Quantum Cellular Automata as extension of Quantum Field Theory

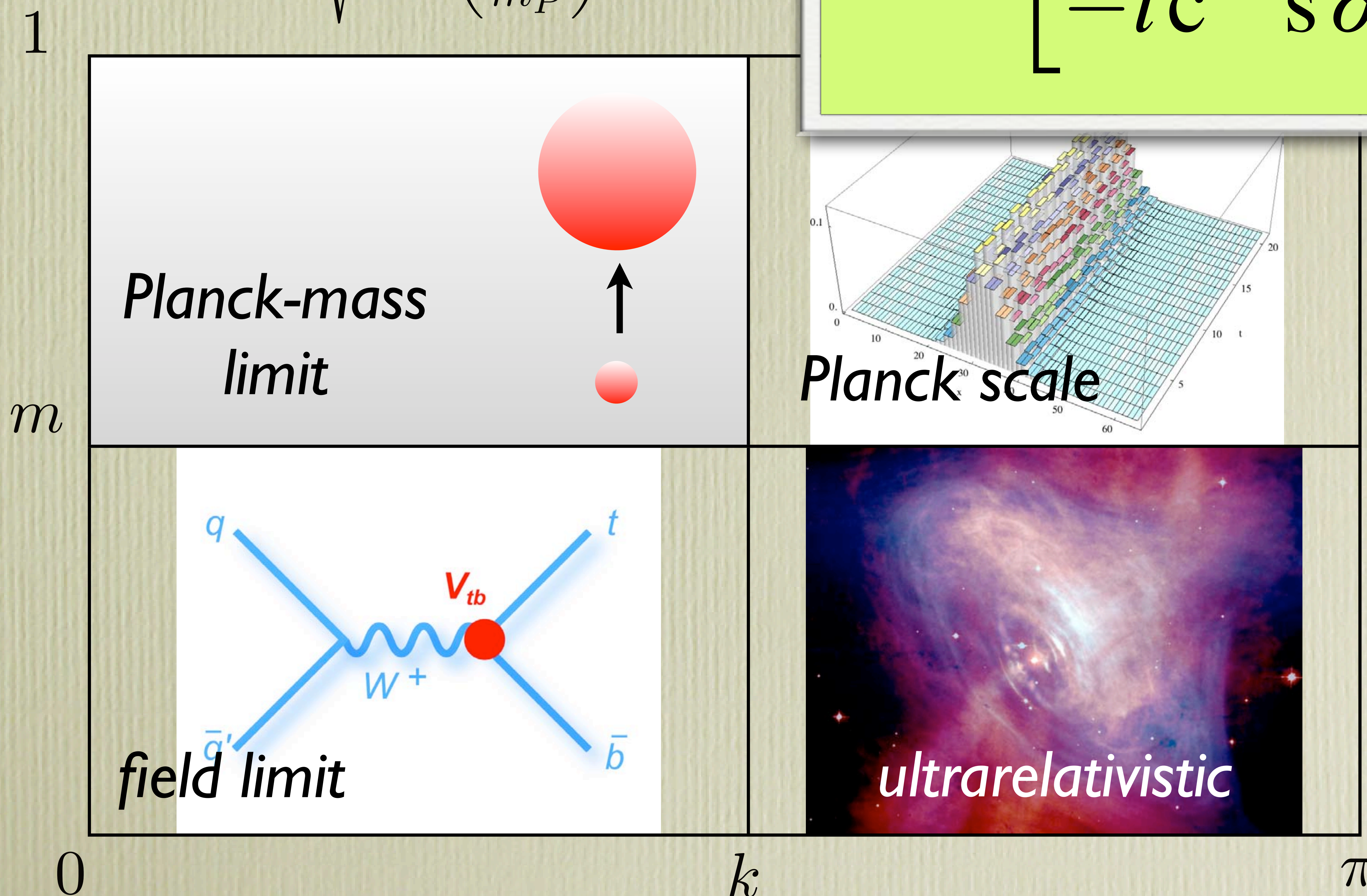
Giacomo Mauro D'Ariano
GE 41, Gruppo IV, Sezione PV INFN

arXiv:1012.0535

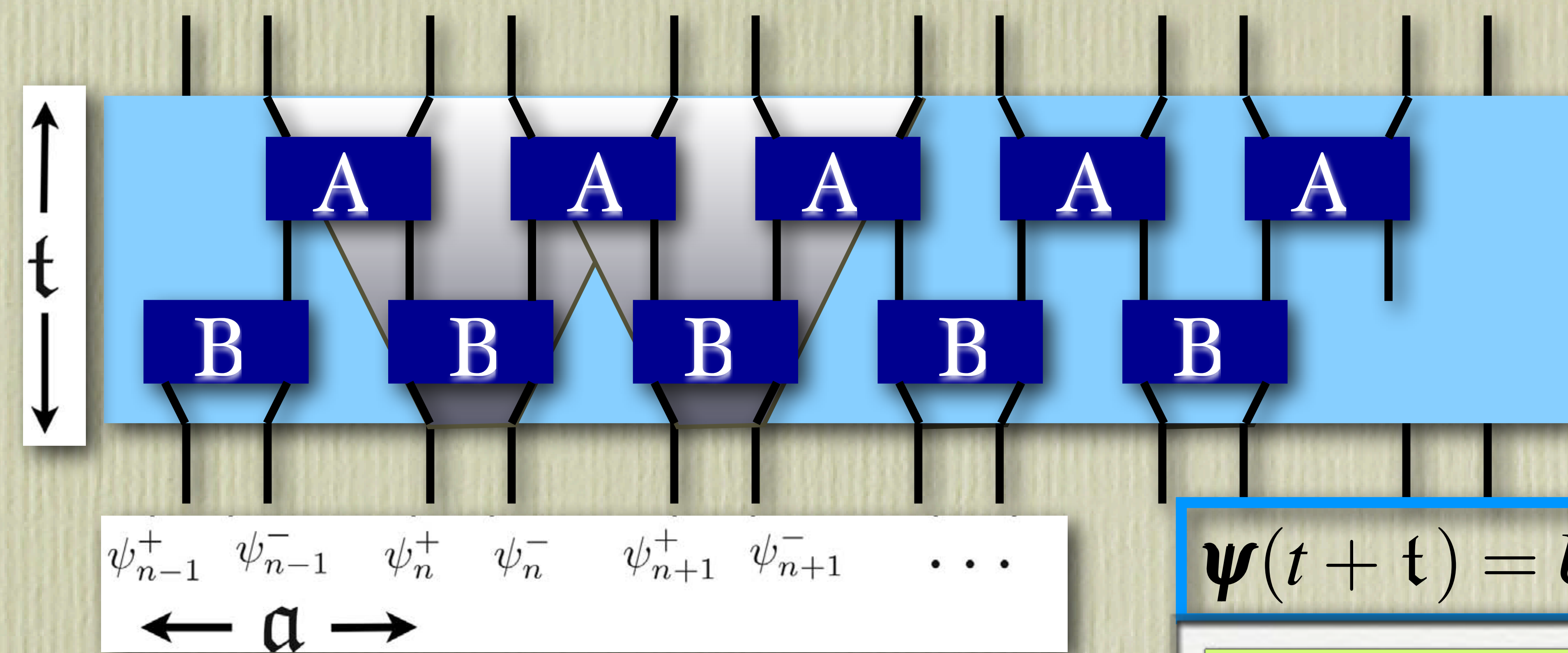
$$\mathrm{c} := \cos\theta = \frac{a}{\lambda} = \frac{m}{m_P}, \quad m_P = \frac{\hbar}{ac}$$

$$\mathrm{s} := \sin\theta = \zeta = \sqrt{1 - \left(\frac{m}{m_P}\right)^2}$$

$$\mathbf{U} = \begin{bmatrix} \mathrm{s}\,\widehat{\partial}_- & -i\mathrm{c} \\ -i\mathrm{c} & \mathrm{s}\,\widehat{\partial}_+ \end{bmatrix}$$



THE DIRAC QU-AUTOMATON



$$\boldsymbol{\psi} := \begin{bmatrix} \dots \\ \boldsymbol{\psi}_n \\ \boldsymbol{\psi}_{n+1} \\ \dots \end{bmatrix}$$

$$\boldsymbol{\psi}_n := \begin{bmatrix} \psi_n^+ \\ \psi_n^- \end{bmatrix}$$

$$\boldsymbol{\psi}(t+\mathfrak{t}) = U^\dagger \boldsymbol{\psi}(t) U = \mathbf{U}\boldsymbol{\psi}(t)$$

Planck length

$$\mathrm{c}^2 + \mathrm{s}^2 = 1$$

$$\mathbf{U} = \begin{bmatrix} \mathrm{s}\,\widehat{\partial}_- & -i\mathrm{c} \\ -i\mathrm{c} & \mathrm{s}\,\widehat{\partial}_+ \end{bmatrix}$$

$$A_n = \exp\left[-i\theta\left(\sigma^-_{2n-1}\sigma^+_{2n} + \sigma^+_{2n-1}\sigma^-_{2n}\right)\right]$$
$$B_n = \exp\left[-i\tfrac{\pi}{2}\left(\sigma^+_{2n}\sigma^-_{2n+1} + \sigma^-_{2n}\sigma^+_{2n+1}\right)\right]$$

$$\mathrm{c} = \cos\theta = \omega\,\mathfrak{t} = \frac{\mathfrak{a}}{\lambda} = \frac{m}{\mathfrak{m}}, \qquad \mathrm{s} = \sin\theta = \zeta = \sqrt{1-\left(\frac{m}{\mathfrak{m}}\right)^2},$$

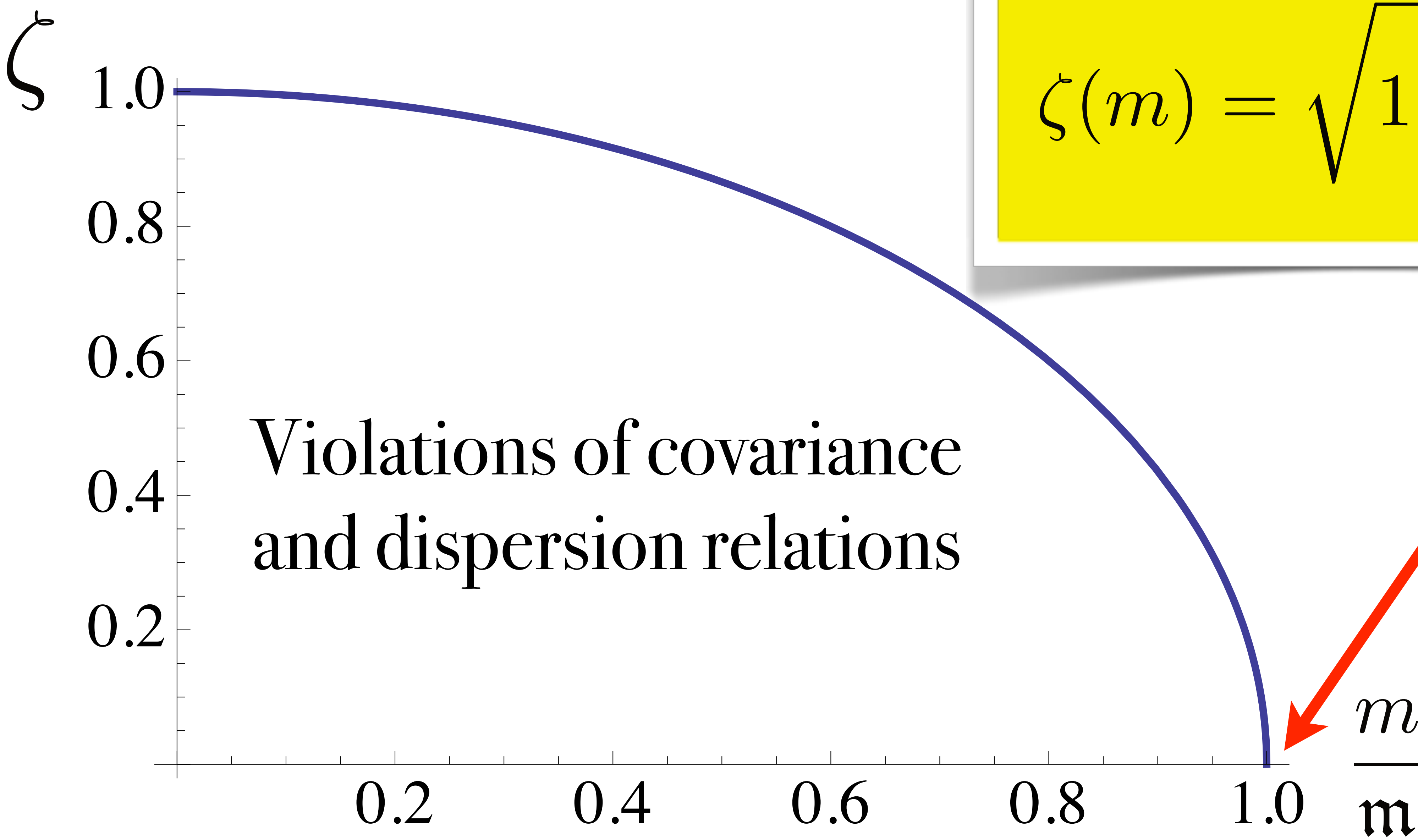
$$\mathfrak{m} := \frac{\hbar}{\mathfrak{a}\,c}$$

THE DIRAC QU-AUTOMATON

MASS-DEPENDENT REFRACTION INDEX OF VACUUM

Unitariety of QCA ➡ $c \to \zeta c, \quad \zeta = \zeta(m)$

$$\zeta(m) = \sqrt{1 - \left(\frac{m}{\mathfrak{m}}\right)^2}$$

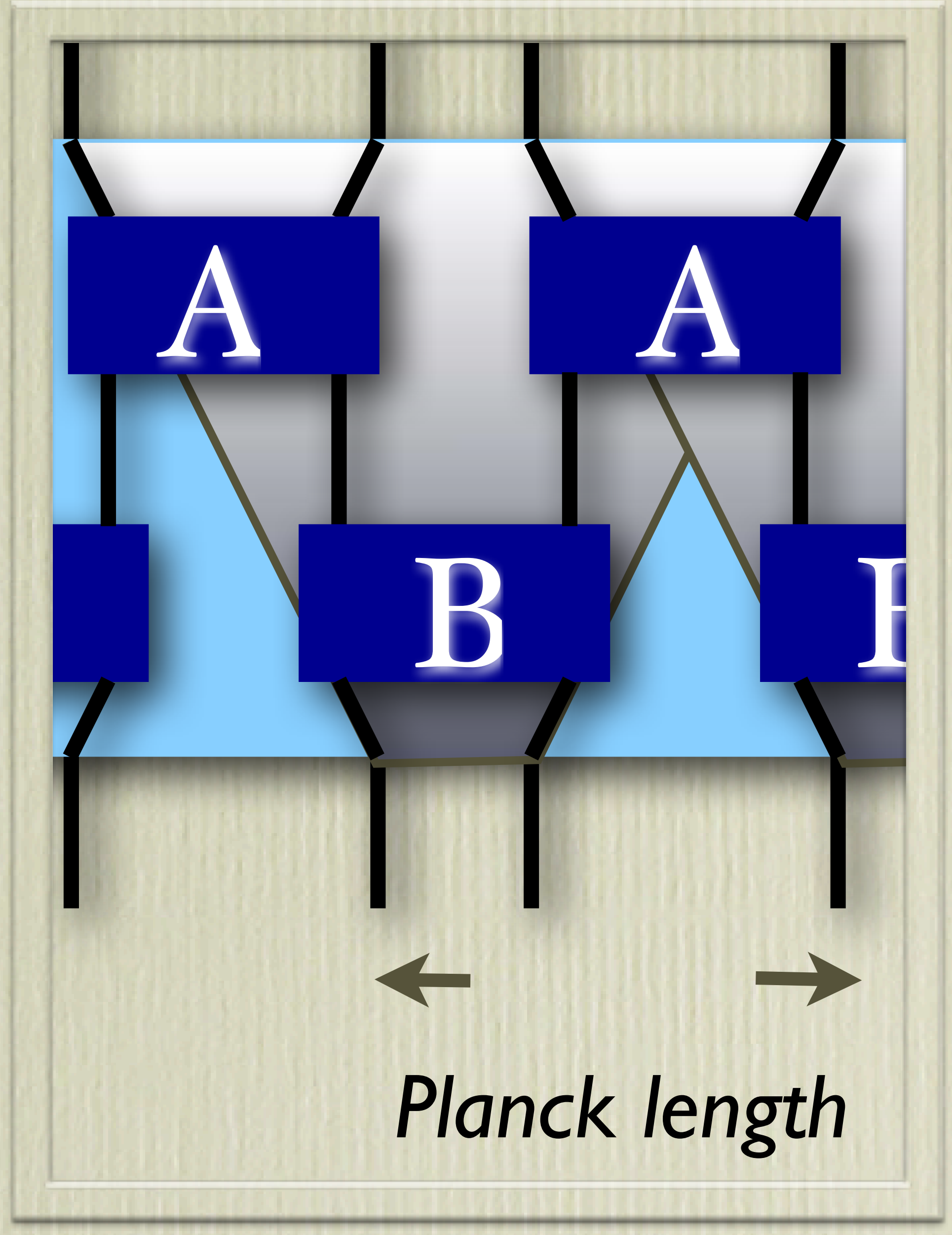


Information halts at the Planck mass

THE DIRAC QU-AUTOMATON

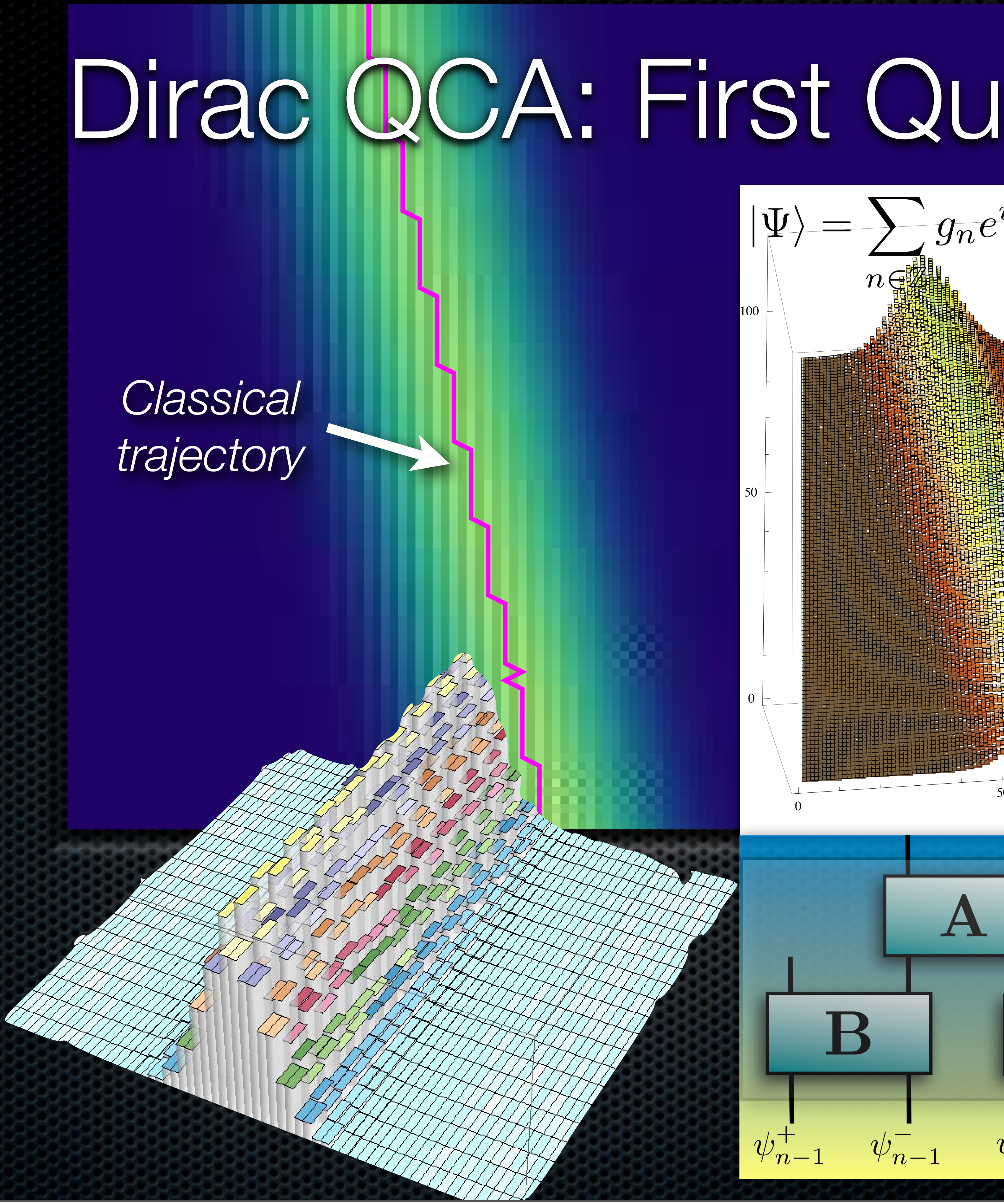
Derived from first (*informational*) principles

- Planck-scale, field limit, ultrarelativistic limit unified in a single simple model
- *Extension* of QFT including localized states and observables
- From first principles [informational (*)]
- Quantum *ab initio*
- *Classicalization* vs *quantization* (¶)
- Symmetries are approximated, and restored in the *field limit*

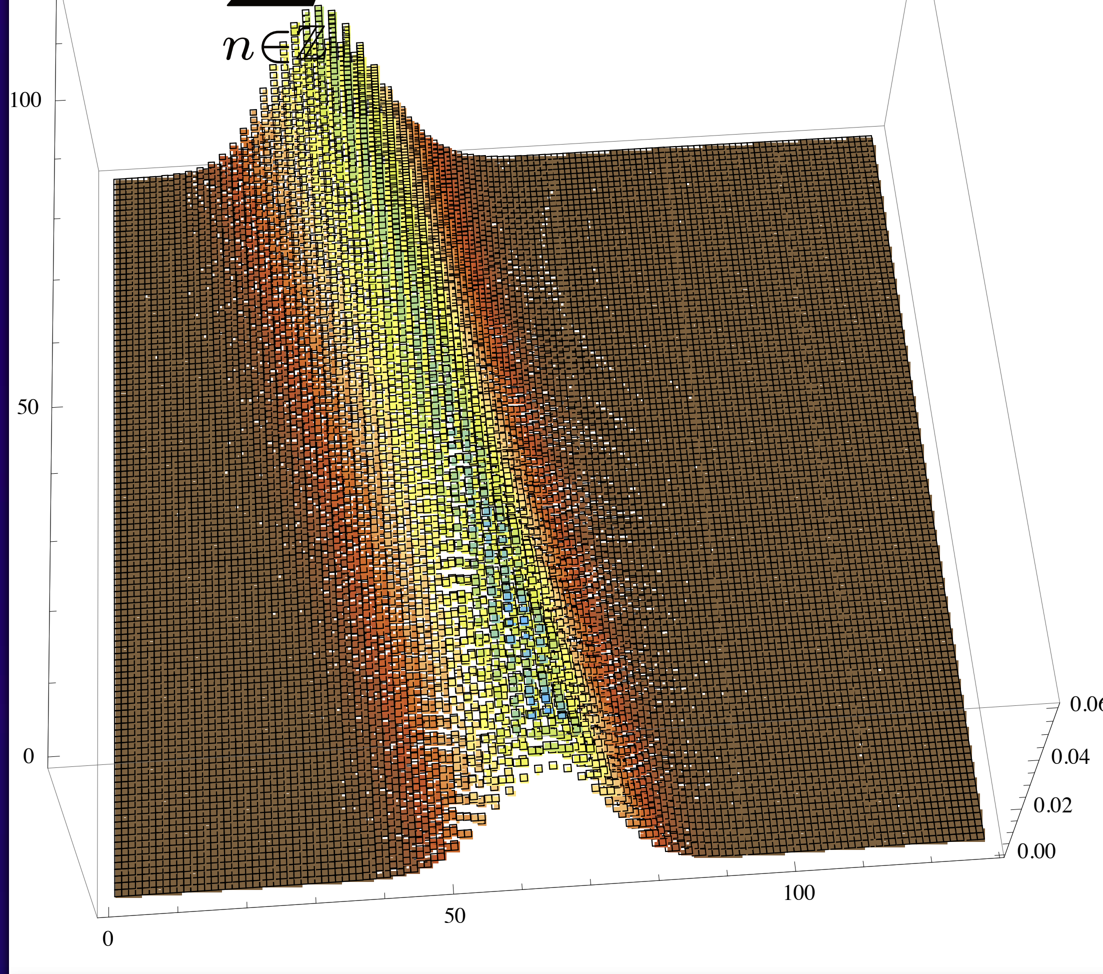


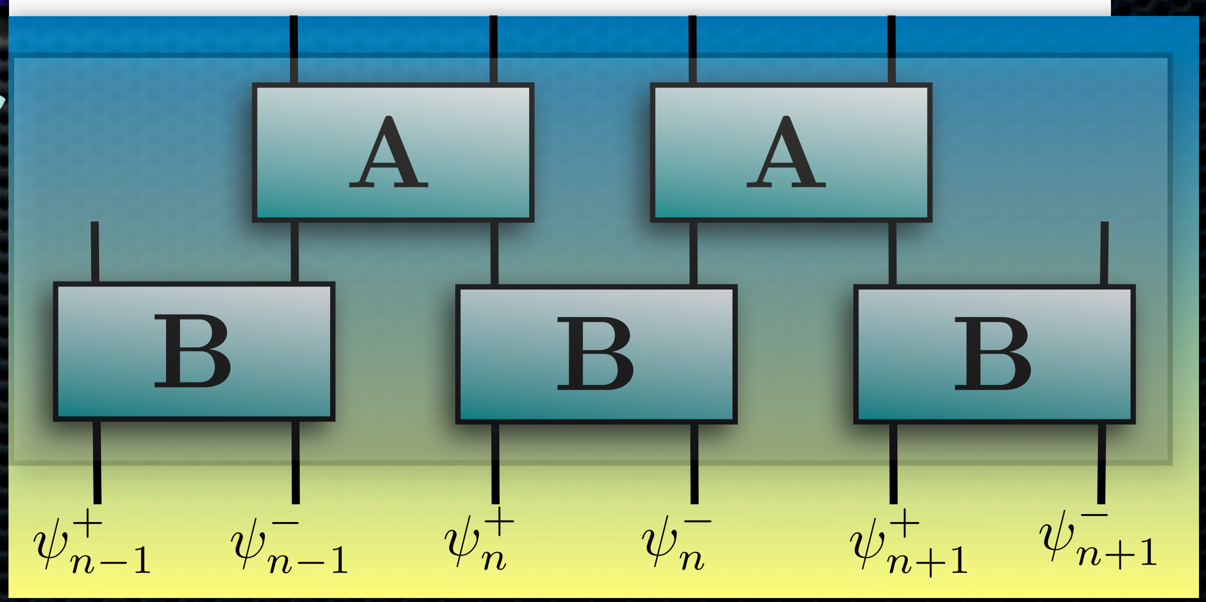
(*) Chiribella, D'Ariano, Perinotti PRA **84** 012311 (2011) ***Viewpoint:*** Č. Brukner, Physics 4, 55 (2011)
(†) D'Ariano, PLA **376** 697 (2012) (arXiv:1012.0756)
(¶) D'Ariano, AIP Conf. Proc. **371** (2012) (arXiv 1012.0535)

Dirac QCA: First Quantization

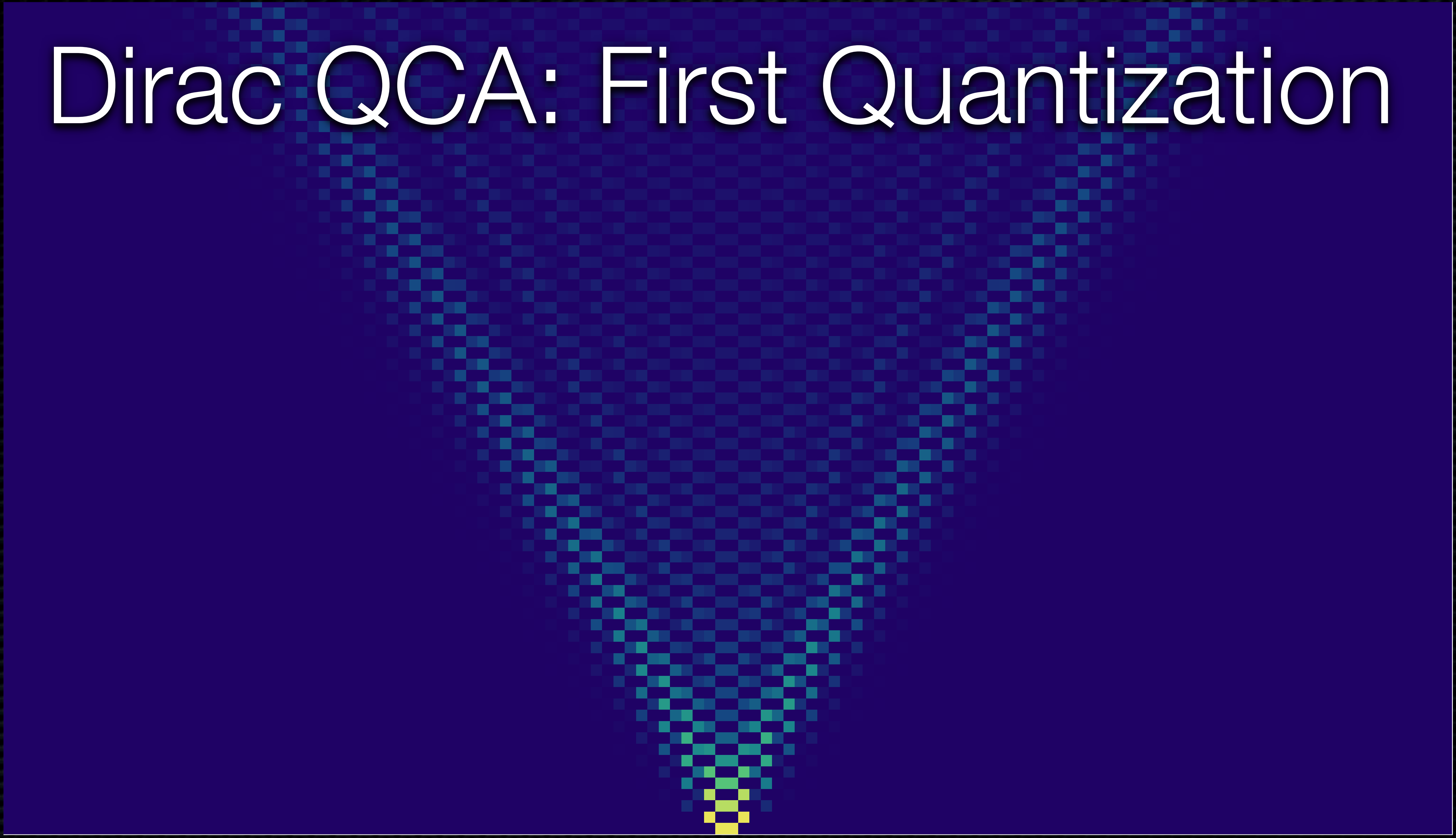


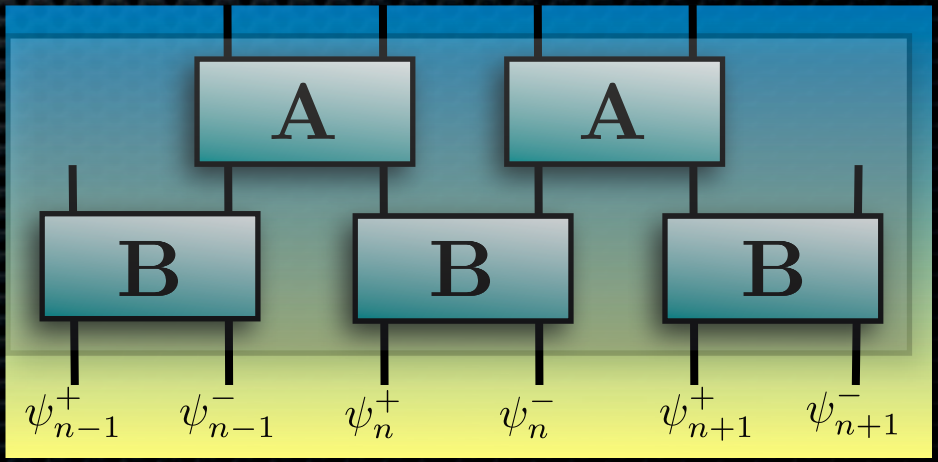
$$|\Psi\rangle = \sum_{n\in\mathbb{Z}} g_n e^{in\phi}(|\psi_n^+\rangle \pm |\psi_n^-\rangle)$$





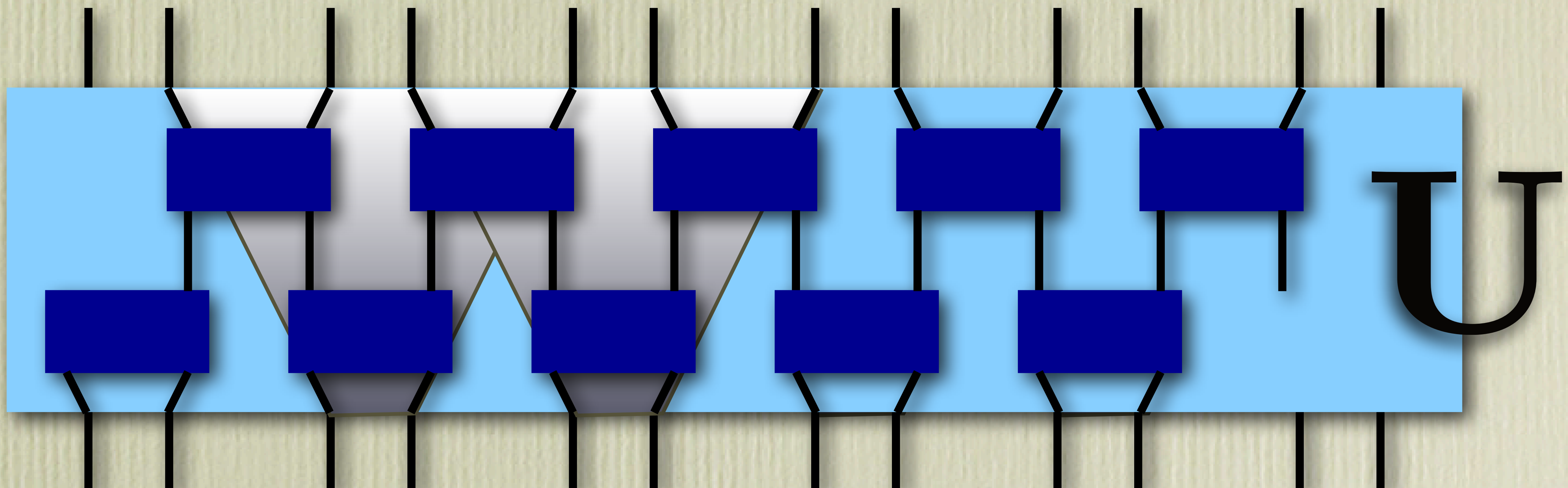
Dirac QCA: First Quantization





CLASSICALIZATION: H FROM U

$$i\hbar\widehat{\partial_t}\boldsymbol{\psi} = [\boldsymbol{\psi}, H]$$



$$H = \frac{i\hbar}{2\mathrm{t}}\boldsymbol{\psi}^\dagger(\mathbf{U} - \mathbf{U}^\dagger)\boldsymbol{\psi}$$

Alessandro Bisio

Planckian Particles

$$\mathbf{U} = \begin{pmatrix} \mathrm{s}\,\widehat{\partial}_- & -i\,\mathrm{c} \\ -i\,\mathrm{c} & \mathrm{s}\,\widehat{\partial}_+ \end{pmatrix} = \begin{pmatrix} \mathrm{s}\,e^{ik} & -i\,\mathrm{c} \\ -i\,\mathrm{c} & \mathrm{s}\,e^{-ik} \end{pmatrix}$$

Alessandro Tosini

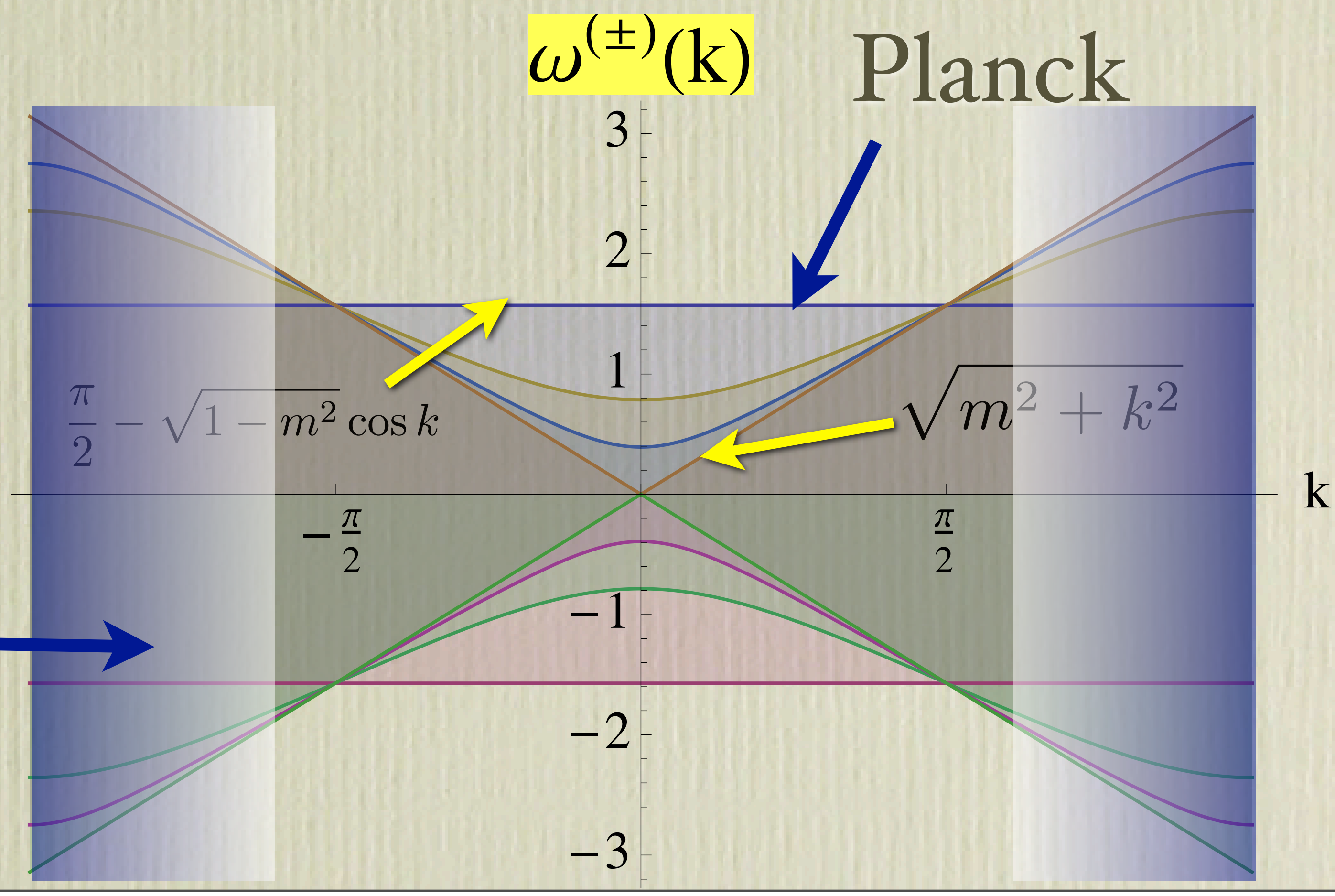
(Foldy-Wouthuysen)

Eigenvectors in *k*-space

$$\frac{1}{2N^{(\pm)}(k)} \begin{pmatrix} i\,\mathrm{c} \\ \mathrm{s}\,e^{ik} - e^{\pm i\omega(k)} \end{pmatrix} \qquad N^{(\pm)}(k) = \sqrt{1 - \mathrm{s}\cos[\mathrm{k} \pm \omega(\mathrm{k})]}$$

Dispersion relation

$$\boxed{\omega(k) = \cos^{-1}(\mathrm{s}\cos\mathrm{k})}$$



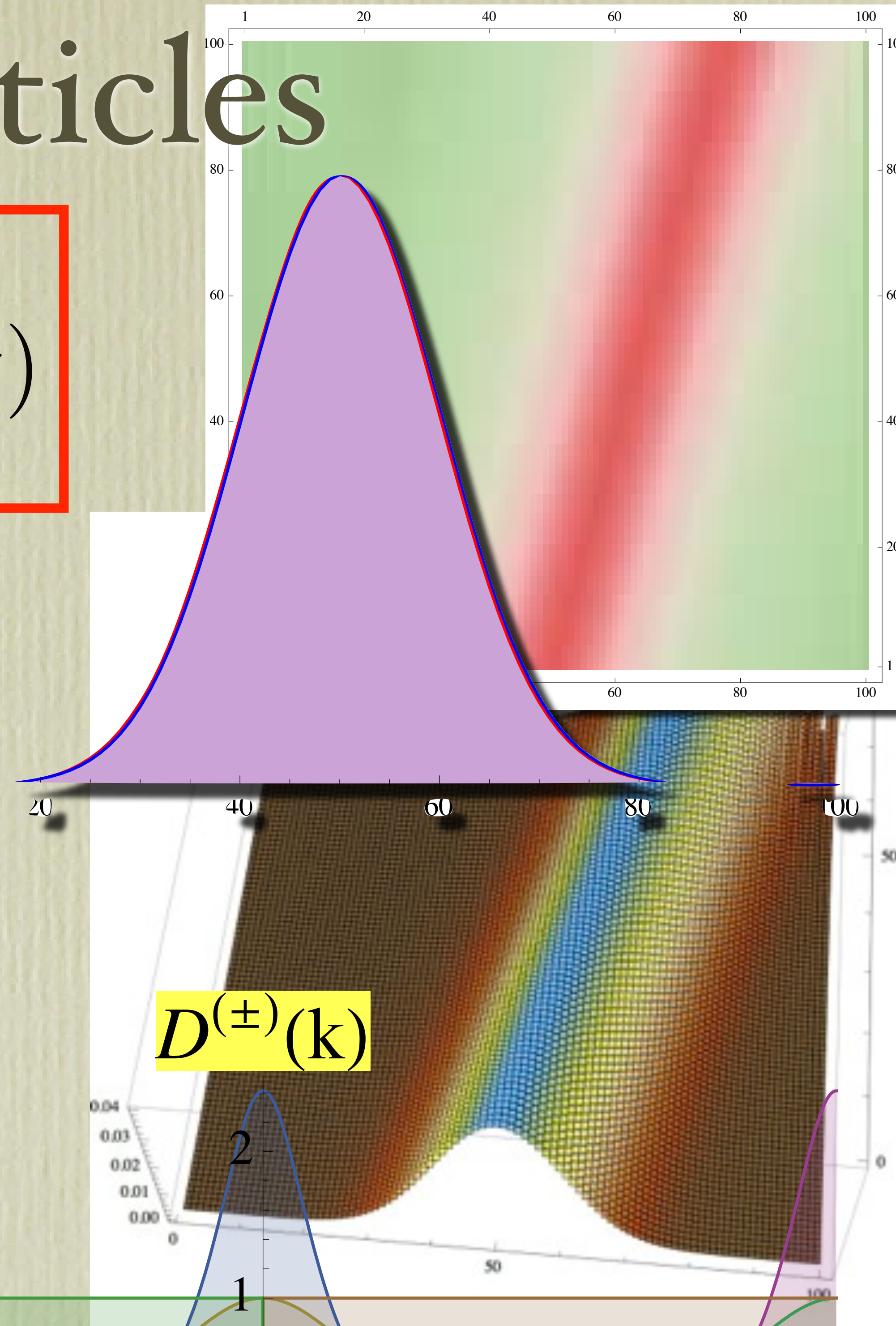
Ultra-relativistic

Planckian Particles

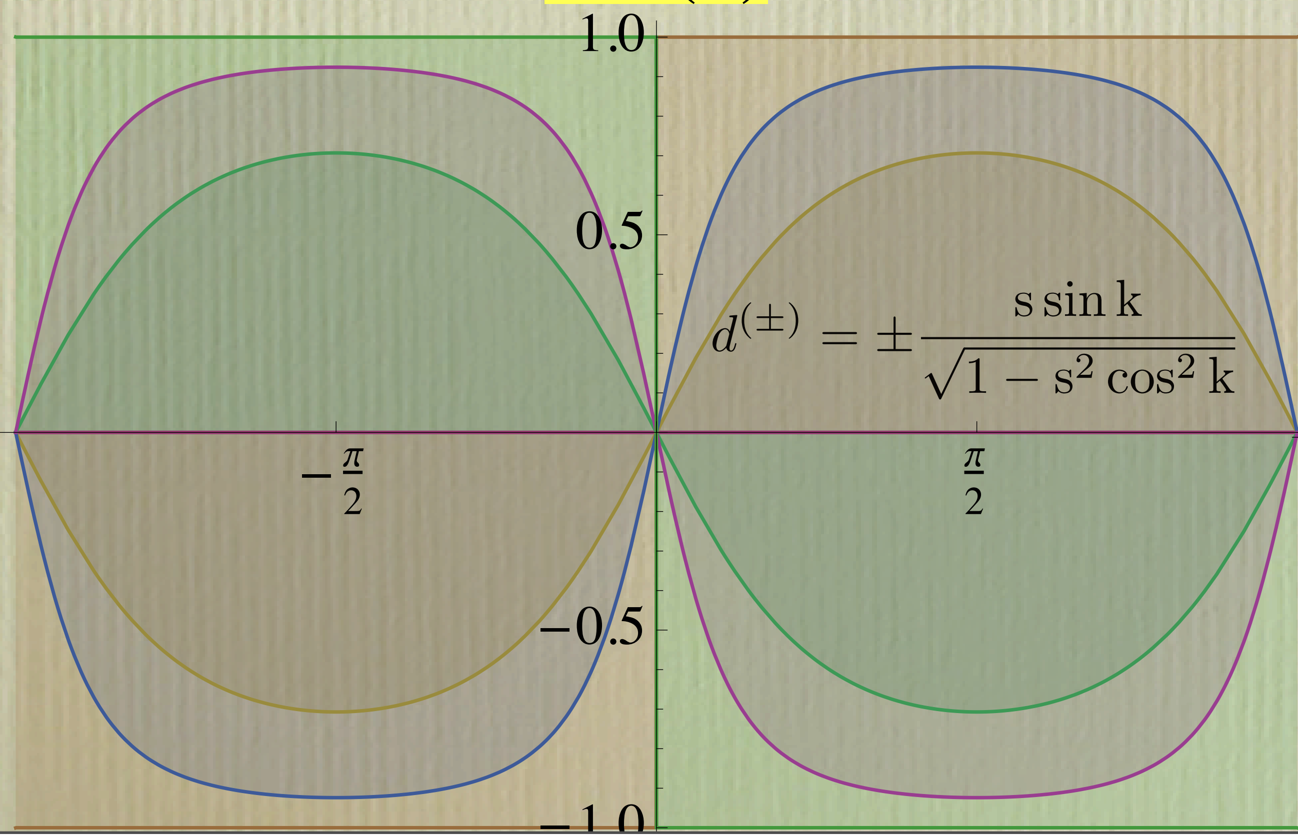
$$\boxed{i\partial_t A(x,t) = \left[-id\partial_x - \frac{D}{2}\partial_x^2\right] A(x,t)}$$

$$A(x,t) = \frac{1}{\sqrt[4]{2\pi\Delta^2(t)}} \exp\left[-\frac{(x-x(t))^2}{4\Delta^2(t)}\right]$$

$$x(t) = d \qquad \Delta(t) = \Delta\sqrt{1 + i\frac{D}{2\Delta^2}t}$$

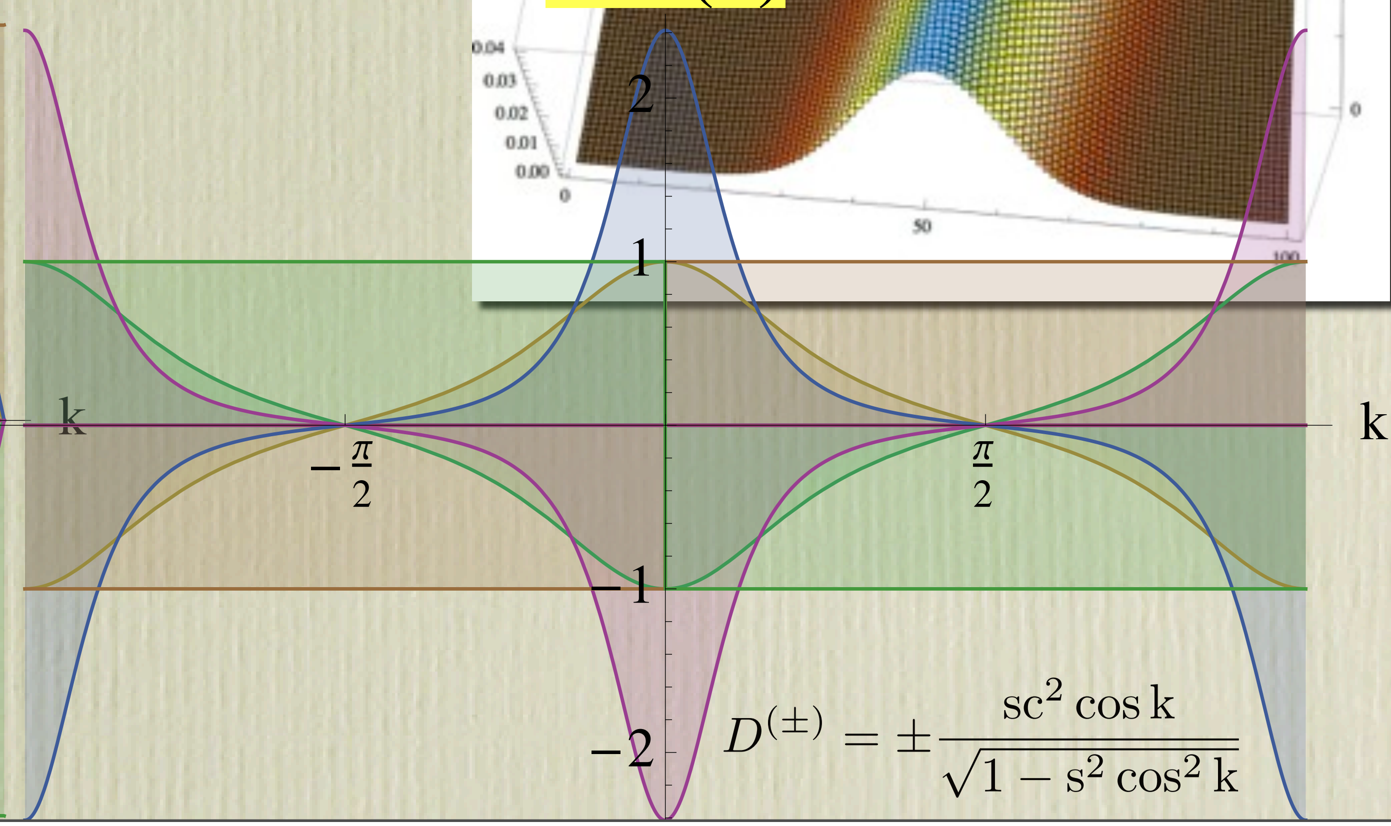


$d^{(\pm)}(\mathrm{k})$

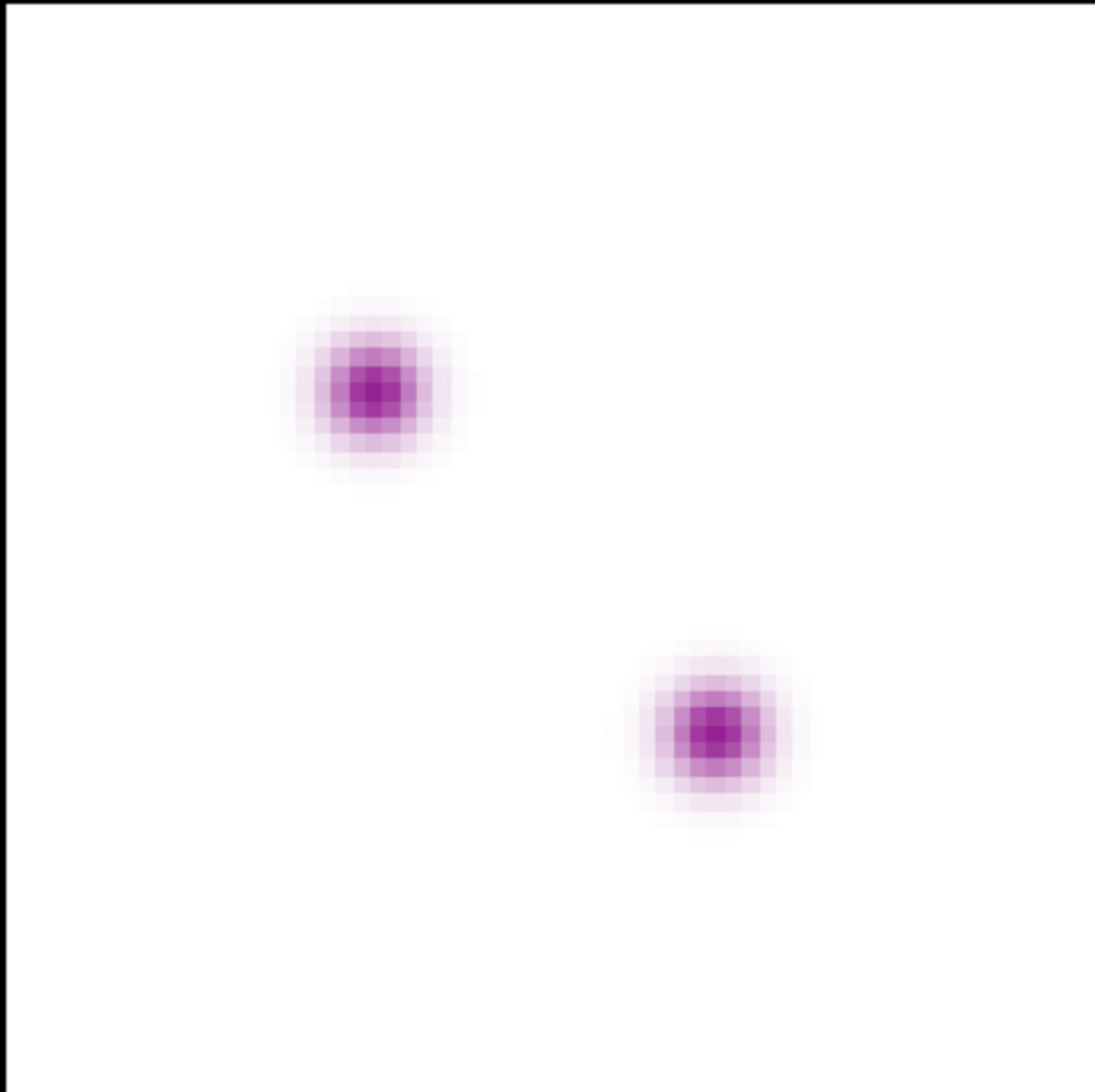


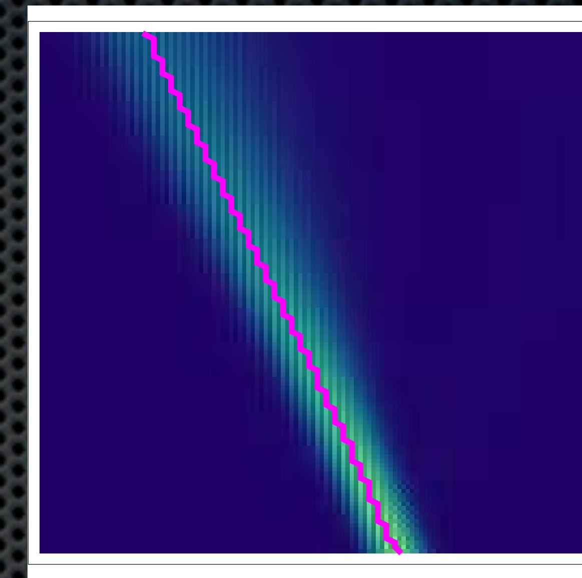
$$d^{(\pm)} = \pm\frac{\mathrm{s}\sin\mathrm{k}}{\sqrt{1-\mathrm{s}^2\cos^2\mathrm{k}}}$$

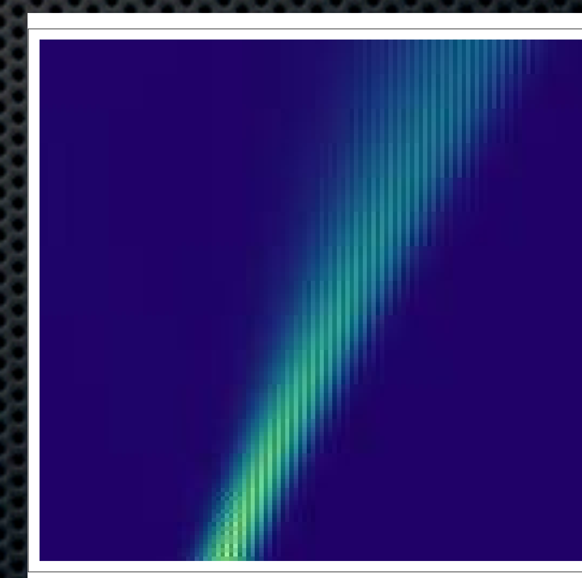
$D^{(\pm)}(\mathrm{k})$



$$D^{(\pm)} = \pm\frac{\mathrm{sc}^2\cos\mathrm{k}}{\sqrt{1-\mathrm{s}^2\cos^2\mathrm{k}}}$$







First Quantization: particle-antiparticle

Are we able to simulate our theory (even with a quantum computer)?

Simulating Physics with Computers

Richard P. Feynman

The question is, if we wrote a Hamiltonian which involved only these operators, locally coupled to corresponding operators on the other space-time points, could we imitate every quantum mechanical system which is discrete and has a finite number of degrees of freedom? I know, almost certainly, that we could do that for any quantum mechanical system which involves Bose particles. I'm not sure whether Fermi particles could be described by such a system. So I leave that open. Well, that's an example of what I meant by a general quantum mechanical simulator. I'm not sure that it's sufficient, because I'm not sure that it takes care of Fermi particles.

Int. J. of Th. Phys. **21 467** (1982)

THE FEYNMAN PROBLEM

$$[\varphi_{\boldsymbol{n}}, \varphi_{\boldsymbol{m}}]_+ = 0, \quad [\varphi_{\boldsymbol{n}}, \varphi^\dagger_{\boldsymbol{m}}]_+ = \delta_{\boldsymbol{mn}}, \quad \boldsymbol{n}, \boldsymbol{m} \in \Lambda$$

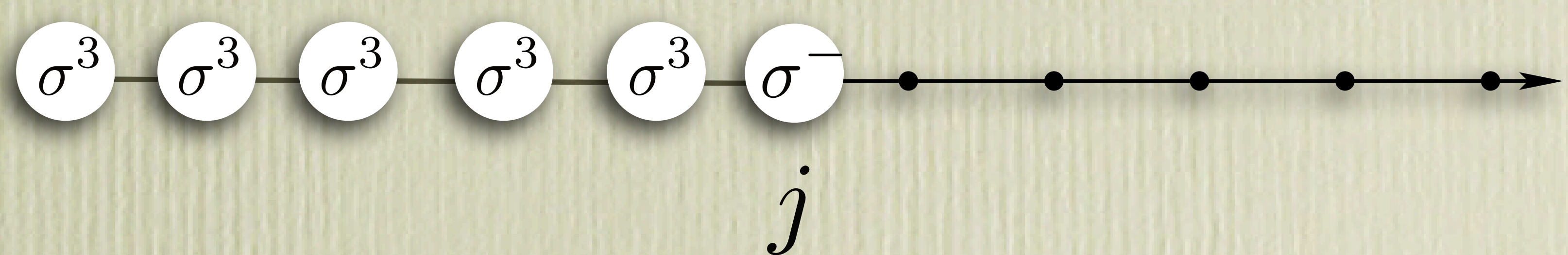
Anticommutation is *nonlocal!*

Problem:

Anticommuting fields $\varphi_{\mathbf{n}}$ $\Longleftrightarrow$ commuting Pauli matrices $\sigma^+_{\mathbf{n}}$

For D=1 dimensions: Jordan Wigner construction

$$\varphi_j = \prod_{k<j} (-\sigma^3_k)\sigma^-_j$$



However:

$$\varphi^\dagger_{j+l}\varphi_j = (-)^l \sigma^-_j \underbrace{\sigma^3_{j+1}\sigma^3_{j+2}\cdots\sigma^3_{j+l-1}}_{l-1} \sigma^+_{j+l}$$

THE FEYNMAN PROBLEM

THE "QUBIT-IZATION" OF THE DIRAC FIELD

$$[\varphi_{\boldsymbol{n}}, \varphi_{\boldsymbol{m}}]_+ = 0, \quad [\varphi_{\boldsymbol{n}}, \varphi^\dagger_{\boldsymbol{m}}]_+ = \delta_{\boldsymbol{mn}}, \quad \boldsymbol{n}, \boldsymbol{m} \in \Lambda$$

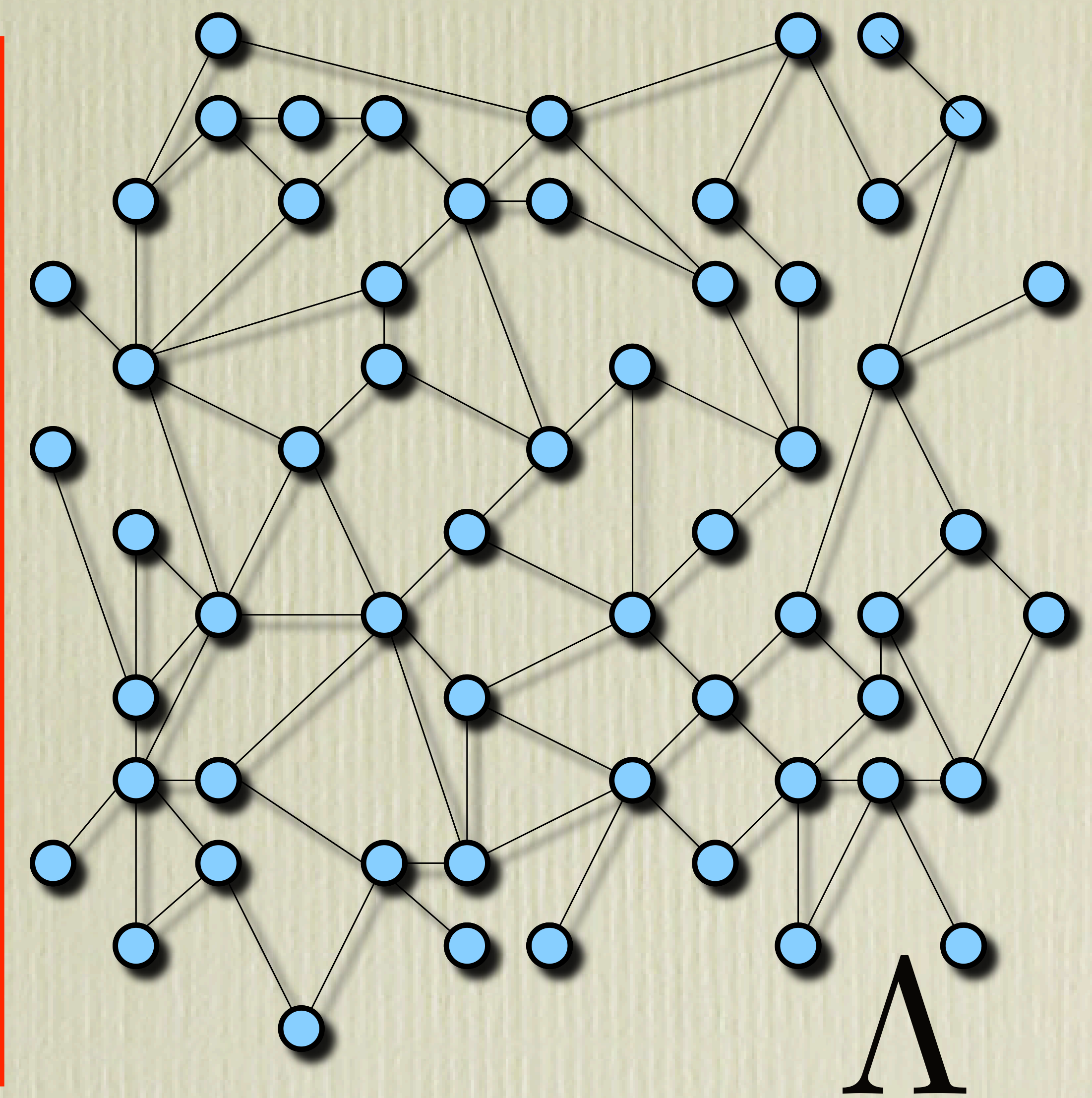
Write $\varphi_{\boldsymbol{n}}$ using Pauli matrices

$$\sigma^\alpha_{\boldsymbol{n}}, \tau^\alpha_{\boldsymbol{n}}, \ldots \qquad \boldsymbol{n} \in \Lambda$$

in such a way that any observable in the field operator

$$\varphi^\dagger_{\boldsymbol{n}} \varphi_{\boldsymbol{m}}$$

contains only Pauli operators in the same locations $\boldsymbol{n}, \boldsymbol{m} \in \Lambda$



Also generalize to many components $\varphi^a_{\boldsymbol{n}}$ mutually commuting/anticommuting

SOLUTION OF THE FEYNMAN PROBLEM

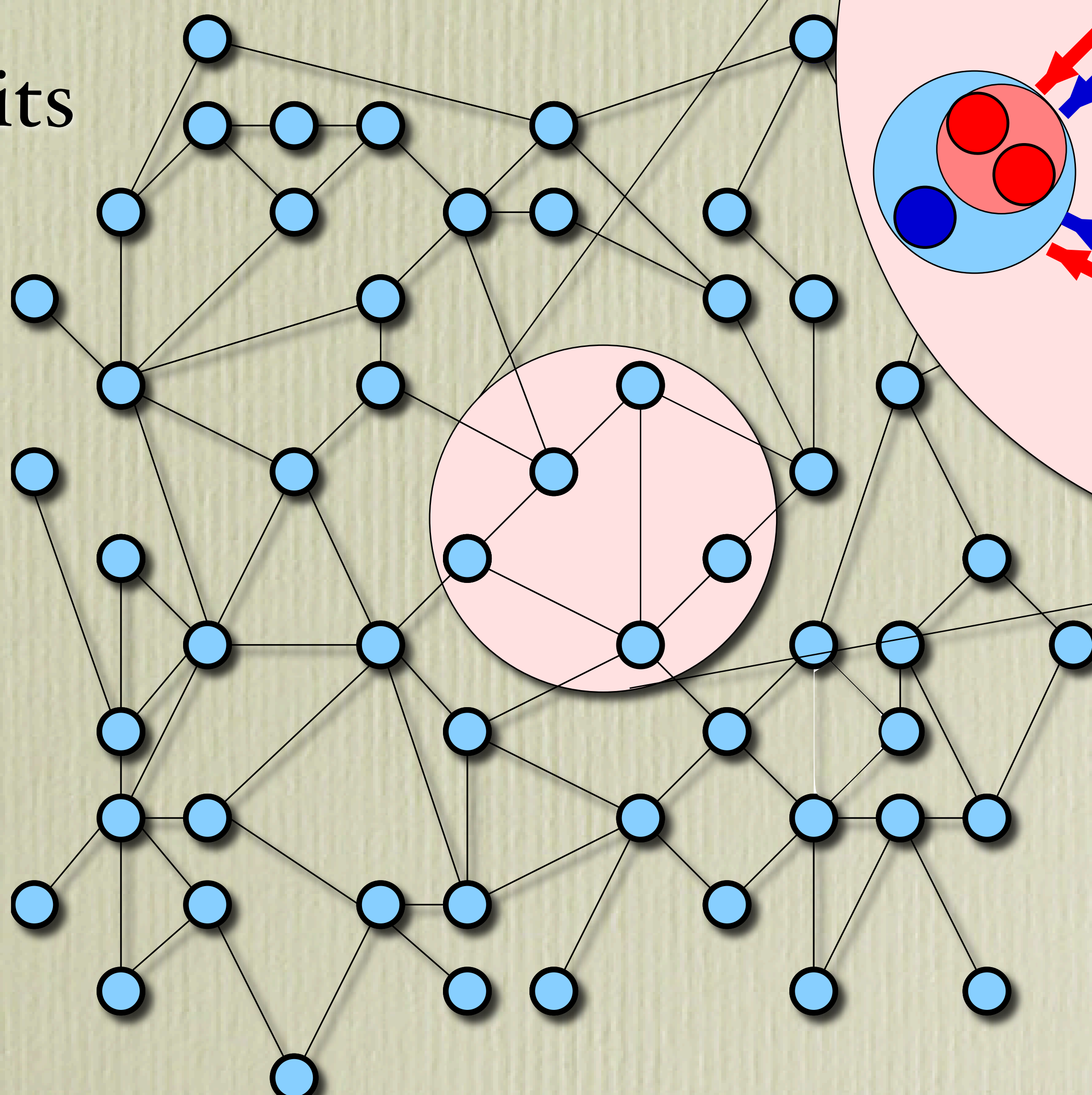
THE SOLUTION

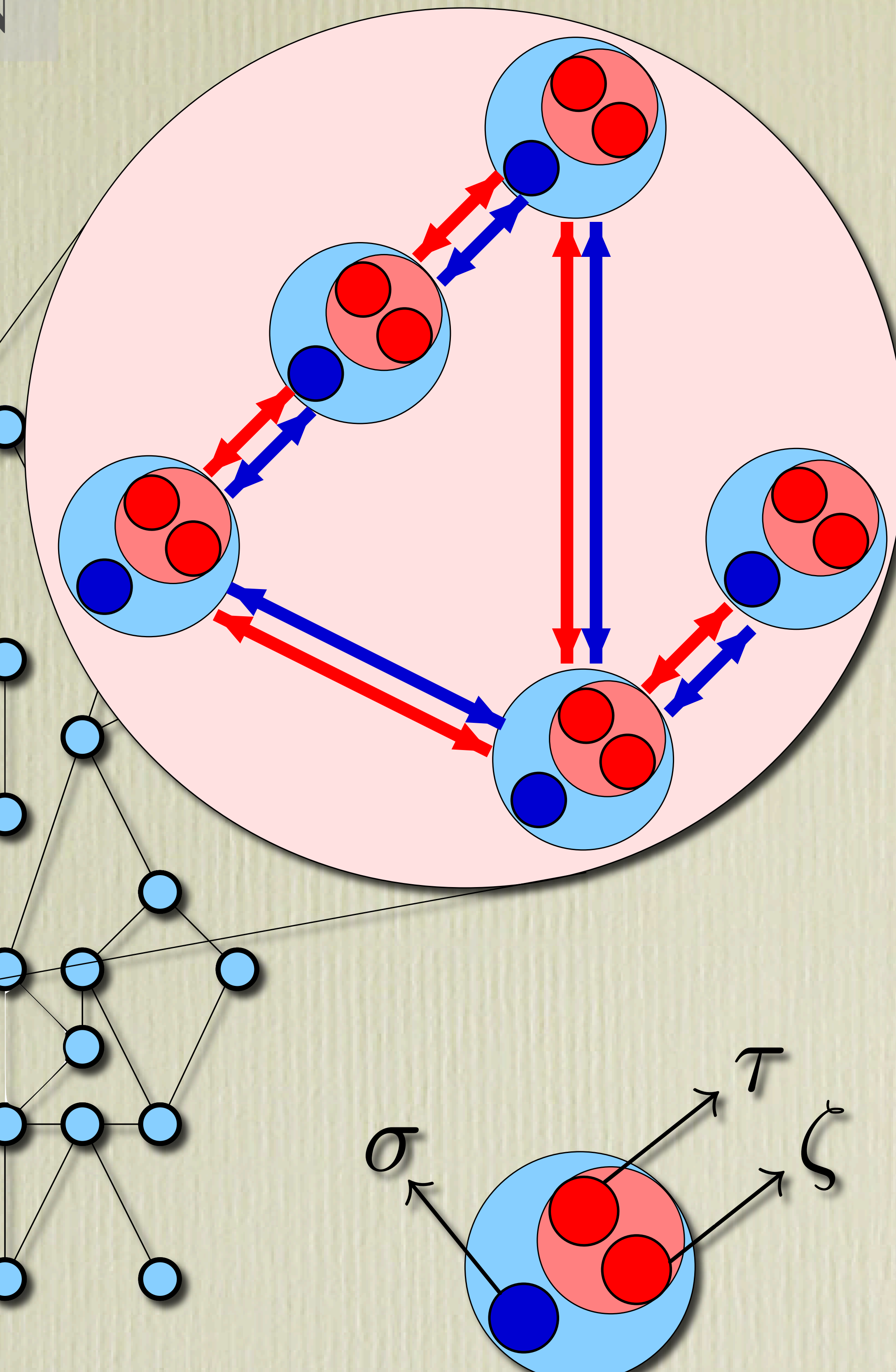
qubits:

σ associated to the Dirac field $\varphi_{\boldsymbol{n}}$

τ associated to Majorana fields $\eta_{\boldsymbol{n}}$, $\chi_{\boldsymbol{n}}$

ζ additional qubits

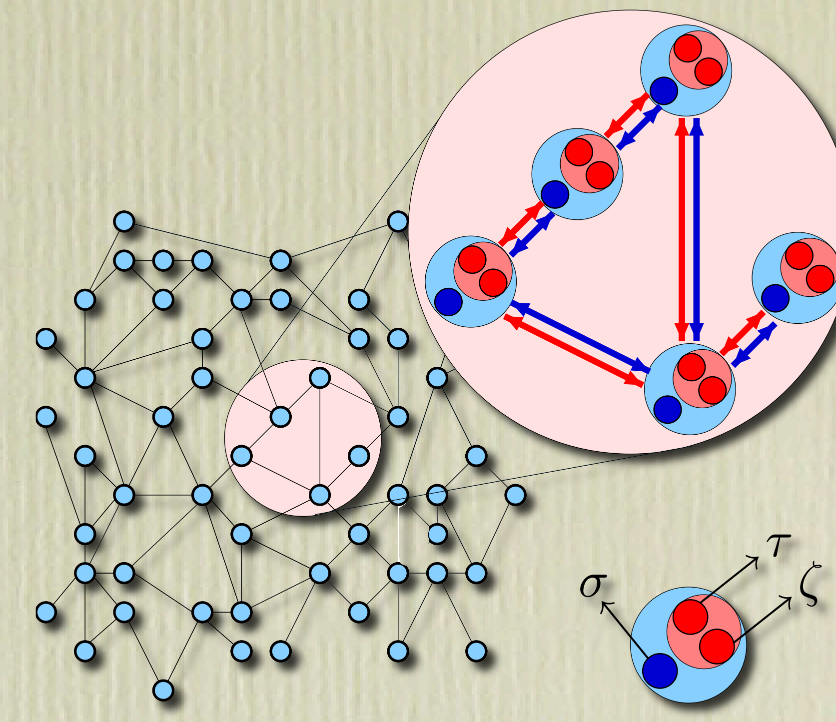




SOLUTION OF THE FEYNMAN PROBLEM

THE SOLUTION

$$\sigma_{\boldsymbol{n}}^{+} = \varphi_{\boldsymbol{n}}^{\dagger}\Phi^{\sigma}(\boldsymbol{n}), \quad \tau_{\boldsymbol{n}}^{+} = \nu_{\boldsymbol{n}}^{\dagger}\Phi^{\tau}(\boldsymbol{n})$$



$$\chi_{\boldsymbol{n}} = \nu_{\boldsymbol{m}}^{\dagger} + \nu_{\boldsymbol{m}} = \Phi(\boldsymbol{n})\tau_{\boldsymbol{n}}^{1}$$

$$\eta_{\boldsymbol{n}} = i(\nu_{\boldsymbol{m}}^{\dagger} - \nu_{\boldsymbol{m}}) = -\Phi(\boldsymbol{n})\tau_{\boldsymbol{n}}^{2}$$

$$\Phi^{\sigma,\tau}(\boldsymbol{n}) = \exp\left[i \sum_{\boldsymbol{m}} \left(\varphi_{\boldsymbol{m}}^{\dagger}\varphi_{\boldsymbol{m}} + \nu_{\boldsymbol{m}}^{\dagger}\nu_{\boldsymbol{m}} + \zeta_{\boldsymbol{m}}^{+}\zeta_{\boldsymbol{m}}^{-} \right) \alpha_{\boldsymbol{m}}^{\sigma,\tau\,(\boldsymbol{n})} \right]$$

$$\alpha^{\sigma}{}_{\boldsymbol{m}}^{(\boldsymbol{n})} = \alpha_{\boldsymbol{m}}^{(\boldsymbol{n})} + \pi\delta_{\boldsymbol{mn}}$$

$$\alpha_{\boldsymbol{m}}^{\tau(\boldsymbol{n})} = \alpha_{\boldsymbol{m}}^{(\boldsymbol{n})} := \pi\theta\left[\sin^{-1}\left(\boldsymbol{k}\cdot\frac{\boldsymbol{n}-\boldsymbol{m}}{|\boldsymbol{n}-\boldsymbol{m}|} \right) \right]$$

SOLUTION OF THE FEYNMAN PROBLEM

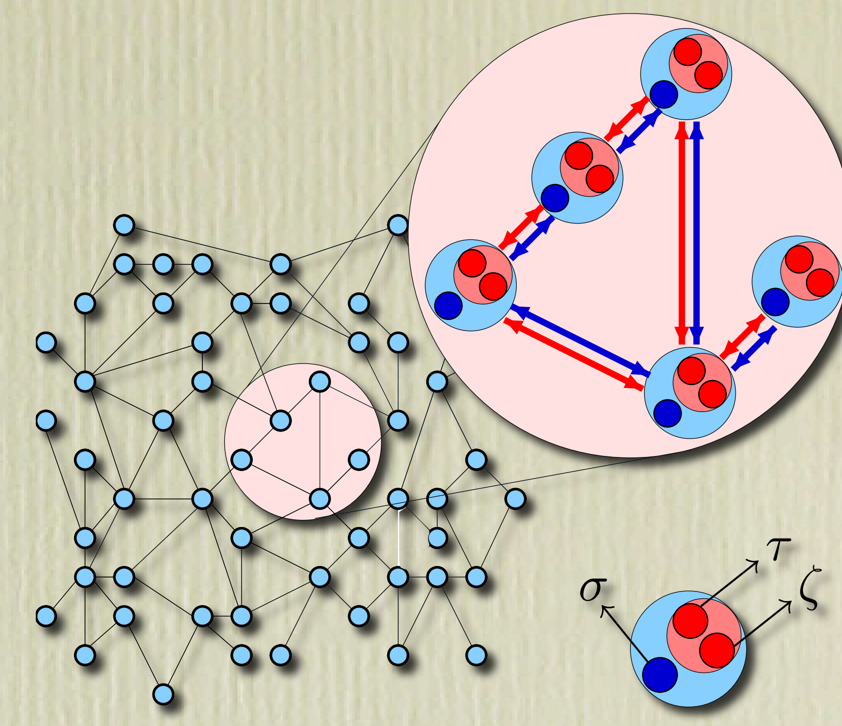
THE SOLUTION

Vacuum

$$|\emptyset\rangle = |\Omega\rangle_\sigma \otimes |W\rangle_{\tau\zeta}, \quad |\Omega\rangle_\sigma = \otimes_{\boldsymbol{n}\in\Lambda}|\downarrow\rangle_{\boldsymbol{n}}^\sigma$$

$$|W\rangle_{\tau\zeta} = \otimes_{\boldsymbol{n}\in\Lambda}|W\rangle_{\boldsymbol{n}}^{\tau\zeta}$$

$$|W\rangle_{\boldsymbol{n}}^{\tau\zeta} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_\tau|\uparrow\rangle_\zeta + |\downarrow\rangle_\tau|\downarrow\rangle_\zeta)$$



Particle states

$$\varphi^\dagger_{\boldsymbol{n}_1}\varphi^\dagger_{\boldsymbol{n}_2}\cdots\varphi^\dagger_{\boldsymbol{n}_N}|\Omega\rangle_\sigma \otimes |W\rangle_{\tau\zeta}$$
$$= s(\boldsymbol{n}_1,\boldsymbol{n}_2,\ldots,\boldsymbol{n}_N)\sigma^+_{\boldsymbol{n}_1}\sigma^+_{\boldsymbol{n}_2}\ldots\sigma^+_{\boldsymbol{n}_N}|\Omega\rangle_\sigma \otimes |W\rangle_{\tau\zeta}$$

Interaction terms

$$\varphi^\dagger_{\boldsymbol{n}}\varphi_{\boldsymbol{m}} \Longrightarrow \varphi^\dagger_{\boldsymbol{n}}\varphi_{\boldsymbol{m}}M_{\boldsymbol{nm}}B_{\boldsymbol{nm}}$$
$$\varphi^\dagger_{\boldsymbol{n}}\varphi_{\boldsymbol{m}}M_{\boldsymbol{nm}}B_{\boldsymbol{nm}}|F\rangle_\sigma \otimes |W\rangle_{\tau\zeta} = \sigma^+_{\boldsymbol{n}}\sigma^-_{\boldsymbol{m}}|F\rangle_\sigma \otimes |W\rangle_{\tau\zeta}$$