

Hydrodynamics and Kinetic Theory: Attractors and Thermalization

Exercise sheet 1

Prof. Dr. S. Schlichting, Dr. C. Werthmann

1 Pressure anisotropy in conformal Bjorken flow

Consider a conformal system ($e = 3P \propto T^4$ and $sT = e + P$) in Bjorken flow that follows the first order Müller-Israel-Stewart-type hydrodynamic equations

$$\frac{de}{d\tau} = -\frac{e + P_L}{\tau}, \quad \tau_\pi \frac{d\pi_\eta^\eta}{d\tau} = -\pi_\eta^\eta - \frac{4\eta}{3\tau},$$

with the longitudinal pressure $P_L = P + \pi_\eta^\eta$, the shear viscosity $\eta = C_\eta s$ and the shear relaxation time $\tau_\pi = C_{\tau\pi} T^{-1}$, where C_η and $C_{\tau\pi}$ are constants. We also introduce the scaled time $\tilde{w} = \frac{T\tau}{4\pi\eta/s}$. Note that we have to assume that $d\tilde{w}/d\tau > 0$ at all times for this to make sense.

i) Show that a defined through $\tau d/d\tau = a(\tilde{w})\tilde{w} d/d\tilde{w}$ satisfies

$$a = \frac{2}{3} - \frac{f_\pi}{4},$$

where $f_\pi = \pi_\eta^\eta/e$.

ii) Show that f_π obeys the following evolution equation in terms of \tilde{w} .

$$\tilde{w} \left(\frac{2}{3} - \frac{f_\pi}{4} \right) \frac{df_\pi}{d\tilde{w}} = -\frac{4\pi C_\eta}{C_{\tau\pi}} \tilde{w} f_\pi - \frac{16C_\eta}{9C_{\tau\pi}} + \frac{4}{3} f_\pi + f_\pi^2$$

Thus, in terms of \tilde{w} , the evolution equation for f_π decouples from the energy density.

2 Asymptotic behaviour

Use the evolution equation for $f_\pi(\tilde{w})$ to find

i) the late time asymptotics for $f_\pi(\tilde{w})$, i.e. expand f_π up to linear order in $1/\tilde{w}$.

ii) possible values for f_π at $\tilde{w} \rightarrow 0$ and their stability at small finite \tilde{w} by using them to reformulate the evolution equation.