BABY STEPS TOWARDS A GENERAL FORMULA

Angelo Esposito

SAPIENZA UNIVERSITÀ DI ROMA

INFN Istituto Nazionale di Fisica Nucleare

PTOLEMY theory meeting, March 2026

WHAT'S THE RIGHT CORRELATOR?

Let's be generic. The initial state of the reaction is:

tritium position

N tritium nuclei, no helium nuclei

$$\Psi_i\left(\mathbf{X}_I, \{\mathbf{X}_T\}_{\neq I}, \{\mathbf{X}_C\}, \{\mathbf{x}_e\}\right) , \quad H(N,0)\,|\,\Psi_i\rangle = E_i(N,0)\,|\,\Psi_i\rangle$$

$$\Psi_f\left(\mathbf{X}_I, \{\mathbf{X}_T\}_{\neq I}, \{\mathbf{X}_C\}, \{\mathbf{x}_e\}\right) , \quad H(N-1,1)\,|\,\Psi_f\rangle = E_f(N-1,1)\,|\,\Psi_i\rangle$$

helium position

$N - 1$ tritium nuclei, 1 helium nucleus

After some massaging, the matrix element for the decay of the I-th tritium is

$$\mathcal{M}_f^I = g\langle\Psi_f|\,e^{-i(\mathbf{k}+\mathbf{q})\cdot\mathbf{X}_I}\,|\Psi_i\rangle$$

WHAT'S THE RIGHT CORRELATOR?

After some massaging, the decay rate can be written as

$$\frac{d\Gamma}{d\mathbf{k}d\mathbf{q}} = |g|^2 \int dt\, e^{i\left(\Delta m + E_i(N,0) - E_\beta - E_\nu\right)t}$$

$$\times \sum_I \langle \Psi_i | e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{X}_I} e^{-iH(N-1,1)t} e^{-i(\mathbf{k}+\mathbf{q})\cdot\mathbf{X}_I} | \Psi_i \rangle$$

This is in principle the correlator one must evaluate.

Note that, written this way, the state is easy (ground state of the initial system), but the operator is complicated. But maybe not completely crazy with DFT?

A SPECULATION

If one were able to justify some sort of "sudden approximation", meaning an expansion in small times, we could rewrite the correlator as a quite evocative expression:

$$\int dt\, e^{i\left(E - \frac{p^2}{2m_{\rm He}}\right)t} \sum_I \langle \Psi_i | \, e^{-iL_I t} \, | \Psi_i \rangle$$

This is somewhat evocative of a path integral… but maybe I'm hallucinating…