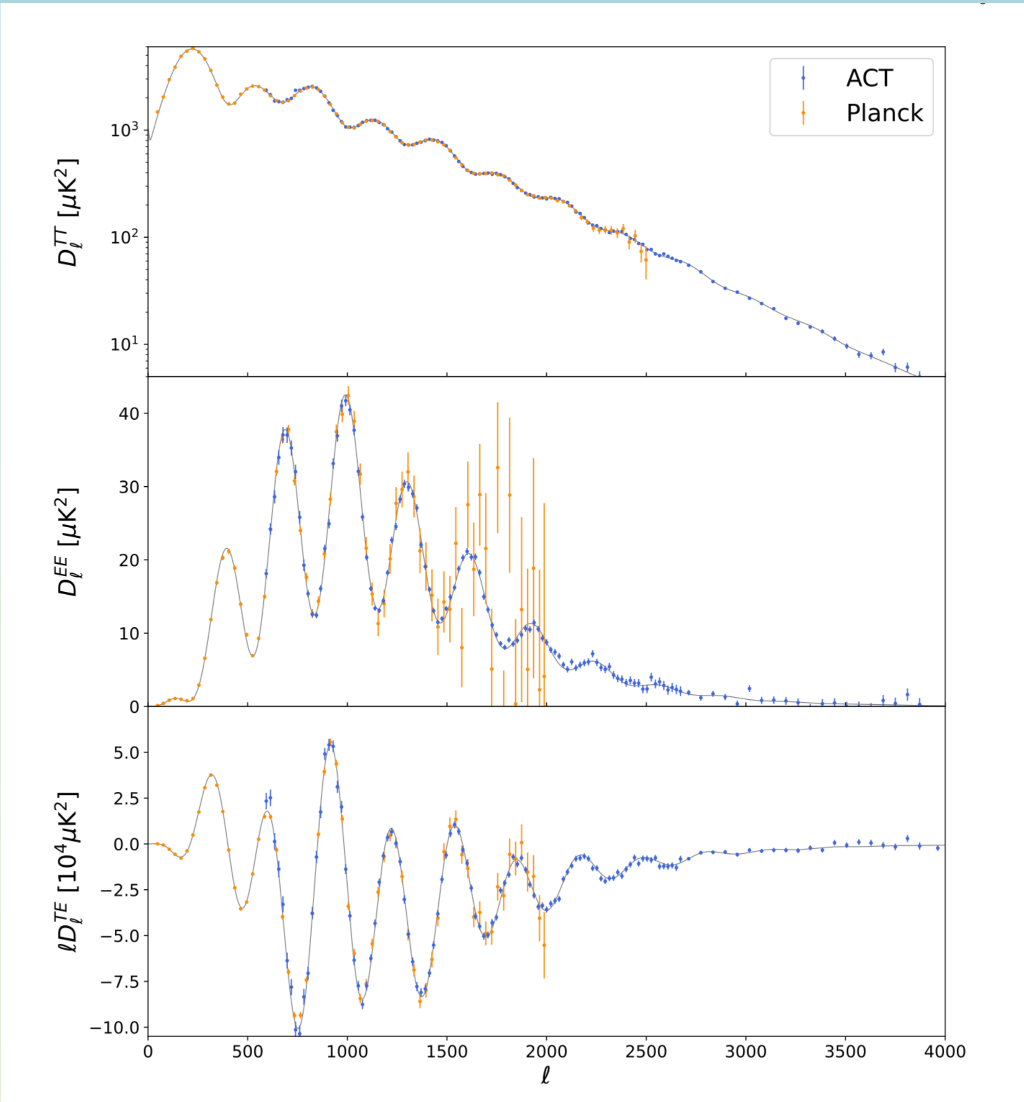

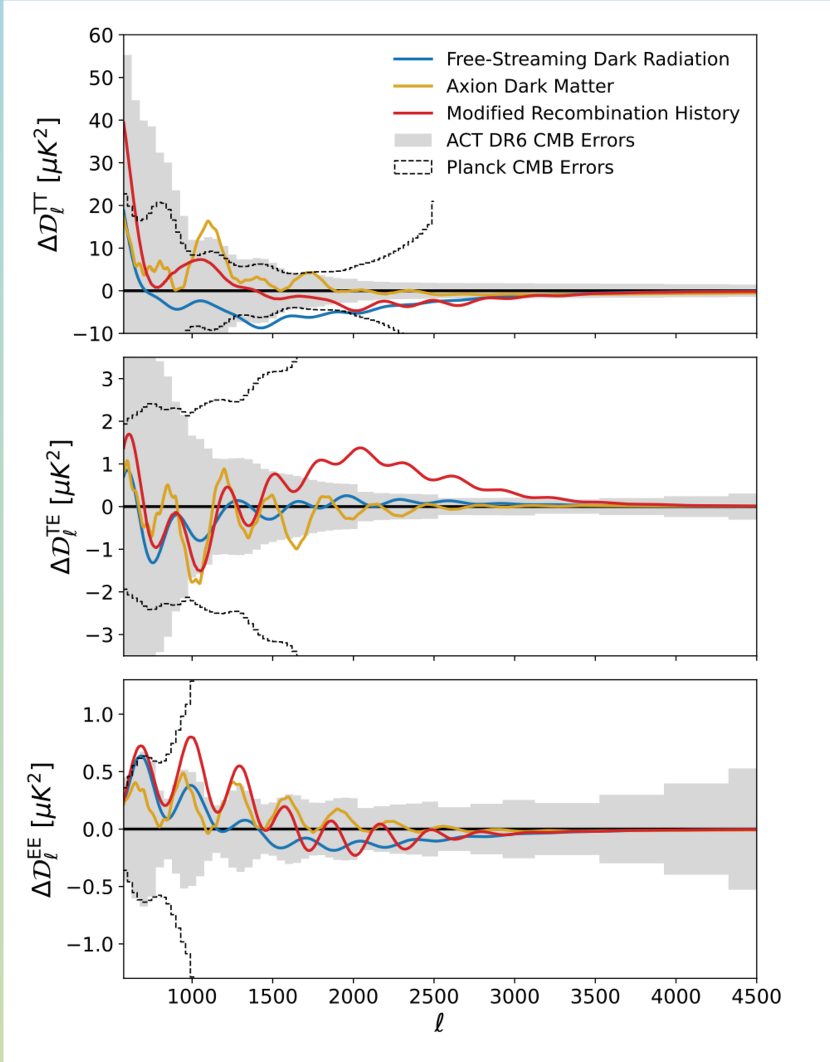

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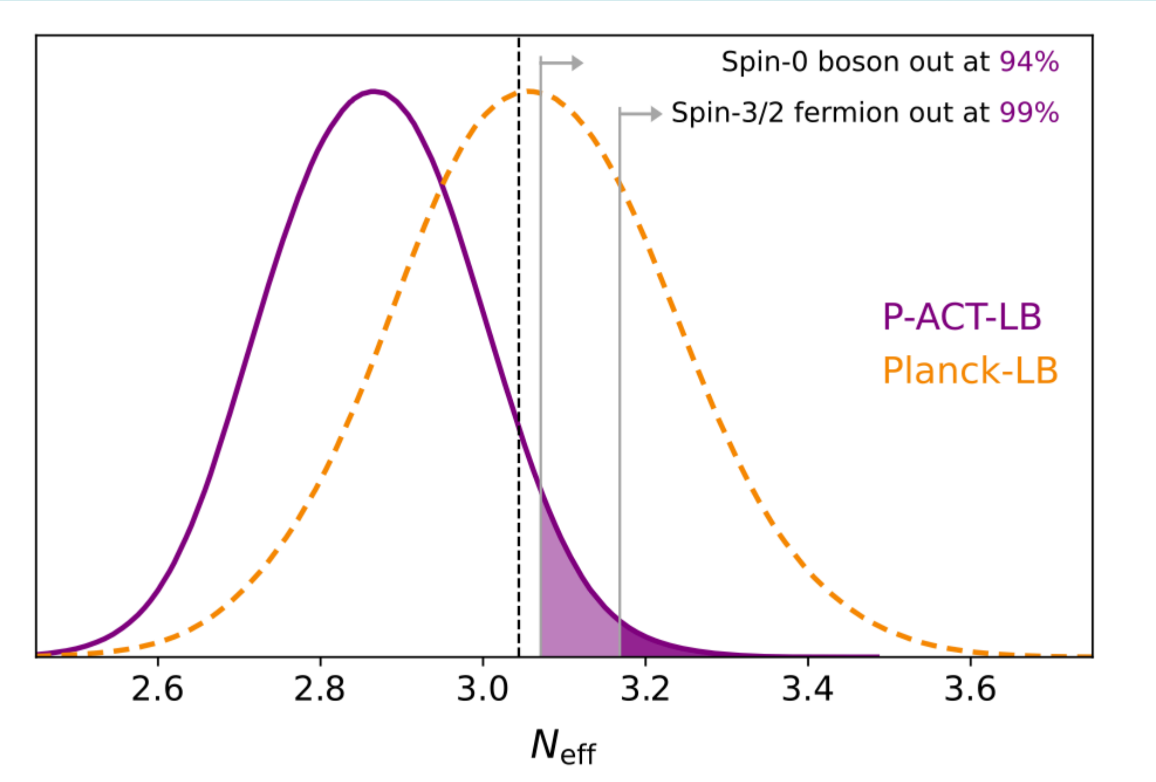

Soon SPT-3G Ext-10k (Prabhu et al., 2024)
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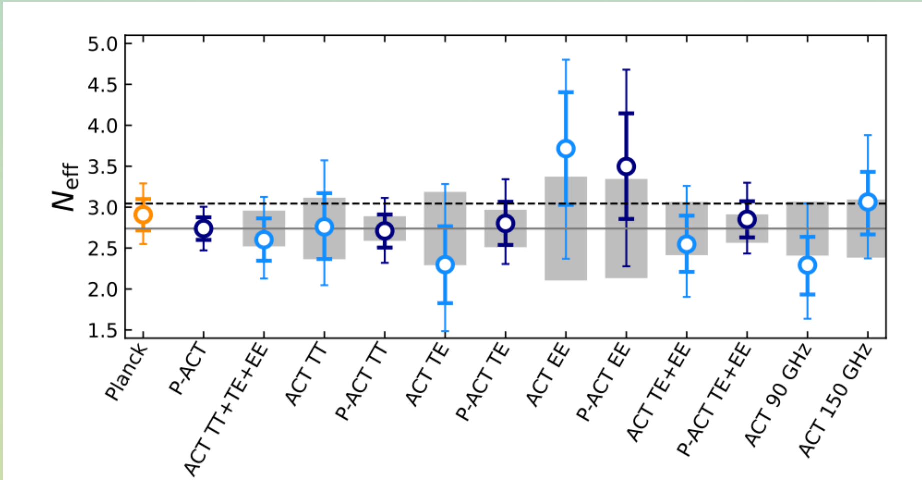

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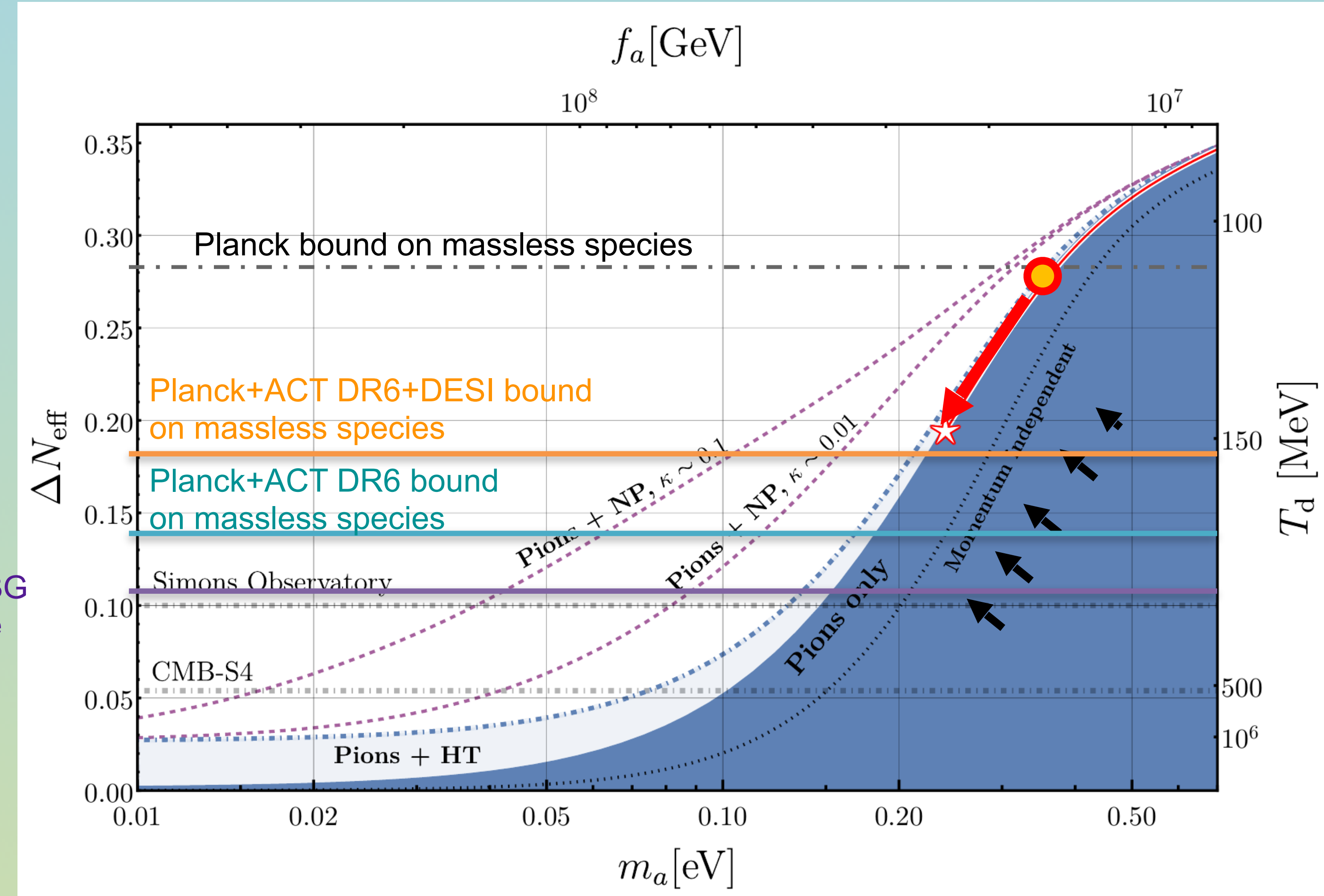

Planck+ACT DR6+SPT-3G + new BBN bound (Large Binocular Telescope) on massless species

(Goldstein, Hill, arXiv:2603.13226)

Conclusions:

- Conservative reliable pion-axion rates m_a (< 0.24 eV) from cosmology (for minimal KSVZ-like QCD axions), Planck+BOSS BAO+Pantheon See
- Importance of momentum dependence on Boltzmann equation @ around QCD scale

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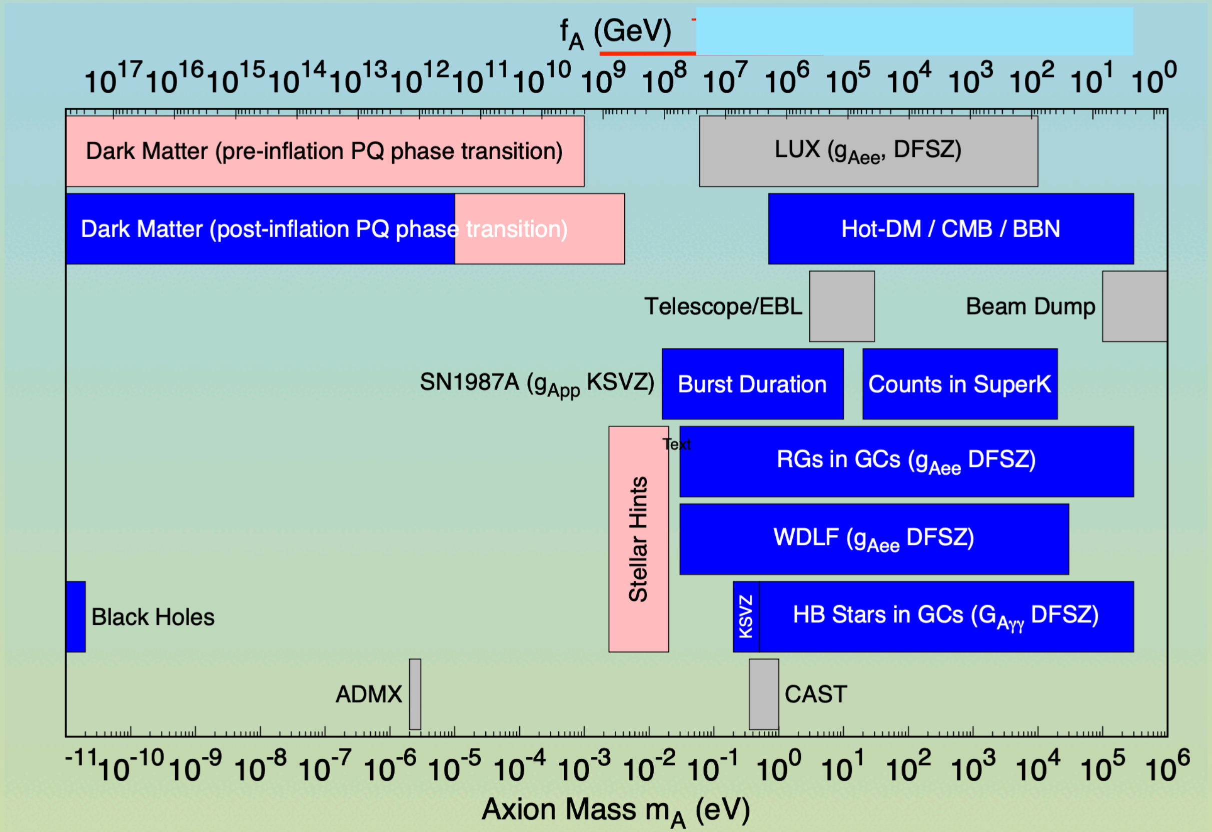
(*If axion couples directly to SM quarks and leptons: more production channels)

Extra question:

Can these large masses (m_a ~ 0.001 - 0.1 eV) overlap with **Dark Matter window**?

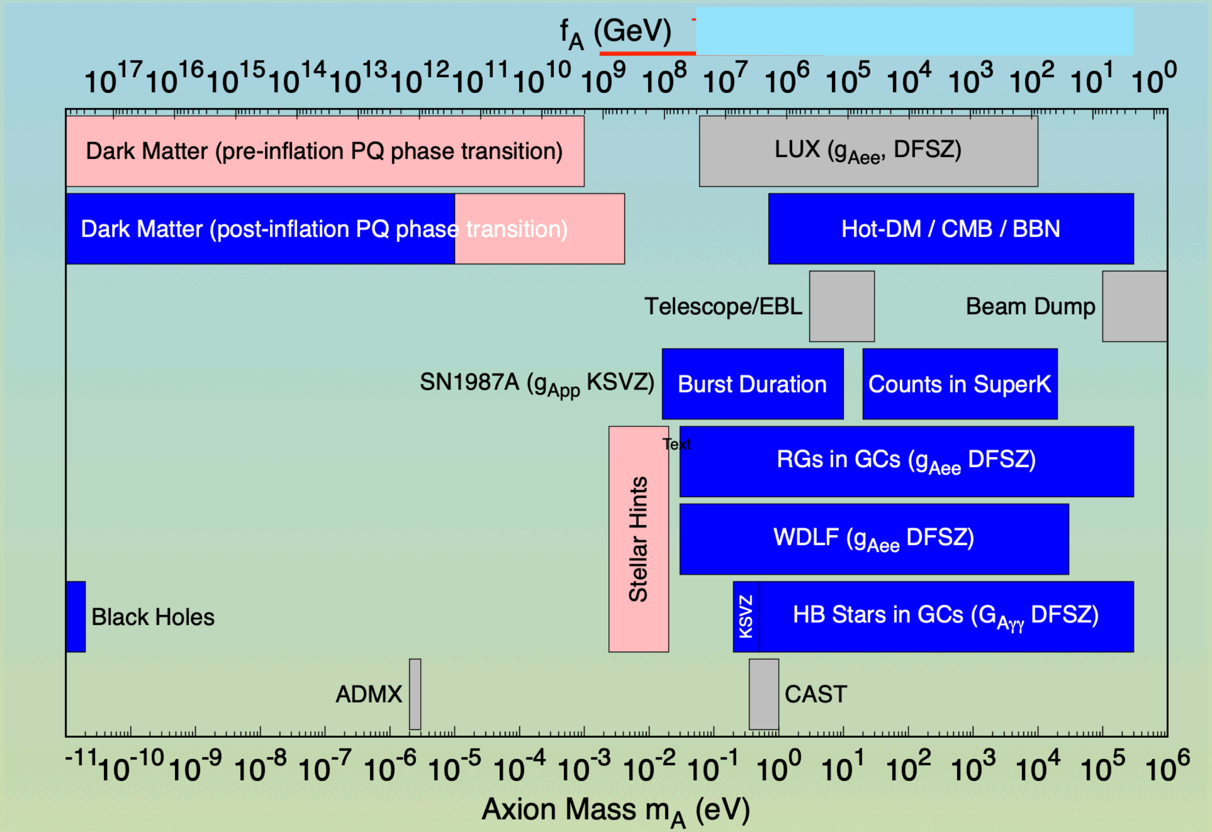


Extra question:

Can these large masses ($m_a \sim 0.001$ - 0.1 eV) overlap with **Dark Matter window?**

Inflationary dynamics, for $H_I \gtrsim f_a$, and lower $T_{\rm RH}$, could create a network of strings that decays "late", enhancing abundance of Dark Matter axions

(M. Gorghetto, E. Hardy, H. Nicolaescu, A. N., M. Redi, JHEP '24)



Back Up

AXION AS COLD DARK MATTER

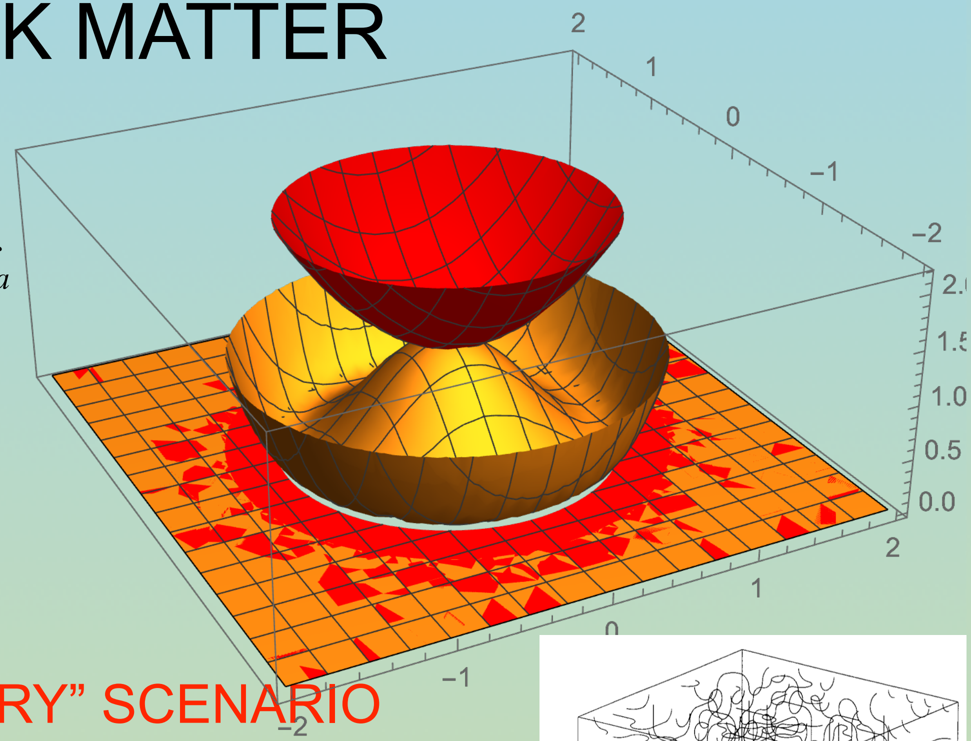
- The axion arises from a Complex Scalar (``KSVZ" models):
- $V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2 \qquad v = f_a \;\; (N_{\rm DW} = 1)$

- The symmetry is broken during inflation $\Phi = ve^{i\theta} = f_a e^{i\frac{a}{v}}$,
- A scalar field in inflation has quantum fluctuations of order H_i
- If very small ($H_i \ll f_a$)

- $\theta(t_i) = a(t_i)/v$ is a random value in $(-\pi, \pi)$, almost homogenous in our horizon

``PRE-INFLATIONARY" SCENARIO

AXION AS COLD DARK MATTER



- Another possible scenario: If the Universe reaches $T = f_a$
- At $T > f_a$ the symmetry is restored $v(\Phi, T) \approx T^2 |\Phi|^2$
- At $T \approx f_a$ the symmetry gets broken

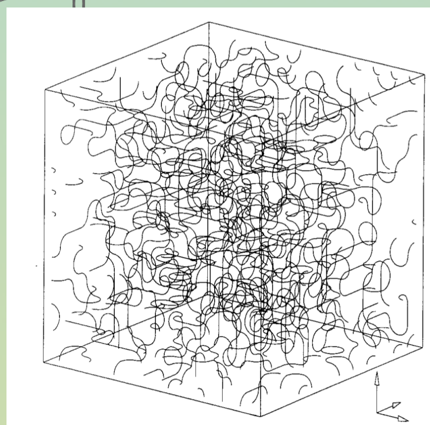
``POST-INFLATIONARY" SCENARIO

The field falls randomly

Strings form when the phase wraps from 0 to 2π

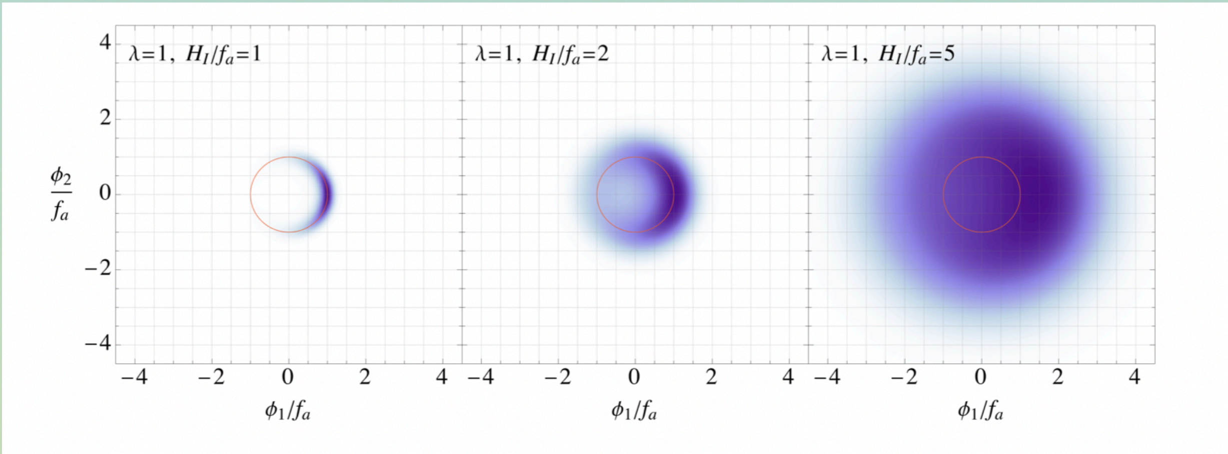
Network of strings forms

After initial transient → "scaling" behavior
O(1) string per Hubble volume



AXION AS COLD DARK MATTER

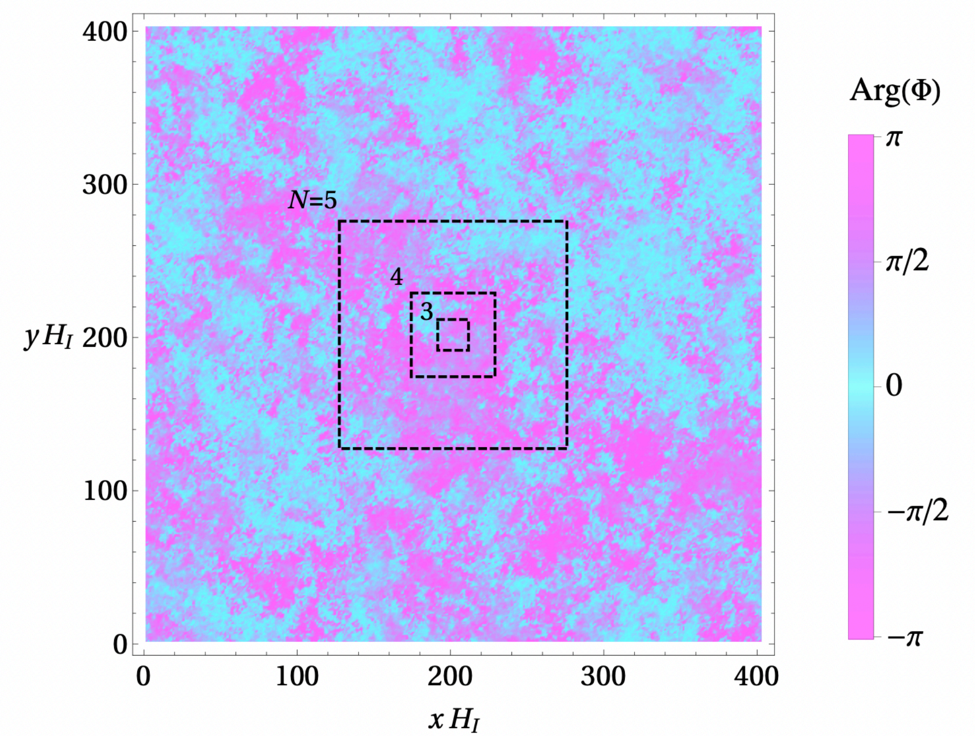
- THIRD POSSIBILITY: "STOCHASTIC INFLATIONARY SCENARIO"
- $H_i \gtrsim f_a$ large fluctuations during inflation (see Lyth 1992, Lyth & Stewart 1992)
- Both the angular and the radial field have large fluctuations



- Strings form due to large inflationary fluctuations
- If Temperature is never large enough after inflation ($T < f_a$) Symmetry is NOT restored after inflation

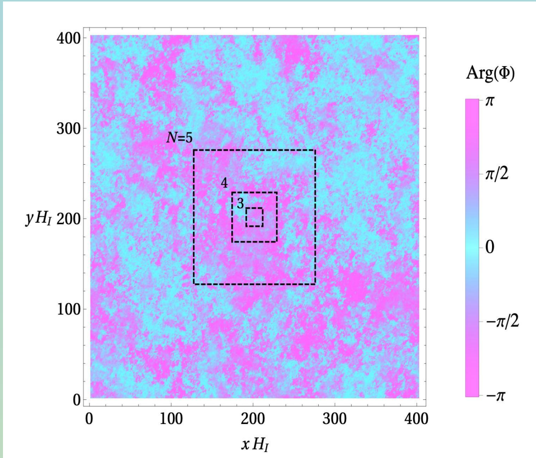
AXION AS COLD DARK MATTER

- On small patches: angle θ almost constant
- On large patches: can wrap from 0 to 2π
- Strings form, separated by a length $d = e^{N_s}/H_I$
- $N_s \approx 10/\sqrt{\lambda}$
- If $N_s \gtrsim 60$ field coherent in our entire horizon
- If $N_s < 60$ Strings separated by macroscopic length d



RANDOMISATION DURING INFLATION

- Once a Fourier mode exits horizon ⇒ becomes classical with amplitude $\frac{H_I}{2\pi}$ ⇒ gives a kick to average field value
- In a coarse-grained region of size H^{-3} : field undergoes random walk, independently in causally disconnected regions, $\mathrm{Var}(\Phi) = \frac{H_I}{2\pi} N$



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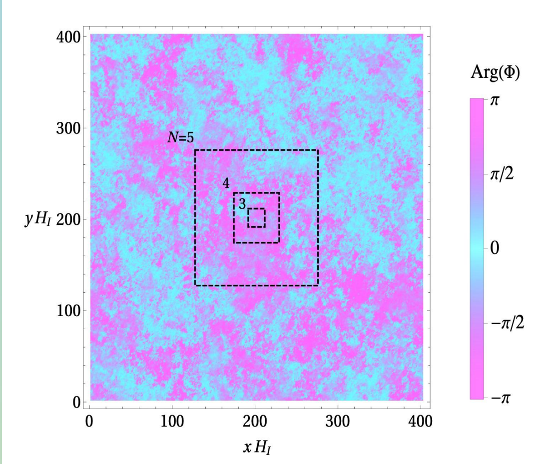
- Φ grows as $\sqrt{N}$ until it reaches quantum vs classical equilibrium:

$$P(\Phi) = \exp\left[-8\pi^2/3V(\Phi)/H^4\right]$$



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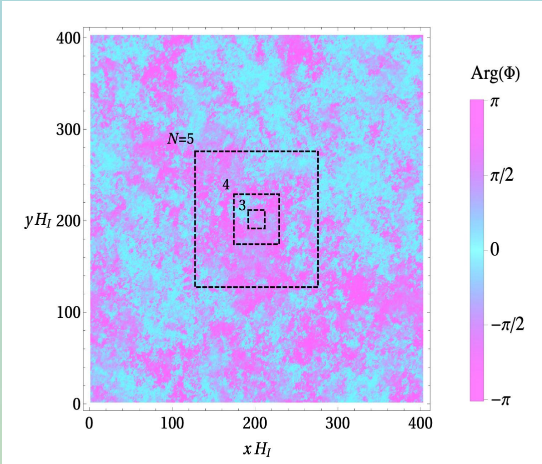

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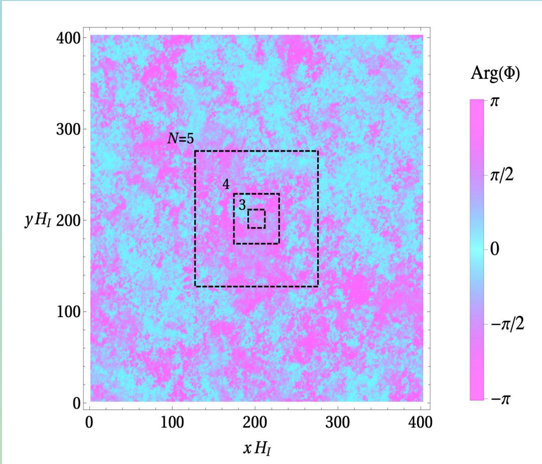

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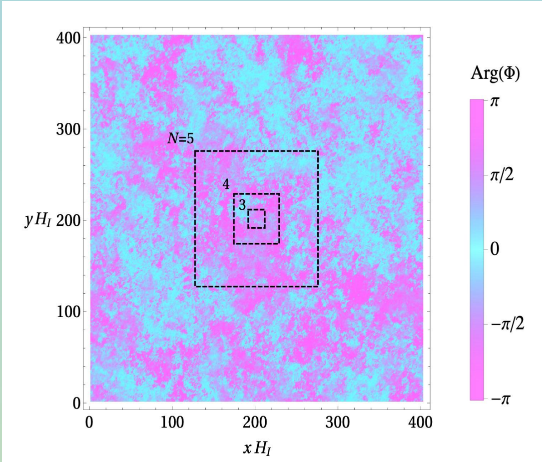

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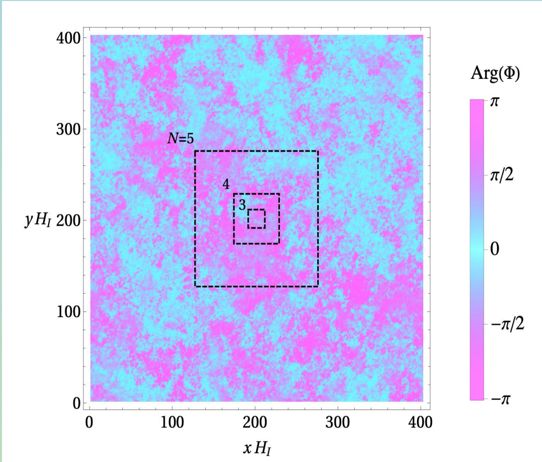
- After N_s Arg(Φ) randomized in $[0, 2\pi]$.

RANDOMISATION DURING INFLATION



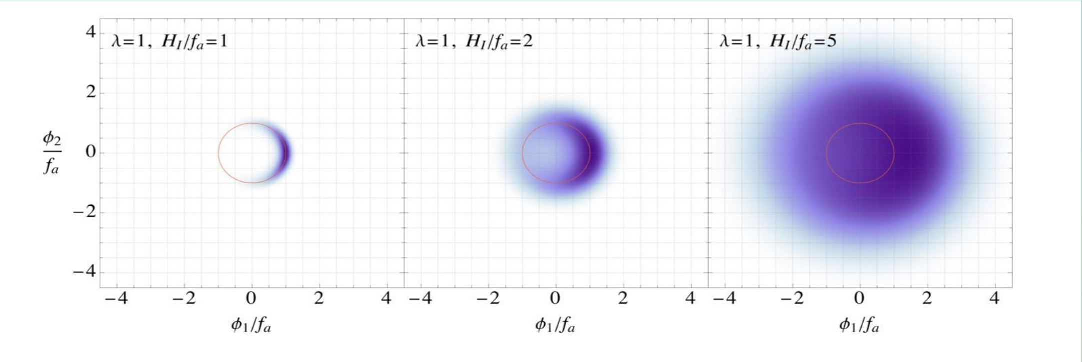
- Once a Fourier mode exits horizon ⇒ becomes classical with amplitude $\frac{H_I}{2\pi}$ ⇒ gives a kick to average field value
- In a coarse-grained region of size H^{-3} : field undergoes random walk, independently in causally disconnected regions, $\mathrm{Var}(\Phi) = \frac{H_I}{2\pi} N$
- Φ grows as $\sqrt{N}$ until it reaches quantum vs classical equilibrium:

$$P(\Phi) = \exp\left[-8\pi^2/3V(\Phi)/H^4\right]$$

- After $N_s \simeq 10/\sqrt{\lambda}$:equilibrium reached (at $N>N_s$ exponential suppression)
- At equilibrium: $\langle|\Phi|\rangle = H_I/\lambda^{1/4}$
- After N_s Arg(Φ) randomized in $[0, 2\pi]$. $N_s = 60$ realized for $\lambda \simeq 0.05$

RANDOMIZATION DURING INFLATION

- $\lambda \simeq 1$



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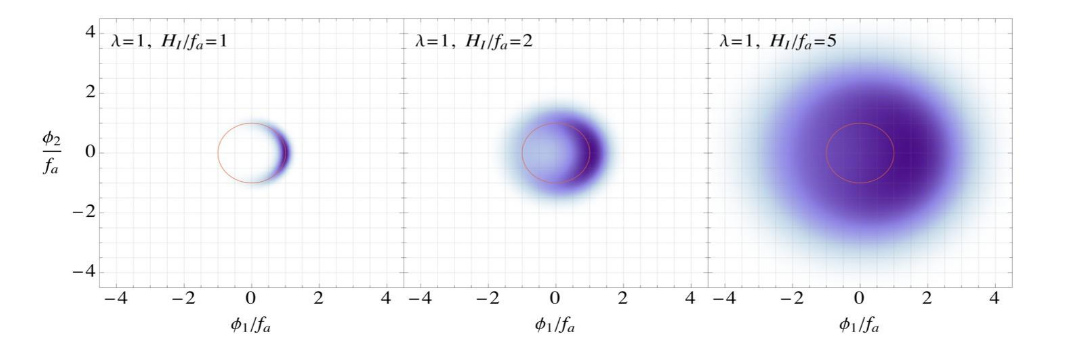
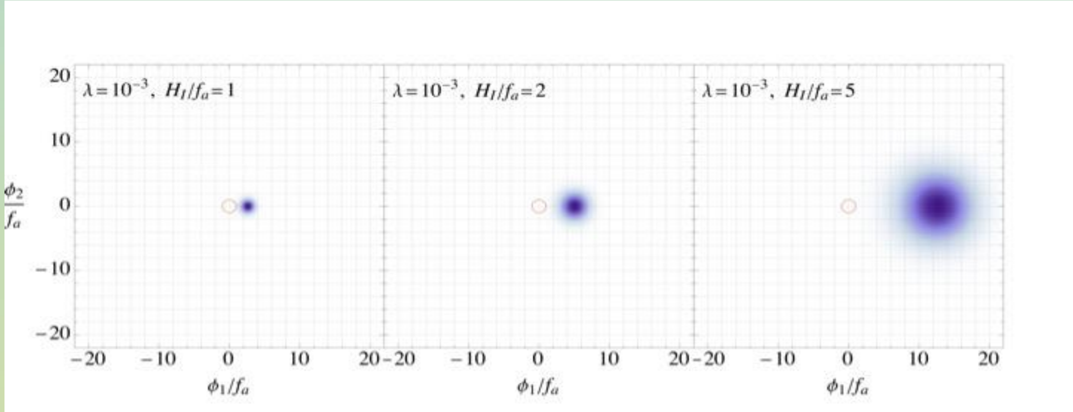
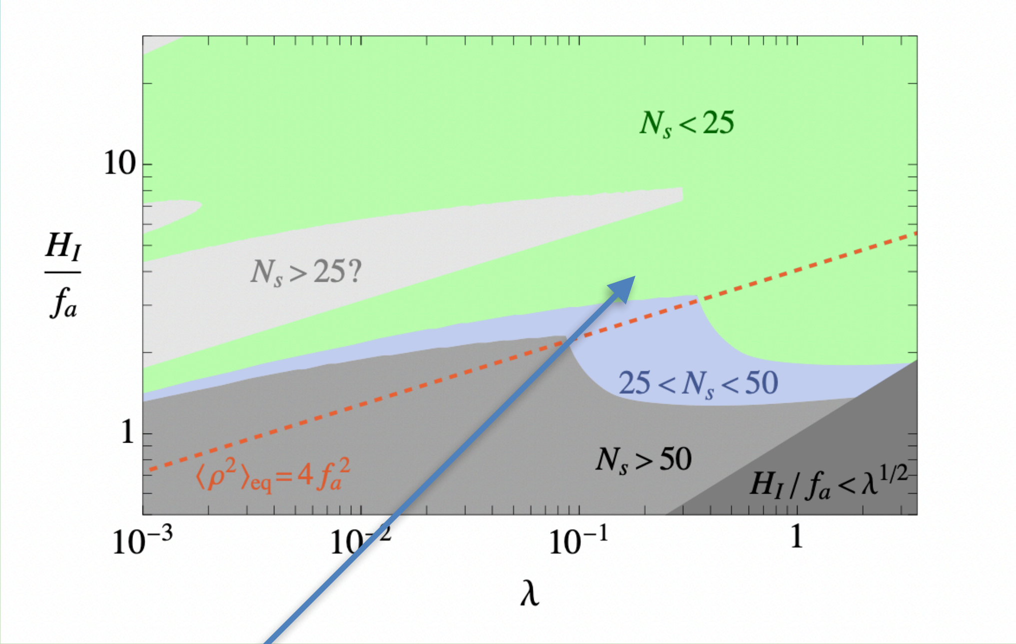


- $\lambda \ll 1$

Radial field: $\langle|\Phi|\rangle = H_I/\lambda^{1/4}$



AXION AS COLD DARK MATTER

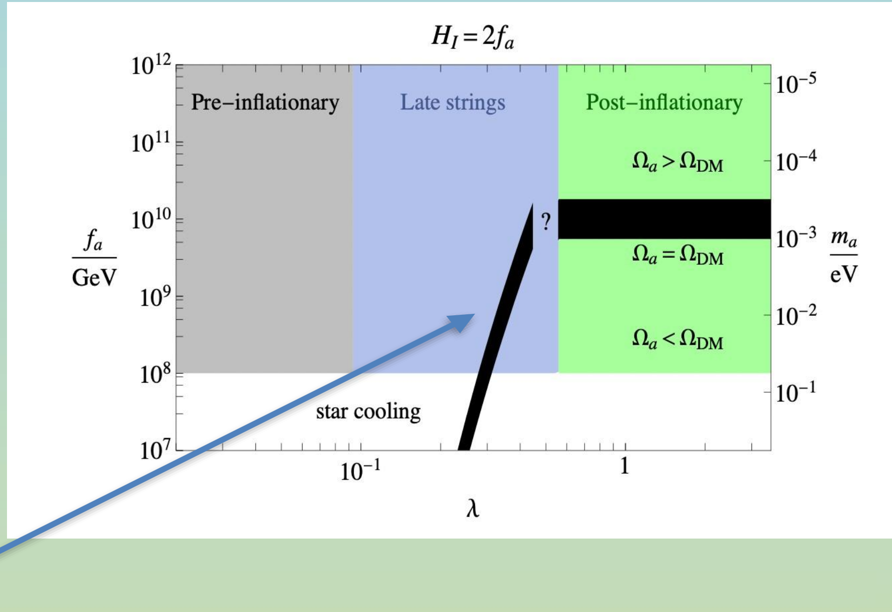


Overshoot mechanism after inflation:
<u>if the field starts high in the potential,</u>
<u>it can roll on the opposite side</u>

⇒ EARLY STRING FORMATION

AXION DARK MATTER at LARGE m_a

Standard post-inflationary: Uncertainty from string simulations, but close to $f_a \sim 10^{10} - 10^{11}\text{GeV}$ ($m_a \sim 10^{-3} - 10^{-4}\text{eV}$)



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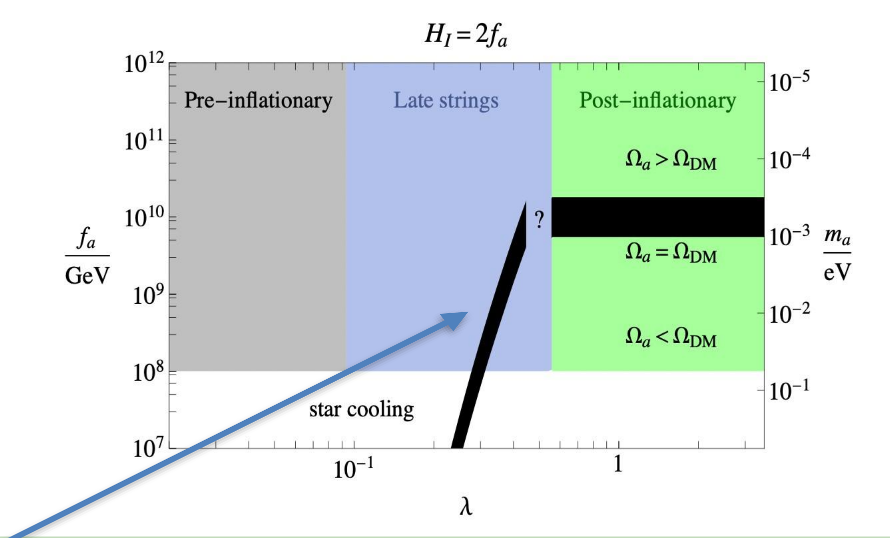

<u>Late strings scenario</u>:

String network born *underdense* (<1 string per Hubble patch)

-Becomes dense (enter horizon) later, even below QCD epoch

-As soon as they enter the horizon: decay into DM axions

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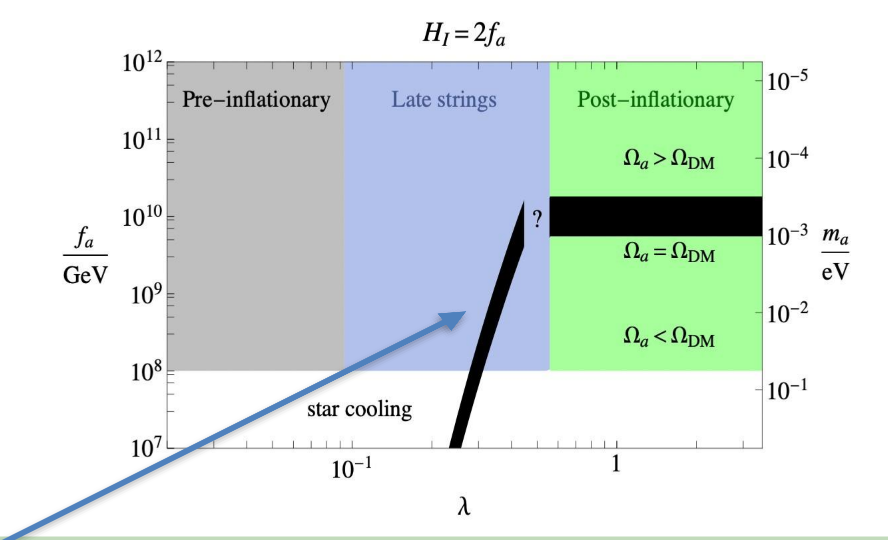

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-Becomes dense (enter horizon) later, even below QCD epoch

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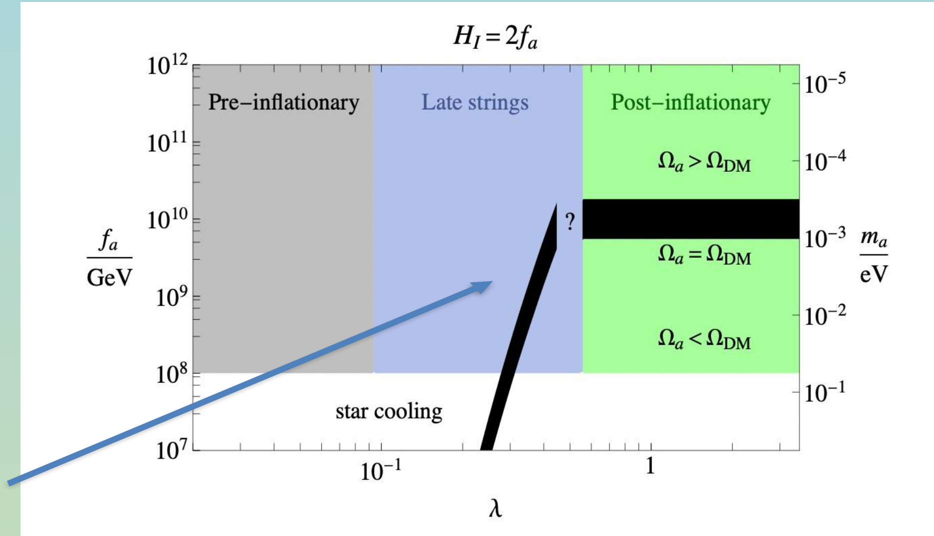
- Smaller f_a can be achieved for correct DM abundance (down to astrophysical bound $f_a \sim 10^8 - 10^{10}$GeV ($m_a \sim 10^{-1} - 10^{-3}$eV)

AXION MINICLUSTERS + ISOCURVATURE

Late strings scenario:

As in Standard post-inflationary:

String-wall system leaves O(1) inhomogeneities in axion DM at length scales H^{-1} at collapse



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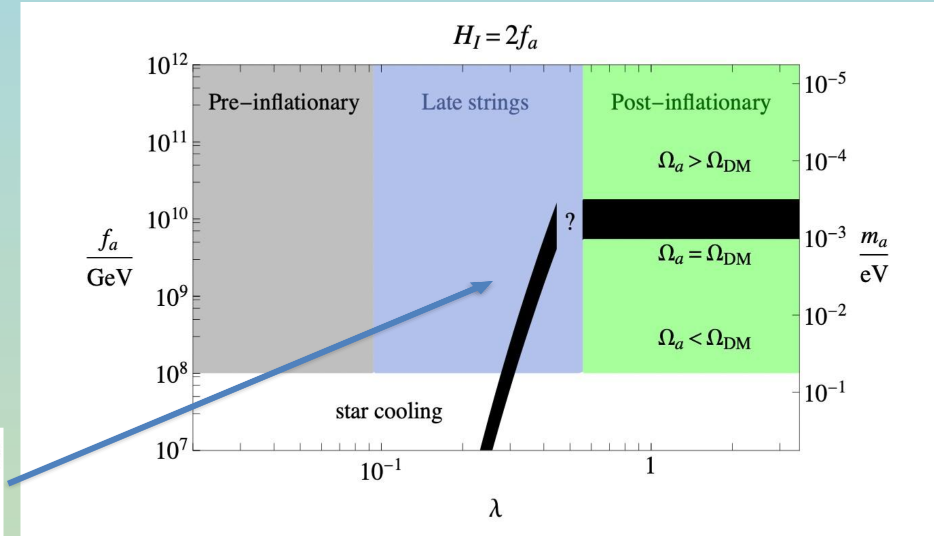
$$M_b \simeq 24 M_\odot \left[\frac{10}{g_*(T_{PQ})}\right]^{\frac{1}{2}} \left[\frac{1\,\mathrm{MeV}}{T_{PQ}}\right]^3$$



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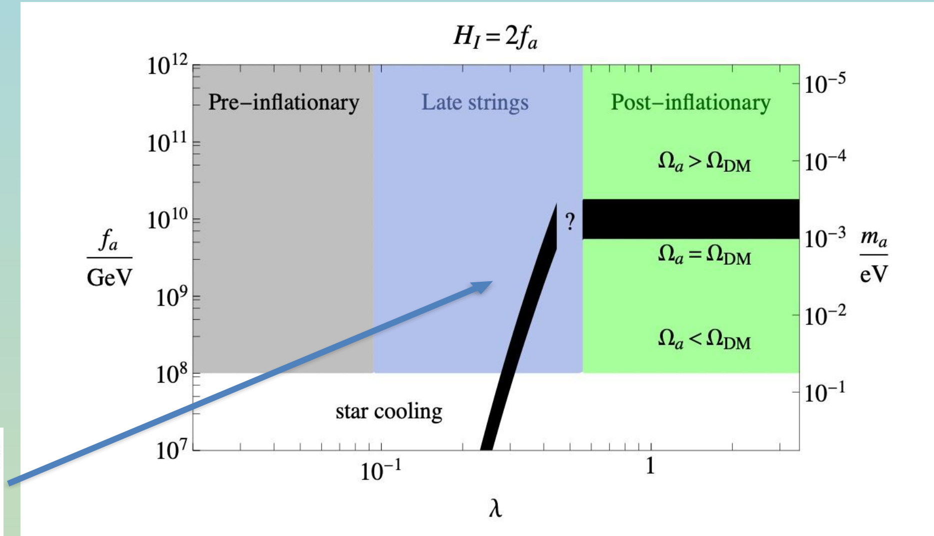


Isocurvature?

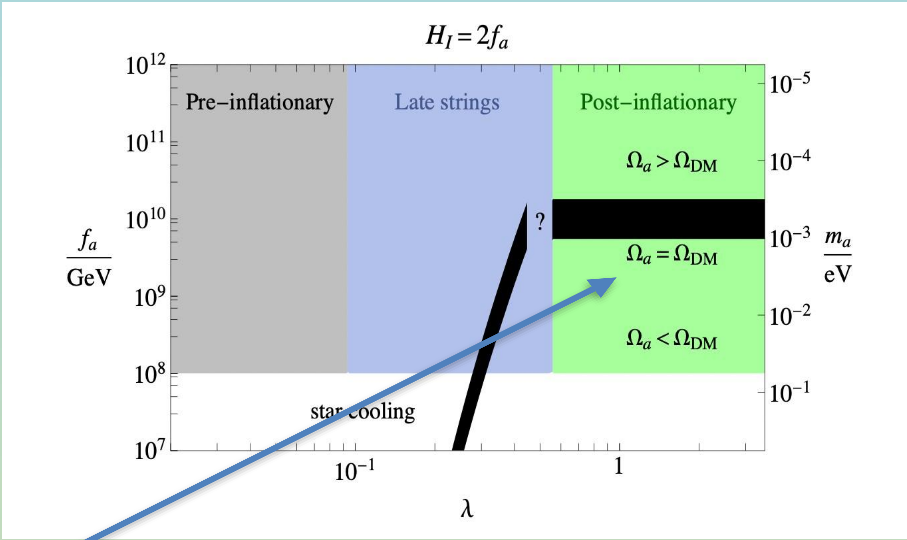
DM axions from misalignment carry large isocurvature (nearly flat), while String-Wall network should have a k^3 IR tail. What happens when DW collapse?

CONCLUSIONS

Late strings scenario:
If $H_I \gtrsim f_a$ and $f_a > T_{max}$
underdense string network could arise from Inflation

- Correct DM abundance could be reached at Smaller f_a (down to astrophysical bound):

$f_a \sim 10^8 - 10^{10}$GeV ($m_a \sim 10^{-1} - 10^{-3}$eV)



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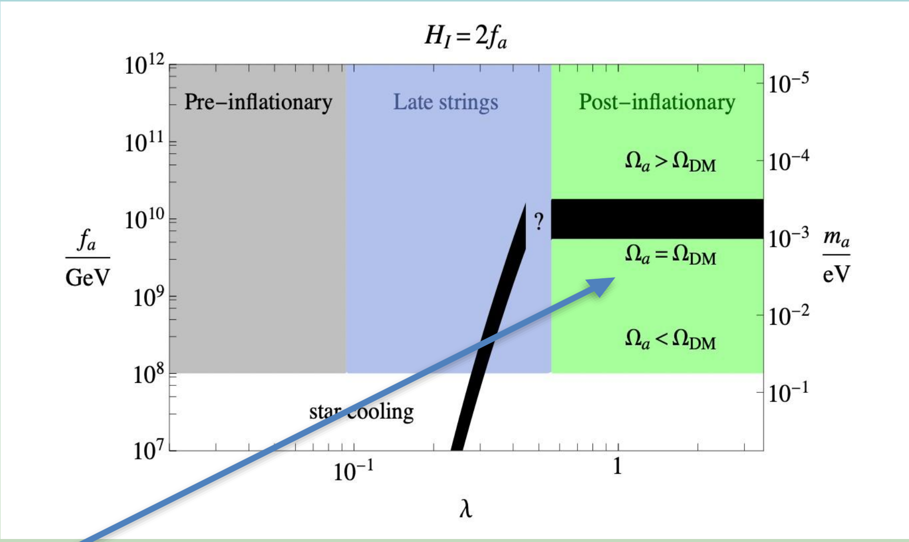
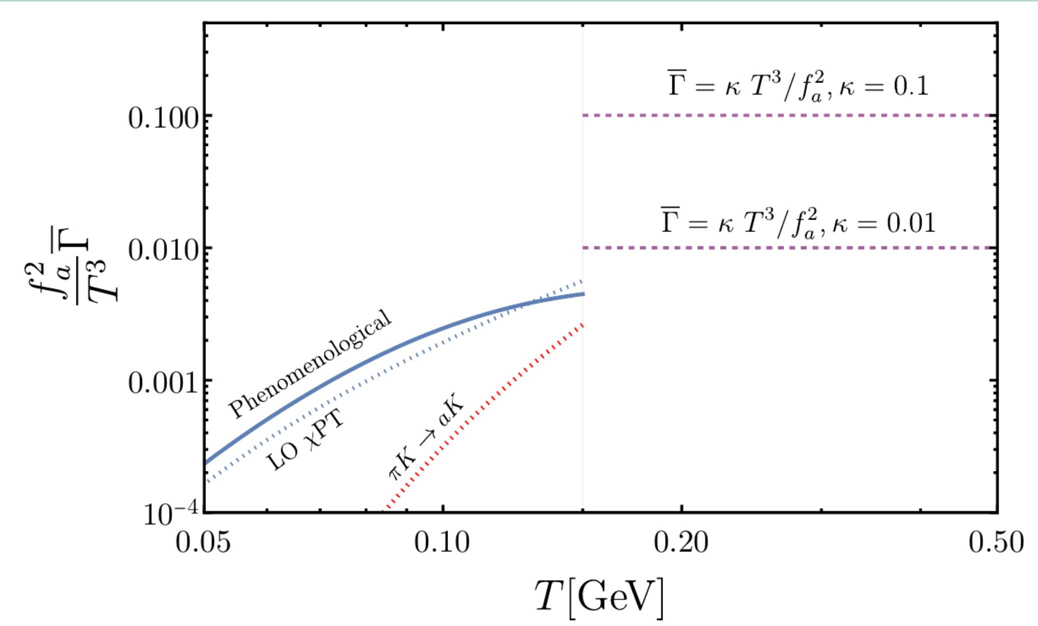


Phenomenological constraints:

- Large miniclusters
- Need to understand better isocurvature constraints

Strong Sphaleron-like contribution to Axion rate

$$\overline{\Gamma}_{\rm sphal} = \frac{1}{n^{\rm eq}} \int^{|\mathbf{k}|<|\mathbf{k}_s|} \frac{d^3\mathbf{k}}{(2\pi)^3 2E} \frac{\Gamma_{\rm sphal}}{f_a^2} e^{-E/T} = \frac{(N_c\alpha_s)^5 T^3}{4\zeta_3 f_a^2} \left(1 - \left(1 + \frac{|\mathbf{k}_s|}{T}\right) e^{-|\mathbf{k}_s|/T}\right)$$



$$\Gamma^{>}_{\rm top}(E = |\mathbf{k}| < |\mathbf{k}_s|) \simeq \Gamma_{\rm sphal} \simeq (N_c\alpha_s)^5 T^4$$

$$|\mathbf{k}_s| \sim N_c \alpha_s T$$

The Thermal Width:

Challenge for Lattice QCD: Compute Γ_k for $T > T_c$

Existing Attempts (at k=0) e.g.

Moore, Tassler '10 : Classical SU(N) simulations

Kotov '18 ,

Altenkort et al. '20,

Mancha, Moore '22 : Quantum Euclidean (plus modeling)

$$\Gamma_{\text{sphal}} = 2T \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

$$G(\tau) = \int \mathrm{d}^3x \, \langle q(\vec{0},0) q(\vec{x},\tau) \rangle$$
$$= -\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \, \rho(\omega) \frac{\cosh[\omega(1/2T - \tau)]}{\sinh(\omega/2T)}$$

Important to exploit upcoming experiments!