Metrology measurements and alignment in MUonE

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MuonE-Pisa meeting

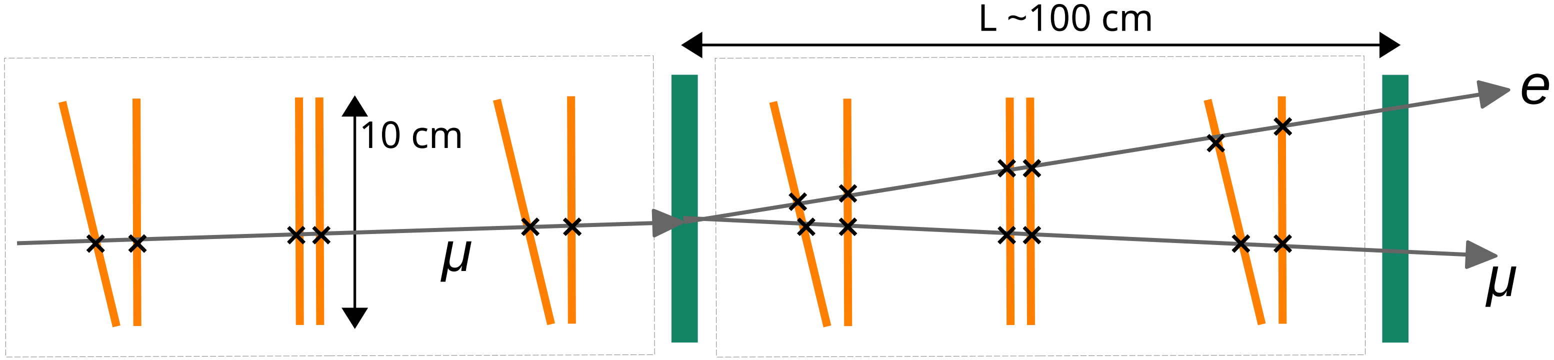
16 March 2026

Outline

- Why does MUonE need a precise alignment?
- Detector alignment workflow
- Detector description
- Metrology measurements in 2023 data reconstruction
- Metrology measurements to improve the MUonE simulation

The need of precise alignment tools in MUonE

MUonE: measurement of $\mu e \rightarrow \mu e$ interactions
at 10ppm systematic accuracy



Observables:
scattering
angles θ_μ, θ_e

The need of precise alignment tools in MUonE

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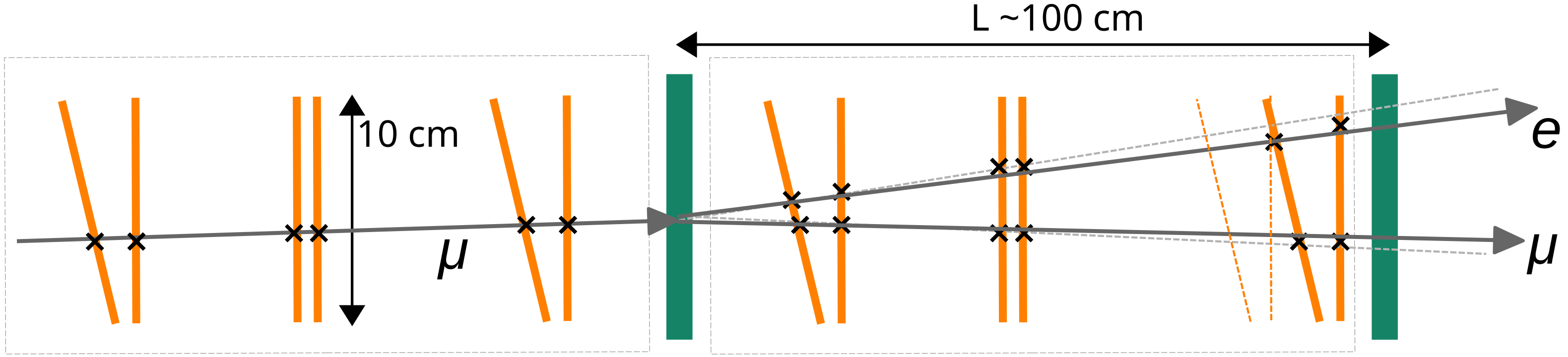

Observables: scattering angles θ_μ, θ_e

If the sensors z-position is wrong by ~10 μm:
10ppm systematic error on θ

$$\theta \sim \frac{x}{L} \quad \rightarrow \quad \frac{\Delta\theta}{\theta} \sim \frac{\Delta L}{L}$$

The need of precise alignment tools in MUonE

MUonE: measurement of $\mu e \rightarrow \mu e$ interactions
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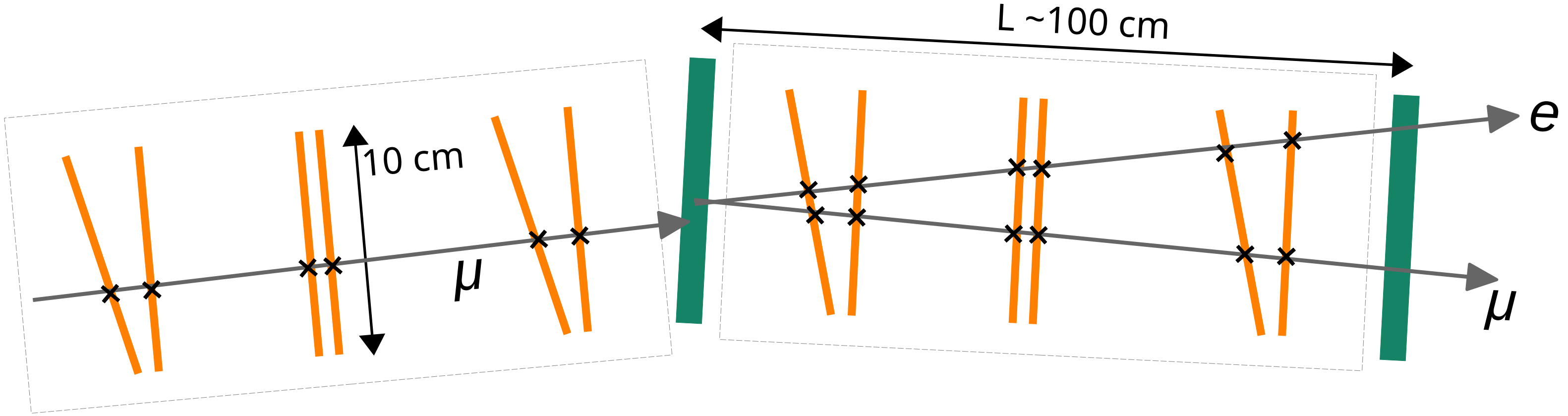

Observables:
scattering
angles θ_μ, θ_e

If the relative position/orientation
between two stations is wrong:
bad vertex reconstruction
(systematic uncertainties to be evaluated)

Precise alignment tools must be developed

MUonE alignment workflow

0. Use the stepper motors to align each station to the beam
   (and align each station relative to the others)

1. Laser survey: determine the position of the stations
   in the experimental hall. *O*(100 μm) precision

2. 3D scanner photogrammetry: determine position and orientation
   of 2S modules and targets (precision better than 100 μm?)

3. Software alignment (iterative local χ^2 minimization):
   use events with one single muon passing through the detector
   to refine the metrology measurements

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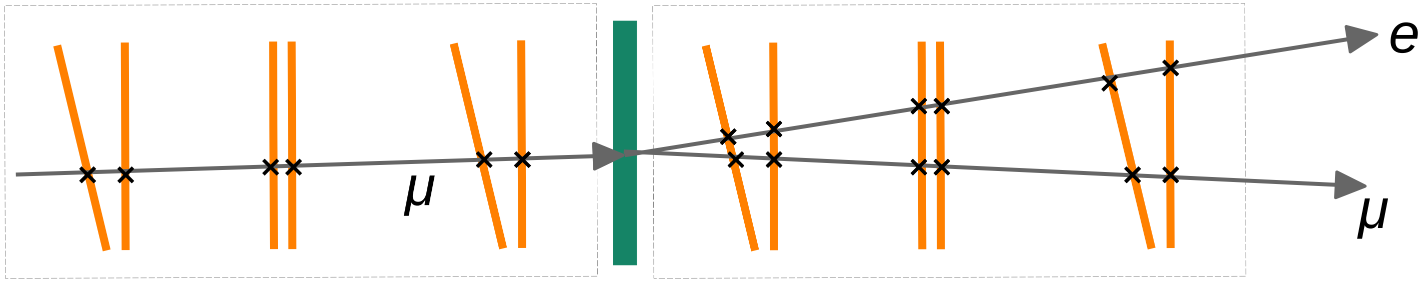
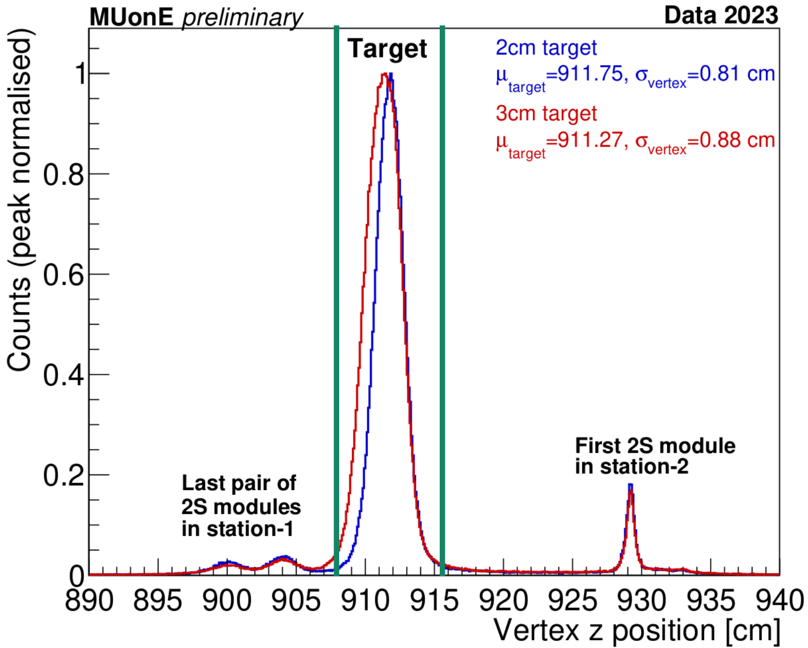
use events with one single muon passing through the detector
to refine the metrology measurements

starting point of the software alignment.
Metrology results must be expressed
in the track reconstruction formalism

Detector description

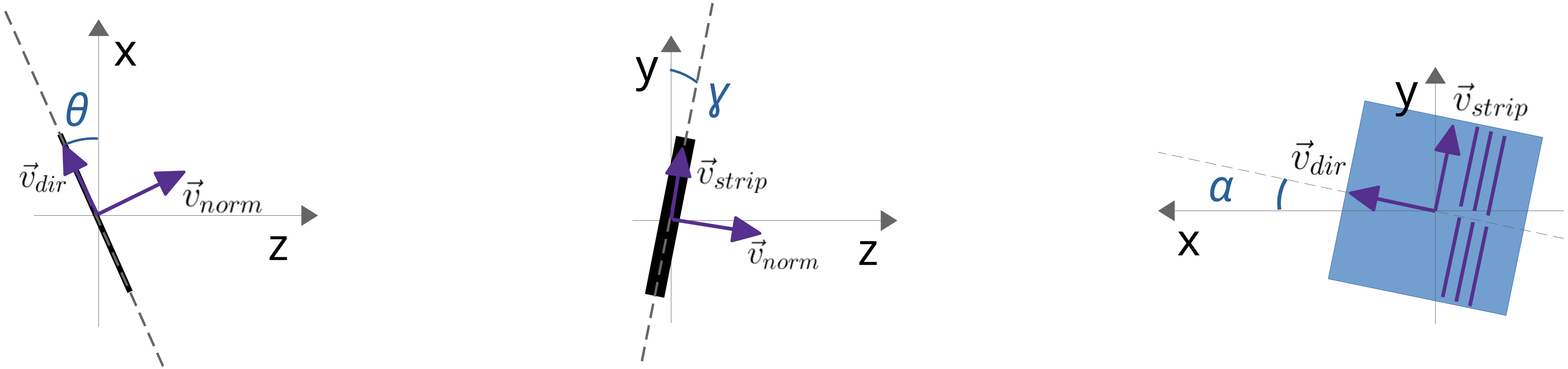
- Target (x, y, z) position. Needed for the event selection: select interactions that actually occurred in the target





Detector description

- 2S modules (x, y, z) position and orientation (3 angular parameters *α*, *θ*, *γ*). Needed for track reconstruction



- $\mathbf{v}_{strip}$ = strips direction
- $\mathbf{v}_{dir}$ = direction measured by the module (orthogonal to $\mathbf{v}_{strip}$)
- $\mathbf{v}_{norm}$ = vector orthogonal to the module surface

$$\alpha = \arctan\left(-\frac{v_{strip,X}}{v_{strip,Y}}\right)$$

$$\gamma = \arcsin\left(v_{strip,Z}\right)$$

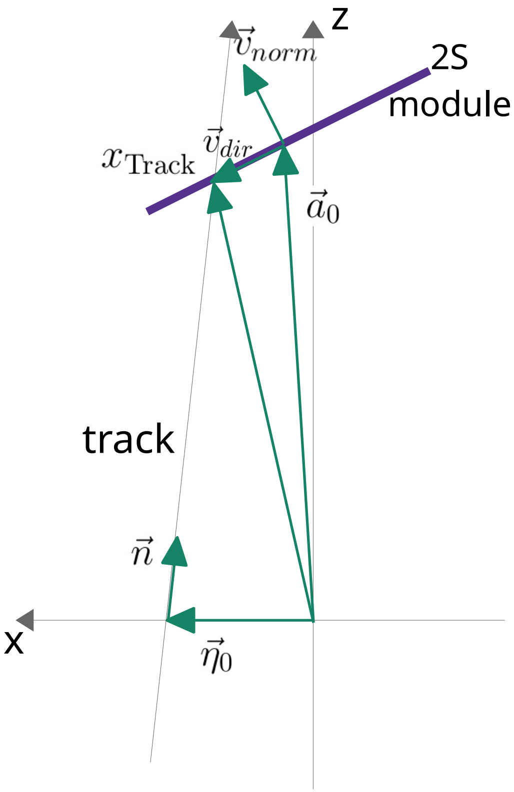
$$\theta = \arctan\left(-\frac{v_{dir,Z}}{v_{norm,Z}}\right)$$

Full calculations
in backup

Detector description in the track reconstruction

$$\chi^2_{\mathrm{Track}} = \sum_{i=0}^{\mathrm{N_{Modules}}} \left(\frac{x_{\mathrm{Track},i}(\vec{p}) - x_{\mathrm{Stub},i}}{\sigma_{\mathrm{Stub}}} \right)^2$$

$$\vec{p} = (x_{\mathrm{Slope}}, x_0, y_{\mathrm{Slope}}, y_0)$$



x_{stub}: stub position on the 2S module

x_{track}: track position along the measurement direction of the 2S module

$$x_{\mathrm{Track},i}(\vec{p}) = (\vec{\eta}_0 + \vec{n}t_i^* - \vec{a}_{0,i}) \cdot \vec{v}_{dir,i}$$

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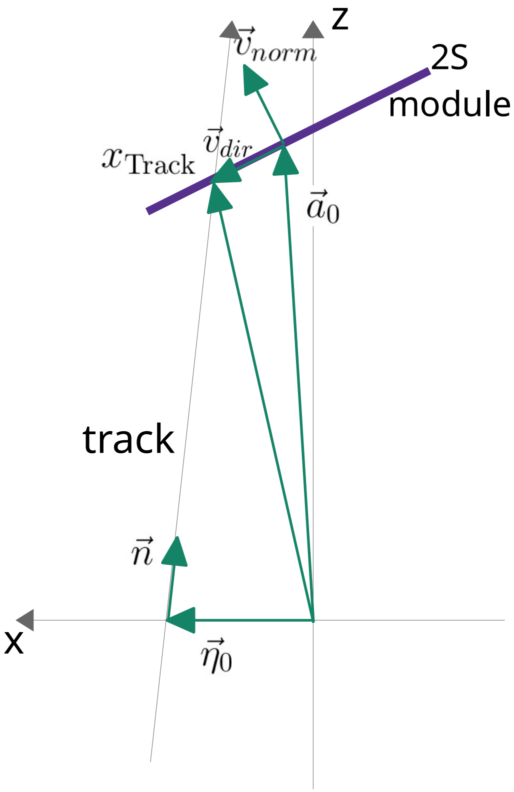
Fit parameters

$$\vec{\eta}_0 = \begin{pmatrix} x_0 \\ y_0 \\ 0 \end{pmatrix} \quad \vec{n} = \begin{pmatrix} x_{\text{Slope}} \\ y_{\text{Slope}} \\ 1 \end{pmatrix}$$



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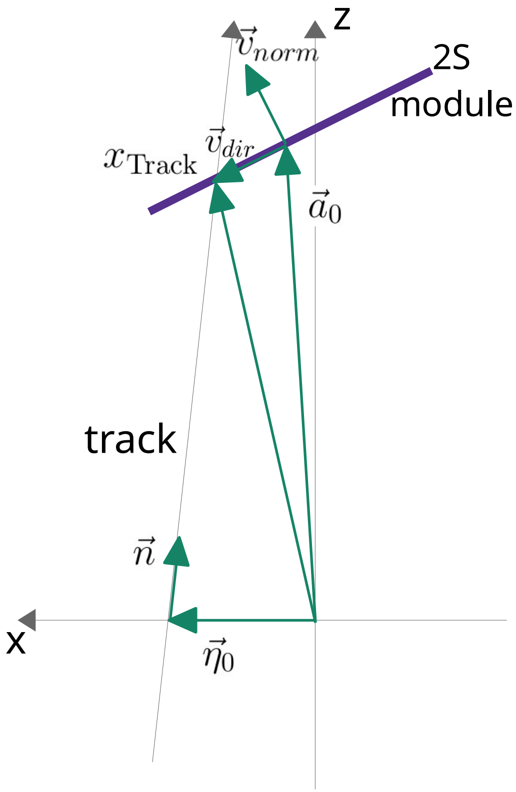
$$x_{\text{Track},i}(\vec{p}) = (\vec{\eta}_0 + \vec{n}t_i^* - \underset{\uparrow}{\vec{a}_{0,i}}) \cdot \underset{\uparrow}{\vec{v}_{dir,i}}$$

2S module
position and orientation

$$\vec{a}_{0,i} = \begin{pmatrix} x_{\text{module}} \\ y_{\text{module}} \\ z_{\text{module}} \end{pmatrix} \qquad \vec{v}_{Dir,i}(\alpha, \theta, \gamma) \qquad \vec{v}_{Norm,i}(\alpha, \theta, \gamma)$$



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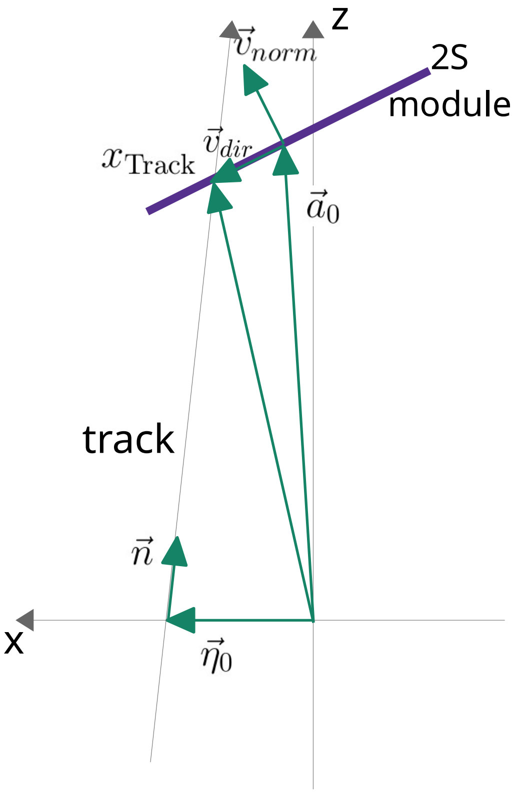
x_{stub}: stub position on the 2S module

x_{track}: track position along the measurement direction of the 2S module

$$x_{\text{Track},i}(\vec{p}) = (\vec{\eta}_0 + \vec{n}t_i^* - \vec{a}_{0,i}) \cdot \vec{v}_{dir,i}$$

$$t_i^* = \frac{(\vec{a}_{0,i} - \vec{\eta}_0) \cdot \vec{v}_{norm,i}}{\vec{n} \cdot \vec{v}_{norm,i}}$$

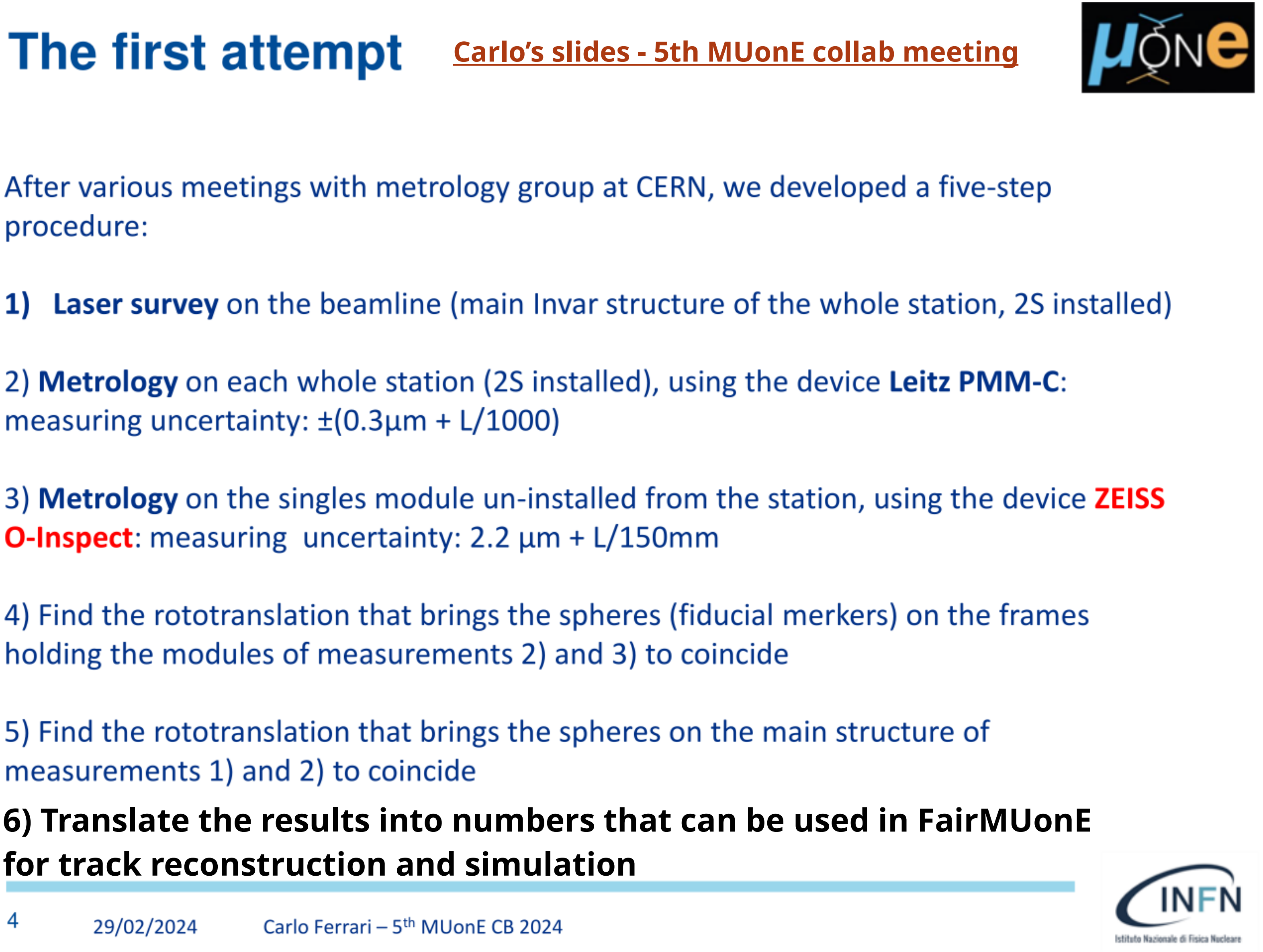
Parameter *t* to evaluate the track position
on the 2S module.
Depends both on the fit parameters
and the 2S module position and orientation



$$\vec{\eta}_0 = \begin{pmatrix} x_0 \\ y_0 \\ 0 \end{pmatrix} \qquad \vec{n} = \begin{pmatrix} x_{\text{Slope}} \\ y_{\text{Slope}} \\ 1 \end{pmatrix}$$

$$\vec{a}_{0,i} = \begin{pmatrix} x_{\text{module}} \\ y_{\text{module}} \\ z_{\text{module}} \end{pmatrix} \qquad \vec{v}_{Dir,i}(\alpha, \theta, \gamma) \qquad \vec{v}_{Norm,i}(\alpha, \theta, \gamma)$$

Metrology measurements in 2023



The first attempt

Carlo's slides - 5th MUonE collab meeting

After various meetings with metrology group at CERN, we developed a five-step procedure:

1) **Laser survey** on the beamline (main Invar structure of the whole station, 2S installed)

2) **Metrology** on each whole station (2S installed), using the device **Leitz PMM-C**: measuring uncertainty: ±(0.3μm + L/1000)

3) **Metrology** on the singles module un-installed from the station, using the device **ZEISS O-Inspect**: measuring uncertainty: 2.2 μm + L/150mm

4) Find the rototranslation that brings the spheres (fiducial merkers) on the frames holding the modules of measurements 2) and 3) to coincide

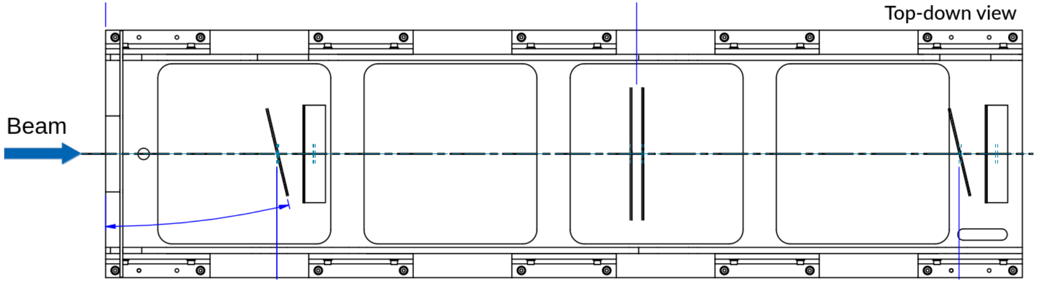
5) Find the rototranslation that brings the spheres on the main structure of measurements 1) and 2) to coincide

6) Translate the results into numbers that can be used in FairMUonE for track reconstruction and simulation

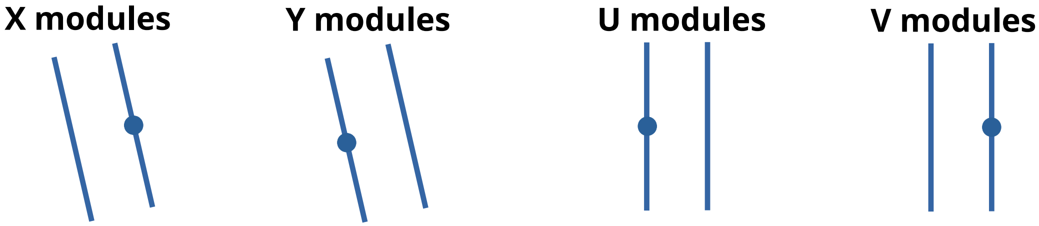
4 29/02/2024 Carlo Ferrari – 5th MUonE CB 2024

Metrology measurements in 2023

2S modules position

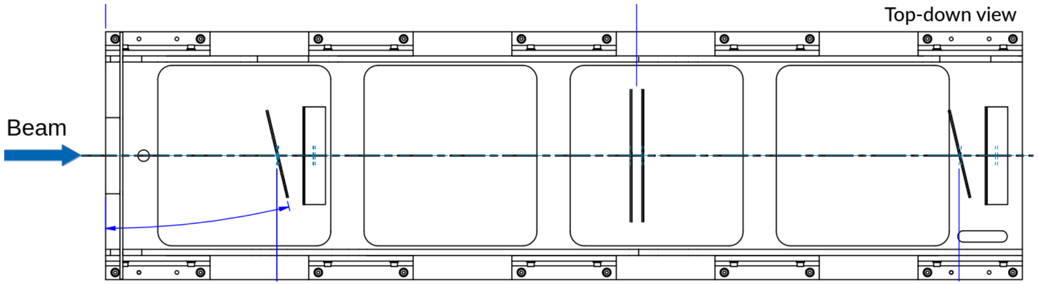


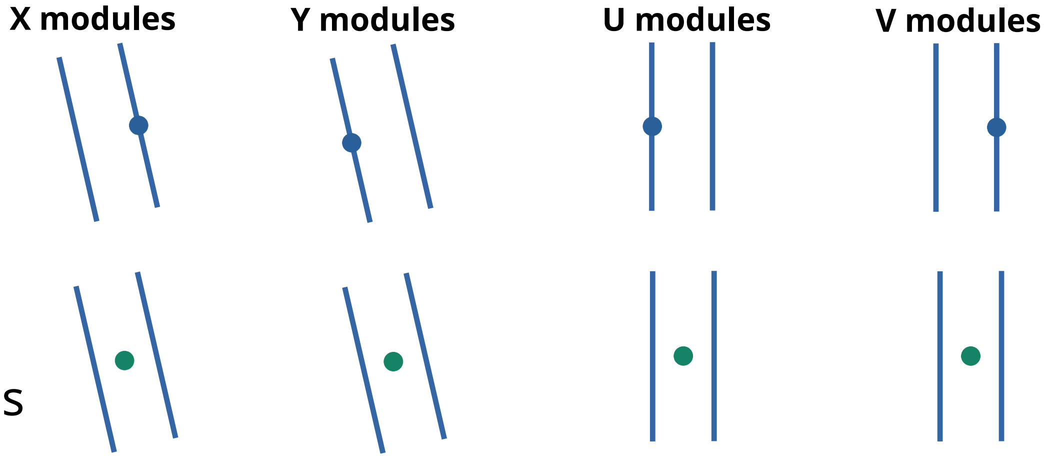
A) Metrology: position of the sensor facing outwards



Metrology measurements in 2023

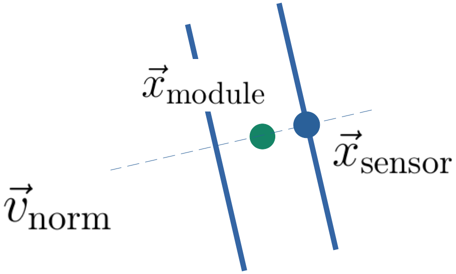
2S modules position





A) Metrology: position of the sensor facing outwards

B) FairMUonE: the module position is averaged between the two sensors positions



$$\vec{x}_{\text{module}} = \vec{x}_{\text{sensor}} \pm \vec{v}_{\text{norm}} \cdot \Delta s$$

$$\Delta s = \frac{\text{sensors spacing} + 320\,\mu\text{m}}{2}$$

Metrology measurements in 2023

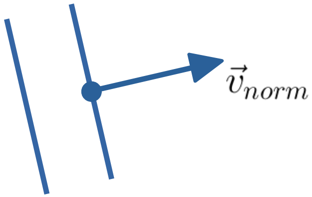
2S modules orientation

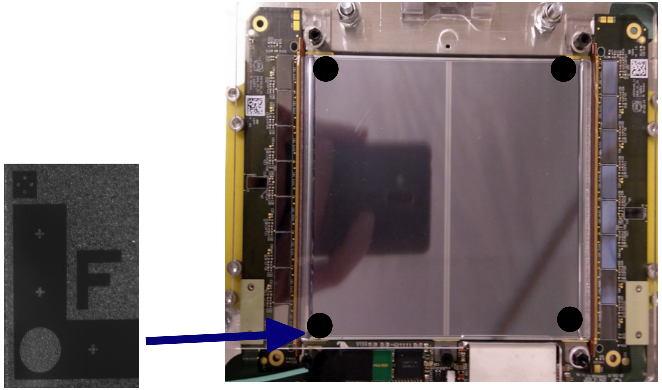
Metrology measured:

A) $\mathbf{v}_{\mathrm{norm}}$

B) reference markers on the modules

To be computed: $\mathbf{v}_{\mathrm{dir}}$ and $\mathbf{v}_{\mathrm{strip}}$





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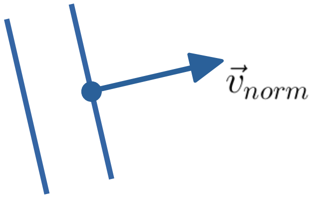
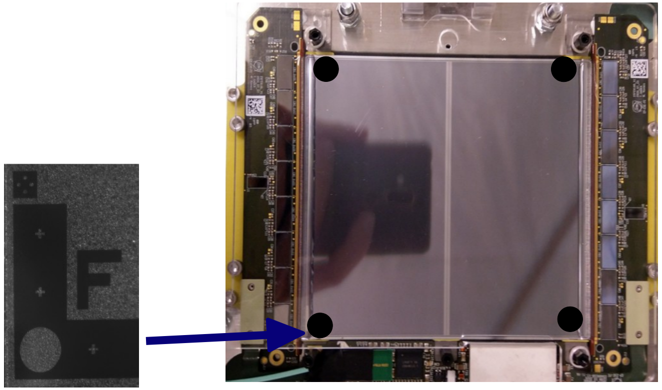
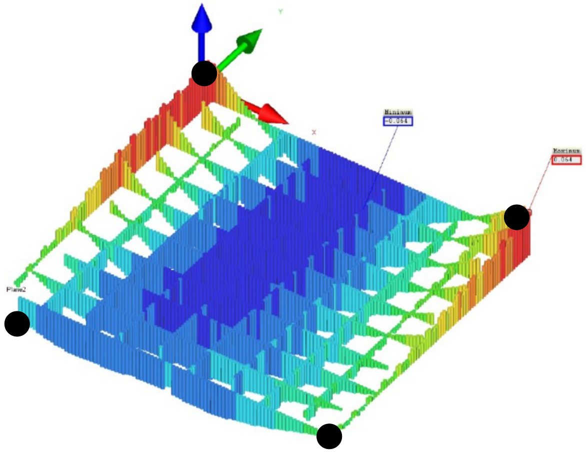
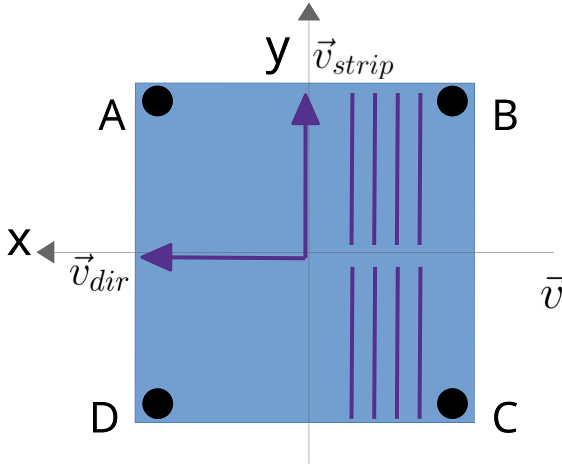




<u>To be computed: $\mathbf{v}_{\mathbf{dir}}$ and $\mathbf{v}_{\mathbf{strip}}$</u>



1) The sensors are not flat (~100 µm deformation): project the four markers on the average plane



$$\vec{v}_{dir} = \vec{A} - \vec{B} = \vec{D} - \vec{C}$$

$$\vec{v}_{strip} = \vec{A} - \vec{D} = \vec{B} - \vec{C}$$

2) Compute $\mathbf{v}_{\mathbf{dir}}$ and $\mathbf{v}_{\mathbf{strip}}$
(take the average of the two possibilities)

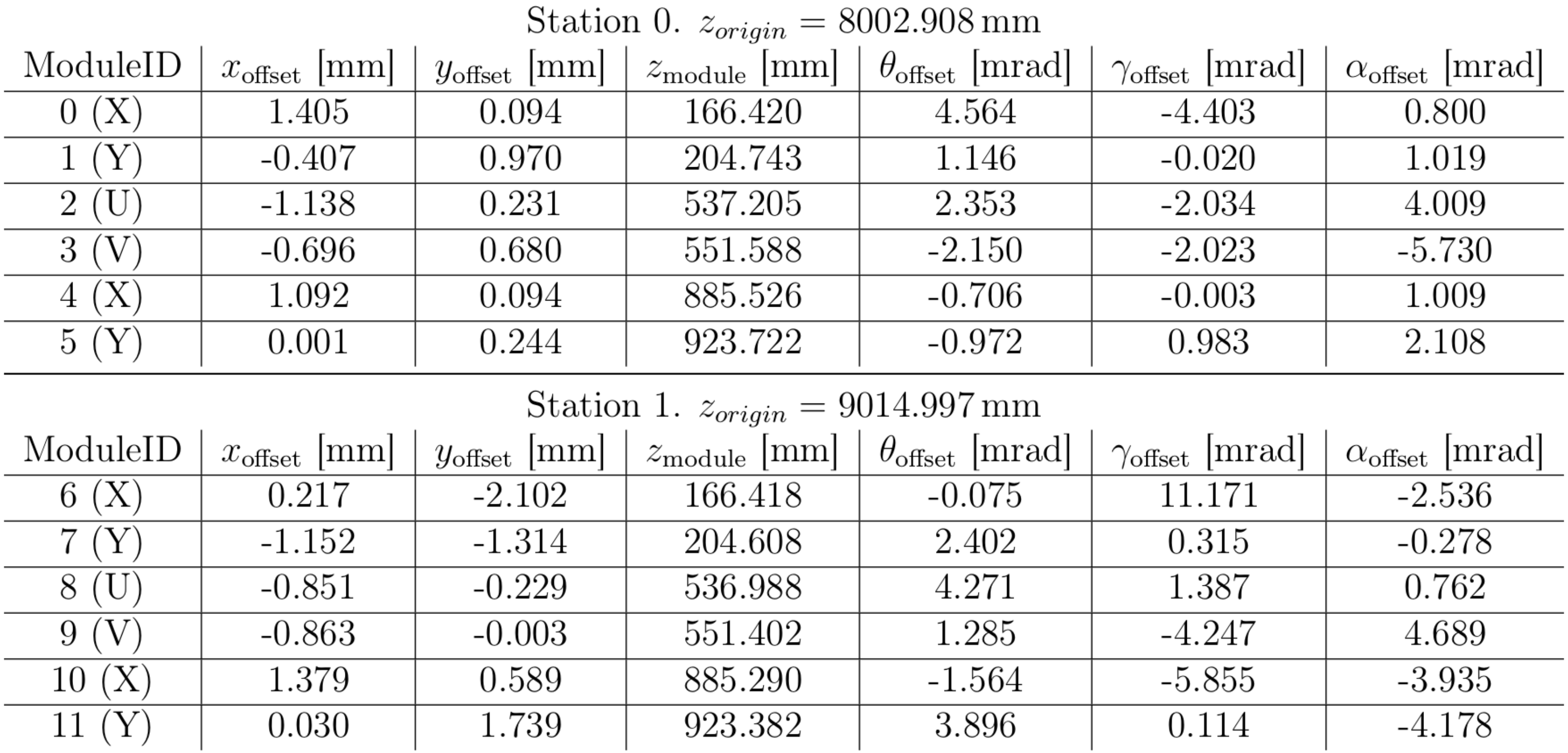
Outcome: modules position and orientation can be used as starting point of the software alignment

Estimated precision on these parameters:

More details in backup

$\Delta x = 200\ \mu m$ $\Delta y = 200\ \mu m$ $\Delta z = 100\ \mu m$

$\Delta \alpha = 1.8$ mrad $\Delta \theta = 1.0$ mrad $\Delta \gamma = 1.0$ mrad



Station 0. $z_{origin} = 8002.908$ mm

ModuleID	x_{offset} [mm]	y_{offset} [mm]	z_{module} [mm]	θ_{offset} [mrad]	γ_{offset} [mrad]	α_{offset} [mrad]
0 (X)	1.405	0.094	166.420	4.564	-4.403	0.800
1 (Y)	-0.407	0.970	204.743	1.146	-0.020	1.019
2 (U)	-1.138	0.231	537.205	2.353	-2.034	4.009
3 (V)	-0.696	0.680	551.588	-2.150	-2.023	-5.730
4 (X)	1.092	0.094	885.526	-0.706	-0.003	1.009
5 (Y)	0.001	0.244	923.722	-0.972	0.983	2.108

Station 1. $z_{origin} = 9014.997$ mm

ModuleID	x_{offset} [mm]	y_{offset} [mm]	z_{module} [mm]	θ_{offset} [mrad]	γ_{offset} [mrad]	α_{offset} [mrad]
6 (X)	0.217	-2.102	166.418	-0.075	11.171	-2.536
7 (Y)	-1.152	-1.314	204.608	2.402	0.315	-0.278
8 (U)	-0.851	-0.229	536.988	4.271	1.387	0.762
9 (V)	-0.863	-0.003	551.402	1.285	-4.247	4.689
10 (X)	1.379	0.589	885.290	-1.564	-5.855	-3.935
11 (Y)	0.030	1.739	923.382	3.896	0.114	-4.178

Software alignment procedure

1) Select events with 1stub/module

2) Track reconstruction using all modules

3) Find the alignment corrections on each module separately:
numerical minimisation of the local χ^2
Start from the metrology and repeat iteratively until convergence

$$\chi^2_{\text{align, module-}j} = \sum_{i=0}^{\mathrm{N_{tracks}}} \left(\frac{x^{ij}_{\mathrm{Track}}(\vec{p}_j) - x^{ij}_{\mathrm{Hit}}}{\sigma^j_{\mathrm{Hit}}} \right)^2$$

p = alignment parameters

- XY modules: 4 parameters aligned: x,y_{offset}, α_{offset}, θ_{offset}
- UV modules: 3 parameters aligned: x,y_{offset}, α_{offset}

Final precision (studied by simulation): $\Delta x,y_{\mathrm{offset}} \sim 5\ \mu\mathrm{m}$ $\Delta\alpha,\theta_{\mathrm{offset}} \sim 2$ mrad

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- XY modules: 4 parameters aligned: x,y_{offset}, α_{offset}, θ_{offset}
- UV modules: 3 parameters aligned: x,y_{offset}, α_{offset}

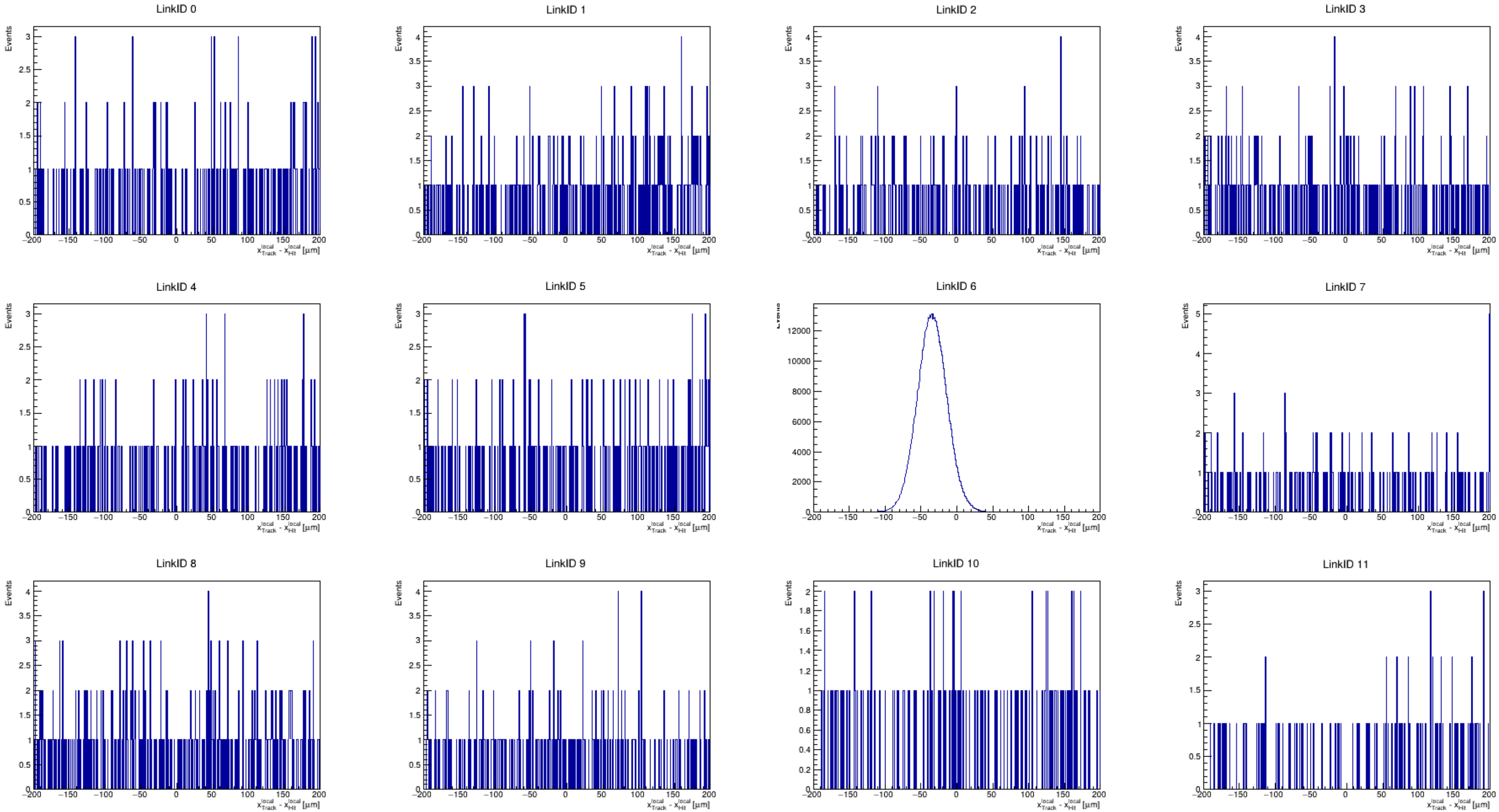
NB: some alignment parameters are not minimised in this procedure. Their value is fixed to the metrology outcome (γ, θ_{UV}, z)

Residuals before the alignment

ideal detector description (CAD)

Residuals $x_{track} - x_{Stub}$ for all the modules

No clear peaks within ±200 μm, except for module 6 → poor detector description

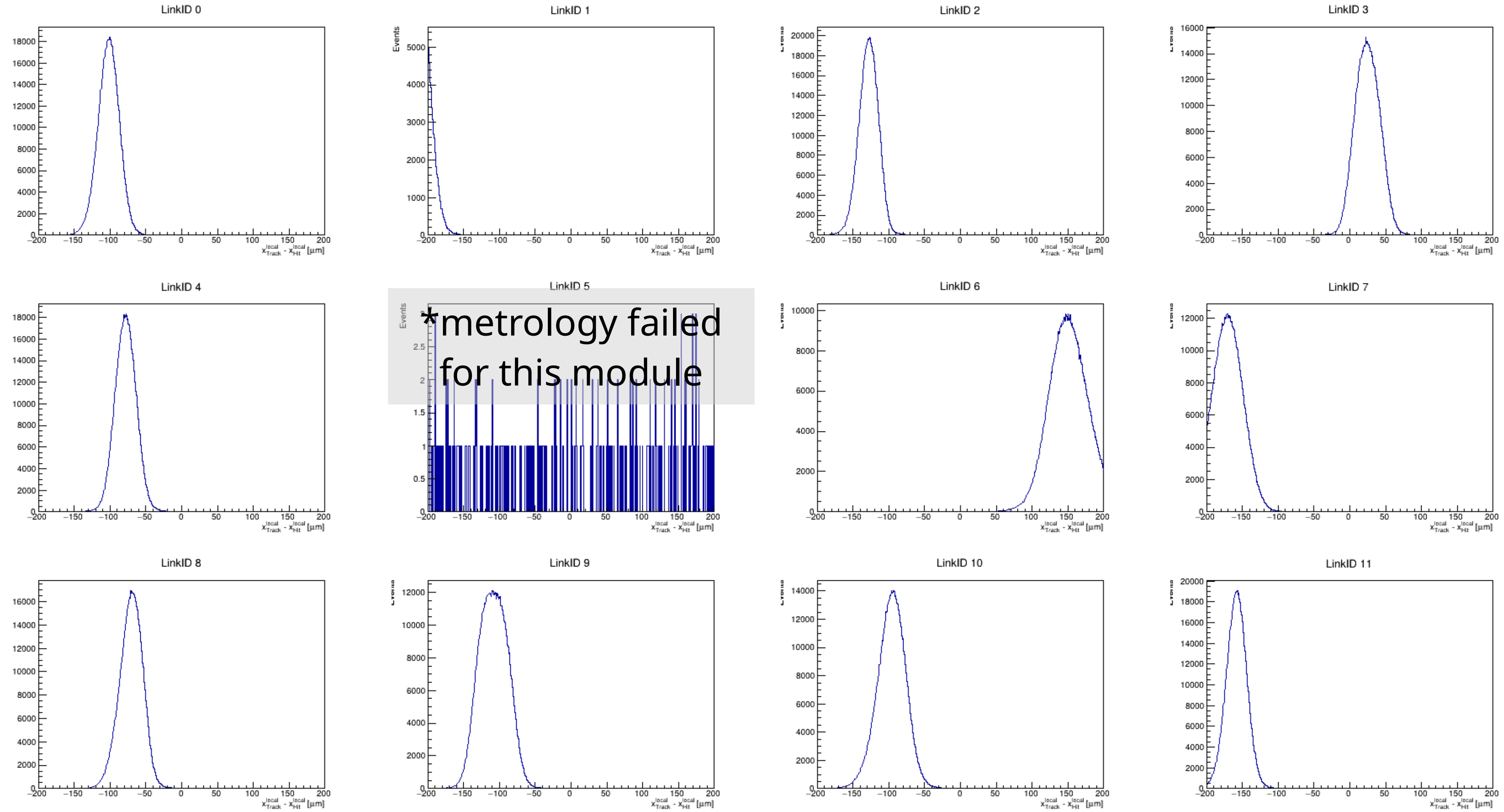


Residuals before the alignment

detector description from metrology measurements

Residuals $x_{track} - x_{Stub}$ for
all the modules

All residuals are centered
within ~200 μm

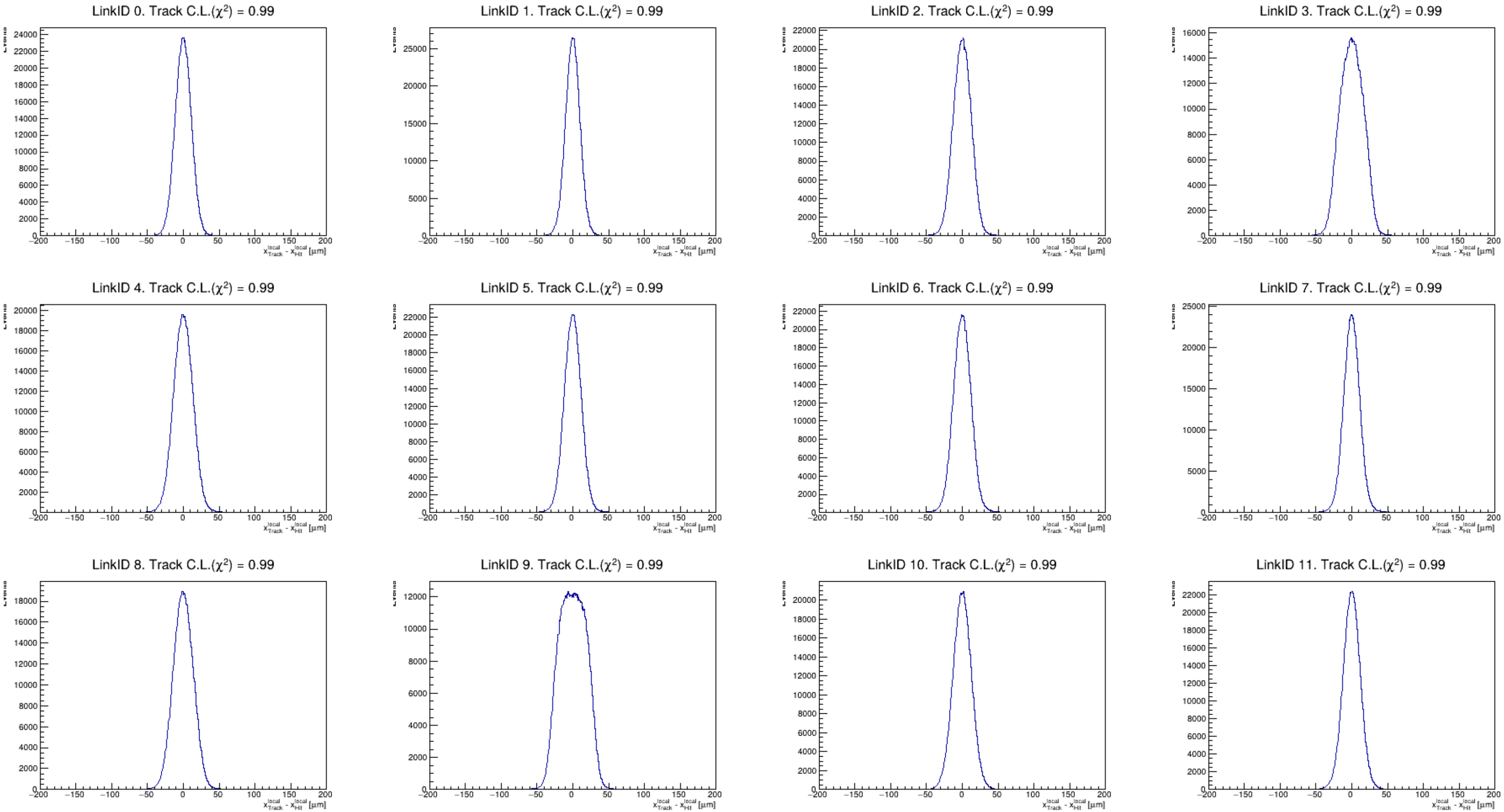


*metrology failed
for this module

Residuals after the alignment

Residuals $x_{track} – x_{Stub}$ for all the modules

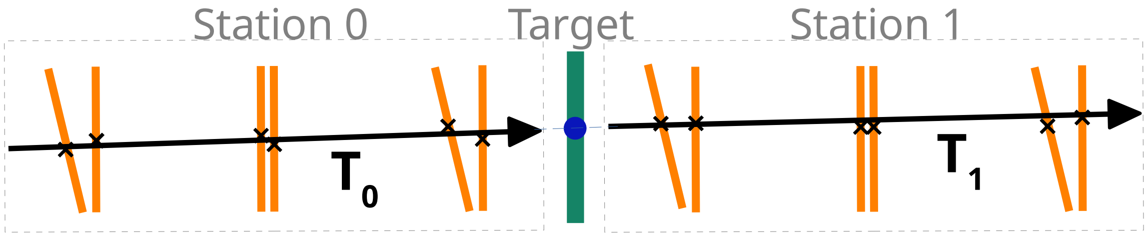
All residuals are well centered within 0.1 µm
Using the result of metrology measurements allows faster convergence towards parameters closer to the real value



Metrology measurements in the MUonE simulation

- MUonE: precise data/MonteCarlo comparison
- To be achieved, a precise simulation of the detector geometry is required
- The metrology measurements are used to simulate a realistic detector geometry, which is then aligned following the same procedure as the real data
- This has a huge impact in reproducing the angular resolution of the real detector

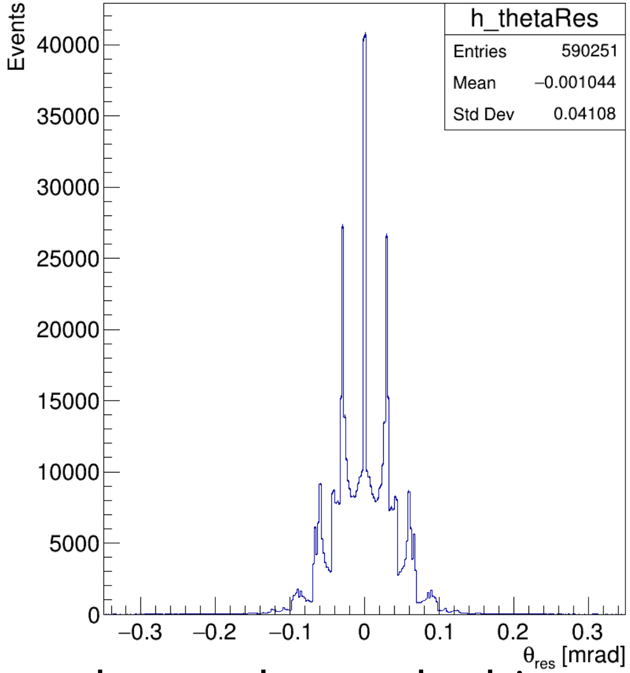
Angular resolution



Define the angular resolution as

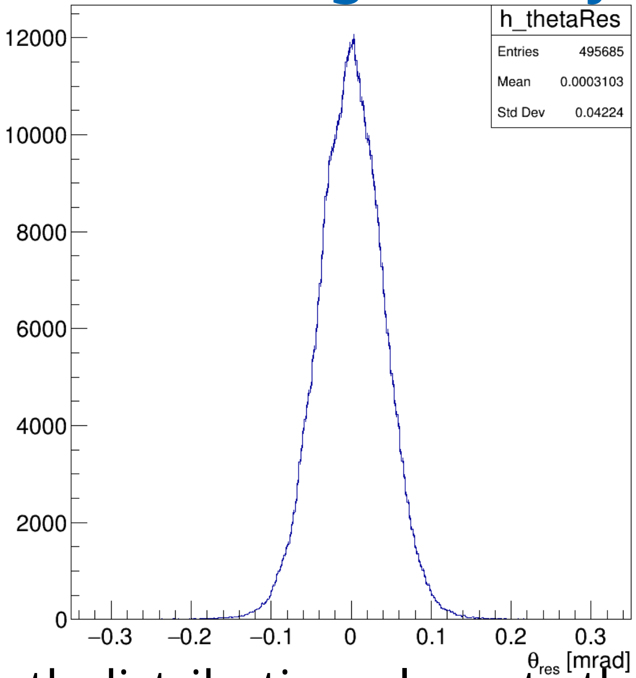
$$\theta_{res} = \theta_0 - \theta_1$$

Ideal geometry



These peaks are due to the binary strip readout of the 2S modules, which are visible in a perfectly aligned apparatus

Realistic geometry



Smooth distribution, closer to the data (the actual angular resolution value depends on the simulation of 2S modules electronics)

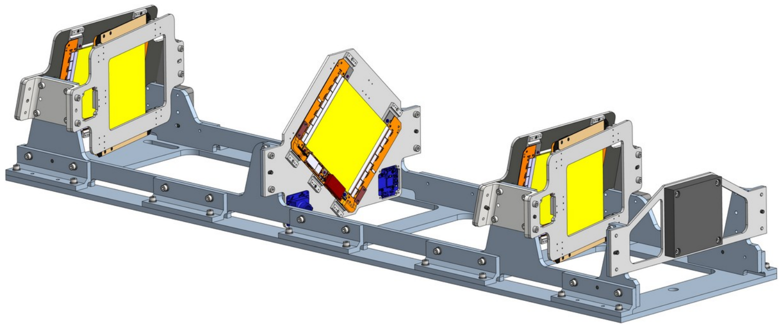
Metrology measurements in the MUonE simulation

Possible improvement

Current simulation

Only target and 2S modules are simulated.
The mechanical structure is missing
(i.e. objects are floating in air)

Improved simulation



Include the invar structure from the 3D scan → simulate muon interaction with the mechanical structre

Conclusions

- Metrology measurements are crucial for MUonE

- **Analysis of real data:**
  metrology is used as starting point of the software alignment.
  Current sw alignment precision: $\Delta x,y_{\text{offset}} \sim 5\ \mu\text{m}$ $\Delta\alpha,\theta_{\text{offset}} \sim 2$ mrad

  Hopefully this can improve in 2025 given
  the new metrology procedure (3D scan).

- **MUonE simulation:**
  metrology is used to simulate a detector which has realistic misalignments

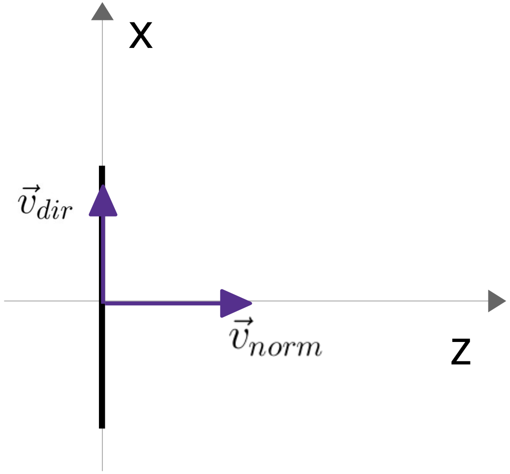
  Possible improvement in 2025: include the invar 3D scan in the simulation
  (and possibly also the target 3D scan)

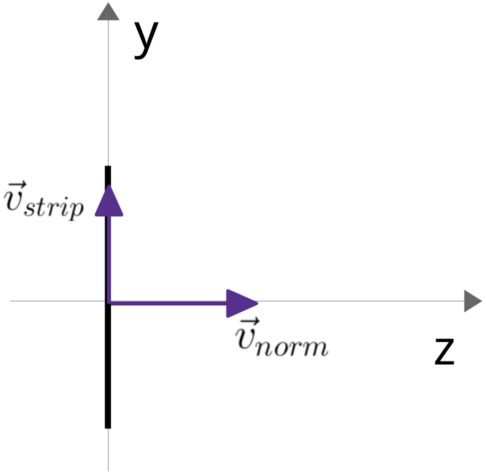
Backup

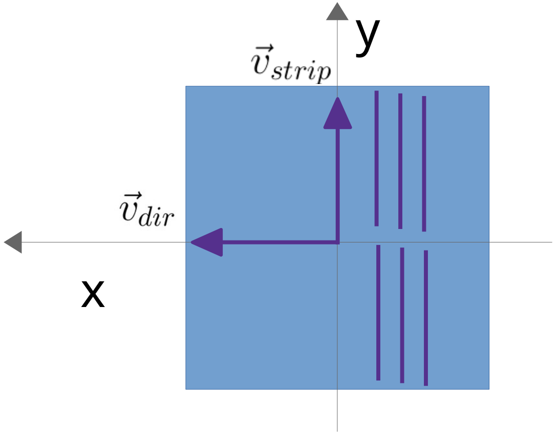
2S modules orientation – calculations

Start with an X module perpendicular to the beam axis

$$\vec{v}_{dir} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \vec{v}_{strip} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \vec{v}_{norm} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$







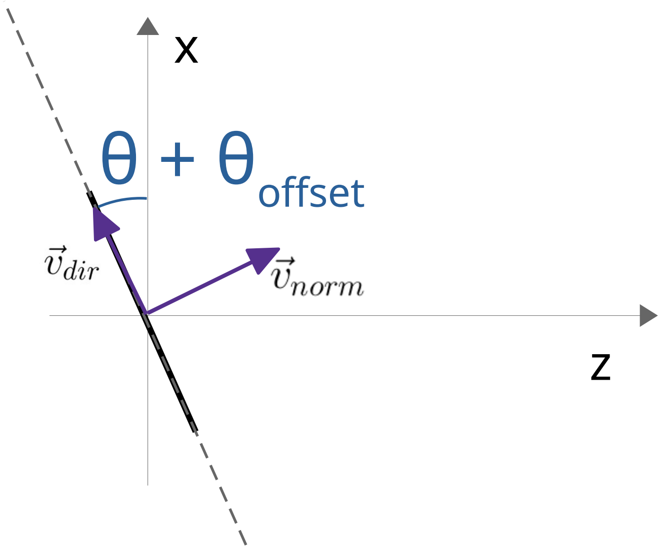
2S modules orientation – calculations

1) Apply the tilt

$$\vec{v}_{dir} = R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{v}_{strip} = R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{v}_{norm} = R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



$\tilde{\alpha} = \alpha + \alpha_{\text{offset}}$ $\quad$ $\tilde{\gamma} = \gamma + \gamma_{\text{offset}}$ $\quad$ $\tilde{\theta} = \theta + \theta_{\text{offset}}$

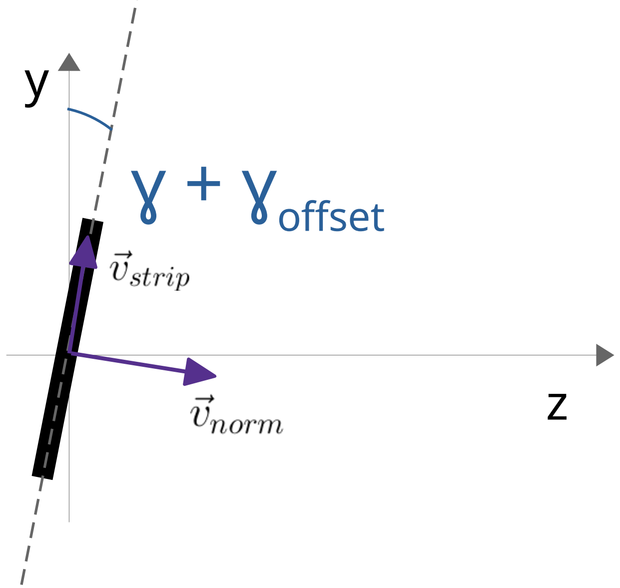
2S modules orientation – calculations

2) Apply the γ rotation

$$\vec{v}_{dir} = R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{v}_{strip} = R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

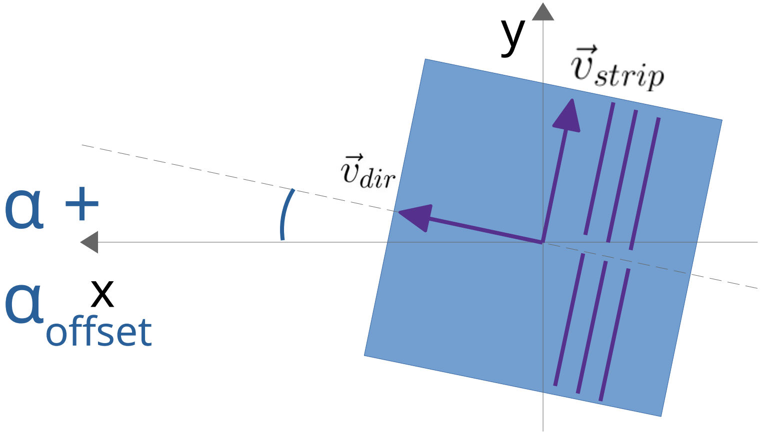
$$\vec{v}_{norm} = R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



$$\tilde{\alpha} = \alpha + \alpha_{\text{offset}} \qquad \tilde{\gamma} = \gamma + \gamma_{\text{offset}} \qquad \tilde{\theta} = \theta + \theta_{\text{offset}}$$

2S modules orientation – calculations

3) Rotate to adjust the strip orientation



$$\vec{v}_{dir} = R_Z(\tilde{\alpha}) \cdot R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{v}_{strip} = R_Z(\tilde{\alpha}) \cdot R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\tilde{\alpha} = \alpha + \alpha_{\text{offset}} \qquad \tilde{\gamma} = \gamma + \gamma_{\text{offset}} \qquad \tilde{\theta} = \theta + \theta_{\text{offset}}$$

$$\vec{v}_{norm} = R_Z(\tilde{\alpha}) \cdot R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

2S modules orientation – calculations

$$\vec{v}_{dir} = R_Z(\tilde{\alpha}) \cdot R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\tilde{\alpha}\cos\tilde{\theta} - \sin\tilde{\alpha}\sin\tilde{\gamma}\sin\tilde{\theta} \\ \sin\tilde{\alpha}\cos\tilde{\theta} + \cos\tilde{\alpha}\sin\tilde{\gamma}\sin\tilde{\theta} \\ \boxed{-\cos\tilde{\gamma}\sin\tilde{\theta}} \end{pmatrix}$$

$$\vec{v}_{strip} = R_Z(\tilde{\alpha}) \cdot R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \left(\boxed{\begin{matrix} -\sin\tilde{\alpha}\cos\tilde{\gamma} \\ \cos\tilde{\alpha}\cos\tilde{\gamma} \\ \sin\tilde{\gamma} \end{matrix}}\right)$$

$$\vec{v}_{norm} = R_Z(\tilde{\alpha}) \cdot R_X(\tilde{\gamma}) \cdot R_Y(\tilde{\theta}) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\tilde{\alpha}\sin\tilde{\theta} + \sin\tilde{\alpha}\sin\tilde{\gamma}\cos\tilde{\theta} \\ \sin\tilde{\alpha}\sin\tilde{\theta} - \cos\tilde{\alpha}\sin\tilde{\gamma}\cos\tilde{\theta} \\ \boxed{\cos\tilde{\gamma}\cos\tilde{\theta}} \end{pmatrix}$$

$$\tilde{\alpha} = \arctan\left(-\frac{v_{strip,X}}{v_{strip,Y}}\right) \qquad \tilde{\gamma} = \arcsin\left(v_{strip,Z}\right) \qquad \tilde{\theta} = \arctan\left(-\frac{v_{dir,Z}}{v_{norm,Z}}\right)$$

2023 survey – determine the errors on modules position and orientation

a) 2S modules position

Carlo's note on metrology

Carlo's slides on metrology 5th collab meeting

Determined from the position of fiducial markers on the frames + optical reference markers on the 2S modules.

3 sets of measurements:
1) laser survey on the beamline
2) metrology in lab with frames mounted ON the station
3) metrology in lab with frames OFF the station

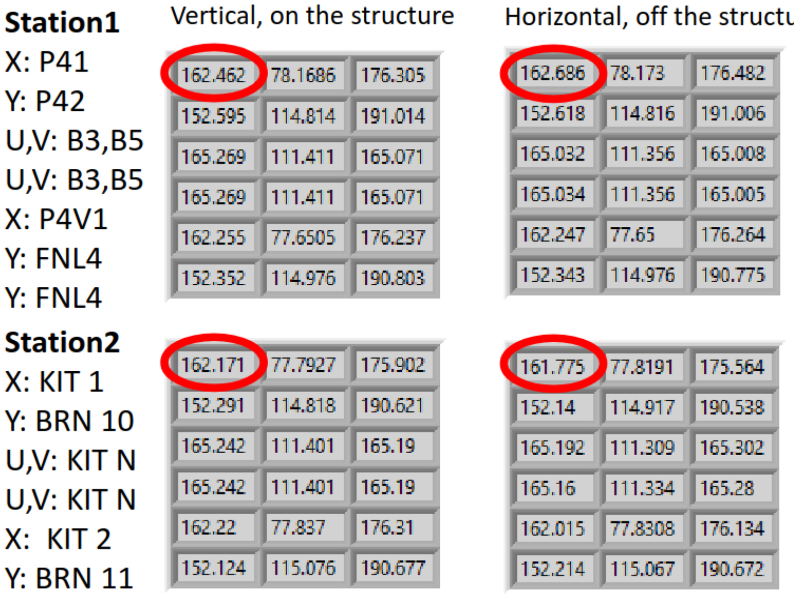
Found differences
up to 400 µm in the measurements
ON and OFF the station

Conservative estimate:

Δx,y = 200 µm

Δz = 100 µm

(see Carlo's presentation)



Station1	Vertical, on the structure			Horizontal, off the structure		
X: P41	162.462	78.1686	176.305	162.686	78.173	176.482
Y: P42	152.595	114.814	191.014	152.618	114.816	191.006
U,V: B3,B5	165.269	111.411	165.071	165.032	111.356	165.008
U,V: B3,B5	165.269	111.411	165.071	165.034	111.356	165.005
X: P4V1	162.255	77.6505	176.237	162.247	77.65	176.264
Y: FNL4	152.352	114.976	190.803	152.343	114.976	190.775
Y: FNL4						
Station2						
X: KIT 1	162.171	77.7927	175.902	161.775	77.8191	175.564
Y: BRN 10	152.291	114.818	190.621	152.14	114.917	190.538
U,V: KIT N	165.242	111.401	165.19	165.192	111.309	165.302
U,V: KIT N	165.242	111.401	165.19	165.16	111.334	165.28
X: KIT 2	162.22	77.837	176.31	162.015	77.8308	176.134
Y: BRN 11	152.124	115.076	190.677	152.214	115.067	190.672

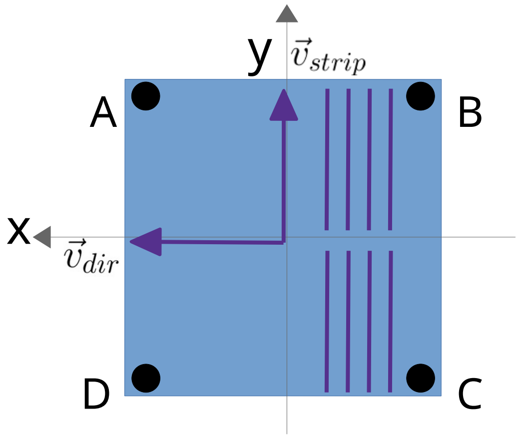
Δ ~ 200 µm

Δ ~ 400 µm

2023 survey – determine the errors on modules position and orientation

b) 2S modules orientation

A, B, C, D: optical markers
on the 2S modules



$$\vec{v}_{dir} = \vec{A} - \vec{B} = \vec{D} - \vec{C}$$
$$\vec{v}_{strip} = \vec{A} - \vec{D} = \vec{B} - \vec{C}$$

1) Δv = 200 μm
(see previous slide)

2) These vectors are normalized, then the modules orientation is parameterized using the angles:

$$\tilde{\alpha} = \arctan\left(-\frac{v_{strip,X}}{v_{strip,Y}}\right)$$

$$\tilde{\gamma} = \arcsin\left(v_{strip,Z}\right)$$

$$\tilde{\theta} = \arctan\left(-\frac{v_{dir,Z}}{v_{norm,Z}}\right)$$

3) Errors on α, θ, γ are then obtained by error propagation

$\Delta\alpha$ = 1.8 mrad

$\Delta\theta$ = 1.0 mrad

$\Delta\gamma$ = 1.0 mrad