QCD Kinetic theory

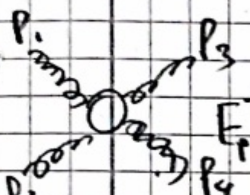
We can consider QCD kinetic theory (Arnold, Moore, Yaffe) where at leading order one needs to include

Inelastic eff. $1\leftrightarrow 2$ processes

$$\mathcal{C} = C_{2\leftrightarrow 2} + C_{1\leftrightarrow 2}$$

elastic $2\leftrightarrow 2$ scattering

If we consider pure glue QCD (see 2012.09079) the elastic process is given by



$$E_p^{-1} C_{gg\leftrightarrow gg} = \frac{1}{2\nu_g}\frac{1}{2E_p}\int d\Omega_{2\leftrightarrow 2}\, |M_{gg\leftrightarrow gg}|^2 \Big[f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4)\Big]$$

where $\int d\Omega_{2\leftrightarrow 2} = \int \frac{d^3p_2}{(2\pi)^3}\frac{1}{2p_2}\int \frac{d^3p_3}{(2\pi)^3}\frac{1}{2p_3}\int\frac{d^3p_4}{(2\pi)^3}\frac{1}{2p_4}$

and the Matrix element is given by

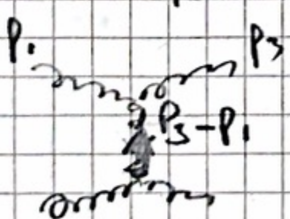
$$|M_{gg\leftrightarrow gg}|^2 = 4g^4 d_A C_A^2 \left(9 + \frac{(s-u)^2}{t^2} + \frac{(s-t)^2}{u^2} + \frac{(t-u)^2}{s^2}\right)$$

with Mandelstam variables

$$s = (p_1+p_2)^2 \qquad t = (p_3-p_1)^2 \qquad u = (p_4-p_1)^2$$

Clear preference for small t, u exchange, in fact leads to divergent rates associated with long-distance Coulomb exchange

t-channel



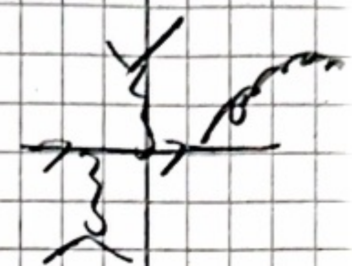
$$p_3 - p_1 = (\omega, \vec{q})$$

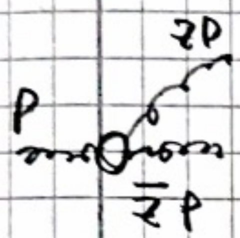
$$\frac{1}{t^2} = \frac{1}{(\omega^2 - q^2)^2}$$

In QCD matters regulated by screening, such that rate of soft interactions is large but finite

$$\Gamma_{soft} \sim g^2 T \qquad \Gamma_{large} \sim g^4 T \;\rightarrow \text{Diffusion approximation}$$

Since soft scattering can also induce radiation these naively higher order processes are promoted to leading order





$\bar z = (1-z)$

$$E_p^{-1} C_{g\leftrightarrow gg} = \frac{-1}{2\nu_g}\int_0^1 dz \Big\{ \frac{d\Gamma^g_{gg}}{dz}(p,z)\Big[f_p (1+f_{pz})(1+f_{p\bar z}) - f_{pz} f_{p\bar z}(1+f_p)\Big]$$

$$- \frac{1}{z^3}\frac{d\Gamma^g_{gg}}{dz}\left(\frac{p}{z}\Big| z\right)\Big[f_{\frac{p}{z}}(1+f_p)(1+f_{\frac{\bar z}{z}p}) - f_p f_{\frac{\bar z}{z}p}(1+f_{\frac{p}{z}})\Big]$$

$$+ (z \leftrightarrow \bar z) \Big\}$$

Since $C[f]=0$ for $f=f_{eq}$ the qualitative behavior is identical to the evolution in RTA but the underlying microscopic dynamics is quite different and very interesting in its own right

Shows characteristic pattern of bottom up thermalization, (see 1808.02133)

Phenomenology of Hydro attractors (1908.02786)

Now if we assume the existence of a pressure attractor, in the sense that

$$\frac{P_L}{\mathcal{E}} = f(\tilde{w})$$

is a universal function of $\tilde{w}$ for a given microscopic theory, we can solve the energy conservation equation

$$\partial_\tau \mathcal{E} = -\frac{\mathcal{E}+P_L}{\tau}$$

to determine how the energy initially deposited in the collision of two nuclei is converted to thermal energy when hydrodynamics becomes applicable.

In order to do so, we re-arrange

$$\frac{1}{\mathcal{E}}\, \tau \partial_\tau \mathcal{E} = -\left(1+\frac{P_L}{\mathcal{E}}\right)$$

now using the identity

$$\tau \partial_\tau = a(\tilde{w})\, \tilde{w}\, \partial_{\tilde{w}} \qquad a(\tilde{w}) = \frac{3}{4} - \frac{1}{4} f(\tilde{w})$$

we get

$$a(\tilde{w})\, \tilde{w}\, \partial_{\tilde{w}} \log(\mathcal{E}) = -\left(1+f(\tilde{w})\right)$$

which we can formally solve as

$$\log\left(\frac{\mathcal{E}(\tau)}{\mathcal{E}_0}\right) = -\int_{\tilde{w}_0}^{\tilde{w}(\tau)} \frac{d\tilde{w}}{\tilde{w}}\; \frac{1+f(\tilde{w})}{\frac{3}{4}-\frac{1}{4}f(\tilde{w})}$$

Next we can consider the late time limit, where the system eventually approaches equilibrium

$p_L = \frac{\varepsilon}{3}$ and $f(\tilde{w}) = \frac{1}{3}$

we get

$$\varepsilon(\tau) = \varepsilon_H \exp\left(- \underbrace{\frac{1+\frac{1}{3}}{\frac{3}{4}-\frac{1}{12}}}_{=2} \log\left(\frac{\tilde{w}_\tau}{\tilde{w}_H}\right)\right)$$

so

$$\varepsilon(\tau) = \varepsilon_H \left(\frac{\tilde{w}_H}{\tilde{w}_\tau}\right)^2 \Rightarrow \varepsilon(\tau)\, \tilde{w}_\tau^2 = \text{const}$$

Now doing the same tricks as before, we can obtain

$$\varepsilon(\tau) = \frac{\varepsilon_H \tau_H^{4/3}}{\tau^{4/3}}$$

which is the result of ideal Bjorken flow.

Now beyond these two limits, the function

$$I(\tilde{w}_\tau, \tilde{w}_0) = \exp\left(- \int_{\tilde{w}_0}^{\tilde{w}_\tau} \frac{d\tilde{w}}{\tilde{w}} \frac{1+f(\tilde{w})}{\frac{3}{4}-\frac{1}{4}f(\tilde{w})}\right)$$

describes the full evolution, so it allows us to relate the asymptotic constant at late times $\varepsilon_H \tau_H^{4/3}$ to the initial energy per unit rapidity $\varepsilon_0 \tau_0$.

So we got

$$\mathcal{E}(\bar\tau) = \mathcal{E}(\bar\tau_0)\,\exp\left(-\int_{\tilde\omega_0}^{\tilde\omega(\tau)} \frac{d\tilde\omega}{\tilde\omega}\,\frac{1+f(\tilde\omega)}{\frac{3}{4}-\frac{1}{4}f(\tilde\omega)}\right)$$

Now to understand this expression, let us first consider the limiting behavior at early and late times

<u>early times ($\tilde\omega \ll 1$):</u> $P_L \simeq 0$ $\quad f(\tilde\omega) = 0$

$$\mathcal{E}(\bar\tau) = \mathcal{E}(\bar\tau_0)\,\exp\left(-\int_{\tilde\omega_0}^{\tilde\omega_\tau} \frac{d\tilde\omega}{\tilde\omega}\,\frac{4}{3}\right)$$

$$\mathcal{E}(\bar\tau) = \mathcal{E}_0 \left(\frac{\tilde\omega_0}{\tilde\omega(\tau)}\right)^{4/3} \quad \Rightarrow \quad \mathcal{E}(\tau)\,\omega_\tau^{4/3} = \text{const}$$

Now we can use $\tilde\omega = \frac{T\tau}{4\pi\eta/s}$ to rewrite

$$\left(\frac{\tilde\omega_0}{\tilde\omega(\tau)}\right)^{4/3} = \left(\frac{\mathcal{E}_0^{1/4}\,\tau_0}{\mathcal{E}(\tau)^{1/4}\,\tau}\right)^{4/3} = \frac{\mathcal{E}_0^{1/3}}{\mathcal{E}(\tau)^{1/3}}\left(\frac{\tau_0}{\tau}\right)^{4/3}$$

Solving for $\mathcal{E}(\tau)$ we got

$$\mathcal{E}(\tau) = \frac{\mathcal{E}_0\,\tau_0}{\tau} \quad \text{consistent with} \quad \partial_\tau \mathcal{E} = -\frac{\mathcal{E}+P_L}{\tau} \quad (P_L \simeq 0)$$

Here the constant

$$\mathcal{E}_0\,\tau_0 = \frac{dE_\perp}{dy\,dx_\perp}$$

describes transverse energy per unit rapidity

In order to work out this matching, let us consider

$$\tau^{4/3}\mathcal{E}(\tau) = \tilde{\omega}_\tau^{4/3}\,\mathcal{E}(\tau)\,\frac{(4\pi\,\eta/s)^{4/3}}{T(\tau)^{4/3}}$$

now using EoS $\mathcal{E}(\tau) = \frac{\pi^2}{30}\nu_{eff}\,T(\tau)^4$

$$= \tilde{\omega}_\tau^{4/3}\,\underbrace{\frac{\mathcal{E}(\tau)}{\mathcal{E}(\tau)^{1/3}}}_{=\mathcal{E}(\tau)^{2/3}}\,(4\pi\,\eta/s)^{4/3}\left(\frac{\pi^2}{30}\nu_{eff}\right)^{1/3}$$

Now use the evolution equation $\mathcal{E}(\tau) = \mathcal{E}_0\,\mathcal{I}(\tilde{\omega}_\tau,\tilde{\omega}_0)$

$$= \tilde{\omega}_\tau^{4/3}\left[\mathcal{E}_0\,\mathcal{I}(\tilde{\omega}_\tau,\tilde{\omega}_0)\right]^{2/3}(4\pi\,\eta/s)^{4/3}\left(\frac{\pi^2}{30}\nu_{eff}\right)^{1/3}$$

we now from our analysis, that for $\tilde{\omega}_0 \to 0$ $\mathcal{I}(\tilde{\omega}_\tau,\tilde{\omega}_0) \propto \tilde{\omega}_0^{4/3}$ so we can introduce this as a factor

$$= \left(\mathcal{E}_0\,\tilde{\omega}_0^{4/3}\right)^{2/3}\left[\frac{\tilde{\omega}_\tau^2}{\tilde{\omega}_0^{4/3}}\,\mathcal{I}(\tilde{\omega}_\tau,\tilde{\omega}_0)\right]^{2/3}(4\pi\,\eta/s)^{4/3}\left(\frac{\pi^2}{30}\nu_{eff}\right)^{1/3}$$

Now the important part to realize is that, since for $\tilde{\omega}_\tau \to \infty$ $\mathcal{I}(\tilde{\omega}_\tau,\tilde{\omega}_0) \propto \frac{1}{\tilde{\omega}_\tau^2}$, the term in brackets approaches a constant in the limit

$$\lim_{\tilde{\omega}_0\to 0}\,\lim_{\tilde{\omega}_\tau\to\infty}\left[\frac{\tilde{\omega}_\tau^2}{\tilde{\omega}_0^{4/3}}\,\mathcal{I}(\tilde{\omega}_\tau,\tilde{\omega}_0)\right]^{2/3} = C_\infty$$

where we match the initial state ($\tilde{\omega}_0 \to 0$) directly after the collision, to the equilibrium QGP ($\tilde{\omega}_\tau \to \infty$)

If we keep $\tilde{\omega}_\tau$ finite, this expression defines an energy attractor

$$C_\infty \mathcal{E}(\tilde{\omega}_\tau) = \lim_{\tilde{\omega}_0 \to 0} \left[\frac{\tilde{\omega}_\tau^2}{\tilde{\omega}_0^{4/3}} I(\tilde{\omega}_\tau, \tilde{\omega}_0) \right]^{2/3}$$

where $\mathcal{E}(\tilde{\omega}_\tau \gg 1) \to 1$

$$\mathcal{E}(\tilde{\omega}_\tau \ll 1) \to \frac{1}{C_\infty} \tilde{\omega}_\tau^{4/9}$$

Now what is left to be done is to work out the prefactors, namely

$$\left(\varepsilon_0 \tilde{\omega}_0^{4/3}\right)^{2/3} = \left(\varepsilon_0 \frac{T_0^{4/3} \tau_0^{4/3}}{(4\pi \eta/s)^{4/3}}\right)^{2/3}$$

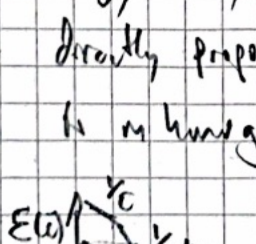
$$= (\varepsilon_0 \tau_0)^{8/9} \left(\frac{\pi^2}{30} \nu_{eff}\right)^{-2/9} (4\pi \eta/s)^{-8/9}$$

Collecting all the pre-factors, we get (2012.09068)

$$\tau^{4/3} \varepsilon(\tau) = C_\infty (\varepsilon_0 \tau_0)^{8/9} (4\pi \eta/s)^{4/9} \left(\frac{\pi^2}{30} \nu_{eff}\right)^{1/9} \mathcal{E}(\tilde{\omega}_\tau)$$

Now what is the meaning of this result

- Energy density at onset of hydrodynamics, not directly proportional to initial energy density due to inhomogeneous cooling



- Earlier equilibration of hot spots leads to faster cooling.
- Highly sensitive to η/s, as this controls time scale for equilibration

Now this can also be turned into multiplicity estimates, by realizing that after the system approaches hydrodynamic regime, the entropy is approximately conserved as viscosity is small

$$Ts = \varepsilon + p \quad \rightarrow \quad (\tau s)_H = \frac{dS_H}{d^2x_\perp d\eta} = \frac{(\varepsilon+p)}{T}\tau \simeq \frac{4}{3}\left(\frac{\pi^2}{30}\nu_{eff}\right)^{1/4}\left(\varepsilon\tau^{4/3}\right)^{3/4}$$

so we get the total entropy per unit rapidity

$$\frac{dS}{d\eta} = \frac{4}{3} C_\infty^{3/4}\left(4\pi\eta/s\right)^{1/3}\left(\frac{\pi^2}{30}\nu_{eff}\right)^{1/3}\int_{x_\perp}\left(\varepsilon_0\tau_0\right)^{2/3}$$

and the final multiplicity is simply given by

$$\frac{dN_{ch}}{d\eta} \simeq \left(\frac{N_{ch}}{S}\right)_{HRG}\frac{dS}{d\eta} \quad \text{with} \quad \left(\frac{S}{N_{ch}}\right)_{HRG} \simeq 6-9$$

→ Can directly predict multiplicity without any sophisticated multi-stage modeling, accuracy is at 10% level compared to full simulation

Can also include transverse dynamics
by including linearized perturbations of $T^{\mu\nu}$
on top of Bjorken background

→ KoMPoST (1805.00961)

$$T^{\mu\nu}(\tau,x_\perp) = \underbrace{\bar{T}^{\mu\nu}(\tau)}_{\text{Bjorken flow}} + \int d^2x'_\perp \, G^{\mu\nu}_{\alpha\beta}(\tau,\tau_0,x_\perp - x'_\perp) \left(T^{\alpha\beta}(x'_\perp) - \bar{T}^{\alpha\beta}(x'_\perp)\right)$$

with non-equilibrium Green's functions calculated in kinetic theory.

Includes development of transverse flow and allows continuous* matching to hydrodynamics
(*except for mismatch in EoS)

<u>Open questions</u> → Breaking symmetries

- Effects of non-conformal EoS
  → multi-dimensional attractors?

- Small systems (2211.14356)
  → breakdown of hydrodynamics when transverse expansion sets in before sufficient degree of equilibration is achieved

- Improved theories of hydrodynamics to include more micro physics?