



FLASH Radiotherapy with high
Dose-rate particle beams

UHDR Intertrack effects analysis: a new method

Lorenzo Castelli



Istituto Nazionale
di Fisica Nucleare

TIFPA

Trento
Institute for
Fundamental
Physics and
Applications



What, why, and How

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(Why) Describe the interaction between tracks a.k.a. **intertrack**

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(How):

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2. Computation of impact (here we focus on heterogeneous chemistry)

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In Practice:

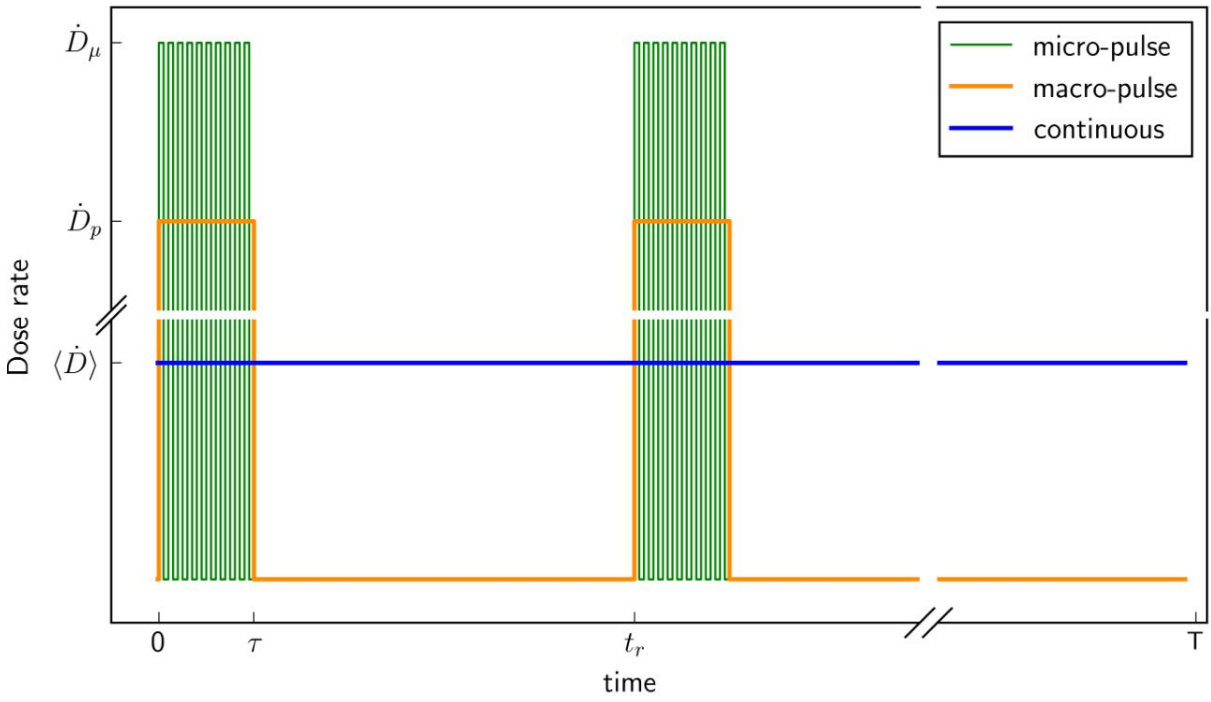
1. Model track arrival as Poisson Random Measure (PRM)
2. Evaluate chemical impact with Monte Carlo (TRAX-CHEM)

From Beam to PRM: Simplified Beam and conditions

Irradiation condition:

- **Dose:** How many track hit the target
- **Dose Rate:** time distribution

$$D = \frac{N_t LET}{\rho S}$$

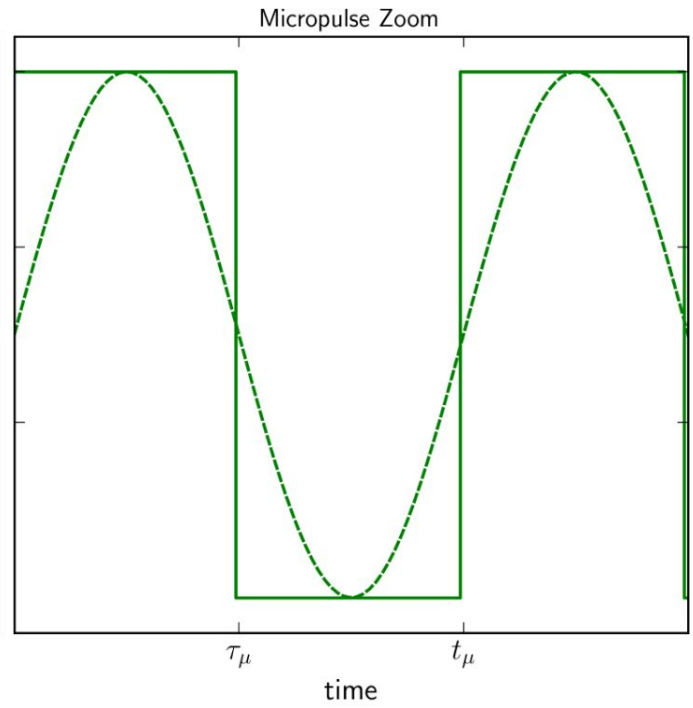
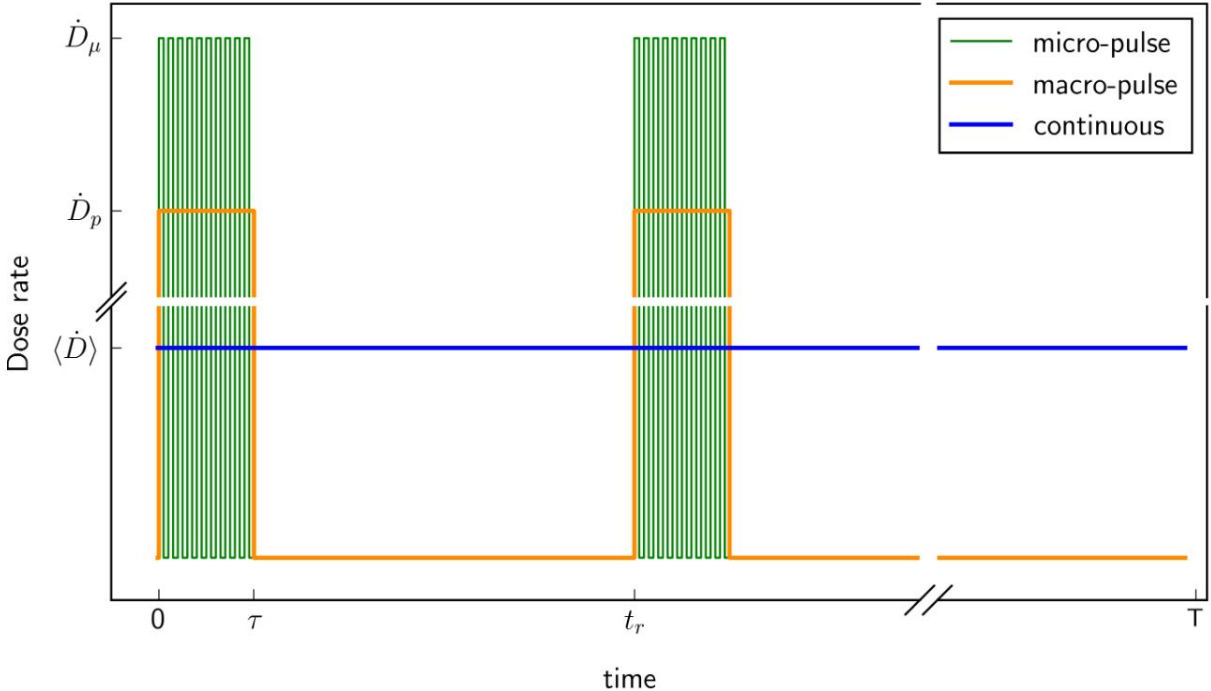


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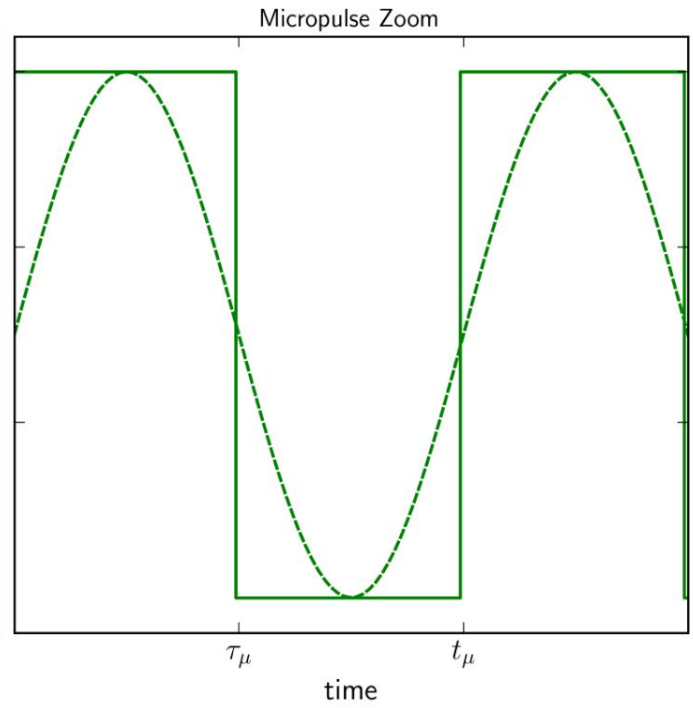
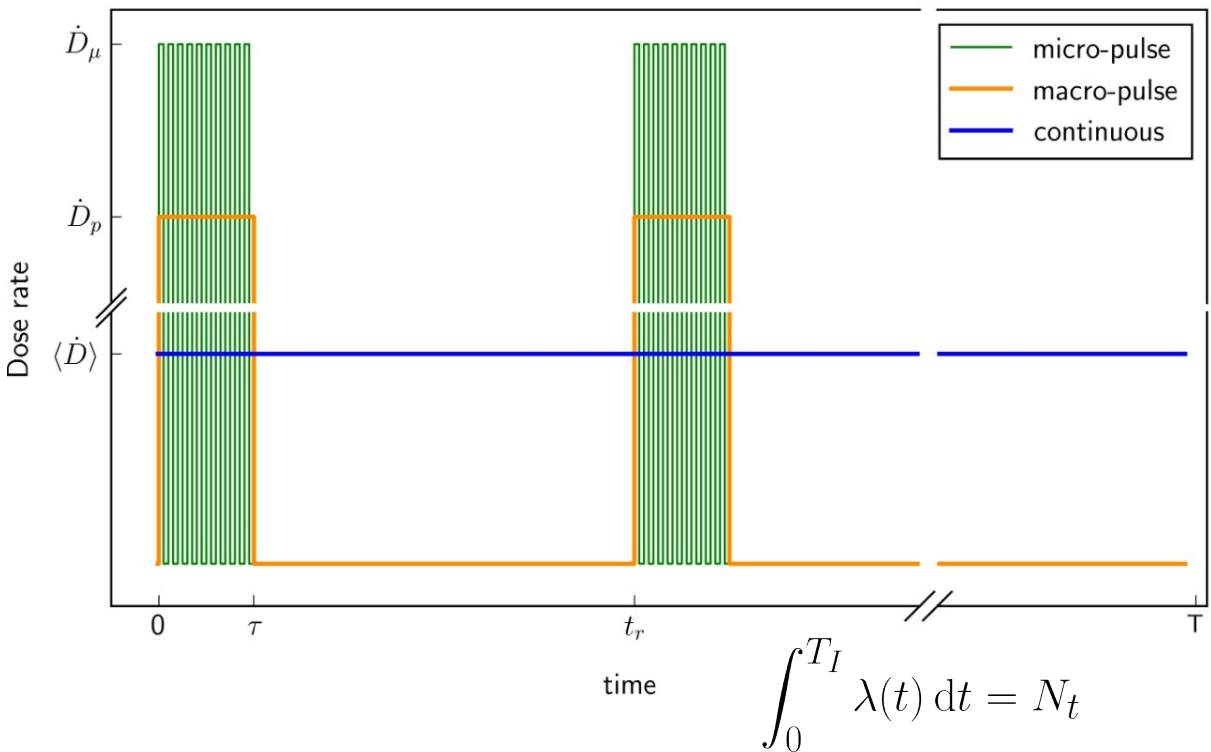


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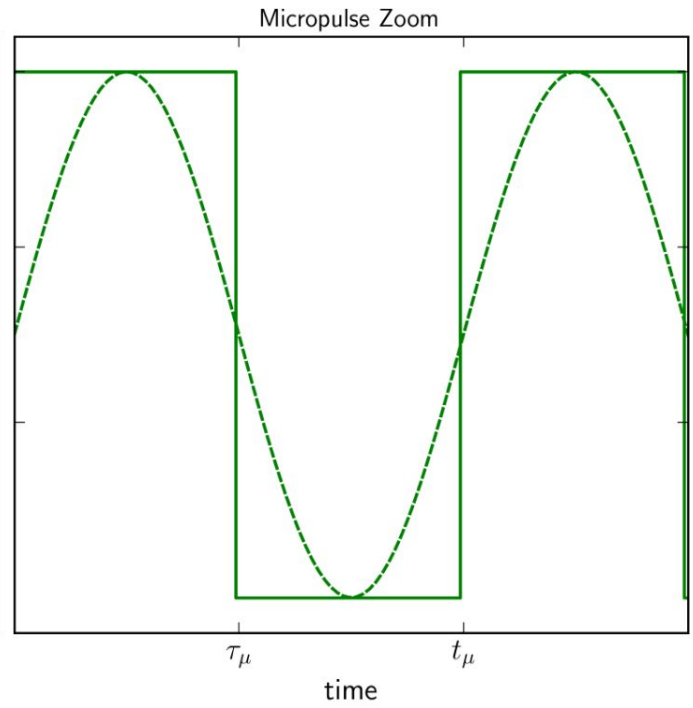
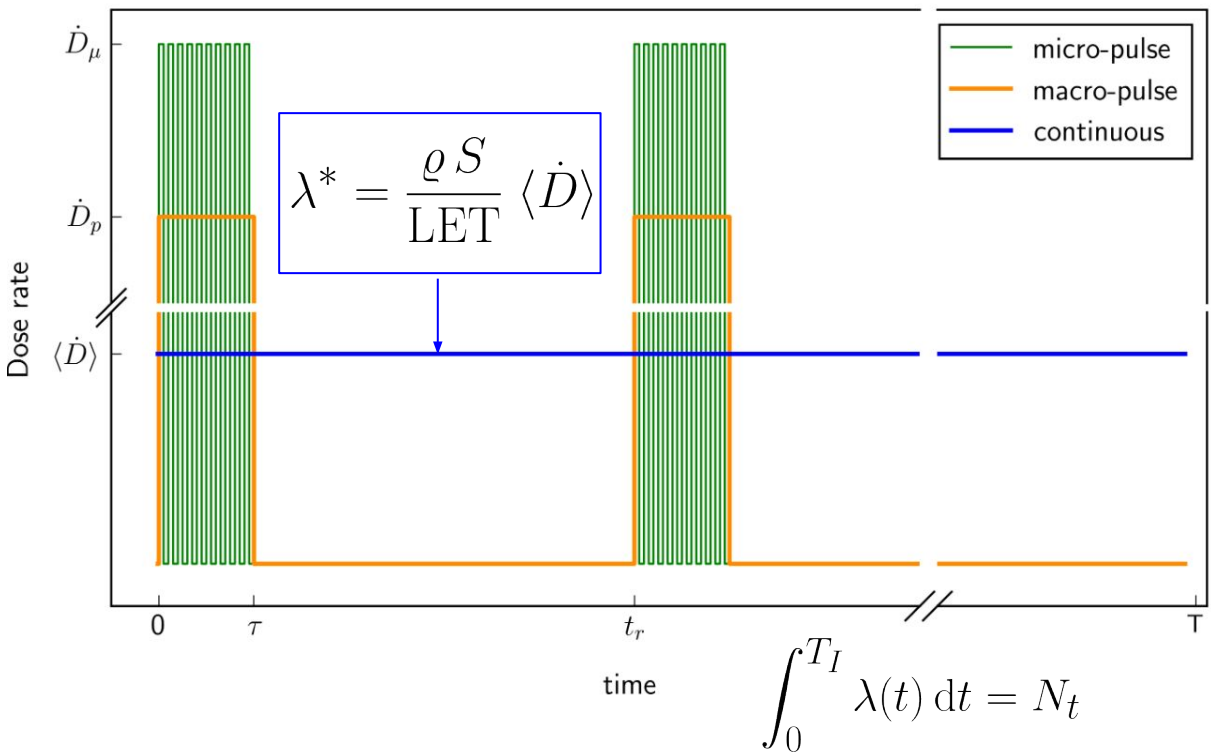


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Assumptions :

- Spatial uniform on B_R (surface S)
- Independent events (Poisson in time)

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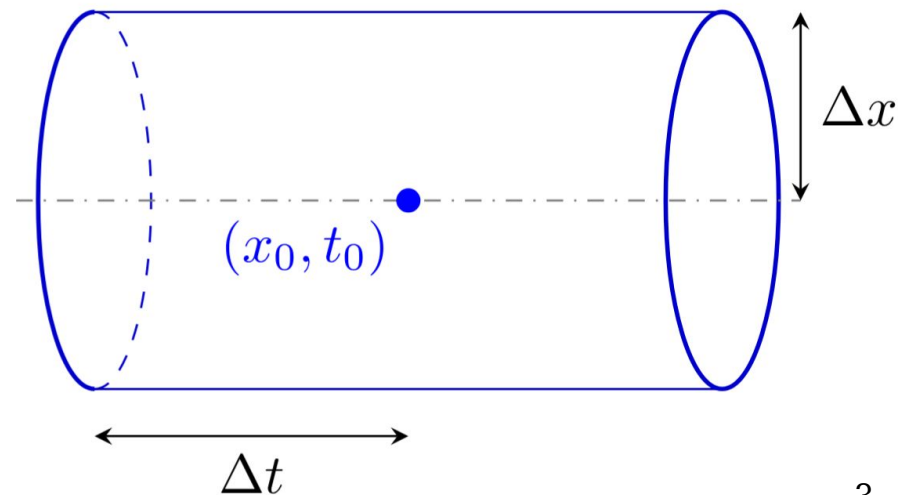
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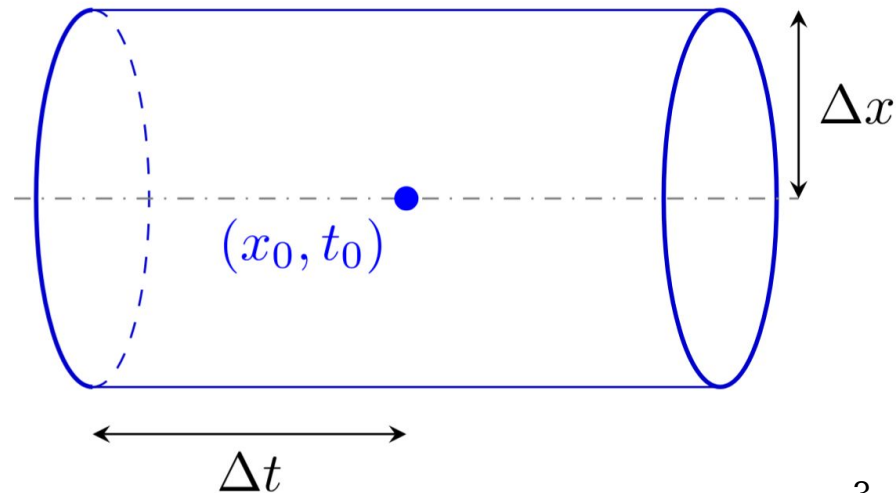
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$$\left\{ \begin{array}{l} \mu(t) := \nu(A_\Delta(x, t)) = \frac{\Delta x^2}{R^2} N_\Delta(t) \\ N_\Delta(t) := \int_0^{T_I} I_{\{|u-t| < \Delta_T\}} \lambda(u) du \\ \mathbb{P}(M(A_\Delta) = 0 \mid (x, t)) = \exp\{-\mu(t)\} \end{array} \right.$$

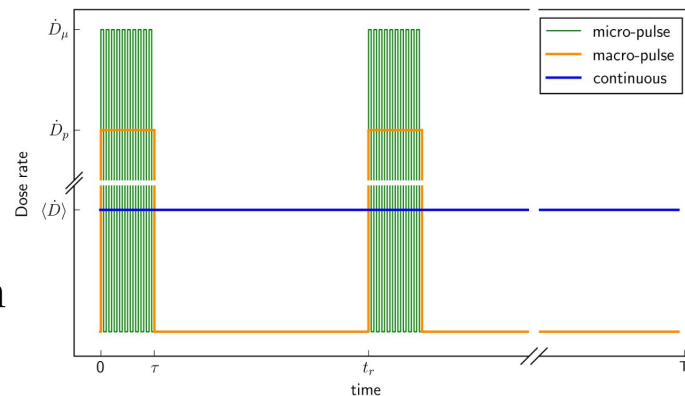


From Beam to PRM: final

Issues:

- Account for different beam structures
- Compute the quantities averages

$$\lambda(t) = \begin{cases} \lambda^* = \langle \dot{D} \rangle \frac{\pi R^2 \rho}{LET}, & \text{continuous beam,} \\ \lambda_p K(t) = \frac{t_r}{\tau} \lambda^* K(t) & \text{macro-pulsed beam,} \\ \lambda_\mu K_\mu(t) = \frac{t_\mu t_r}{\tau_\mu \tau} \lambda^* K_\mu(t), & \text{micro-structured beam} \end{cases}$$



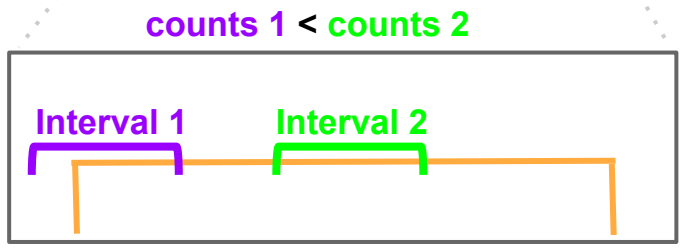
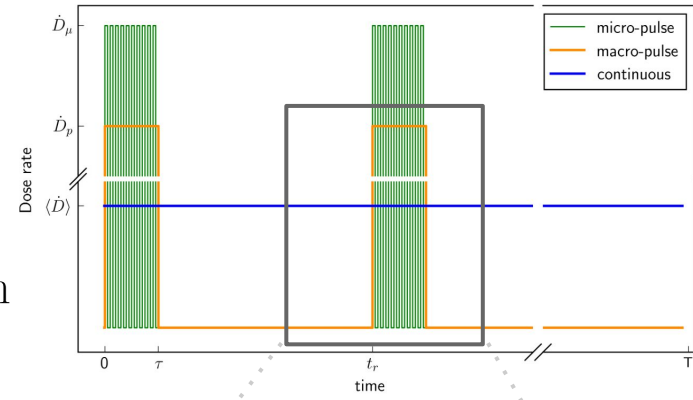
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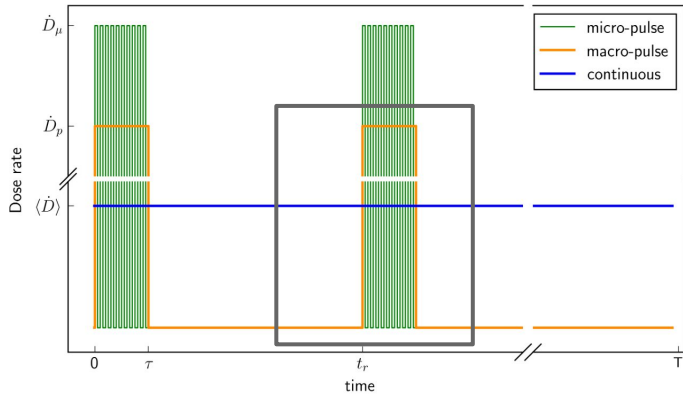


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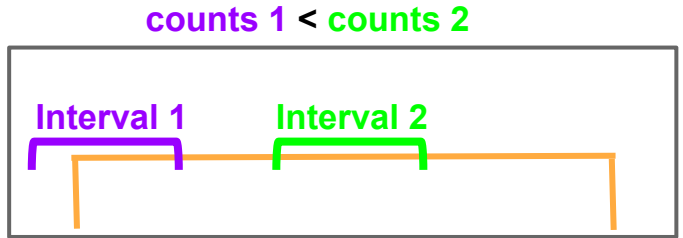
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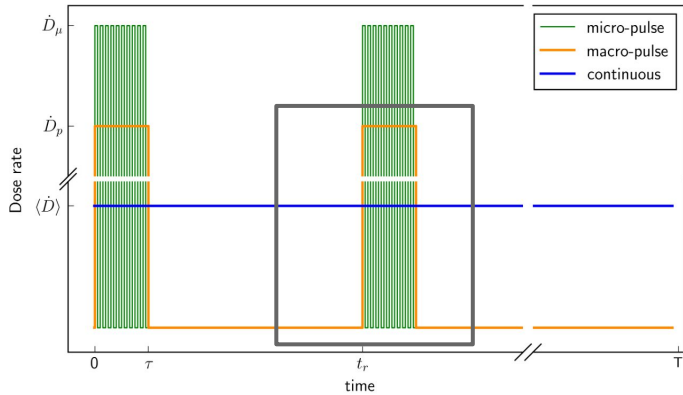


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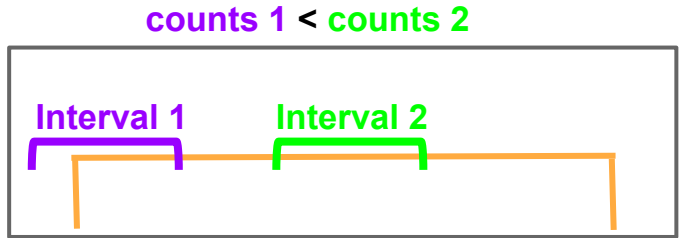
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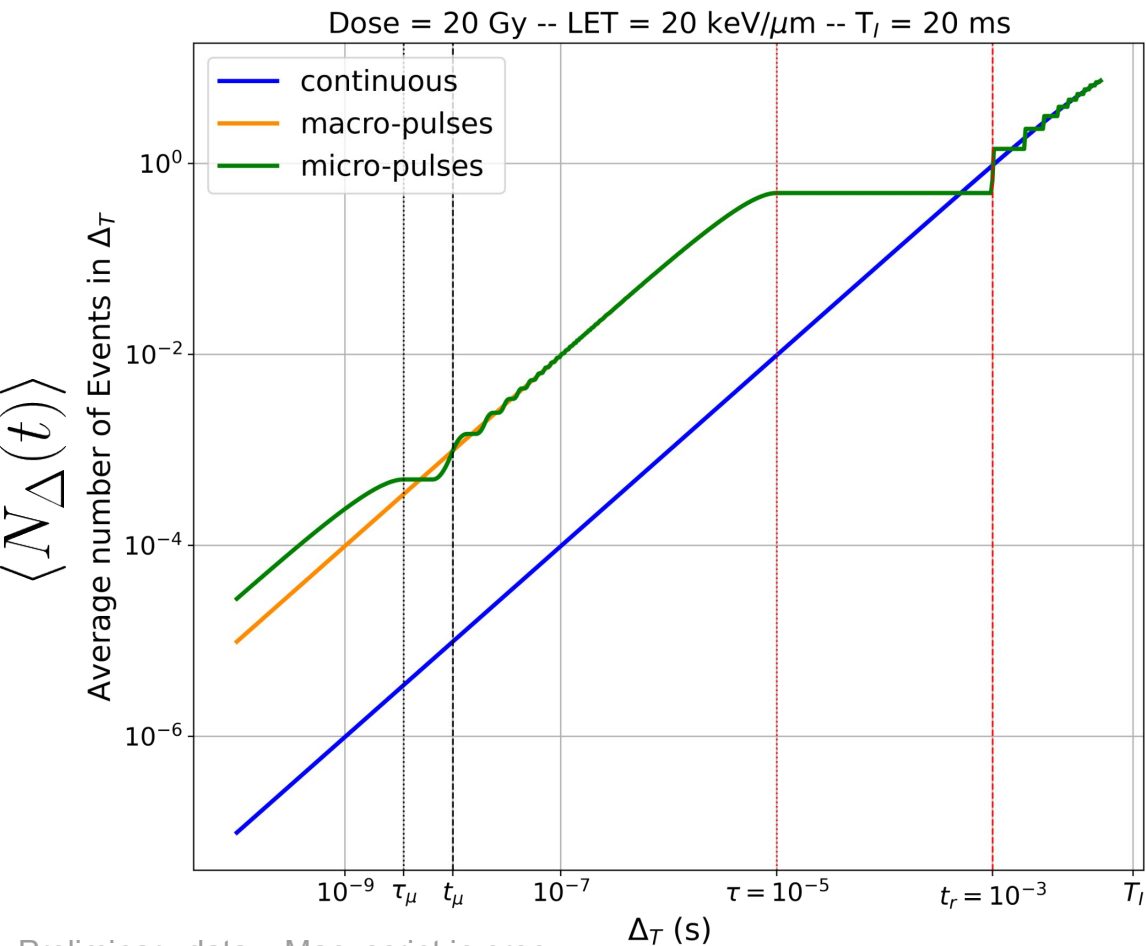


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$$\langle N_\Delta(t) \rangle = \begin{cases} \frac{\lambda^*}{T} (2\Delta_T T - \Delta_T^2), & \text{continuous beam,} \\ \left(\frac{t_r}{\tau}\right)^2 \frac{\lambda^*}{T} \mathbb{I}(\Delta_T) & \text{macro-pulsed beam,} \\ \left(\frac{t_\mu}{\tau_\mu}\right)^2 \left(\frac{t_r}{\tau}\right)^2 \frac{\lambda^*}{T} \mathbb{I}_\mu(\Delta_T) & \text{micro-pulsed beam} \end{cases}$$



Results: Beam parameters impact on $\langle N \rangle$



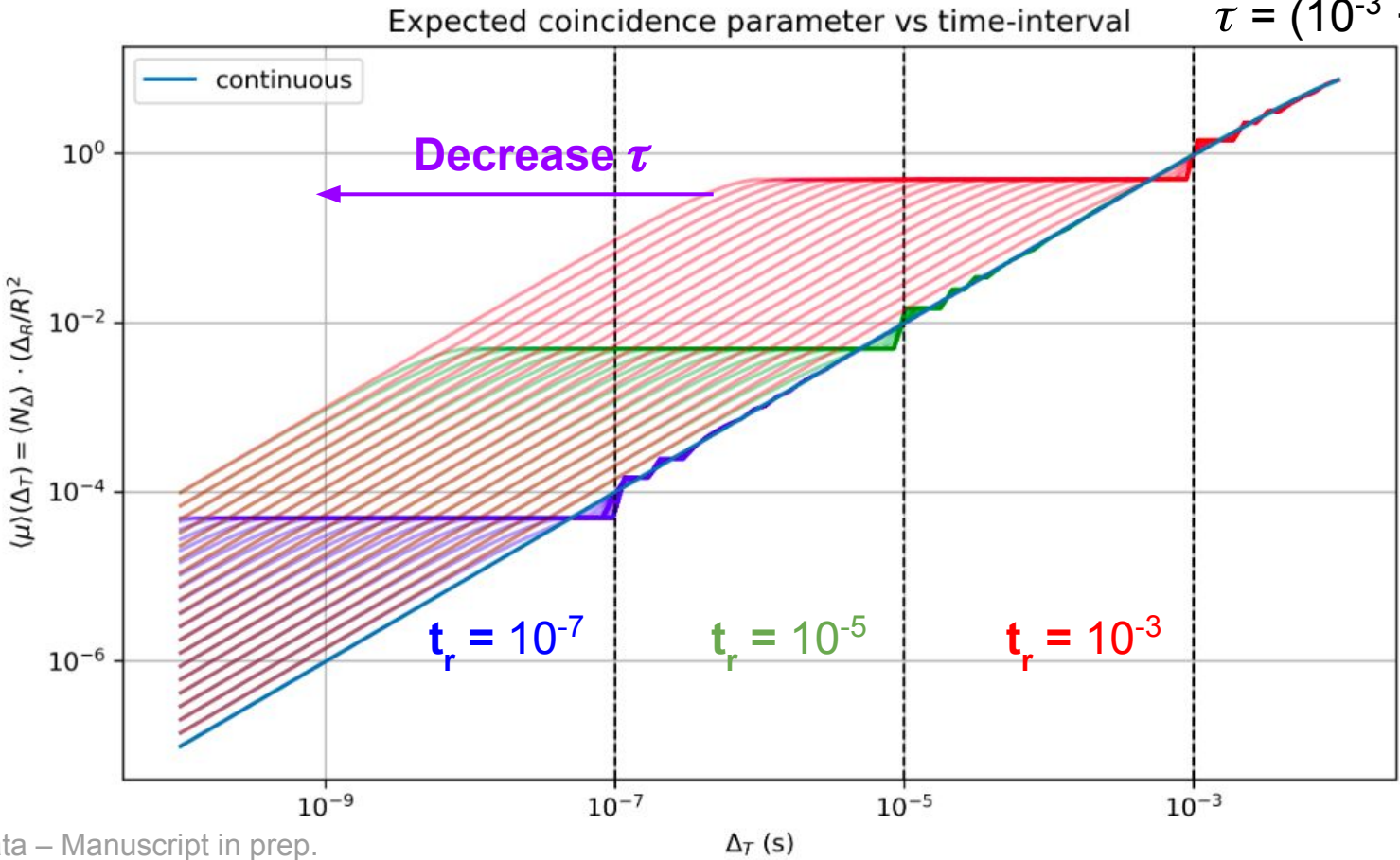
Differences:

- **Micropulse** structure "collapse" to **macropulse** after surpassing the repetition time (t_{μ})
- **Macropulse** does the same with **continuous** version after t_r
- Counting saturate and then rise with same slope as the continuous case

Results: Macropulse with different t_r and τ

- Same D , LET and T as before

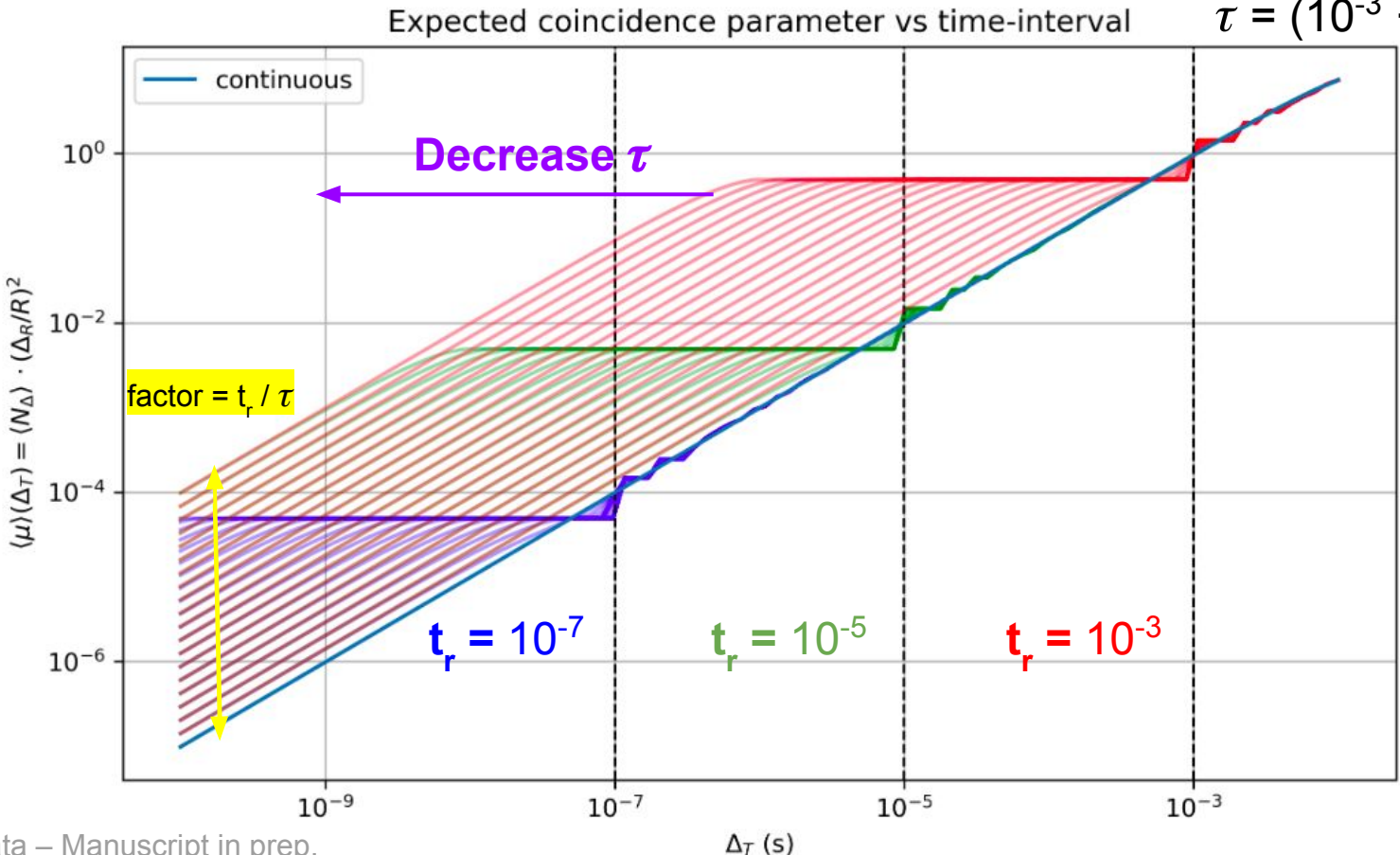
$$t_r = (10^{-7}, 10^{-5}, 10^{-3}) \text{ s}$$
$$\tau = (10^{-3} \cdot t_r, \dots, t_r) \text{ s}$$



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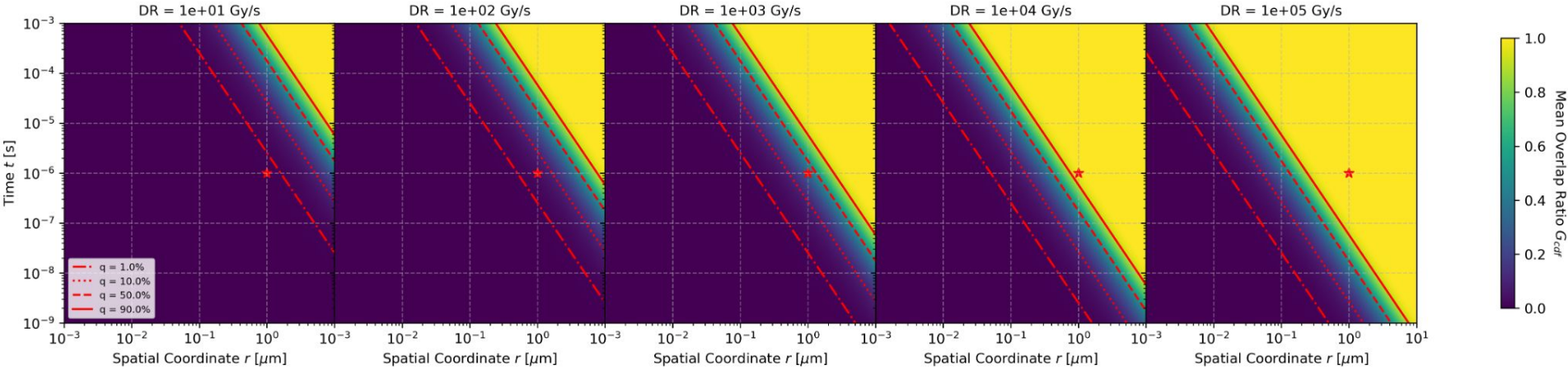
Results: Continuous Beam LET a impact

$\square = (\Delta x = \mu\text{m} ; \Delta t = \mu\text{s})$

$$\langle e^{-\mu} \rangle = \frac{(T - 2\Delta_T)e^{-2C\Delta_T}}{T} + \frac{2}{TC}e^{-2C\Delta_T}(1 - e^{-2C\Delta_T})$$

$$C = \lambda^* \frac{\Delta x^2}{R^2}$$

Mean Overlap Ratio for Varying DR (LET = 0.1 keV/ μm)



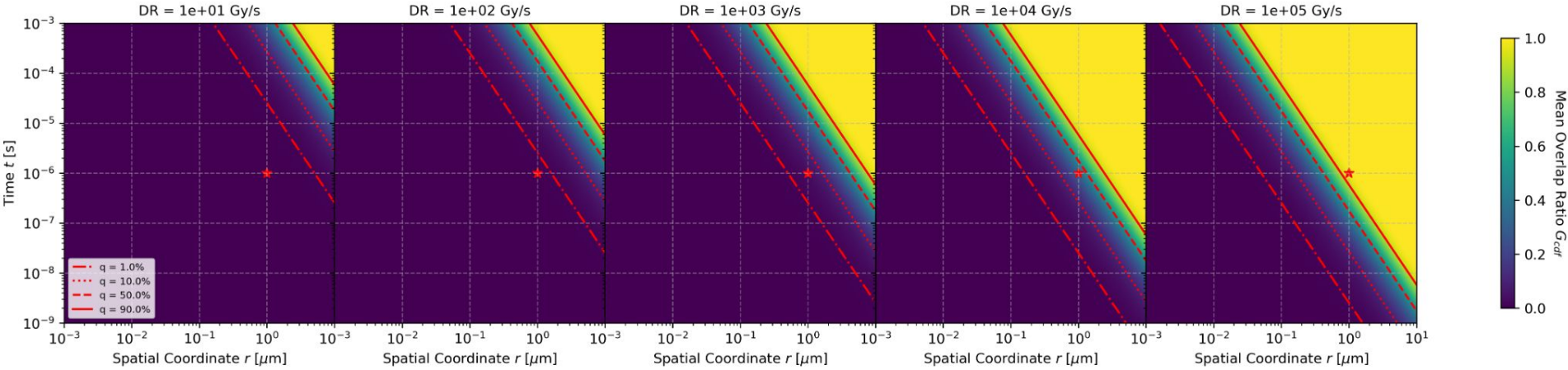
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Mean Overlap Ratio for Varying DR (LET = 1 keV/ μm)



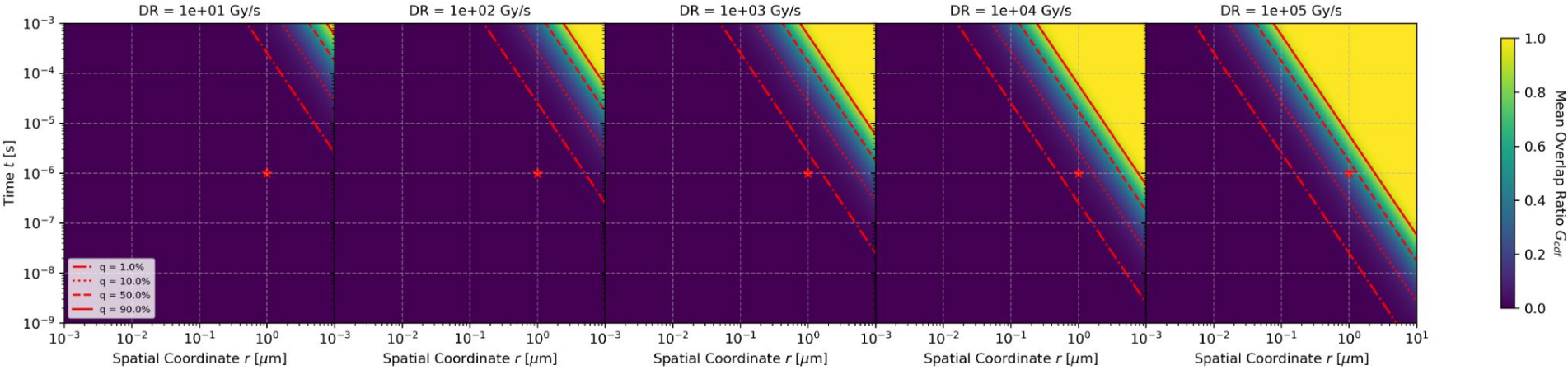
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Mean Overlap Ratio for Varying DR (LET = 10 keV/μm)



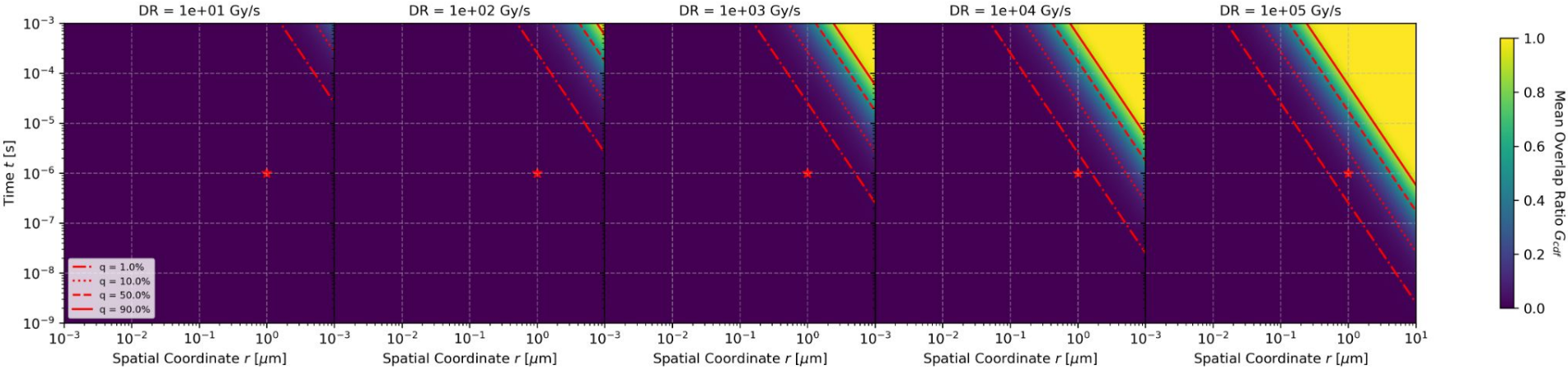
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Mean Overlap Ratio for Varying DR (LET = 100.0 keV/μm)



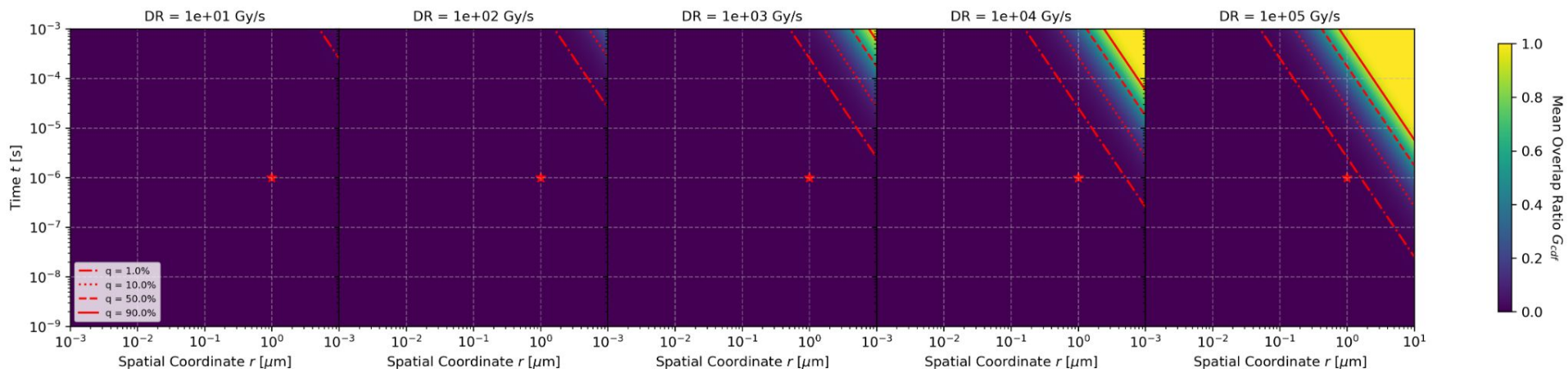
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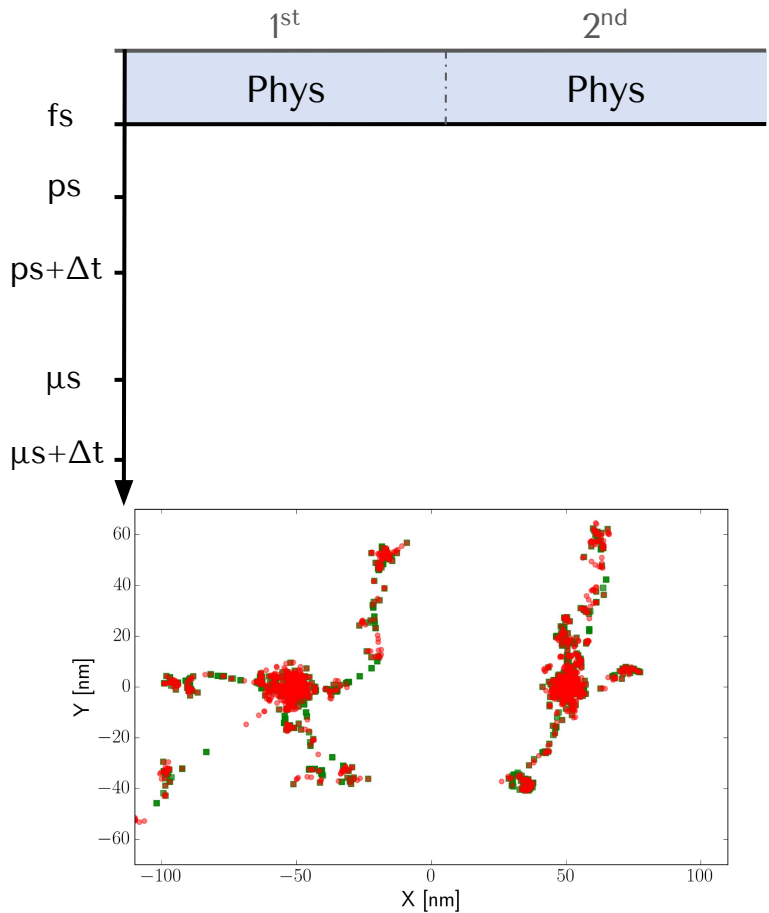
Mean Overlap Ratio for Varying DR (LET = 1000.0 keV/ μm)



- Increase LET \rightarrow decrease probability of intertrack
- Increase Dose rate \rightarrow increase probability of intertrack

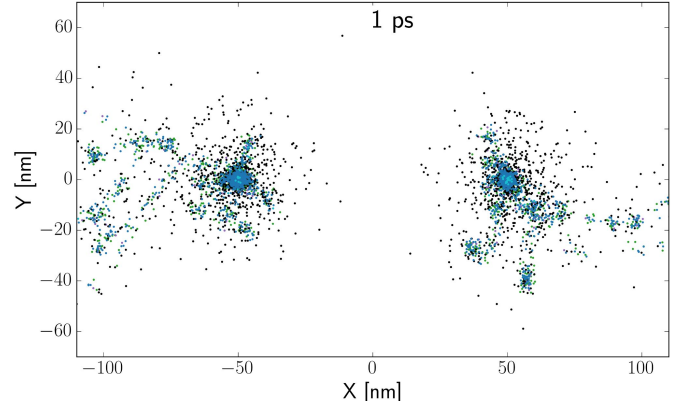
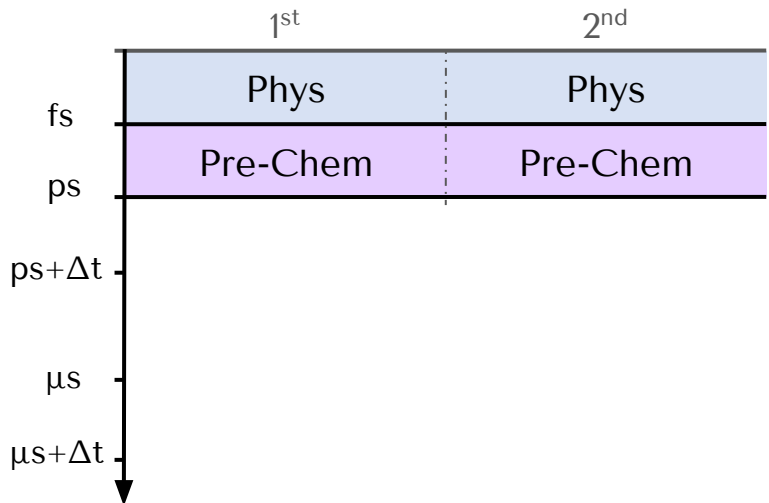
Monte Carlo Track Structure: Intertrack at Heterogeneous stage

INTERTRACK : 2 Proj. $\Delta x = 100$ nm ; $\Delta t = 1$ ns



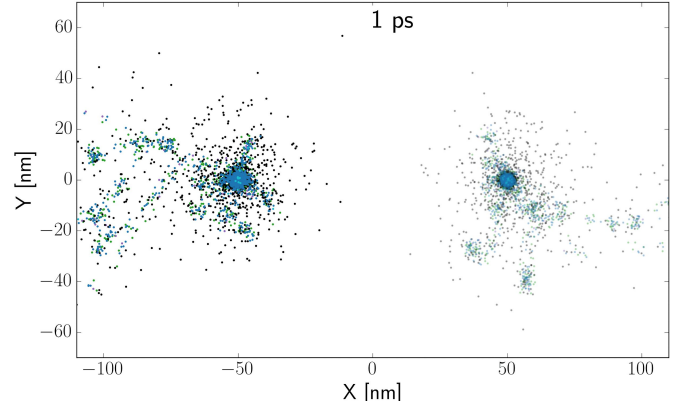
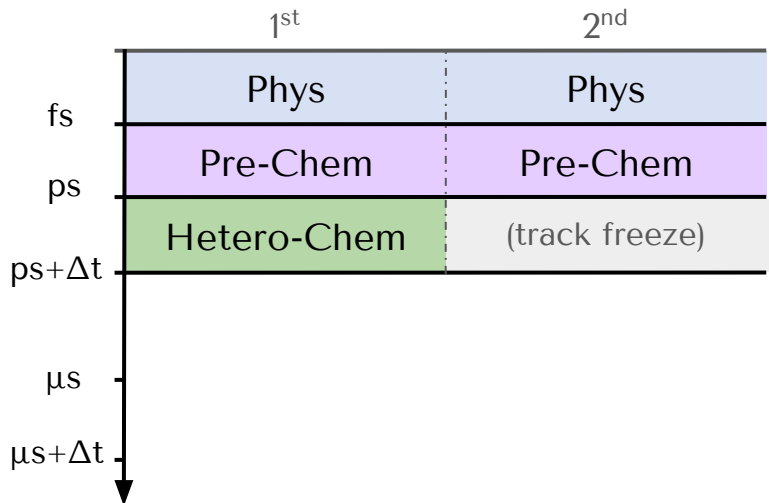
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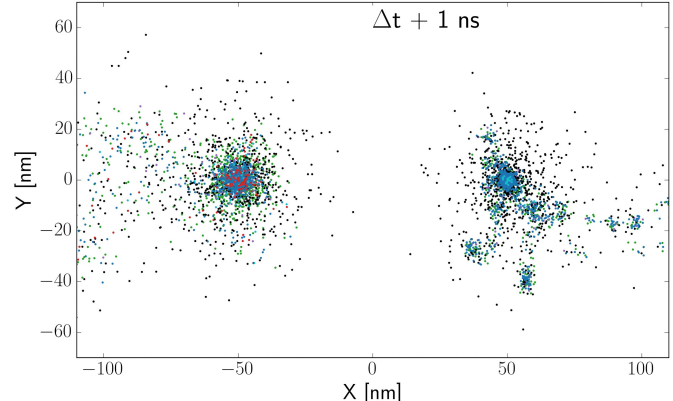
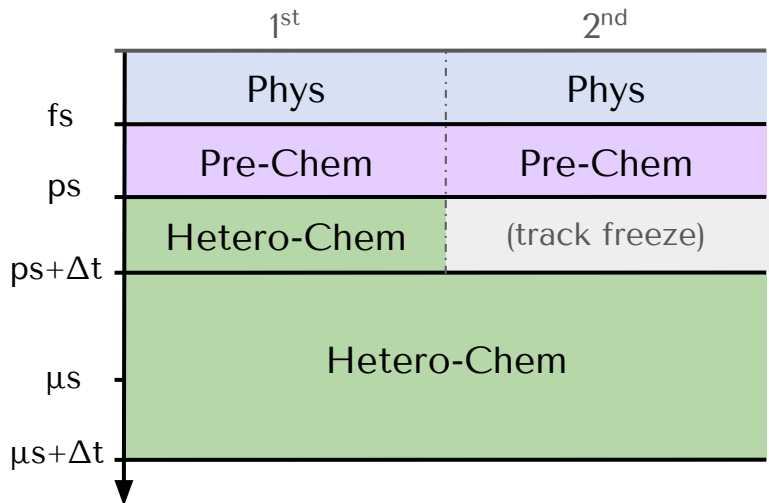
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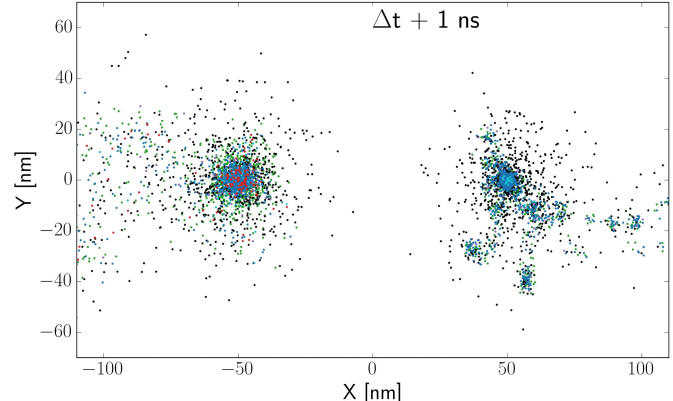
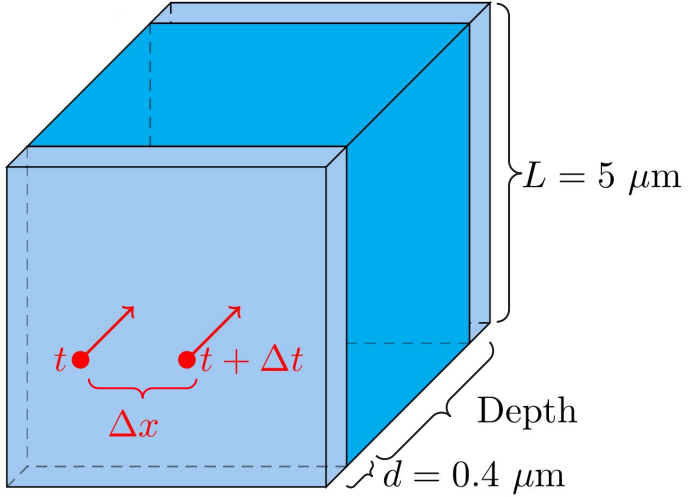
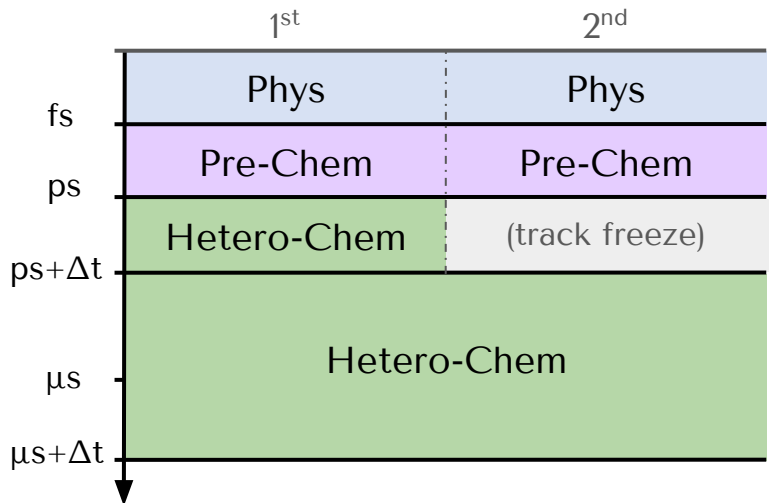
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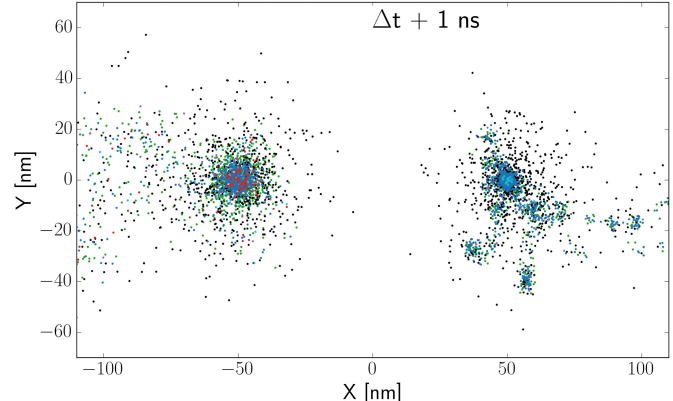
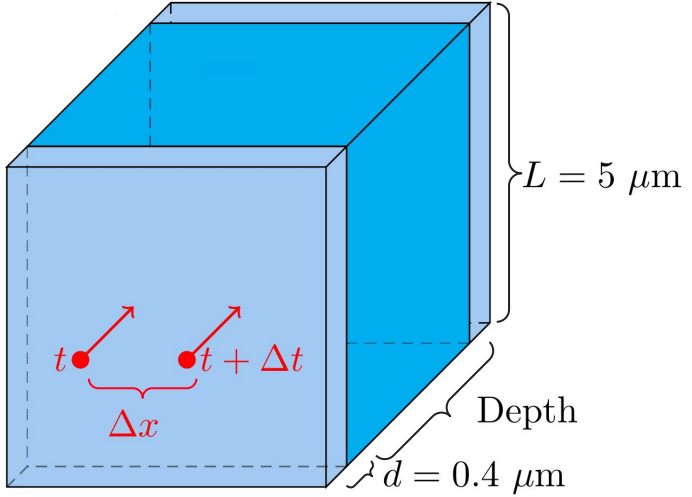
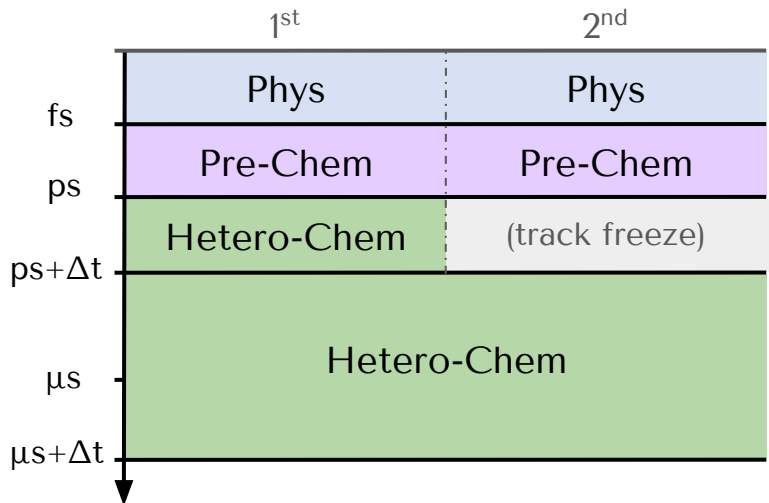


Projectile:

- Proton 20 MeV
- Helium 5 MeV/u
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- $\Delta t = (\text{ps}, \text{ns}, \mu\text{s})$
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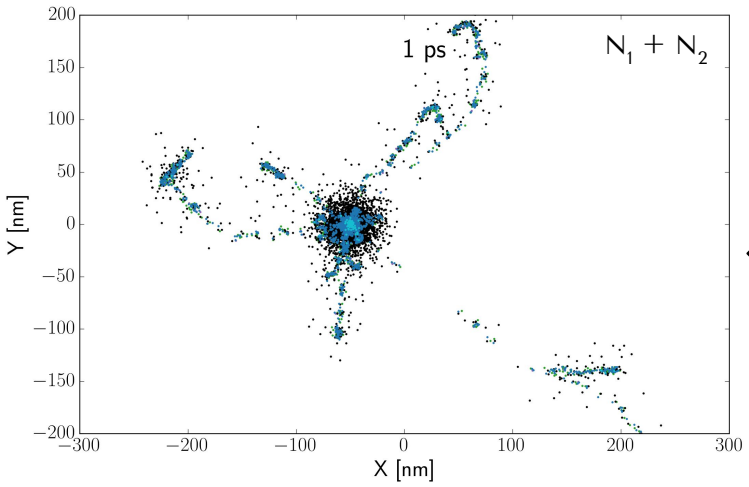
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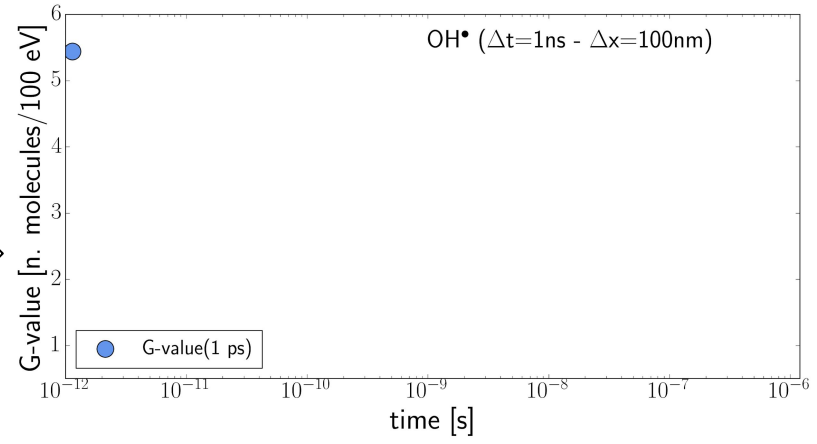
pO₂:

- 0 and 5

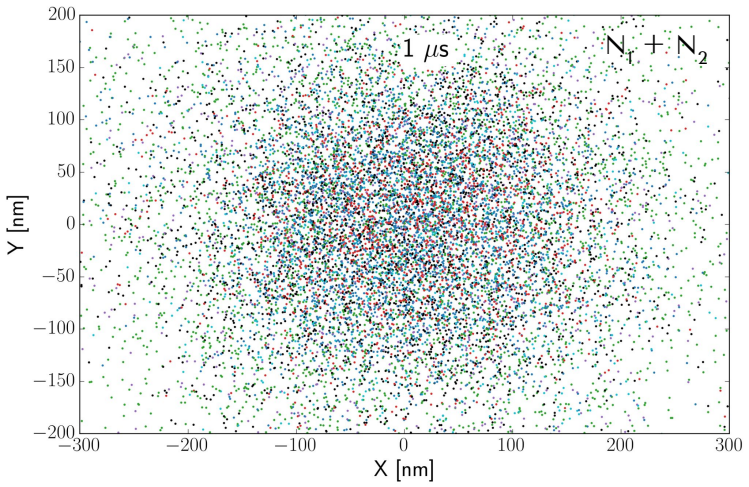
Intertrack: Quantities and their time evolution



$$G\text{-value} = \frac{N_1 + N_2}{E_1 + E_2}$$

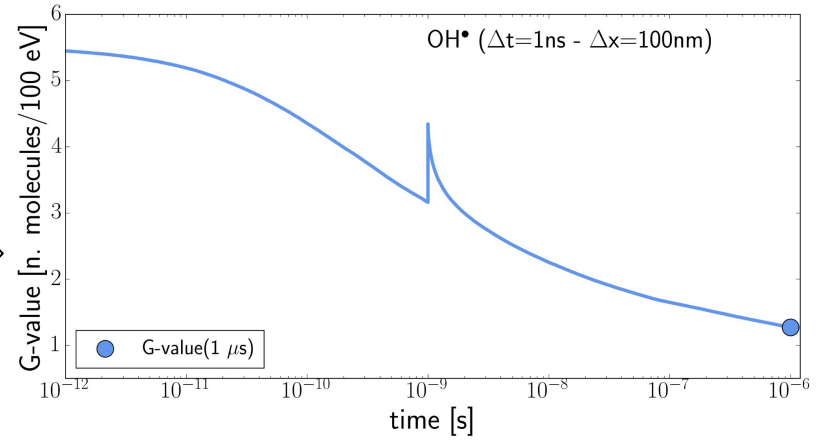


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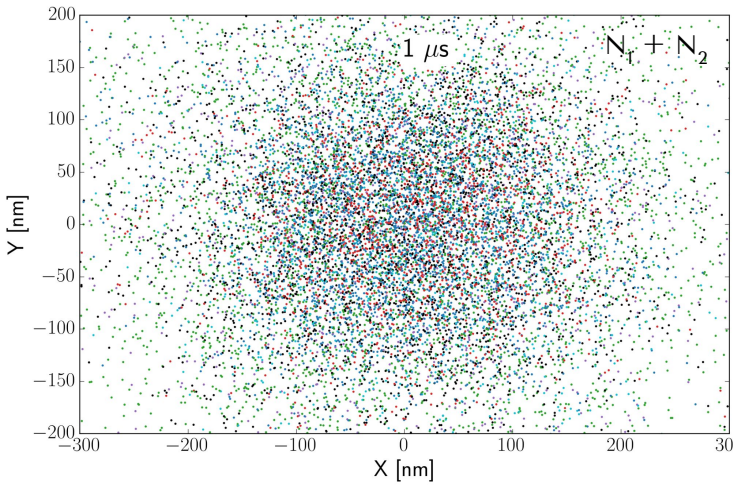


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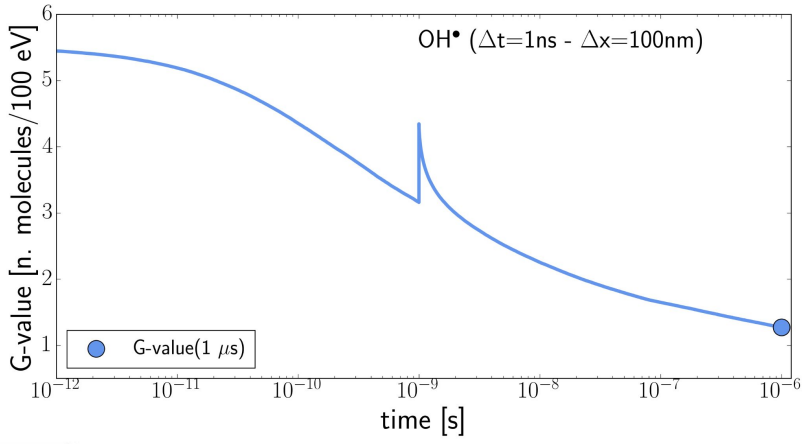
Two large horizontal arrows, one pointing left and one pointing right, are positioned below the equation.



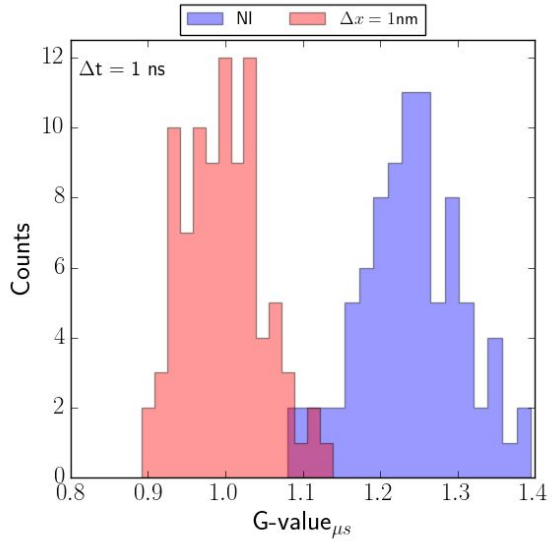
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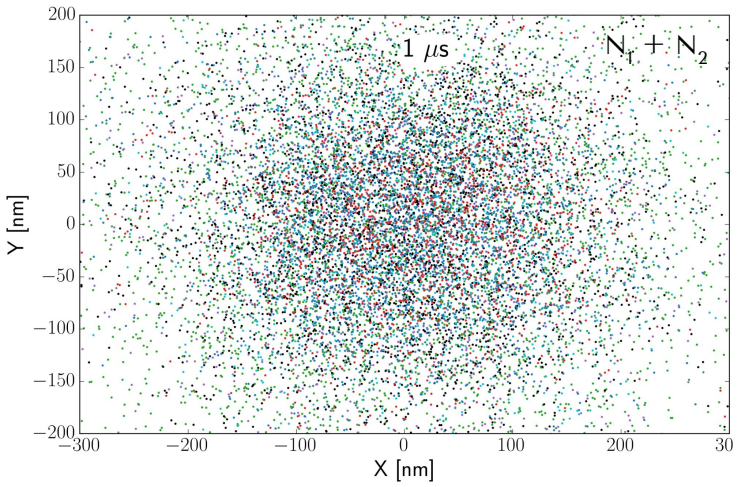
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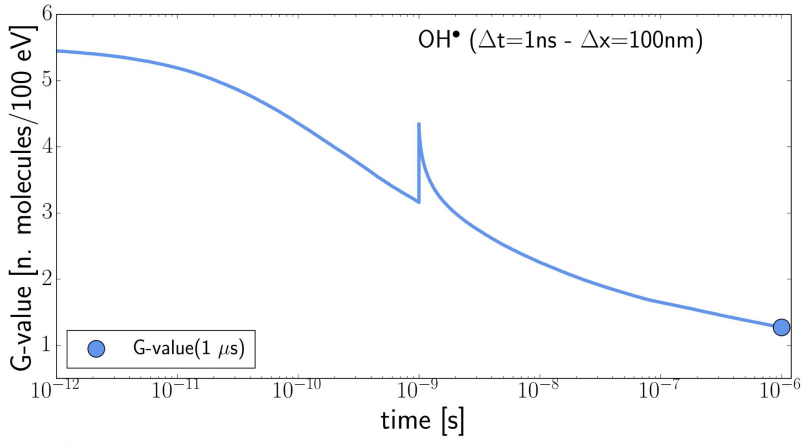
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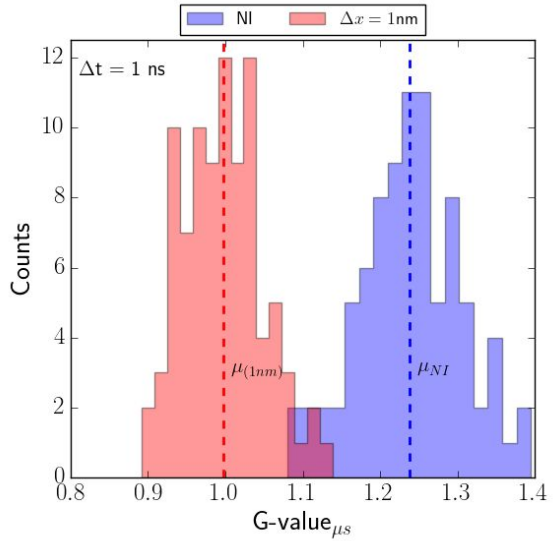


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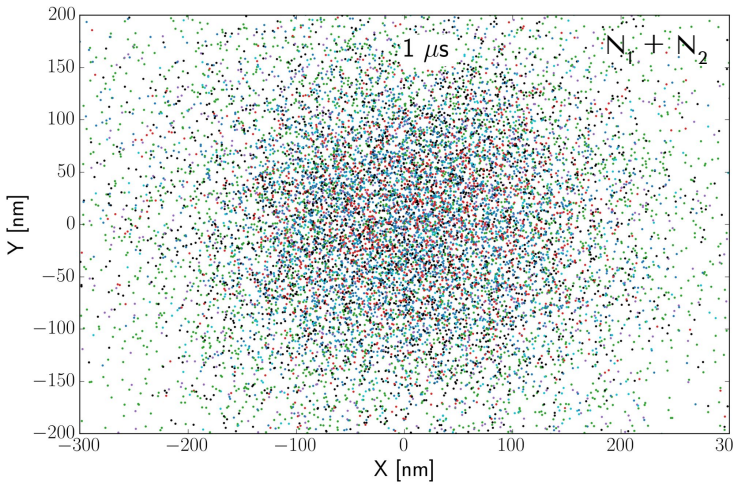


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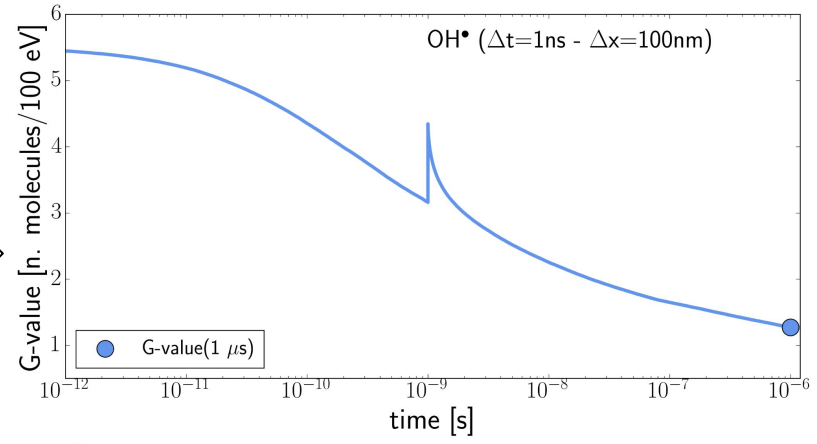
$$\delta = \mu_{\text{Inter}} - \mu_{\text{NI}}$$



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$$G\text{-value} = \frac{N_1 + N_2}{E_1 + E_2}$$



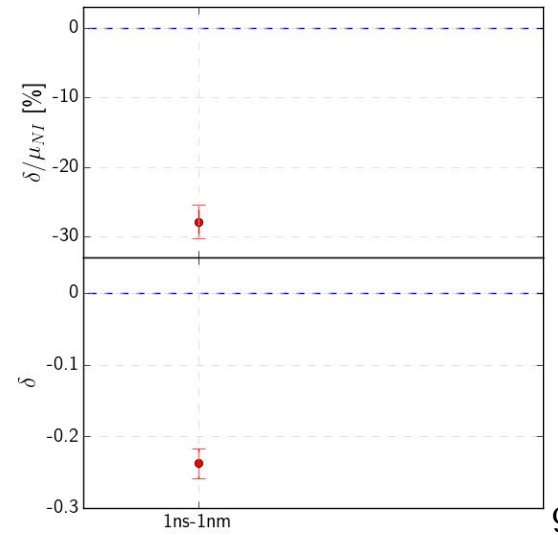
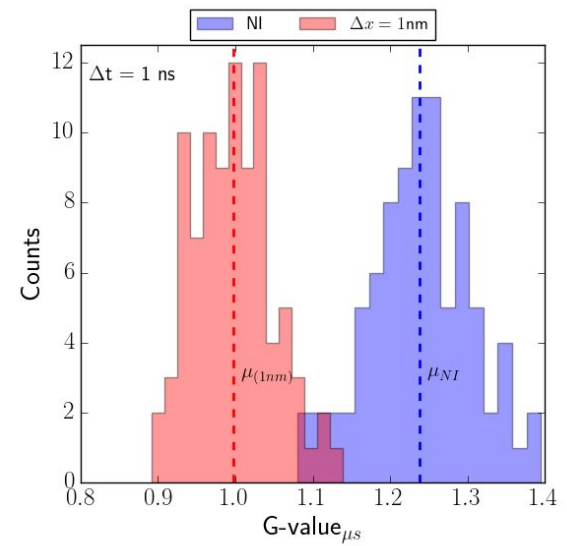
$$G\text{-value}(\mu s) = G\text{-value}_{\mu s}$$

$$\delta = \mu_{Inter} - \mu_{NI}$$

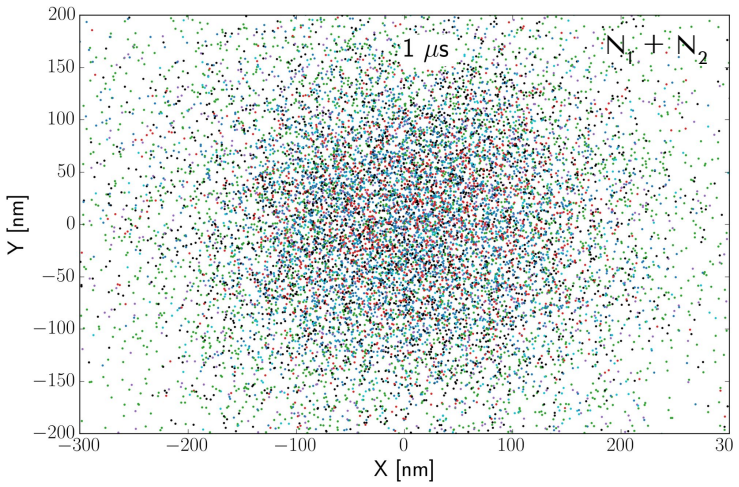
• **Welch Test:**

$$H_0 : \delta = 0 \quad ;$$

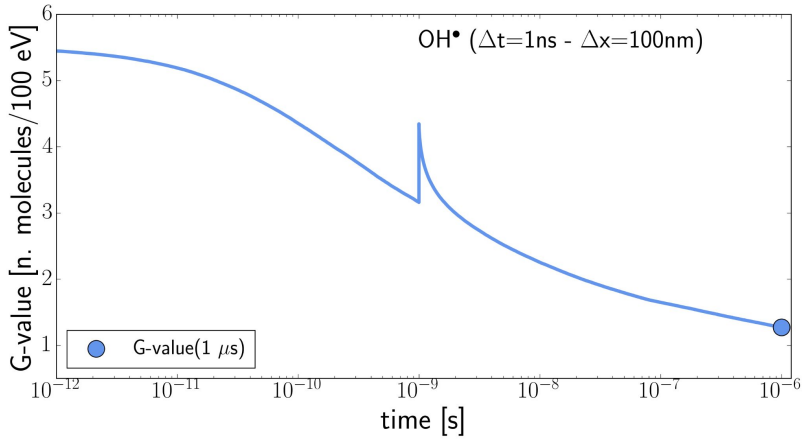
$$H_1 : \delta \neq 0$$



Intertrack: Quantities and their time evolution



$$G\text{-value} = \frac{N_1 + N_2}{E_1 + E_2}$$



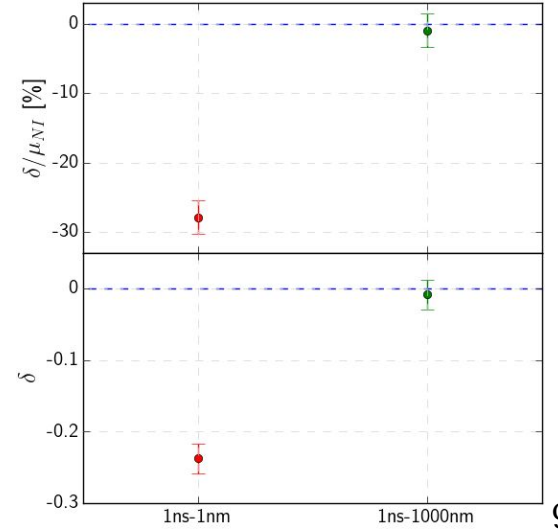
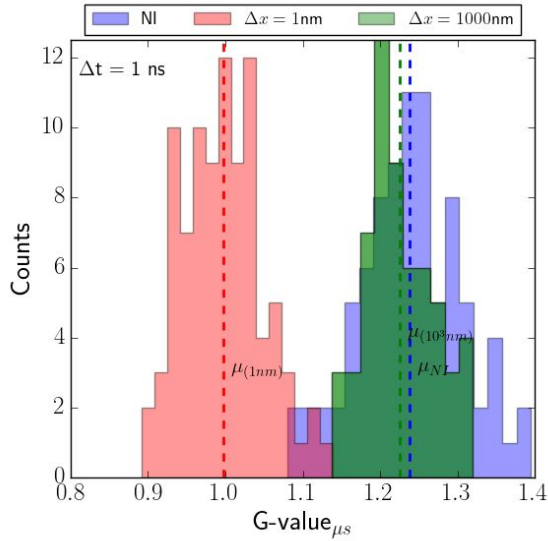
$$G\text{-value}(\mu s) = G\text{-value}_{\mu s}$$

$$\delta = \mu_{Inter} - \mu_{NI}$$

• **Welch Test:**

$$H_0 : \delta = 0 \quad ;$$

$$H_1 : \delta \neq 0$$



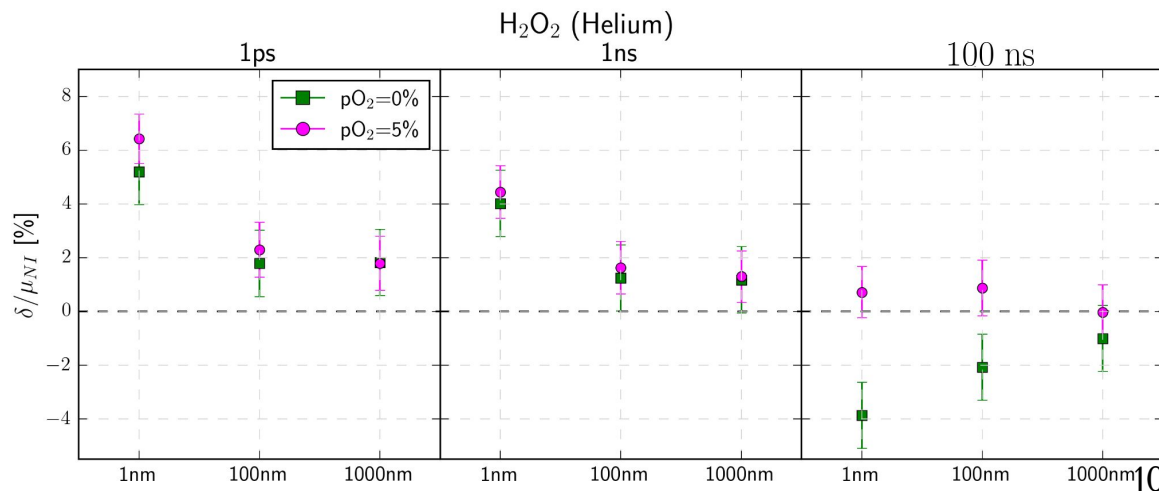
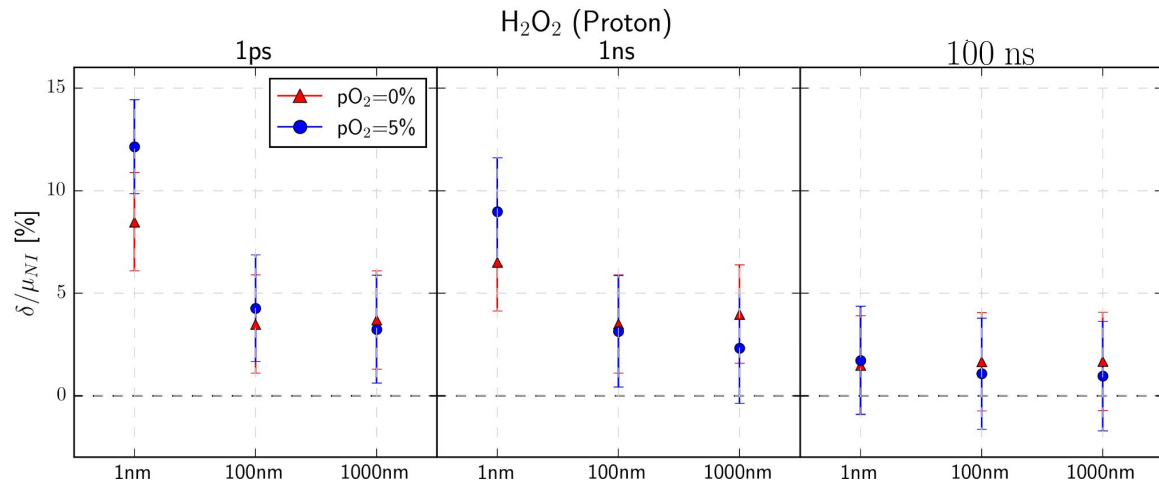
Results: Monte Carlo impact on chemical endpoint

INTERTRACK:

- Δx - Δt track overlap and number of available species
- **Intra-track** influence intertrack

OXYGENATION:

- Production of HO_2^\bullet and $\text{O}_2^{\bullet -}$ by consuming e^- and H^\bullet
- Effects only after 1 ns



Results: Monte Carlo impact on chemical endpoint

INTERTRACK:

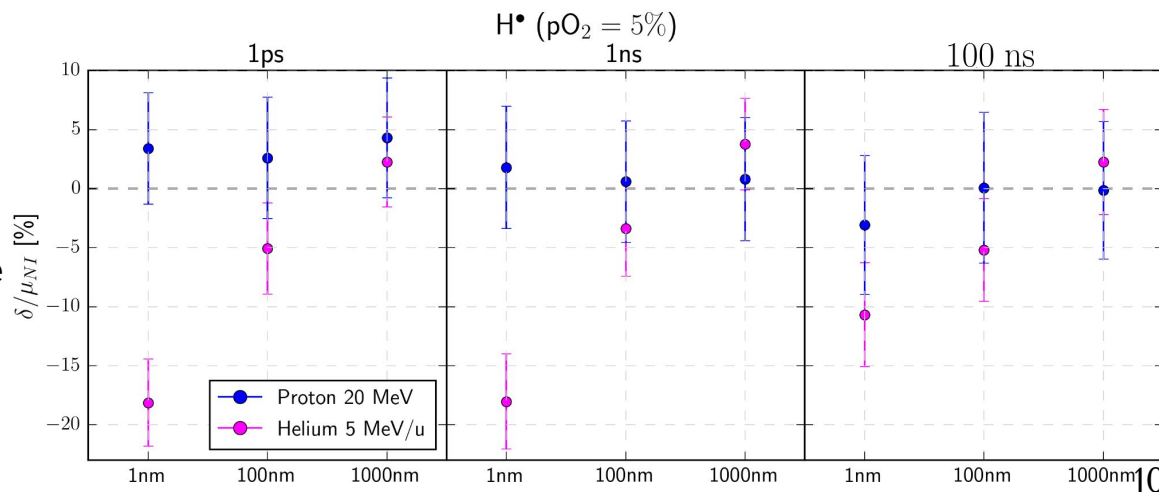
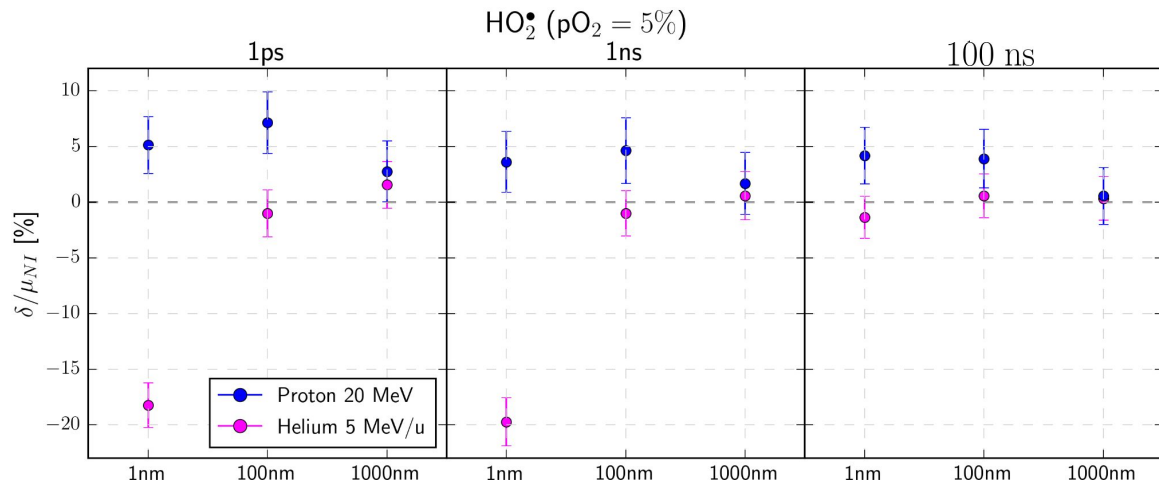
- Δx - Δt track overlap and number of available species
- **Intra-track** influence intertrack

OXYGENATION:

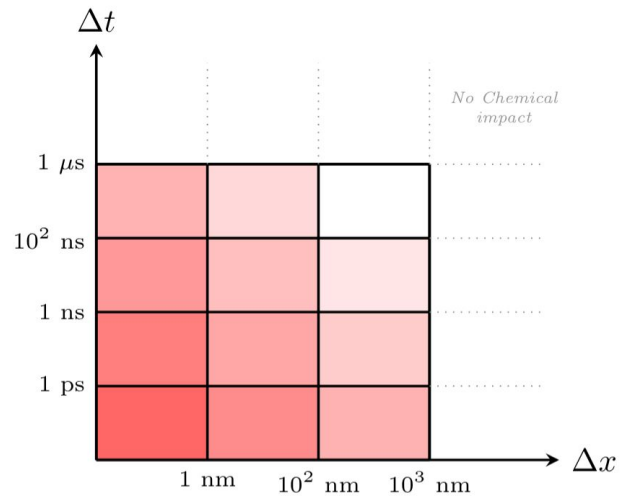
- Production of HO_2^\bullet and $\text{O}_2^{\bullet-}$ by consuming e^- and H^\bullet
- Effects only after 1 ns

OXYG + INTER

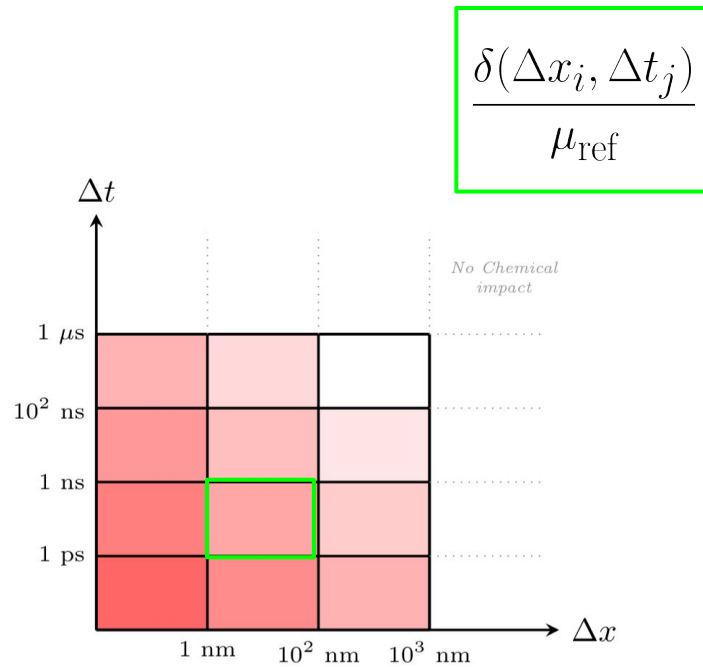
- Difference from 0% only at $\Delta t > 1\text{ns}$
- e^- consumption cause depletion of $\text{O}_2^{\bullet-}$
- H^\bullet consumption at high LET cause depletion of HO_2^\bullet



Chemical Impact estimation



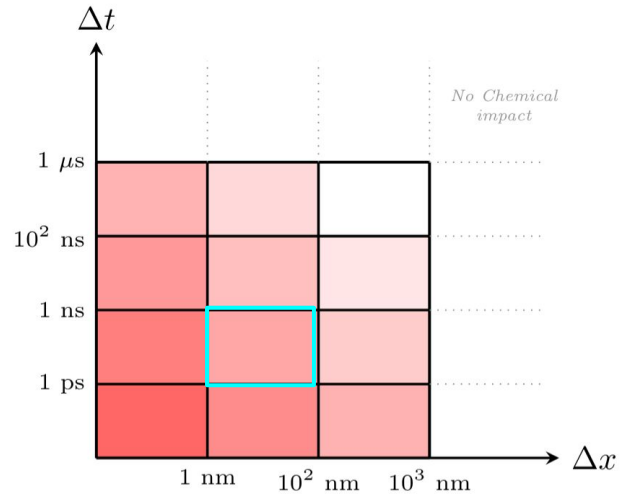
Chemical Impact estimation



Chemical Impact estimation

$$\Pr(\Delta x_i, \Delta t_j) = P(\Delta x_i, \Delta t_j) - P(\Delta x_{i-1}, \Delta t_j) - P(\Delta x_i, \Delta t_{j-1}) + P(\Delta x_{i-1}, \Delta t_{j-1}).$$

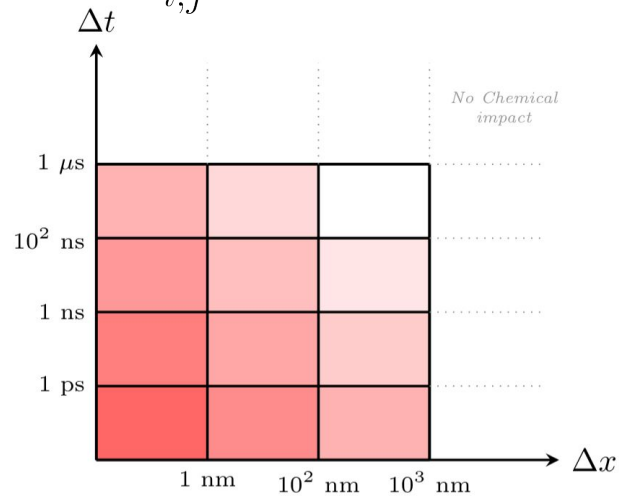
$$\frac{\delta(\Delta x_i, \Delta t_j)}{\mu_{\text{ref}}}$$



Chemical Impact estimation

$$\Pr(\Delta x_i, \Delta t_j) = P(\Delta x_i, \Delta t_j) - P(\Delta x_{i-1}, \Delta t_j) - P(\Delta x_i, \Delta t_{j-1}) + P(\Delta x_{i-1}, \Delta t_{j-1}).$$

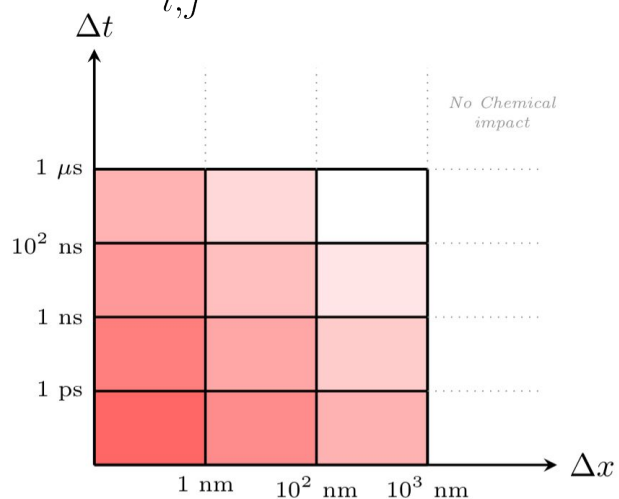
$$\Delta \text{Chem}_{\mu s} \approx \sum_{i,j} \Pr(\Delta x_i, \Delta t_j) \frac{\delta(\Delta x_i, \Delta t_j)}{\mu_{\text{ref}}}$$



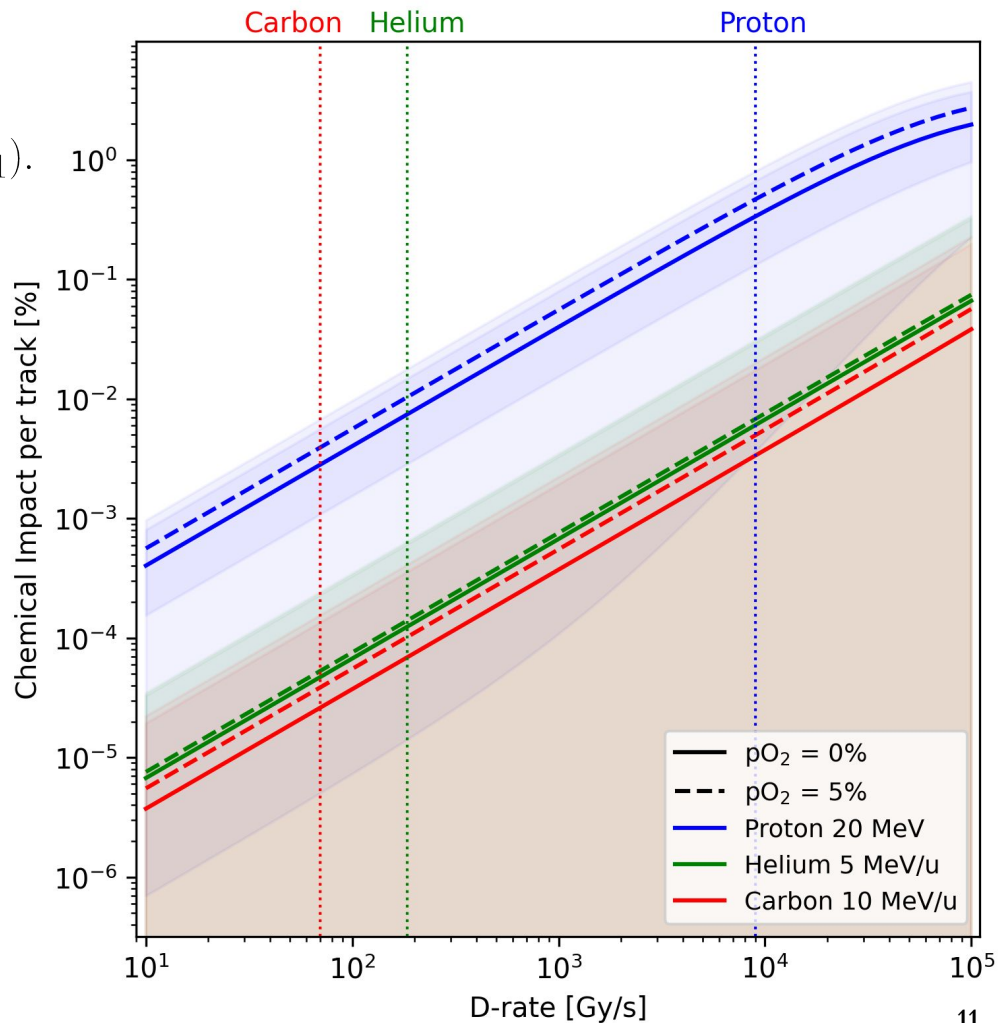
Chemical Impact estimation

$$\Pr(\Delta x_i, \Delta t_j) = P(\Delta x_i, \Delta t_j) - P(\Delta x_{i-1}, \Delta t_j) - P(\Delta x_i, \Delta t_{j-1}) + P(\Delta x_{i-1}, \Delta t_{j-1}).$$

$$\Delta \text{Chem}_{\mu s} \approx \sum_{i,j} \Pr(\Delta x_i, \Delta t_j) \frac{\delta(\Delta x_i, \Delta t_j)}{\mu_{\text{ref}}}$$



- Proton 20 MeV **small effect**
- Helium 5 MeV/u **no effect**
- Carbon 10 MeV/u **no effect**



Conclusions

MCTS (TRAX-CHEM)

- New implementation of TRAX-CHEM with intertrack and oxygen
- Small $\Delta x - \Delta t$ \square high effect
- Big $\Delta x - \Delta t$ \square small effect (nothing after $2\mu\text{m}$)
- **Intrack** effect impact on **intertrack**
- O_2 can reduce and change intertrack effect for high Δt (rad. dependent)
- O_2 species undergo strong intertrack effect

PRM Model

- Develop PRM model for track proximity quantification
- **Micro-** structure "collapse" to **macro-** after surpassing the repetition time (t_μ)
- **Macro-** does the same with **continuous** version after t_r
- Factor = t_r / τ between continuous and macropulse
- Increase LET \rightarrow decrease probability of intertrack
- Increase Dose rate \rightarrow increase probability of intertrack

- No potential impact in heterogeneous chemistry of intertrack (e-