# Reggeization of the pion exchange

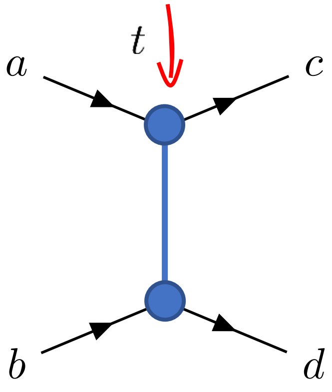
Glòria Montaña

Based on *Phys. Rev. D* 110, 114012 (2024)

**Workshop JPAC and JLab12**
University of Genova, February 18-20, 2026

# Recap of Regge Theory

At high energy, the scattering amplitude in the physical region of the s-channel is related to t-channel exchanges



$$A(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell(t) P_\ell(z_t)$$

t-channel partial wave amplitudes

**Regge limit**

$$s \gg -t, m^2$$

$$A(s,t) \sim s^{\ell_{\text{eff}}}$$

$$z_t = \cos\theta_t = 1 + \frac{2s}{t-4m^2}$$

$$\lim_{z\to\infty} P_\ell(z) \sim z^\ell$$

T.Regge, *Nuovo Cim.* 18, 947 (1960)

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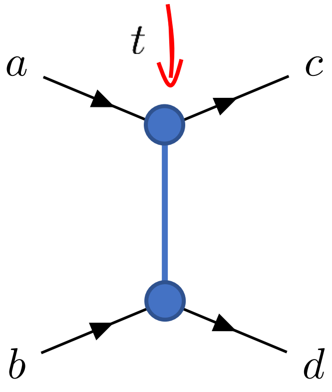

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The concept of partial-wave amplitude can be extended to complex values of angular momentum

$$\{f_\ell(t)\} \longrightarrow f(\ell,t) \quad \text{with} \quad f(\ell,t) \to f_\ell(t),\ \ell \in \{0,1,2,\ldots\}$$

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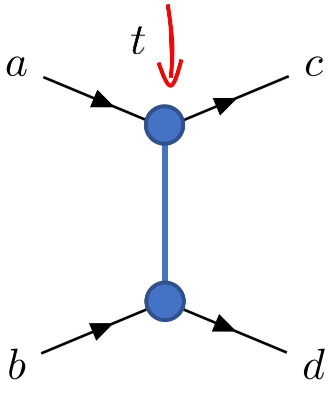

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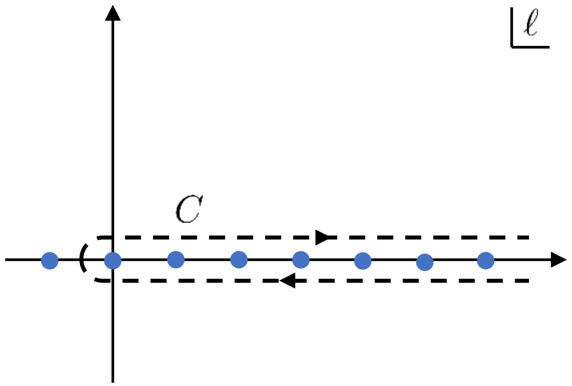
**Do the sum**: Sommerfeld-Watson transform

$$A(s,t) = -\frac{1}{2i}\int_C \mathrm{d}\ell \, \frac{(2\ell+1)\,P_\ell(-z_t)\,f(\ell,t)}{\sin\pi\ell}$$



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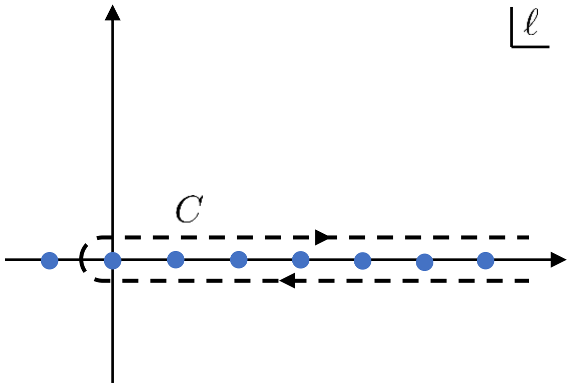

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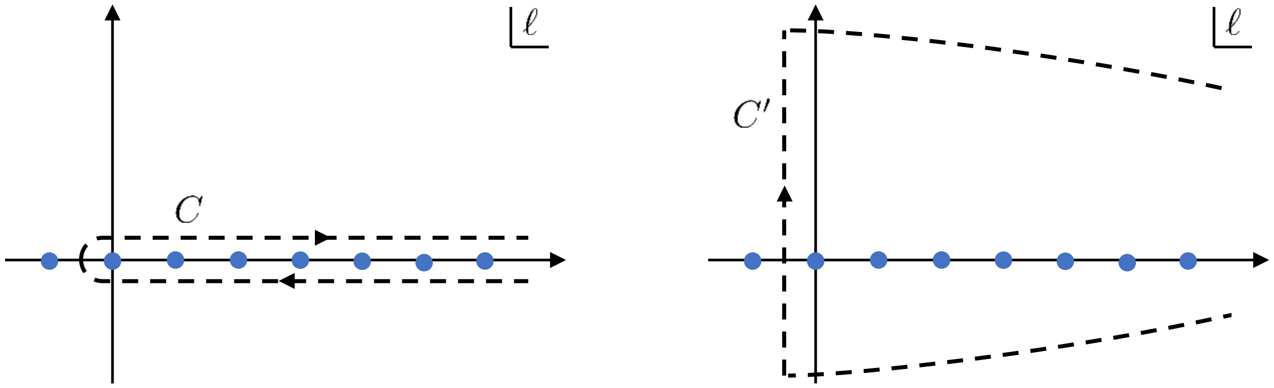
∗ Deform the contour



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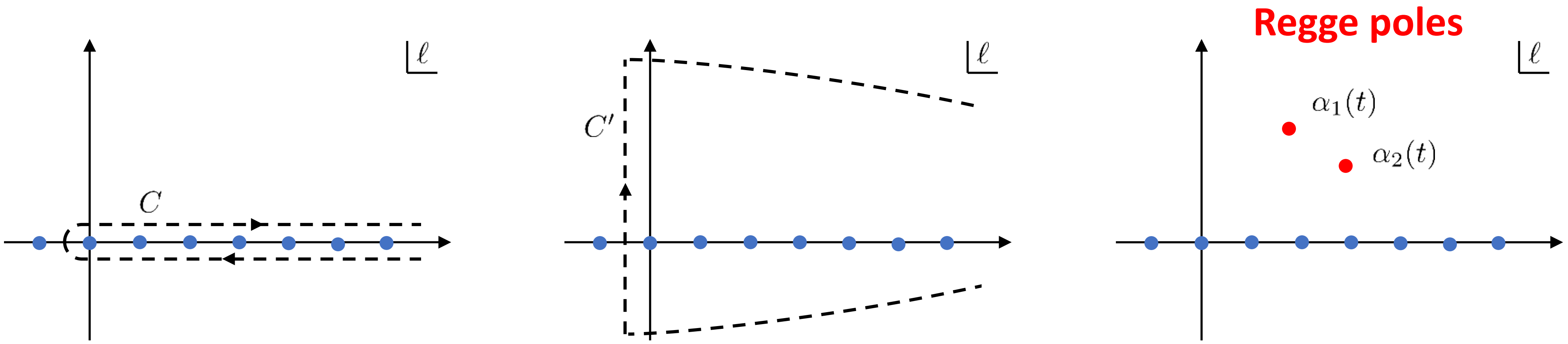
✳ Deform the contour

→ Assuming the partial waves are analytic ℓ, the only singularities are poles $f_\ell(t) \sim \frac{\beta(t)}{\ell - \alpha(t)}$



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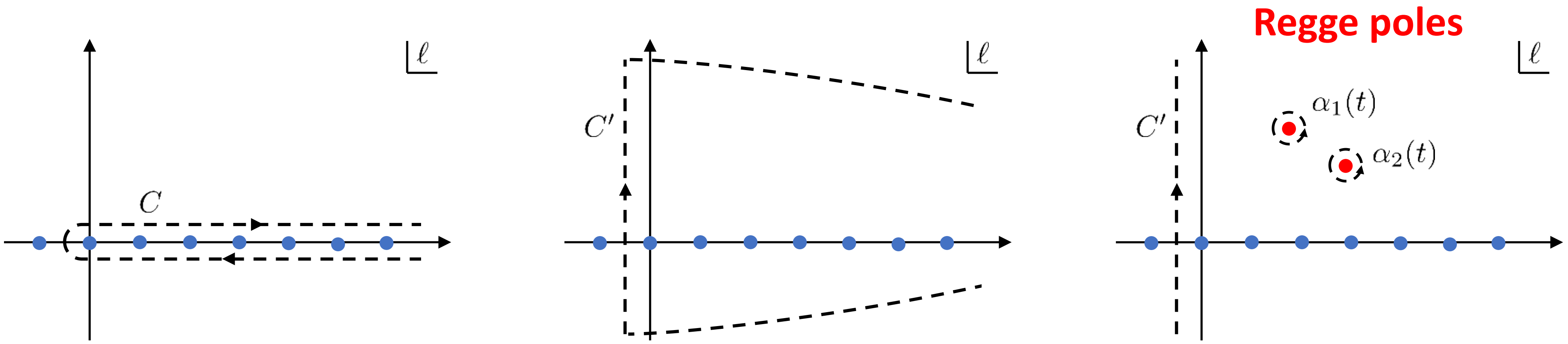
$$A(s,t) = \underbrace{-\frac{1}{2i}\int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \mathrm{d}\ell \ldots}_{\text{background } \sim\, s^{-1/2}} \underbrace{- \sum_i \frac{\pi(2\alpha_i(t)+1)\beta_i(t)}{\sin\left(\pi\alpha_i^{\pm}(t)\right)}\frac{1}{2}P_{\alpha_i}(-z_t)}_{\text{pole contributions } \sim\, s^{\alpha(t)}}$$



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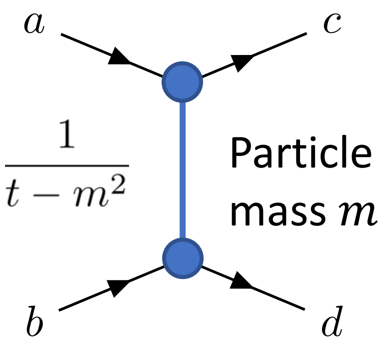
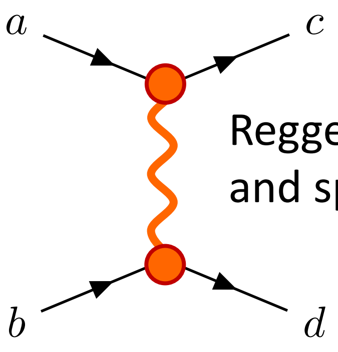


$$\frac{1}{t-m^2}$$

Particle with mass $m$ and spin $J$

→



Reggeon with mass $\sqrt{t}$ and spin $\alpha(t)$

$$\mathcal{P}_{\mathrm{Regge}} = \frac{\pi\alpha'}{\sin\left(\pi\alpha(t)\right)} \frac{1+\eta e^{-i\pi\alpha(t)}}{2} \left(\frac{s}{s_0}\right)^{\alpha(t)} \frac{1}{\Gamma\left(1+\alpha(t)\right)}$$

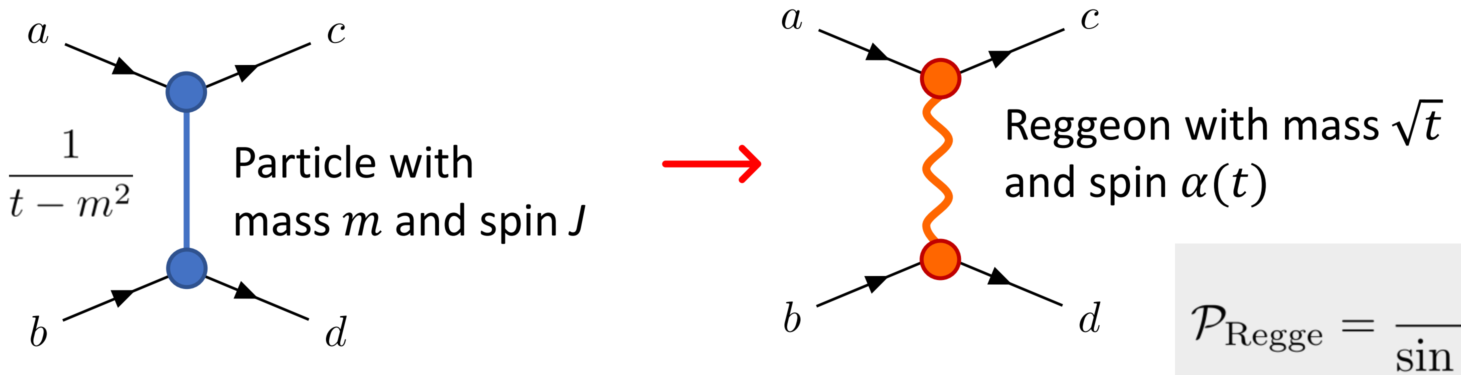
poles for integer $\alpha(t)$

signature factor

asymptotic behavior

cancel non-physical poles

# Recap of Regge Theory



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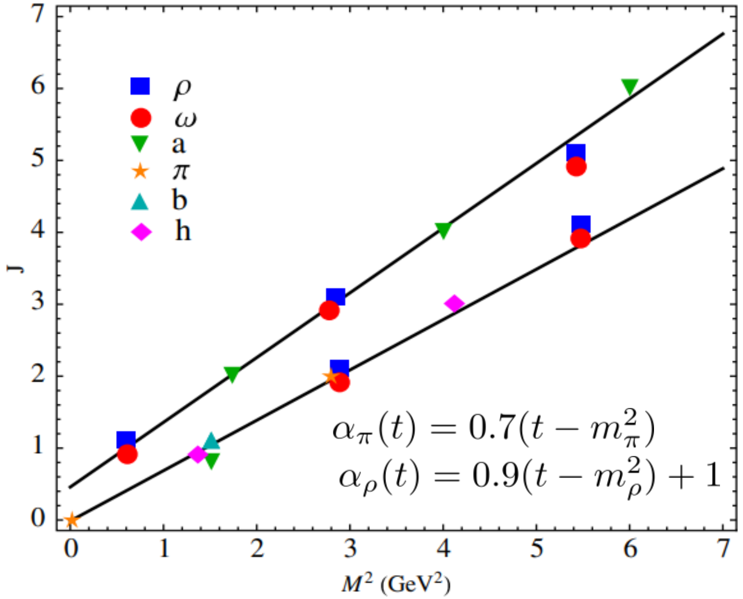
V.Mathieu et al., *Phys.Rev.D* 98, 014041 (2018)



**Resonances appear simultaneously as poles in energy and spin!**

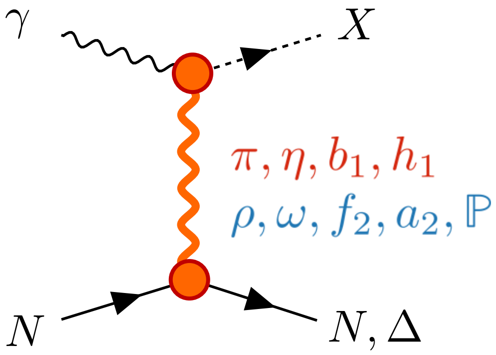
✳ **Regge trajectories:** families with same quantum numbers but different spin

✳ Almost straight lines (Chew-Frautschi plot)

✳ In standard Regge theory parameterized by $\alpha(t) = \alpha' t + \alpha_0$

# Regge amplitude for meson photoproduction

We "only" need to determine the (dominant) crossed-channel exchanges



→ **Unnatural** parity $(P(-1)^J = -1)$ : $0^-, 1^+, 2^-, 3^+, \ldots$

→ **Natural** parity $(P(-1)^J = +1)$ : $0^+, 1^-, 2^+, 3^-, \ldots$

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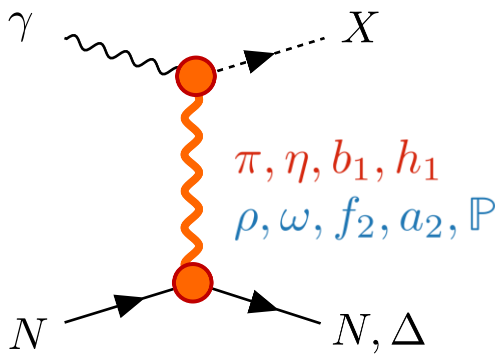

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$\ast$ Neutral-exchange reactions:

$$\gamma p \to \pi_1^0(1600) p$$

$$\gamma p \to b_1^0 p \ , \ \gamma p \to a_0^0 p \ , \ \gamma p \to a_2^0 p \ ,$$

$$\gamma p \to \eta^{(\prime)} \pi^0 p \ \ldots$$

→ Natural parity exchanges dominate

$\ast$ Charge-exchange reactions:

$$\gamma p \to \pi_1^-(1600) \Delta^{++}$$

$$\gamma p \to b_1^- \Delta^{++} \ , \ \gamma p \to \pi^- \Delta^{++} \ , \ \gamma p \to a_0^- \Delta^{++} \ , \ \gamma \Delta^{++} \to a_2^- p \ ,$$

$$\gamma p \to \eta^{(\prime)} \pi^- \Delta^{++} \ \ldots$$

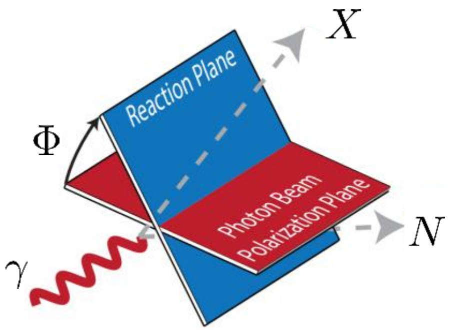
→ Small -t: unnatural (pion) exchanges favored

→ Large -t: natural exchanges favored

# Polarization observables to access the production mechanism

Linearly polarized photon beam helps disentangle the production mechanisms

- ∗ Beam asymmetry



$$\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel} \approx \frac{d\sigma_N - d\sigma_U}{d\sigma_N + d\sigma_U}$$

V.Mathieu et al., Phys.Rev.D 92, 074013 (2015)

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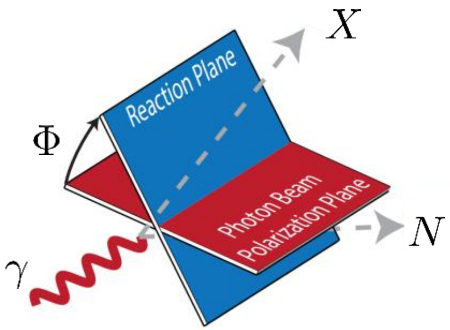

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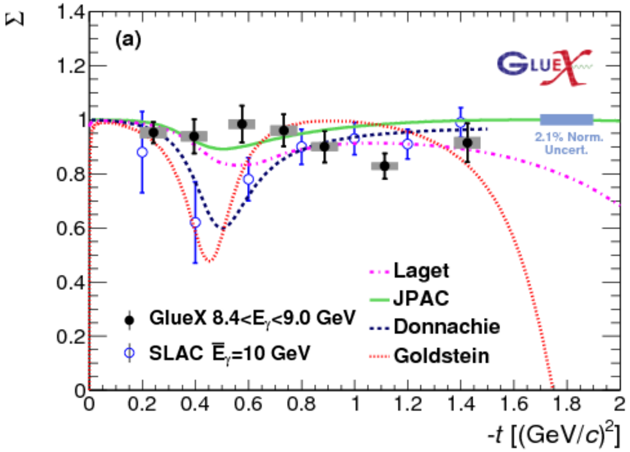
→ Neutral- vs charge-exchange reactions:

$\gamma p \to \pi^0 p$



Phys.Rev.C 95, 042201 (2017)

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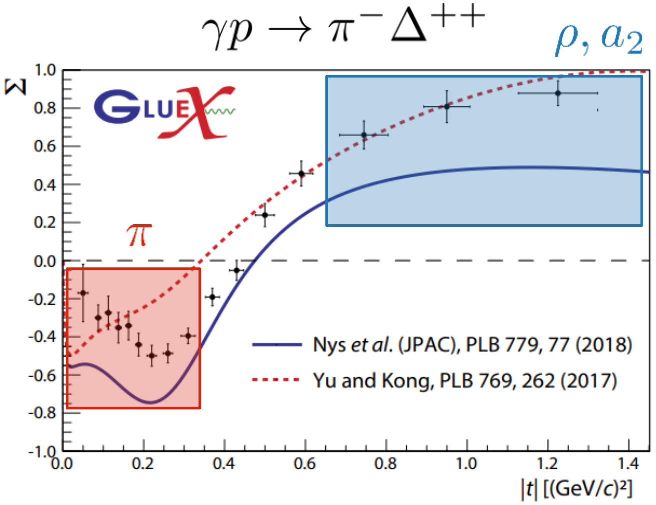

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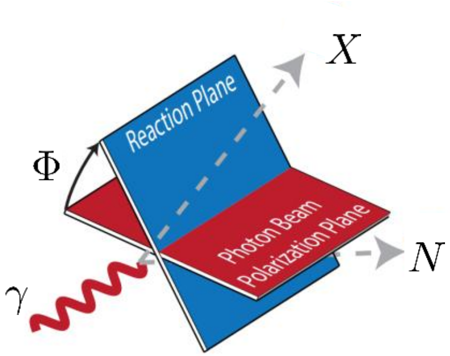

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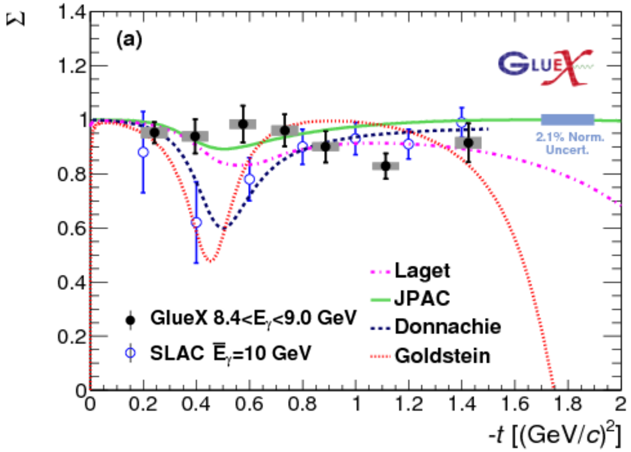

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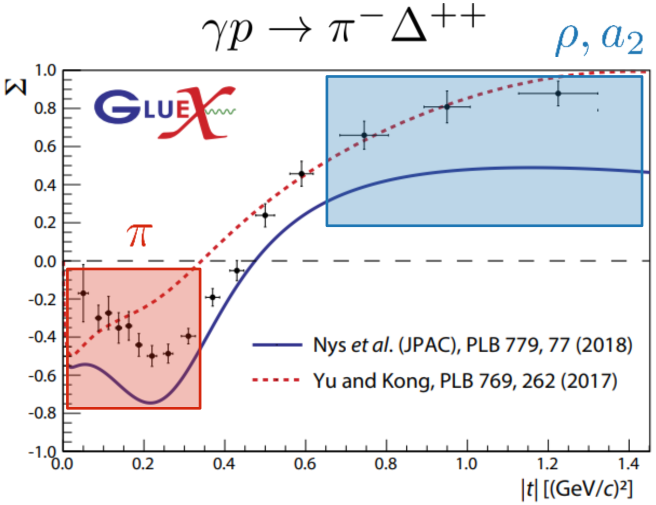


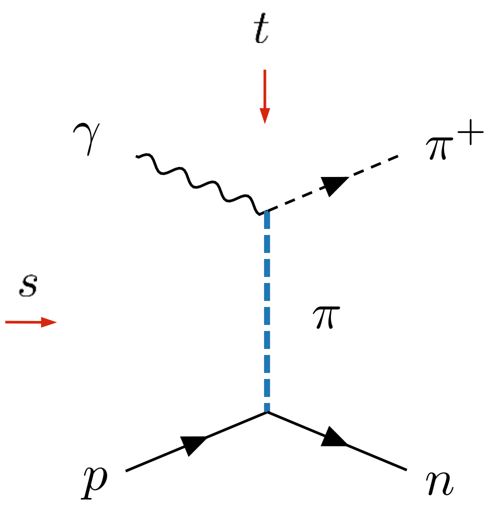
Phys.Rev.C 103, L022201 (2021)

- ✳ SDMEs (Vanamali's talk)

# Understanding pion exchange

What do we know?

- Dominates in charge exchange reactions at small momentum transfer
- Low energies: constrained by effective Lagrangians of QCD
- High energies: Regge theory



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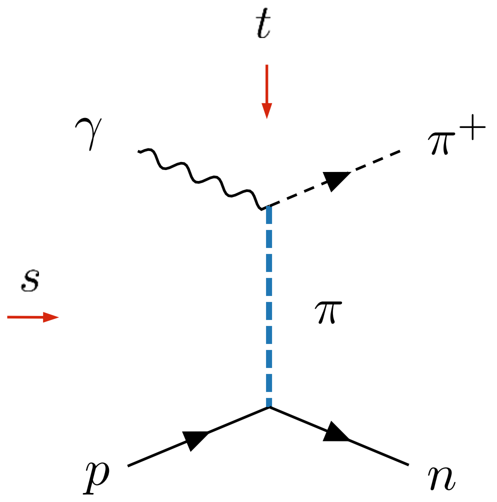

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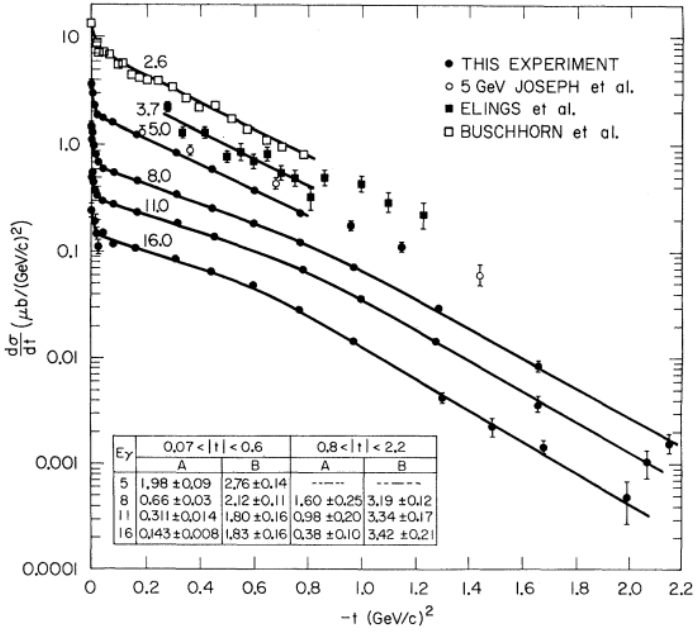

| Eγ | 0.07 < \|t\| < 0.6 | | 0.8 < \|t\| < 2.2 | |
|---|---|---|---|---|
| | A | B | A | B |
| 5 | 1.98 ±0.09 | 2.76 ±0.14 | ------ | ------- |
| 8 | 0.66 ±0.03 | 2.12 ±0.11 | 1.60 ±0.25 | 3.19 ±0.12 |
| 11 | 0.311 ±0.014 | 1.80 ±0.16 | 0.98 ±0.20 | 3.34 ±0.17 |
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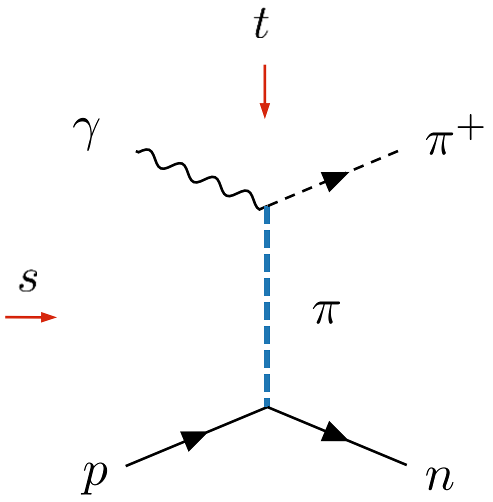

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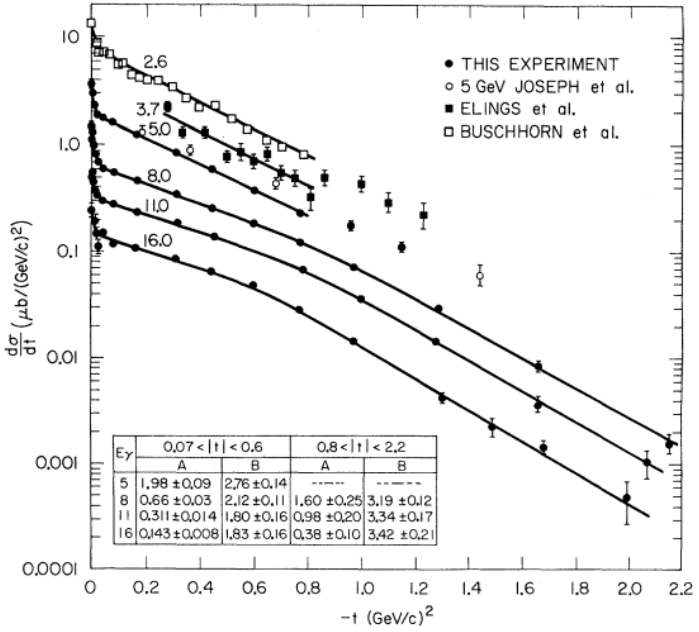

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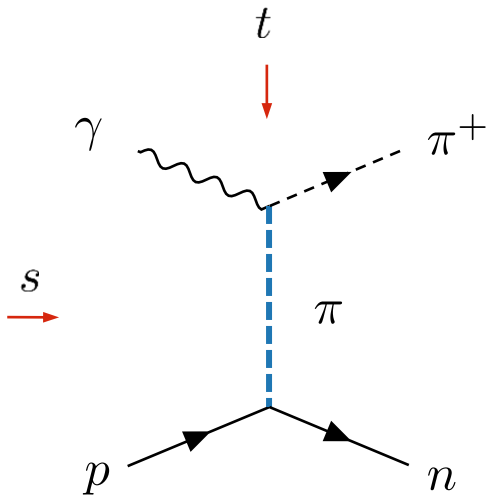

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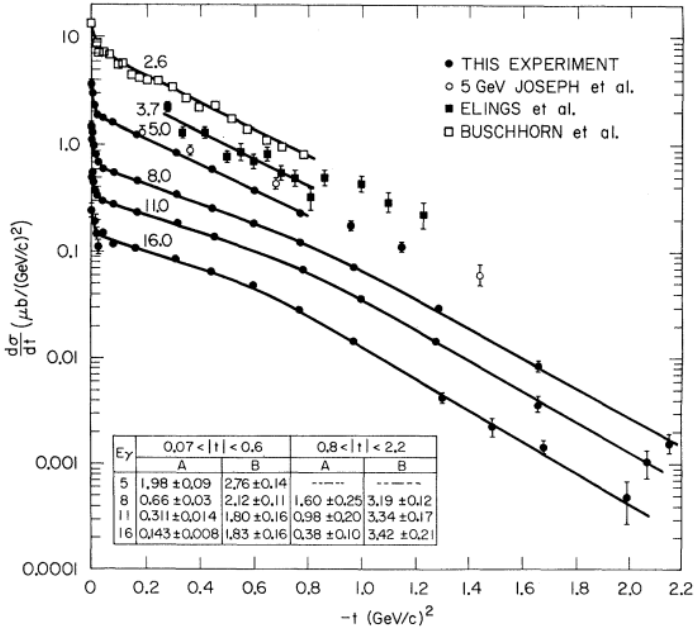

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Early solutions:

☒ Parity-doublet conspirator of the pion

Ball, Frazer and Jacob, *Phys.Rev.Lett.* 20, 518 (1968)

? Regge cuts and absorption (final state interactions)

Henyey, Kane, Pumplin, *Phys.Rev.* 182, 1579 (1969)

☑ Nucleon Born terms

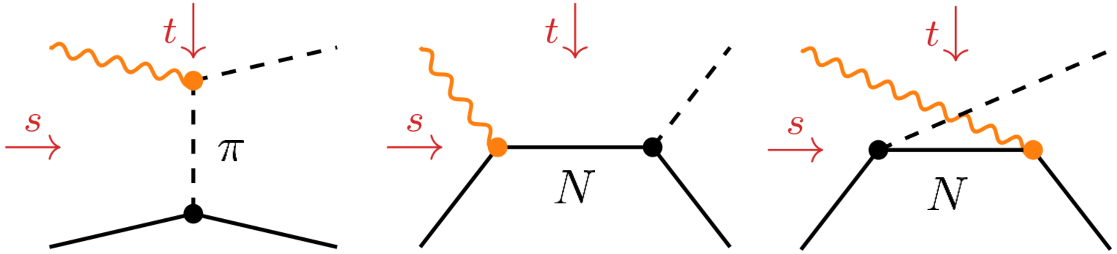
Jones, *Rev.Mod.Phys.* 52, 545 (1980) 545

# Born amplitudes

In terms of Born diagrams, **current conservation** relates t-channel diagram (pion exchange) and s-, u-channel diagrams (nucleon exchanges)

$$A_{\mu_\gamma \mu_i \mu_f} = \epsilon_{\mu_\gamma}(k) \cdot J_{\mu_i \mu_f}$$

$$J^\mu_{\mu_i \lambda_f} = J^\mu_{\mu_i \mu_f, t} + J^\mu_{\mu_i \mu_f, s} + J^\mu_{\mu_i \mu_f, u}$$



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$$J^\mu_{\mu_i \mu_f, s} = \sqrt{2} e_N g_{\pi N N} \bar{u}_{\mu_f}(p_f) \gamma_5 \frac{\not{q}_s + M}{s - M^2} \gamma^\mu u_{\mu_i}(p_i)$$

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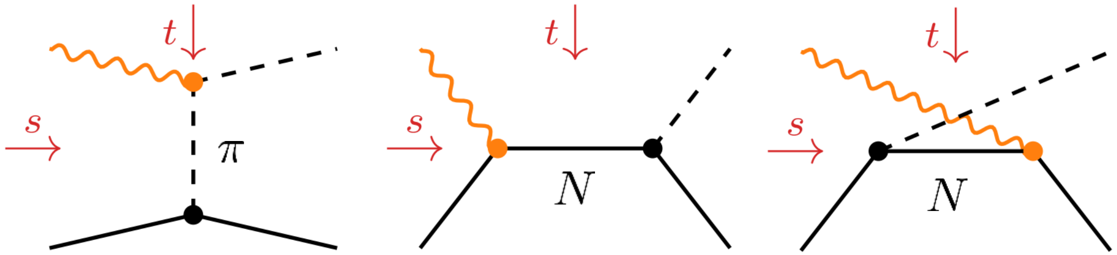

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Separate electric and magnetic contributions (gauge invariant alone)

$$A_{\mu_\gamma \mu_i \mu_f} = A^{\mathrm{e}}_{\mu_\gamma \lambda_i \mu_f} + A^{\mathrm{m}}_{\mu_\gamma \mu_i \mu_f} \quad , \qquad A^{\mathrm{e}}_{\mu_\gamma \mu_i \mu_f} = 2\sqrt{2} g_{\pi NN} \left[ e_\pi \frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{t - \mu^2} + e_{N_i} \frac{(\epsilon_{\mu_\gamma} \cdot p_i)}{s - M^2} + e_{N_f} \frac{(\epsilon_{\mu_\gamma} \cdot p_f)}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

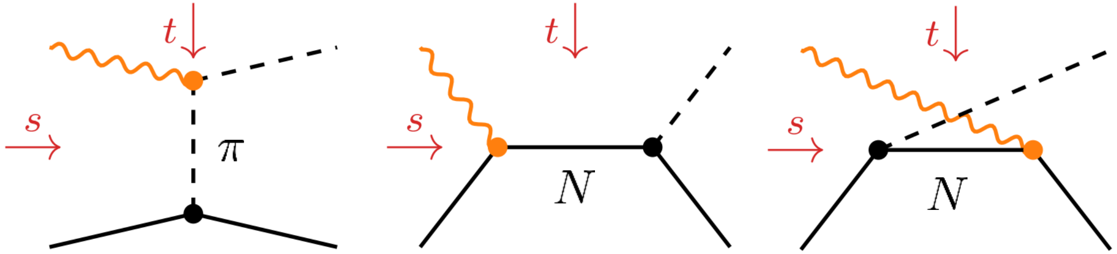
$$A^{\mathrm{m}}_{\mu_\gamma \mu_i \mu_f} = \sqrt{2} g_{\pi NN} \left[ \frac{e_{N_i}}{s - M^2} + \frac{e_{N_f}}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 \not{k} \not{\epsilon}_{\mu_\gamma} u_{\mu_i}(p_i)$$

# Born amplitudes

In terms of Born diagrams, **current conservation** relates t-channel diagram (pion exchange) and s-, u-channel diagrams (nucleon exchanges)

$$A_{\mu_\gamma \mu_i \mu_f} = \epsilon_{\mu_\gamma}(k) \cdot J_{\mu_i \mu_f}$$

$$J^\mu_{\mu_i \lambda_f} = J^\mu_{\mu_i \mu_f, t} + J^\mu_{\mu_i \mu_f, s} + J^\mu_{\mu_i \mu_f, u}$$



$$J^\mu_{\mu_i \mu_f, t} = -\sqrt{2} e_\pi g_{\pi NN} \frac{q_t^\mu - p_\pi^\mu}{t - \mu^2} \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

$$J^\mu_{\mu_i \mu_f, s} = \sqrt{2} e_N g_{\pi NN} \bar{u}_{\mu_f}(p_f) \gamma_5 \frac{\not{q}_s + M}{s - M^2} \gamma^\mu u_{\mu_i}(p_i)$$

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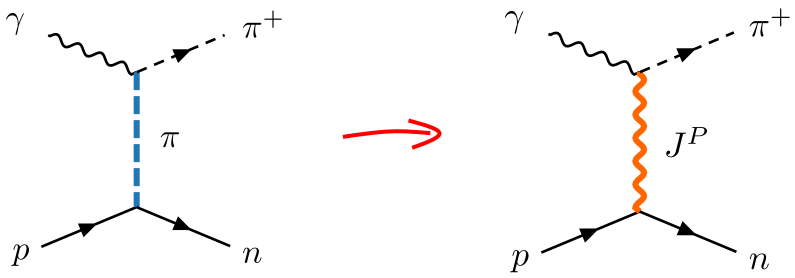
vanishes in t-channel CM

vanishes in s-channel CM

$$A^{\mathrm{m}}_{\mu_\gamma \mu_i \mu_f} = \sqrt{2} g_{\pi NN} \left[ \frac{e_{N_i}}{s - M^2} + \frac{e_{N_f}}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 \not{k} \not{\epsilon}_{\mu_\gamma} u_{\mu_i}(p_i)$$

# Reggeization of pion exchange

In the Regge-pole approximation:



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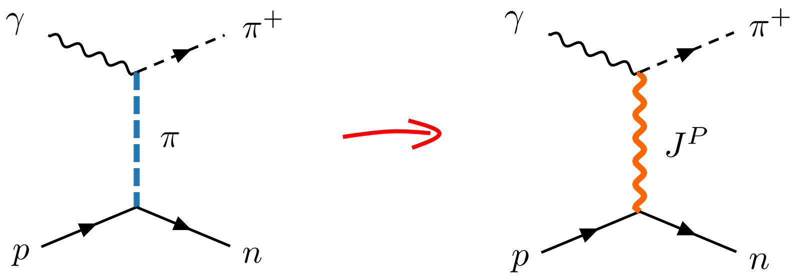

⁎ Empirical prescription by Vanderhaegen, Guidal and Laget

- Preserves gauge invariance and describes forward peak)

$$A^{\mathrm{VGL}}_{\lambda_\gamma\lambda_i\lambda_f}(s,t) = A^{\mathrm{Born}}_{\lambda_\gamma\lambda_i\lambda_f}(s,t)(t-m_\pi^2)\,\alpha'\frac{1+e^{-i\pi\alpha(t)}}{2}\Gamma(-\alpha(t))\left(\frac{s}{s_0}\right)^{\alpha(t)}$$

*Nucl.Phys.A* 627, 645 (1997)

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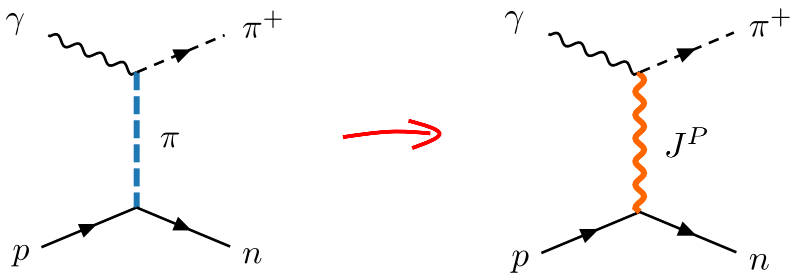

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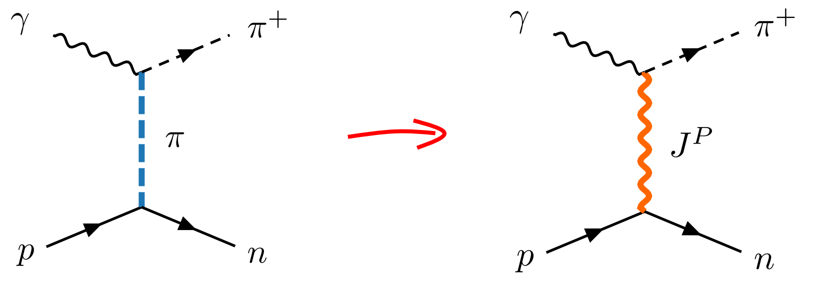

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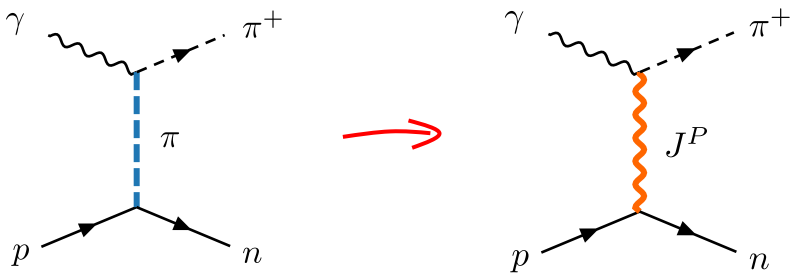

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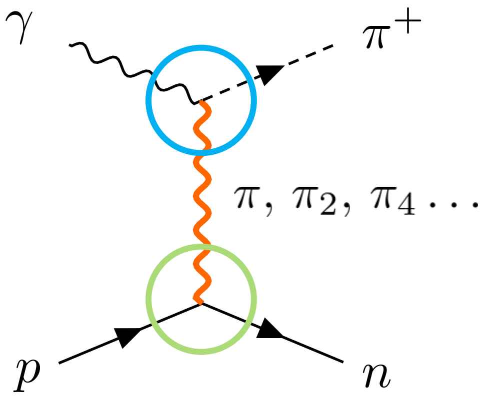
**No pion pole?**

# Reggeization of pion exchange

GM, D.Winney, et al. (JPAC), Phys.Rev.D 110, 114012 (2024)



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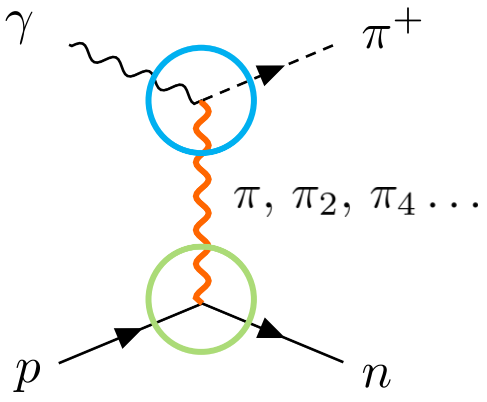

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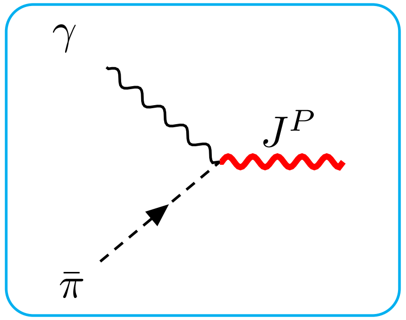

$$1^-\otimes 0^- = 1^+ \left\{ \begin{array}{ll} L=1 & \quad J=0 \\ L=\{J-1,J+1\} & \quad J\geq 2 \end{array}\right\}$$

one *L* vs two *L*’s

$$V^J_{\lambda_\gamma}(\sigma_J) = 2\sqrt{2}\, g_{\gamma\pi}\, \epsilon^*_{\nu_1\cdots\nu_J}(\sigma_J)\, \epsilon_\mu(k,\lambda_\gamma)\, (k^{\nu_1}\cdots k^{\nu_{J-1}}) \left[k^{\nu_J}\, p^\mu_\pi - g^{\nu_J\mu}\,(k\cdot p_\pi)\right]$$

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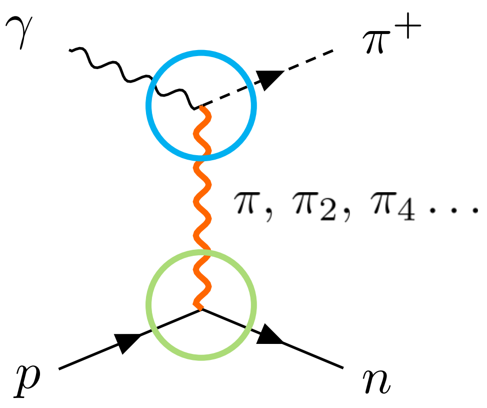

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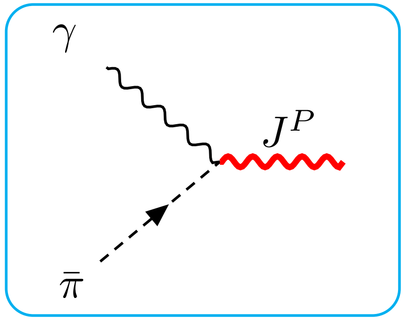

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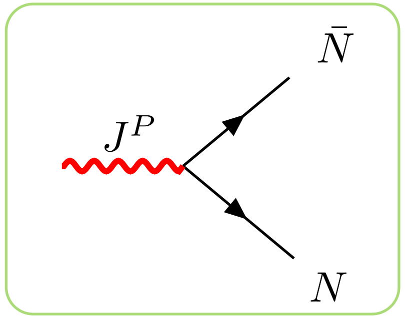

$$\tfrac{1}{2}^+\otimes\tfrac{1}{2}^- = 0^-\oplus 1^- \quad \rightarrow \quad L=J$$

$$V^J_{\lambda_i\lambda_f}(\sigma_J) = g_{N\bar{N}}\,(P^{\nu_1}\cdots P^{\nu_J})\,\epsilon_{\nu_1\cdots\nu_J}(\sigma_J)\bar{u}(p_f,\lambda_f)\,\gamma_5\, v(-p_i,\lambda_i) \qquad (P^\nu = p_i^\nu + p_f^\nu)$$

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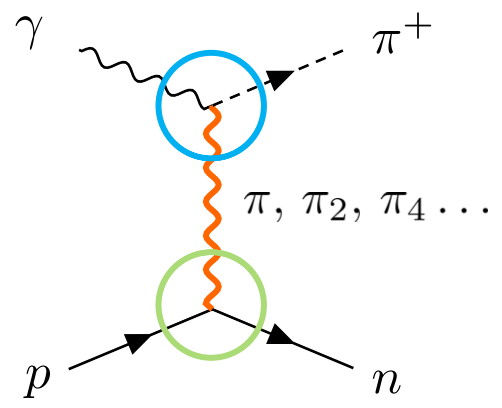

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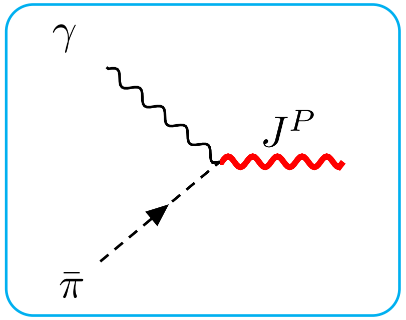

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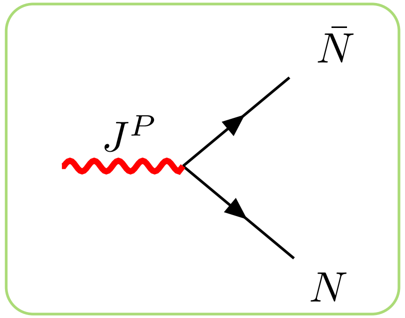

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→ spin-*J* amplitude:

$$a^J_{\lambda_\gamma\lambda_i\lambda_f}(t) \equiv \frac{2e_\pi g_J t}{J-\alpha_\pi(t)}(2\lambda_i\delta_{\lambda_i\lambda_f})c^2_J\sqrt{\frac{J+1}{J}}(-2p_t k_t)^J$$

**for *J* > 0**

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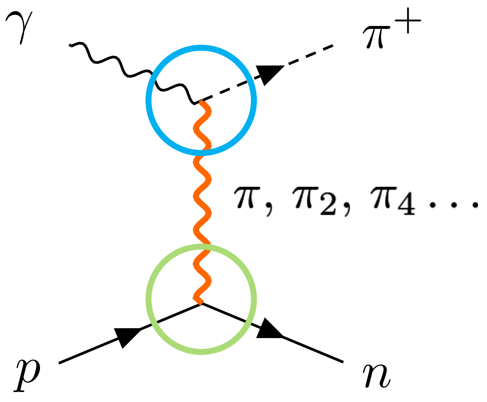

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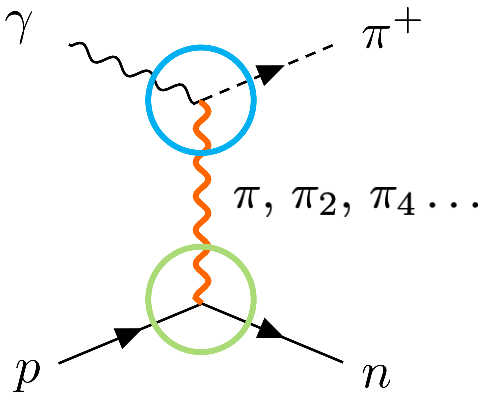

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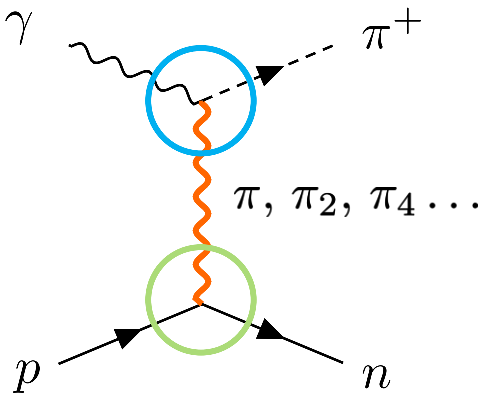

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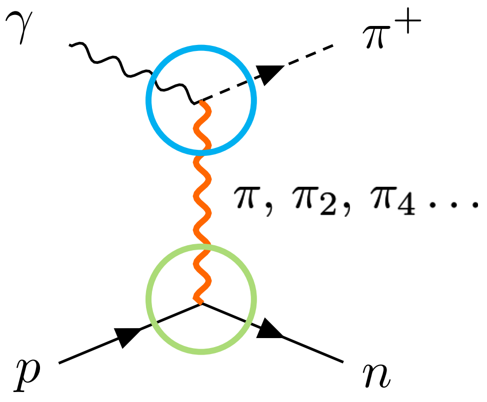

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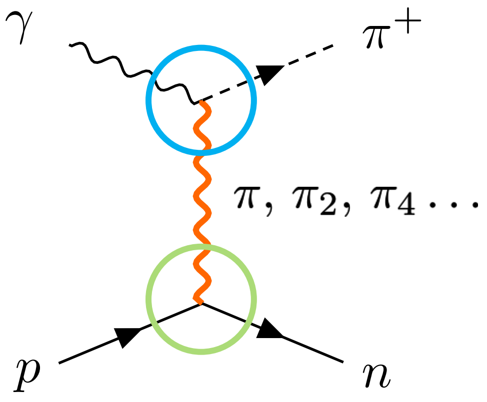

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vs $\mathcal{P}^{\text{Regge}}_\pi \propto \alpha'\,\tau\,\Gamma(-\alpha(t))\left(\frac{s}{s_0}\right)^{\alpha(t)}$

# Interpretation of electric Born amplitude at large s

- ⁕ In the s-channel CM frame

$$A^{\mathrm{e}}_{\mu_\gamma \mu_i \mu_f} = 2\sqrt{2} g_{\pi NN} \left[ e_\pi \frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{t - \mu^2} + \underbrace{e_{N_i} \cancel{\frac{(\epsilon_{\mu_\gamma} \cdot p_i)}{s - M^2}} + e_{N_f} \frac{(\epsilon_{\mu_\gamma} \cdot p_f)}{u - M^2}}_{1/s} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

$$\sim \sqrt{t}$$

- ⁕ In the t-channel CM frame

$$A^{\mathrm{e}}_{\lambda_\gamma \lambda_i \lambda_f} = 2\sqrt{2} g_{\pi NN} \left[ e_\pi \cancel{\frac{(\epsilon_{\lambda_\gamma} \cdot p_\pi)}{t - \mu^2}} + e_{N_i} \frac{(\epsilon_{\lambda_\gamma} \cdot p_i)}{s - M^2} + e_{N_f} \frac{(\epsilon_{\lambda_\gamma} \cdot p_f)}{u - M^2} \right] \bar{u}_{\lambda_f}(p_f) \gamma_5 v_{\lambda_i}(-p_i)$$

# Interpretation of electric Born amplitude at large s

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$$A^{\mathrm{e}}_{\mu_\gamma \mu_i \mu_f} = 2\sqrt{2} g_{\pi N N} \left[ e_\pi \underset{\sim \sqrt{t}}{\underbrace{\frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{t - \mu^2}}} + \underbrace{e_{N_i} \cancel{\frac{(\epsilon_{\mu_\gamma} \cdot p_i)}{s - M^2}} + e_{N_f} \frac{(\epsilon_{\mu_\gamma} \cdot p_f)}{u - M^2}}_{1/s} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

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Re-write as 3 gauge-invariant terms proportional to the mass of the exchanged particle

$$A^{\mathrm{e}}_{\mu_\gamma \mu_i \mu_f} = 2\sqrt{2} g_{\pi N N} \Bigg[ e_\pi \left( \frac{\epsilon \cdot (p_\pi - k/2)}{t - m_\pi^2} + \frac{\epsilon \cdot P}{s - u} \right)$$

$$+ \frac{1}{2} e_{N_i} \left( \frac{\epsilon \cdot p_\pi}{s - m_N^2} + \frac{\epsilon \cdot P}{s - u} \frac{t - m_\pi^2 - k^2}{s - m_N^2} \right) - \frac{1}{2} e_{N_f} \left( \frac{\epsilon \cdot p_\pi}{u - m_N^2} + \frac{\epsilon \cdot P}{s - u} \frac{t - m_\pi^2 - k^2}{u - m_N^2} \right) \Bigg] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

# Interpretation of electric Born amplitude at large s

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$$A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} = 2\sqrt{2}g_{\pi NN}\left[\underset{\sim\sqrt{t}}{e_\pi\frac{(\epsilon_{\mu_\gamma}\cdot p_\pi)}{t-\mu^2}} + \underbrace{e_{N_i}\cancel{\frac{(\epsilon_{\mu_\gamma}\cdot p_i)}{s-M^2}} + e_{N_f}\frac{(\epsilon_{\mu_\gamma}\cdot p_f)}{u-M^2}}_{1/s}\right]\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i)$$

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$$A^{\mathrm{e}}_{\lambda_\gamma\lambda_i\lambda_f} = 2\sqrt{2}g_{\pi NN}\left[e_\pi\cancel{\frac{(\epsilon_{\lambda_\gamma}\cdot p_\pi)}{t-\mu^2}} + e_{N_i}\frac{(\epsilon_{\lambda_\gamma}\cdot p_i)}{s-M^2} + e_{N_f}\frac{(\epsilon_{\lambda_\gamma}\cdot p_f)}{u-M^2}\right]\bar{u}_{\lambda_f}(p_f)\gamma_5 v_{\lambda_i}(-p_i)$$

Re-write as 3 gauge-invariant terms proportional to the mass of the exchanged particle

$$A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} = 2\sqrt{2}g_{\pi NN}\left[e_\pi\left(\frac{\epsilon\cdot(p_\pi-k/2)}{t-m_\pi^2} + \frac{\epsilon\cdot P}{s-u}\right)\right.$$

vanishes in t-channel CM

only contribution s-channel CM, subleading in the t-channel CM

$$\left. +\frac{1}{2}e_{N_i}\left(\frac{\epsilon\cdot p_\pi}{s-m_N^2} + \frac{\epsilon\cdot P}{s-u}\frac{t-m_\pi^2-k^2}{s-m_N^2}\right) - \frac{1}{2}e_{N_f}\left(\frac{\epsilon\cdot p_\pi}{u-m_N^2} + \frac{\epsilon\cdot P}{s-u}\frac{t-m_\pi^2-k^2}{u-m_N^2}\right)\right]\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i)$$

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Re-write as 3 gauge-invariant terms proportional to the mass of the exchanged particle

$$A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} = 2\sqrt{2}g_{\pi NN}\left[e_\pi\left(\frac{\epsilon\cdot(p_\pi-k/2)}{t-m_\pi^2} + \frac{\epsilon\cdot P}{s-u}\right)\right.$$

➜ At large s, this is the *J=0* partial wave (π + *N* contributions)

vanishes in t-channel CM

only contribution s-channel CM, subleading in the t-channel CM

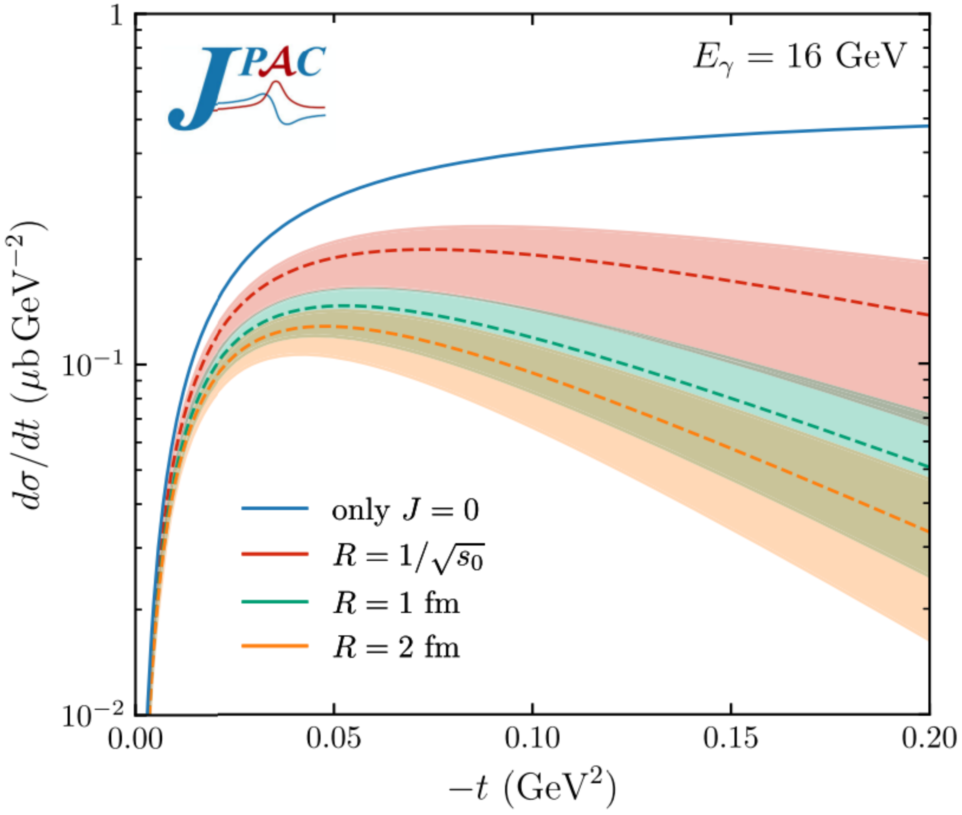
$$\left. + \frac{1}{2}e_{N_i}\left(\frac{\epsilon\cdot p_\pi}{s-m_N^2} + \frac{\epsilon\cdot P}{s-u}\frac{t-m_\pi^2-k^2}{s-m_N^2}\right) - \frac{1}{2}e_{N_f}\left(\frac{\epsilon\cdot p_\pi}{u-m_N^2} + \frac{\epsilon\cdot P}{s-u}\frac{t-m_\pi^2-k^2}{u-m_N^2}\right)\right]\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i)$$

# Results for pion photoproduction

GM, D.Winney, et al. (JPAC), Phys.Rev.D 110, 114012 (2024)

⁕ Reggeized pion exchange

$$A^{\text{Regge}}_{\lambda_\gamma \lambda_i \lambda_f}(s,t) \propto t(sR^2)^{\alpha(t)}$$



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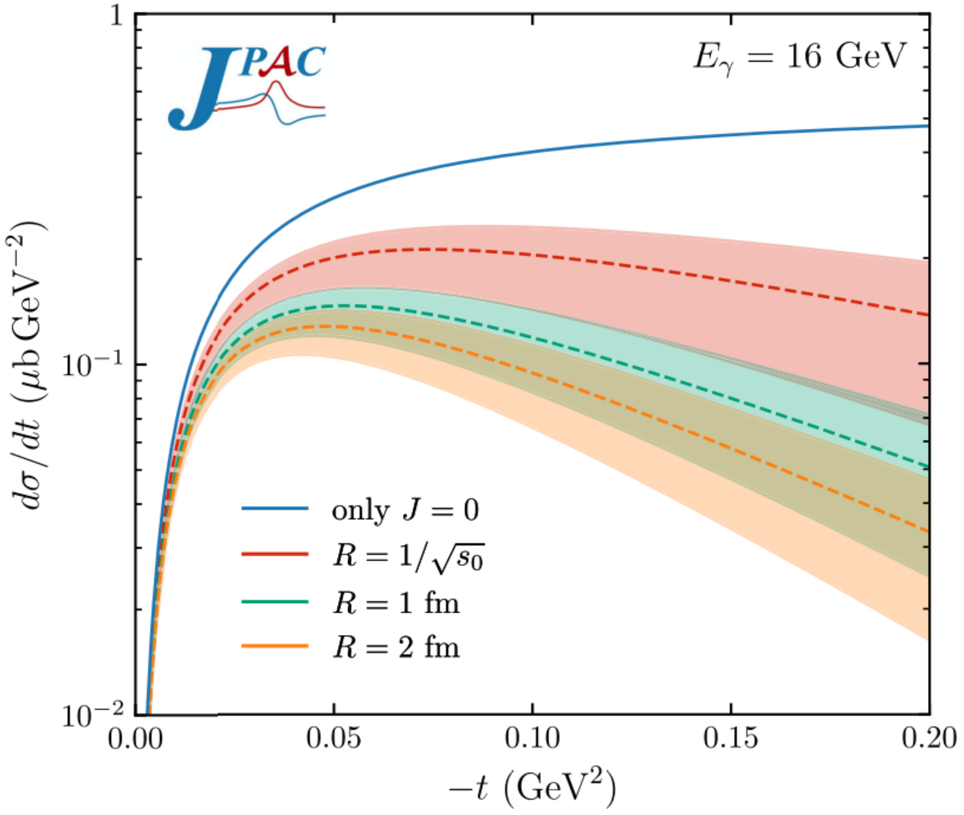
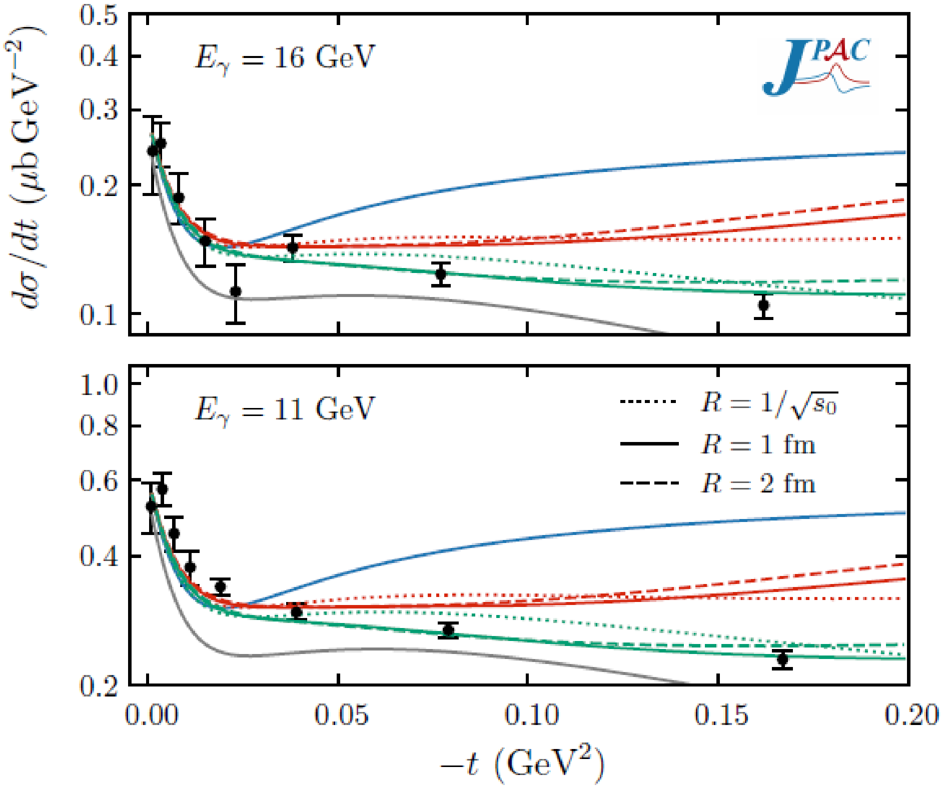


⁕ Adding the nucleon magnetic term

- With a form factor (missing Regge exchanges)

Comparison with VGL approach (gray line)



# Summary and outlook

- ✳ Understanding pion exchange is crucial to describe charge exchange reactions at GlueX/CLAS at small momentum transfer

- ✳ Our approach reconciles gauge invariance and Regge theory

  Key step → analytical continuation to *J=0* gives a finite contribution

  Interpretation in terms of Born diagrams: contribution of π + *N* Exchange diagrams

- ✳ Next step: extension to electroproduction and pion form factor