Unitarity and Universality

from Kaons with KLOE

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History

1947: Rochester and Butler, $K^0 \to \pi^+\pi^-$, $K^+ \to \pi^+\pi^0$

1953: Gell-Mann (Nisijima) strangeness

1958: Gell-Mann and Feynman Universal V-A interaction.
$\Lambda \to pe\nu \sim 1.6\%$, $\Sigma^- \to ne^-\nu \sim 5.6\%$

1958: Columbia-Pisa: should see 6 events each, none found

1963: Cabibbo, $s-d$ mixing. $\sin\theta_c = .26$

1970: GIM, 2 quark family mixing

1973: Kobayashi and Maskawa, 3 quark family mixing

$--$: GWS, the standard model

2004: PDG, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 < 1??$

Standard Model

$$\frac{g}{\sqrt{2}} W_\alpha^+ (\bar{\mathbf{U}}_L \mathbf{V}_{\text{CKM}} \gamma^\alpha \mathbf{D}_L + \bar{e}_L \gamma^\alpha \nu_{e\,L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu\,L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau\,L}) + \text{h.c.}$$

There is just one gauge coupling g, $G_F = g^2/(4\sqrt{2}\,M_W^2)$

The quarks are mixed by a unitary matrix **V**

Quarks and leptons have the same weak charge

At the opposite extreme:

$g_q, \quad g_e, \quad g_\mu, \quad g_\tau$

V is not unitary

(But probabilities are conserved!)

$$\mathbf{U} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \end{pmatrix}$$

Decay Width

From $\langle f|J_\alpha J^\alpha|i\rangle \Rightarrow$

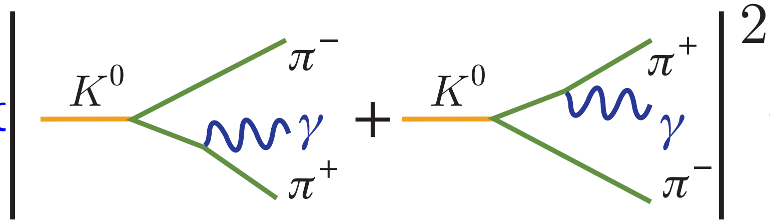
$$\Gamma(|\Delta S|=1)=|V_{us}|^2\times G^2\times F_1(\text{masses})\times F_2(\text{emi})\times F_3(\text{si}).$$

1. F_1: kinematics, spinor algebra, phase space, OK
2. F_2: Cannot turn off electromagnetism. Lengthy but necessary. Few % correction, to 0.1% accuracy
3. F_3: Strong interactions are hardest to compute, can have large effects, choose best processes, lattice to the rescue!

F_2 – Radiation must be included

$m(\gamma) = 0 \Rightarrow$ charge particles radiate.

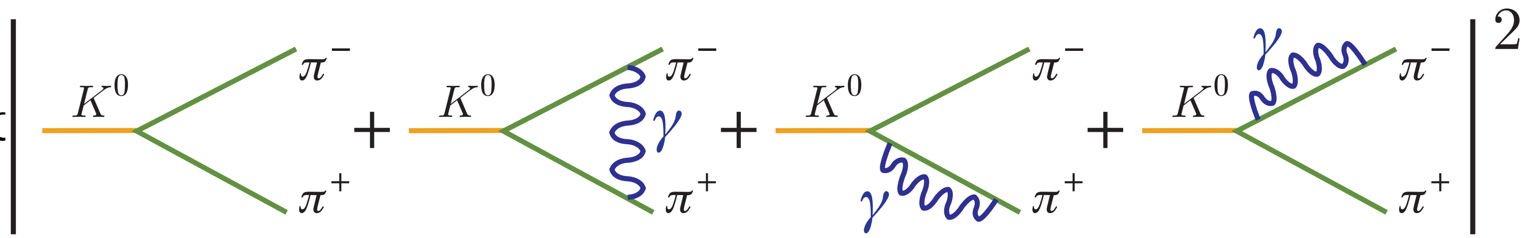
Take for example $K_S \to \pi^+\pi^-$. In the real world the em interaction gives $\Gamma_1 \propto$



$\to \infty$ for $\omega \to 0$. .

The infinity is cancelled by an opposite sign contribution:

$\Gamma_2 \propto$



.

$\Gamma_1 + \Gamma_2$ is finite but contains a correction of $\mathcal{O}\left(\frac{\alpha}{\pi} \times \log \frac{\omega_0}{m}\right)$.

ω_0 is finite and experimental acceptance dependent.

Radiation inclusive branching ratios

Inclusion (or exclusion) of photons does affect event acceptance. It is modern practice to give results fully inclusive of radiation up to the kinematic limit. This requires correct accounting of radiation in the Monte Carlo detector simulation program.

In the KLOE MC simulation, Geanfi, radiation is included at the event generation level, event by event. In the following, even if not explicitly stated, BRs are totally inclusive of radiation.

BR($K\to f(\gamma)$) stands for BR($K\to f,\ f+\gamma, 0<\omega<\omega_{\rm max}$)

(I will also just use BR($K\to f$))

F_3 – SI – Hadrons vs Quarks

Nuclear β-decay. Look at $0^+ \to 0^+$ transitions. No axial current.

CVC helps with nuclear matrix elements.

$|\Delta S| = 1$ decays. Corrections to

$$\langle f|\bar{u}\gamma_\alpha s|i\rangle$$

appear only at 2$^{\rm nd}$ order in $m_s - m_d$. A-G theorem.

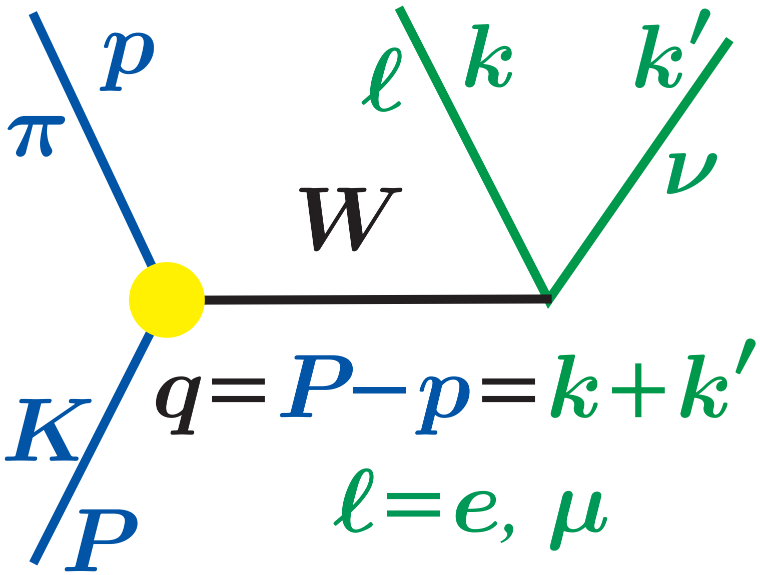
Kaon semileptonic decays. $K \to \pi\ell\nu \ \Rightarrow 0^- \to 0^-$.

Only vector part of current contributes to

$$\langle \pi|\bar{u}_{\rm L}\gamma_\alpha s_{\rm L}|K\rangle$$

$$\langle \pi|\bar{u}\gamma_\alpha s|K\rangle = f_+(t)(P+p)_\alpha + f_-(t)(P-p)_\alpha$$

Vector Current is protected



$$\langle\pi|J_\alpha|K\rangle = f_+(0)\times\Big((P+p)_\alpha\,\tilde f_+(t) + (P-p)_\alpha\,\big(\tilde f_0(t)-\tilde f_+(t)\big)\,\frac{\Delta}{t}\Big)$$

$$\Delta = M^2 - m^2$$

Form factor because K, π are not point like

Scalar and vector FFs equal at $t{=}0$ (by construction)

Factor out $f_+(0)$. $f_+(0) < 1$ because $K \neq \pi$

$f_+(0) - 1 \ll 1$ (0.04) – small SU(3) breaking

$\tilde f_+(0) = \tilde f_0(0) = 1$

Decay Width

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{768\,\pi^3} S_{\rm EW}\, |V_{us}|^2\, f_+(0)^2\, I_{K\ell} \times \left(1 + \delta_K^{\rm SU(2)} + \delta_{K\ell}^{\rm EM}\right)$$

192

128

$C_K^2 = 1(1/2)$ for $K^0(K^\pm)$; $S_{\rm EW} = 1.0232(3)$, EW corr, Sirlin

$\Downarrow$ Measure: τ, BR, $\tilde{f}_+$, $\tilde{f}_0$ $\Downarrow\Uparrow$ Compute: χPT, Quark M, Lattice

Define

$f_+(0)$ as

$f_+^{K^0\to\pi^\pm}(0)$

	$\delta_K^{SU(2)}$(%)	$\delta_{K\ell}^{\rm EM}$(%)
K^0_{e3}	0	+0.57(15)
K^+_{e3}	2.36(22)	+0.08(15)
$K^0_{\mu 3}$	0	+0.80(15)
$K^+_{\mu 3}$	2.36(22)	+0.05(15)

Phase space integral, K_{e3}

$$\rho(E_e,\ E_\nu) \propto \sum_{\text{spins}} |\mathfrak{M}|^2 \propto G^2\, C_K^2\, M^4 \left[\frac{E_e}{M}\,\frac{E_\nu}{M} + \frac{\vec{k}\cdot\vec{k}'}{M^2}\right]$$

$$I_{K\ell} \propto \iint \rho(E_e\, E_\nu)\, \mathrm{d}E_e\, \mathrm{d}E_\nu$$

$$\Gamma = \frac{G^2\, C_K^2\, M_K^5}{768\,\pi^3} \underbrace{\int_{zm}^{zM} \mathrm{d}z\ f(t)^2 \int_{xm(z)}^{xM(z)} 24\,((z+x-1)(1-x)-\alpha)\,\mathrm{d}x}_{I_{K\ell}}$$

With 768 in the denominator, $f(t)=1$, all final state masses zero:

$$I_{K\ell} = 1.0$$

768 corresponds to 192 in muon decay rate. $I_{K\ell}$ is dimensionless.

$$x = \frac{2E_e}{M},\ \ y = \frac{2E_\nu}{M},\ \ z = \frac{2E_\pi}{M},\ \ \alpha = \frac{m}{M}$$

End of introduction

DAΦNE

An e^+e^- collider operated (mostly) at $W = M_\phi$

$$e^+e^- \to \phi \to \begin{cases} K_S + K_L & 34.0\% \\ K^+ + K^- & 49.3\% \end{cases}$$

$\Rightarrow$ Tagged, monochromatic, pure K_S, K_L, K^+, K^- beams

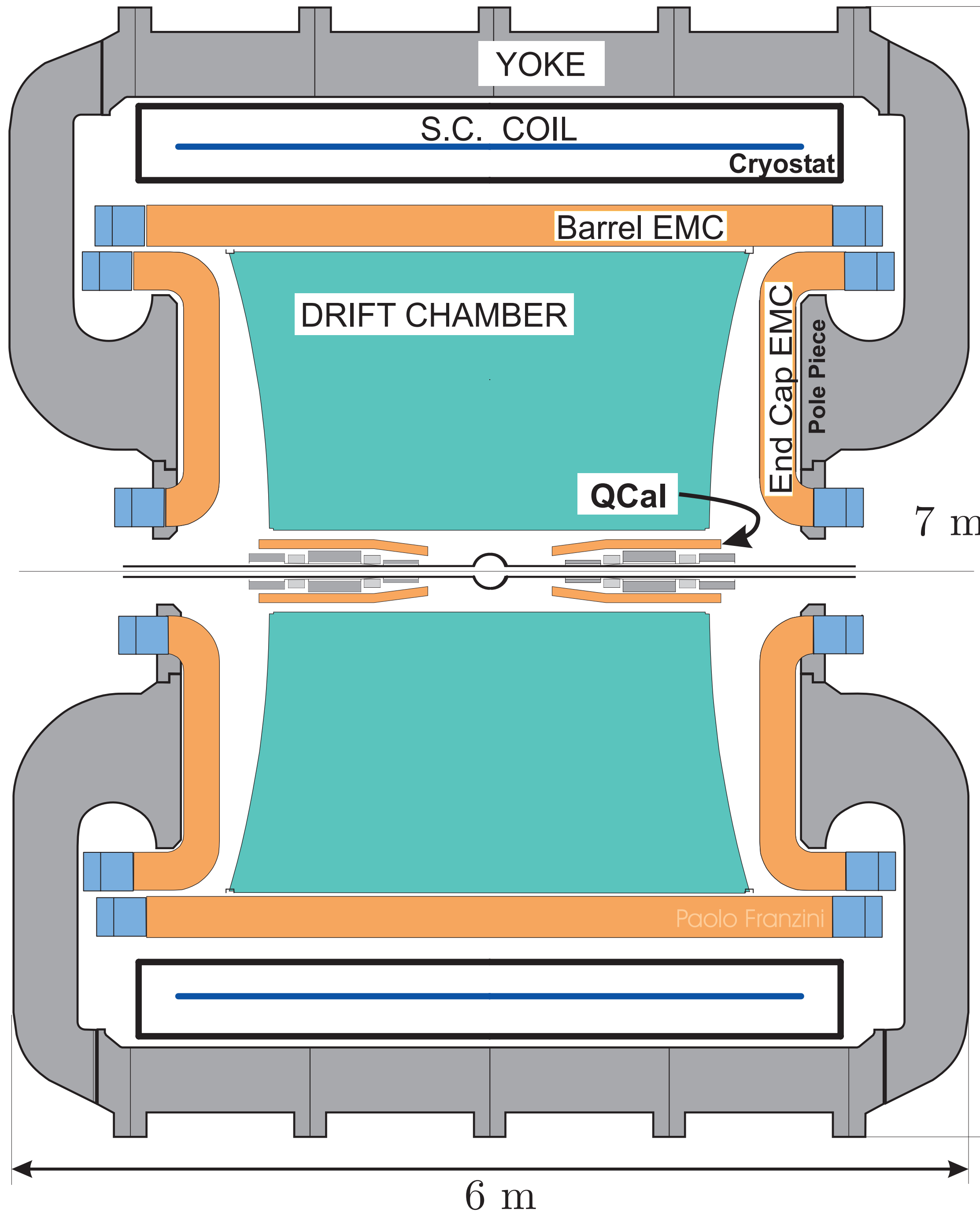
$$\mathrm{BR}(K_S \to \pi\ell\nu) = (7.04 + 4.69) \times 10^{-4} \quad \text{“difficult”}$$

$$\mathrm{BR}(K^\pm \to \pi\ell\nu) = (4.98 + 3.32) \times 10^{-2} \quad \text{“so so”}$$

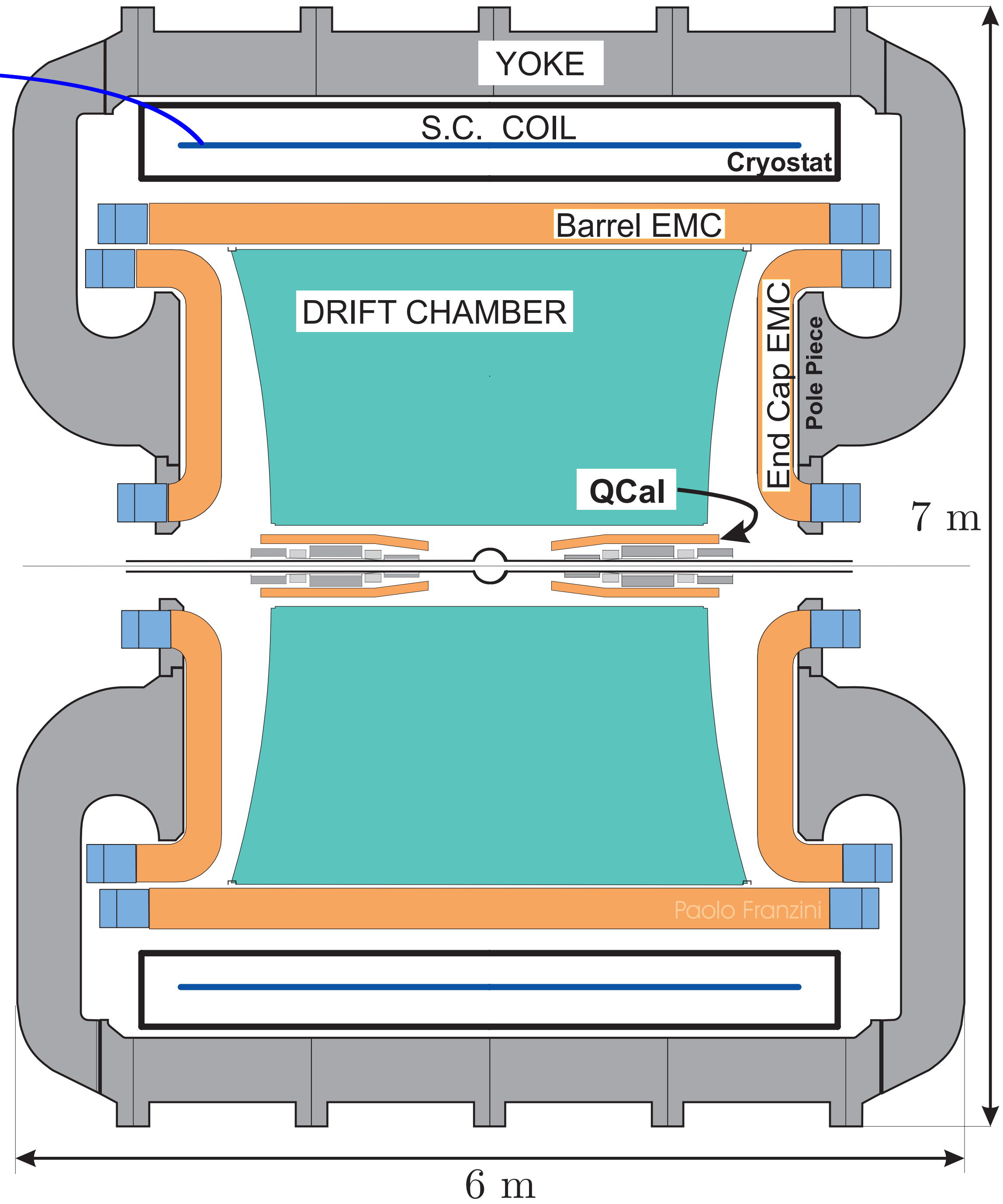
$$\mathrm{BR}(K_L \to \pi\ell\nu) = (4.06 + 2.71) \times 10^{-1} \quad \text{“best”}$$

$$e \qquad \mu$$

KLOE


YOKE
S.C. COIL
Cryostat
Barrel EMC
DRIFT CHAMBER
End Cap EMC
Pole Piece
QCal
Paolo Franzini
7 m
6 m

Magnet
SC Coil, B=0.6 T



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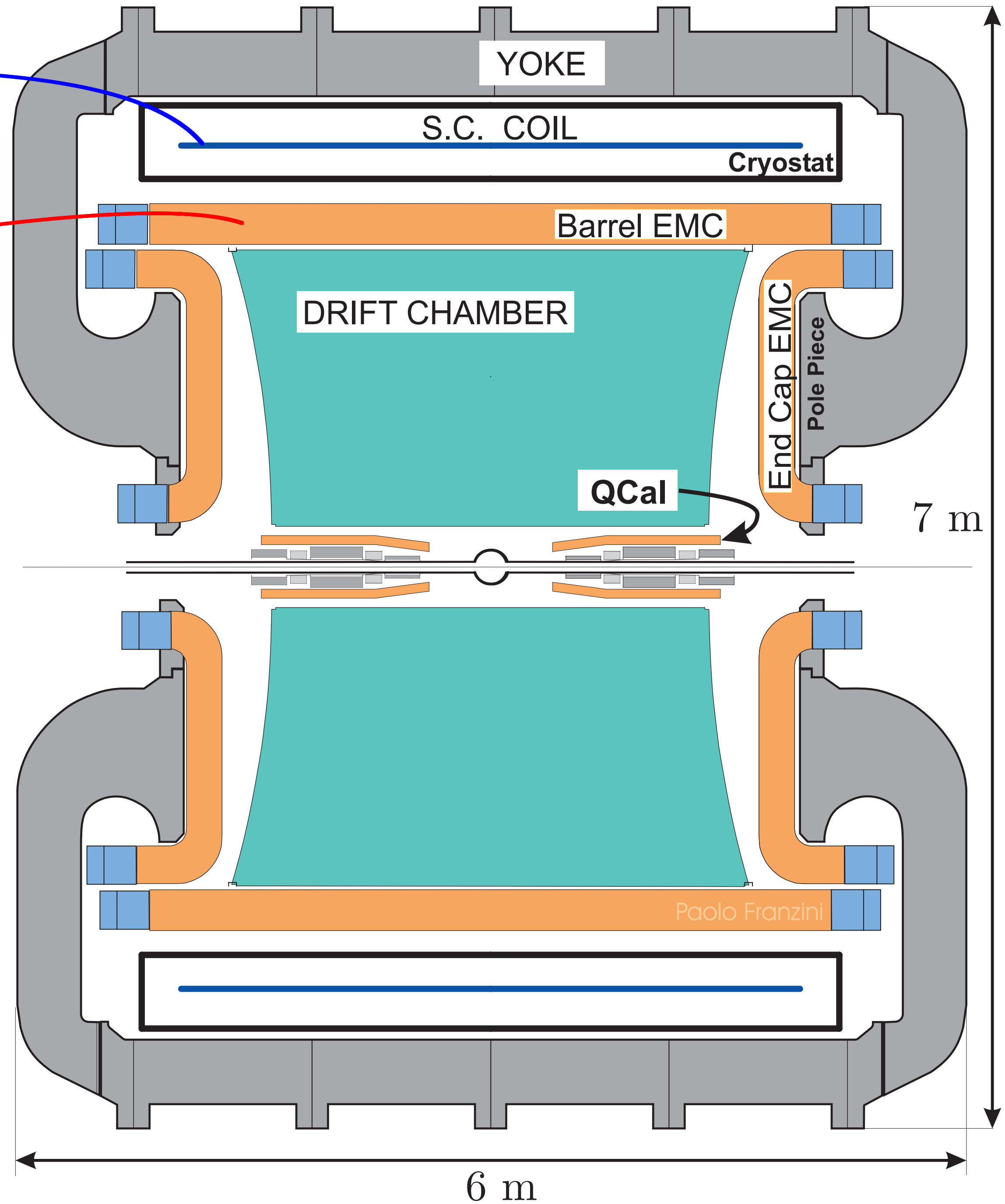
Magnet
SC Coil, B=0.6 T

EM Calor.
Pb–scint fiber
4880 pm, 2440 cells



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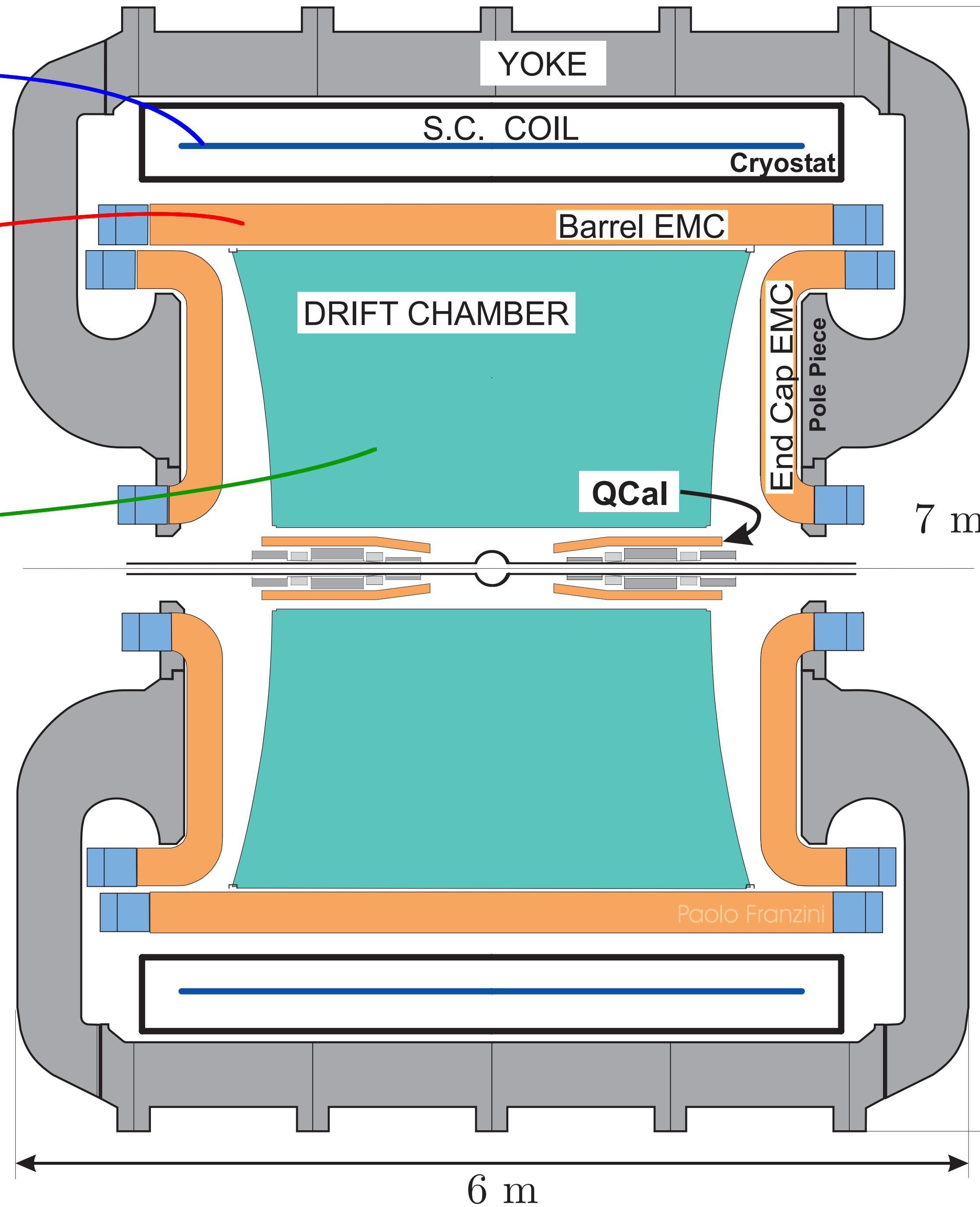
Magnet
SC Coil, B=0.6 T

EM Calor.
Pb–scint fiber
4880 pm, 2440 cells

Drift Ch.
12582 sense wires
52140 tot wires
Carbon fiber walls



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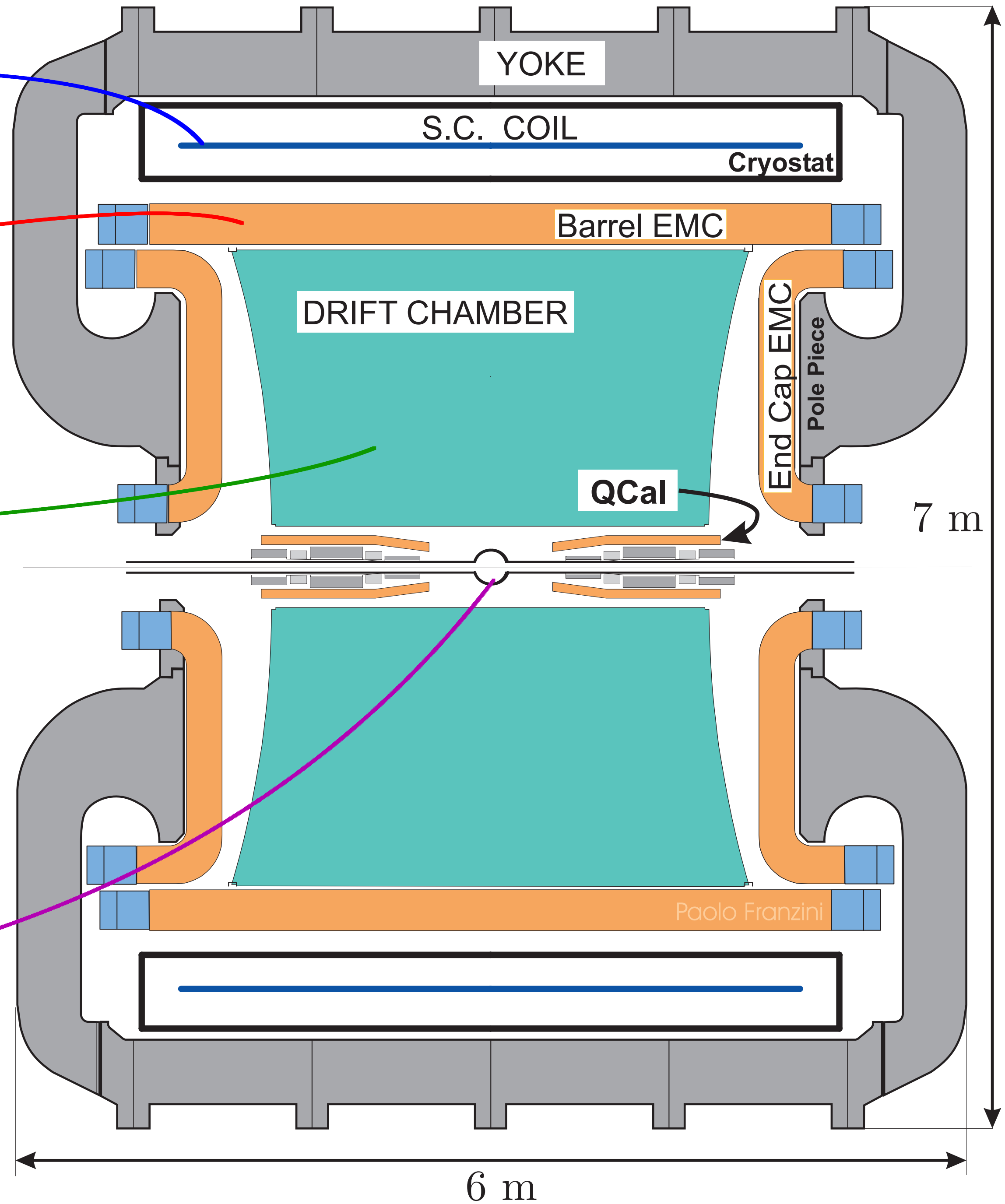
Magnet
SC Coil, B=0.6 T

EM Calor.
Pb–scint fiber
4880 pm, 2440 cells

Drift Ch.
12582 sense wires
52140 tot wires
Carbon fiber walls

Al-Be beam pipe
r=10 cm, 0.5 mm thick



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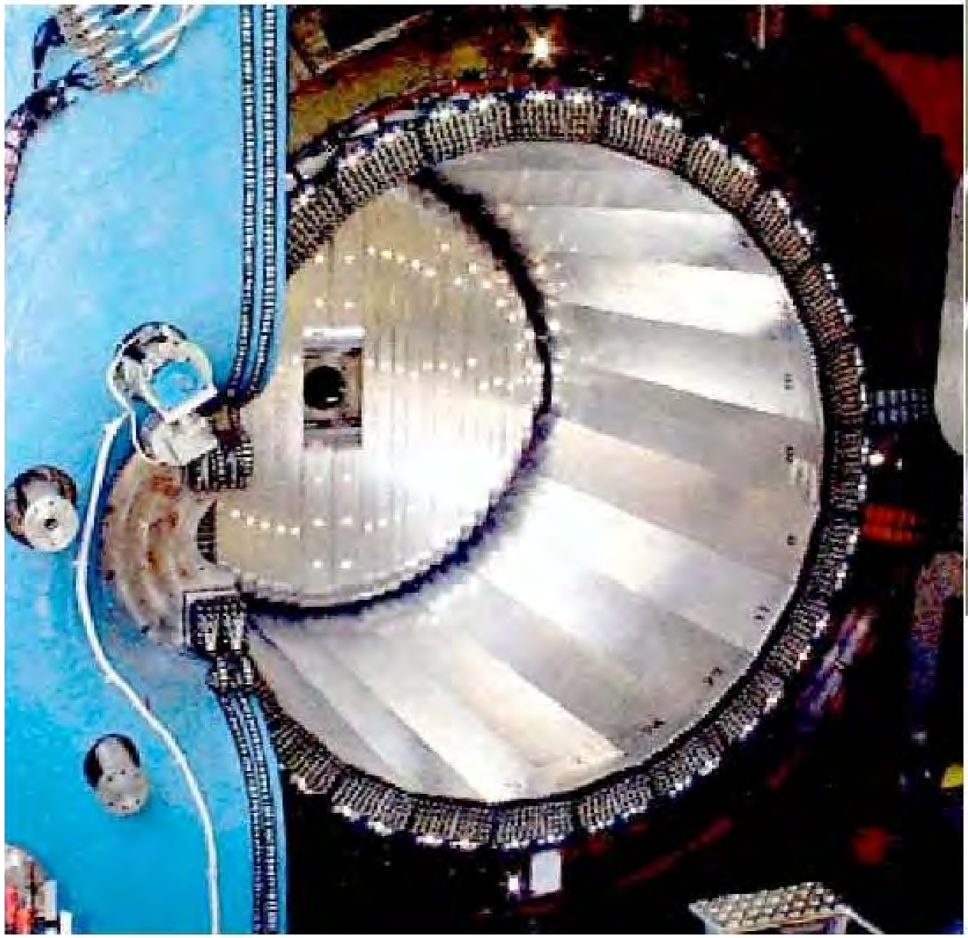
$$\sigma_E/E = 5.7\%/\sqrt{E\ (\mathrm{GeV})}$$
$$\sigma_t = 54/\sqrt{E\ (\mathrm{GeV})} \oplus 100\text{-}150\ \mathrm{ps}$$

Drift chamber

$$\sigma(p_\perp)/p_\perp = 0.4\%$$
$$\sigma_{x,y} = 150\ \mu\mathrm{m};\ \sigma_z = 2\ \mathrm{mm}$$



EM Calorimeter

KLOE precision measurements

$$M(K^0) = 497,583 \pm 5 \pm 20 \text{ keV}$$
$$M(\eta) = 547,874 \pm 7 \pm 29 \text{ keV}$$

Using $M(\phi)$ measurements based on electron anomaly

a_e=0.0011596521859±0.0000000000038, Novosibirsk

KLOE calibration

KLOE is continuously calibrated with data

Time, energy and position scales are calibrated every 1-2 hours

Machine energy, collision point, $p(\phi)$ in lab

Together with $\mathcal{L}$ and $\int\mathcal{L}\mathrm{d}t$, all provided to DAΦNE

Analysis

Prefilter. Reconstruction ∼2 hours after data taking

Initial classification

Massive Monte Carlo production. MC includes run by run
background simulation

Tracking efficiency measured from data

Photon efficiency measured from data

Signal shapes and backgrounds from control samples

Continuous improvement of MC with feedback from
physics analisys

Particle ID by time of flight

Particle ID by shape of energy release in EMC cells

Measuring BRs

We only measure "absolute" BR, *i.e.* Γ_i/Γ, not ratios (Γ_i/Γ_j) of partial rates. We have three ways to do it:

1. Measure single BR's
2. Measure (almost) all BR
3. Measure all partial rates: $\Rrightarrow$ BRs & $\Gamma = 1/\tau$

We only use tagged kaons. Trigger efficiency must not depend on decay mode of tagged kaon. For every event, we verify the trigger was due to the tagging kaon.

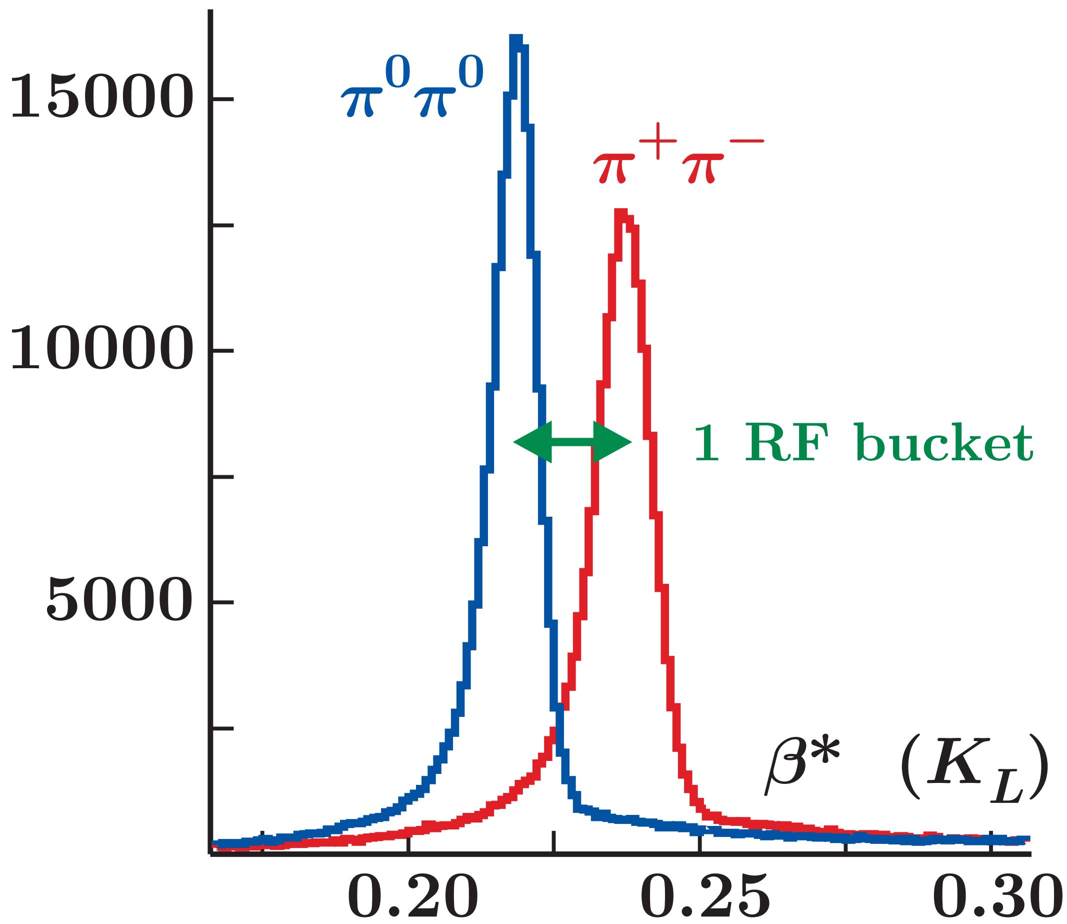
There are differences between K_S, K_L and $K^{\pm}$:

1. 1 vs 2 charged decay products
2. Tag by 1 or 2 body decays
3. Tag by time of flight

The K_S beam

First level, crude K_S tag by K_L TOF

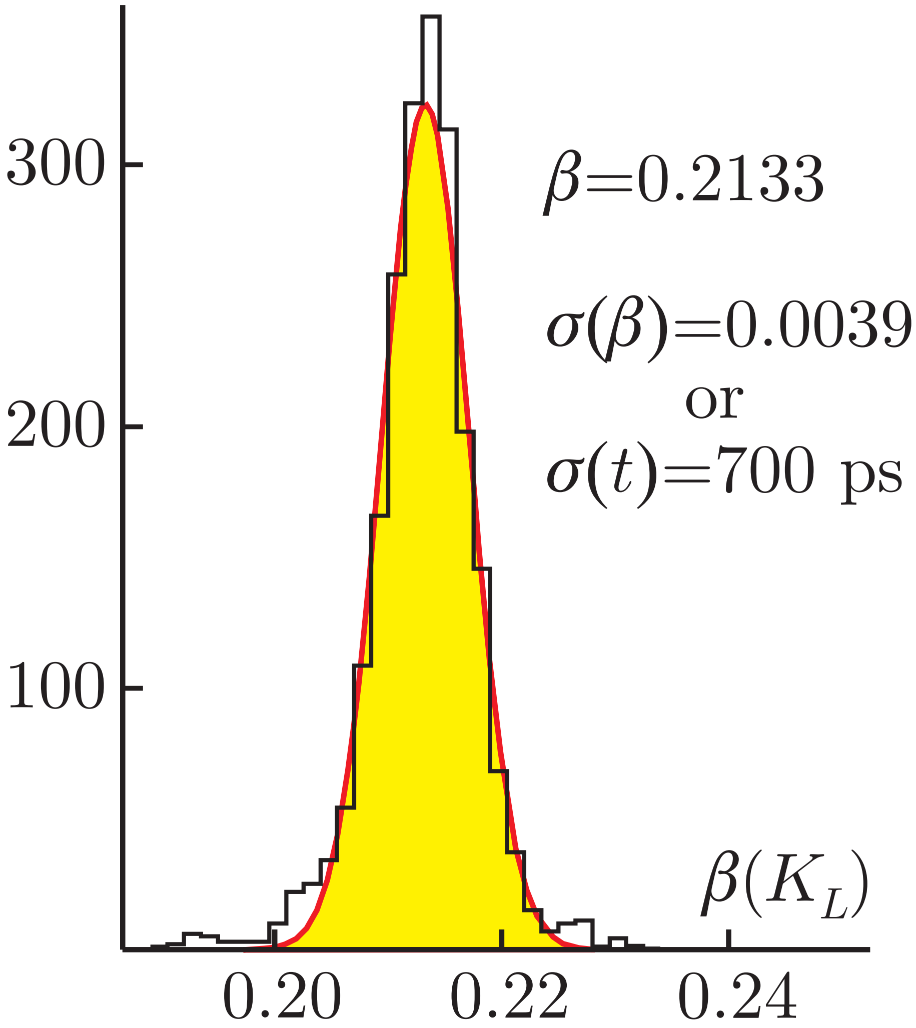
$\sim$50% of the K_L-mesons reach the calorimeter where most interact. Since $\beta(K^0)\sim$0.2, we identify kaon interactions by TOF.



K_L interacting in the calorimeter give an ideal K_S tag, almost independent of K_S decay mode. The value of T_0 here is obtained from the first particle to reach the calorimeter.

π, μ, γ *or* e

$K_S \to \pi^{\pm} e^{\mp} \nu$



$\delta W(\mathrm{DA\Phi NE})$=1 MeV
gives $\delta\beta$=0.004

After event reconstruction very clean signal tags the presence of K_S. Search for two track decays near IP.

Reject $K_S \to \pi^+ \pi^-$.

Use TOF difference for two tracks taken as $\pi - e$ or $e - \pi$.

Provides electron ID and charge of lepton.

Use $|E_{\rm miss} - p_{\rm miss}|$ to isolate $K_{S,\,e3}$ events.

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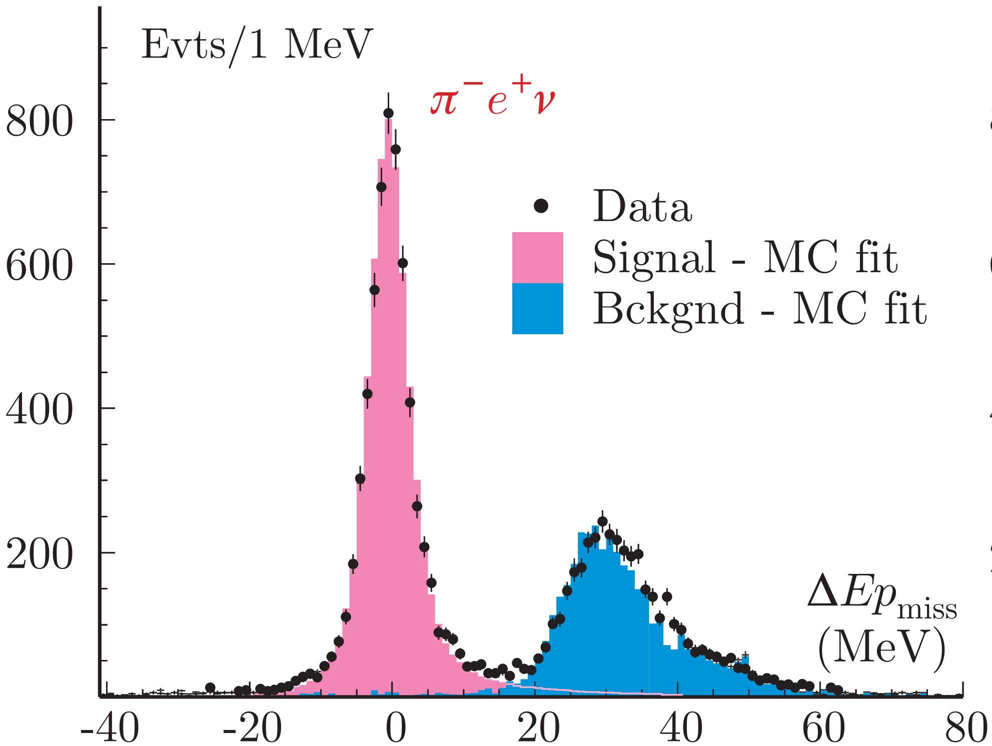
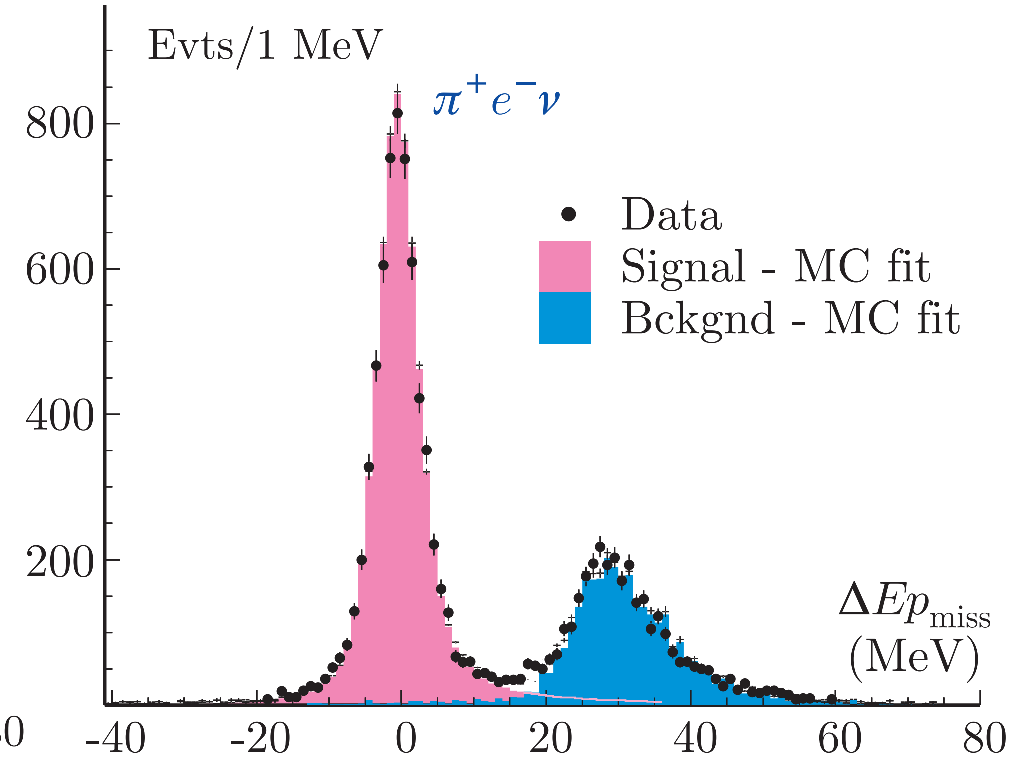
$K_S \to \pi e \nu$, 2005

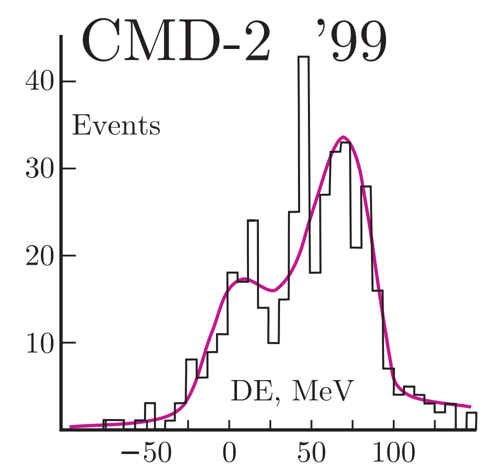




~13,000 signal events

BR($K_S \to \pi e \nu$)=$(7.046 \pm 0.091) \times 10^{-4}$

$|V_{us}|$ to 0.7%



Dominant K_L decay modes

We know (KLOE, PDG, etc)

$$\Sigma_{\rm B} = {\rm BR}(K_{Le3}) + {\rm BR}(K_{L\mu3}) + {\rm BR}(K_L \to 3\pi^0)$$
$$+ {\rm BR}(K_L \to \pi^+\pi^-\pi^0) = 1 - 0.0034(\pm 0.00004)$$

We assume

$$\tau(K_L) = 51.7 \text{ ns}$$

We measure:

$$\Sigma_{\rm B} = 1.0104 - 0.0034$$

GOOD!

$$\text{We can solve for : } \begin{cases} \text{4 BRs} \\ K_L \text{ lifetime} \end{cases}$$

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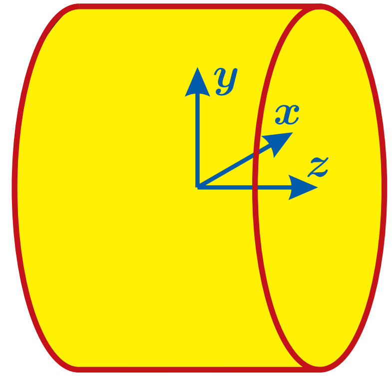
A step at a time



Detect $K_S \to \pi^+\pi^-$, $\Rightarrow$ tagged K_L beam

Find: a) $K_L \!\to$ 2 charged particles: b) $K_L \!\to\! 3\pi^0$

Classify a) as $e\pi\nu$, $\mu\pi\nu$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-$

Determine "tag bias" corrections

Determine FV acceptance & all other efficiencies

Count in FV. FV: $35 < x,\ y < 150$ cm, $|z| < 120$ cm

All the above requires knowledge of τ. Use old value τ^0=51.7 ns.

In KLOE,

$$\mathcal{A}_{\mathrm{FV}} = \mathcal{A}^0_{\mathrm{FV}} \times \Big(1 + 0.0128\ \mathrm{ns}^{-1}(\tau^0 - \tau)\Big)$$

$\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow\Downarrow$

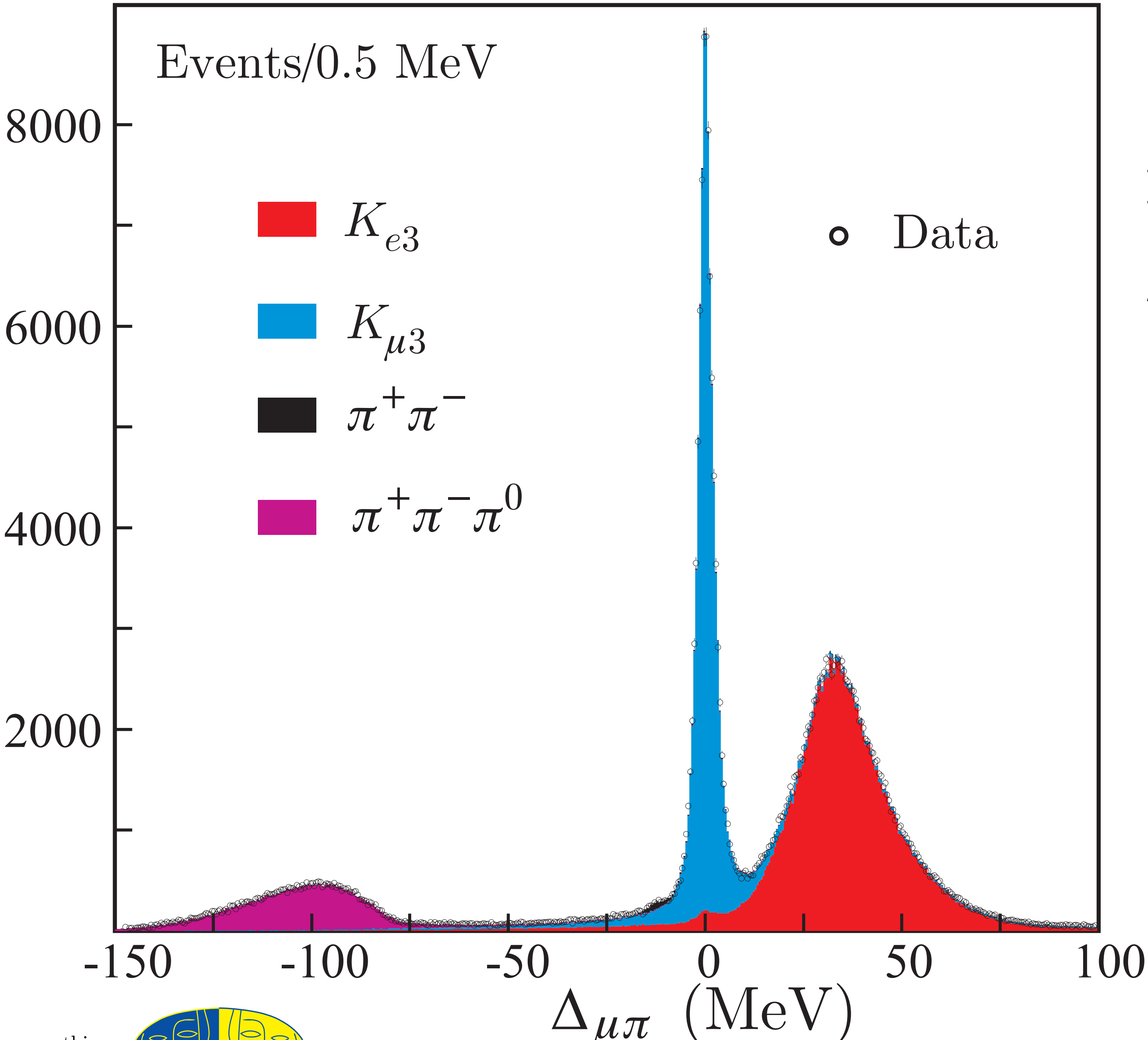
Solve for BRs and lifetime

$$\mathrm{BR}(K_{Le3}) = 0.4009 \pm 0.0015$$
$$\mathrm{BR}(K_{L\mu3}) = 0.2699 \pm 0.0014$$
$$\mathrm{BR}(K_L \to 3\pi^0) = 0.1996 \pm 0.0020$$
$$\mathrm{BR}(K_L \to {}^{+-0}) = 0.1261 \pm 0.0011$$
$$\tau(K_L) = 50.72 \pm 0.37 \text{ ns}$$

There are of course correlations

$$\begin{pmatrix} -0.25 & -0.56 & -0.07 & 0.25 \\ & -0.43 & -0.20 & 0.33 \\ & & -0.39 & -0.21 \\ & & & -0.38 \end{pmatrix}$$

$K_L \to$ two track events



For $K_L \to \pi^\pm \ell^\mp \nu$:

$E_{\text{miss}} \neq 0$

$\mathbf{p}_{\text{miss}} \neq 0$

$m_\nu = 0 \to$

$\Delta = E_{\text{miss}} - |\mathbf{p}_{\text{miss}}| = 0$

$\Delta_{\mu\pi}$=smallest of

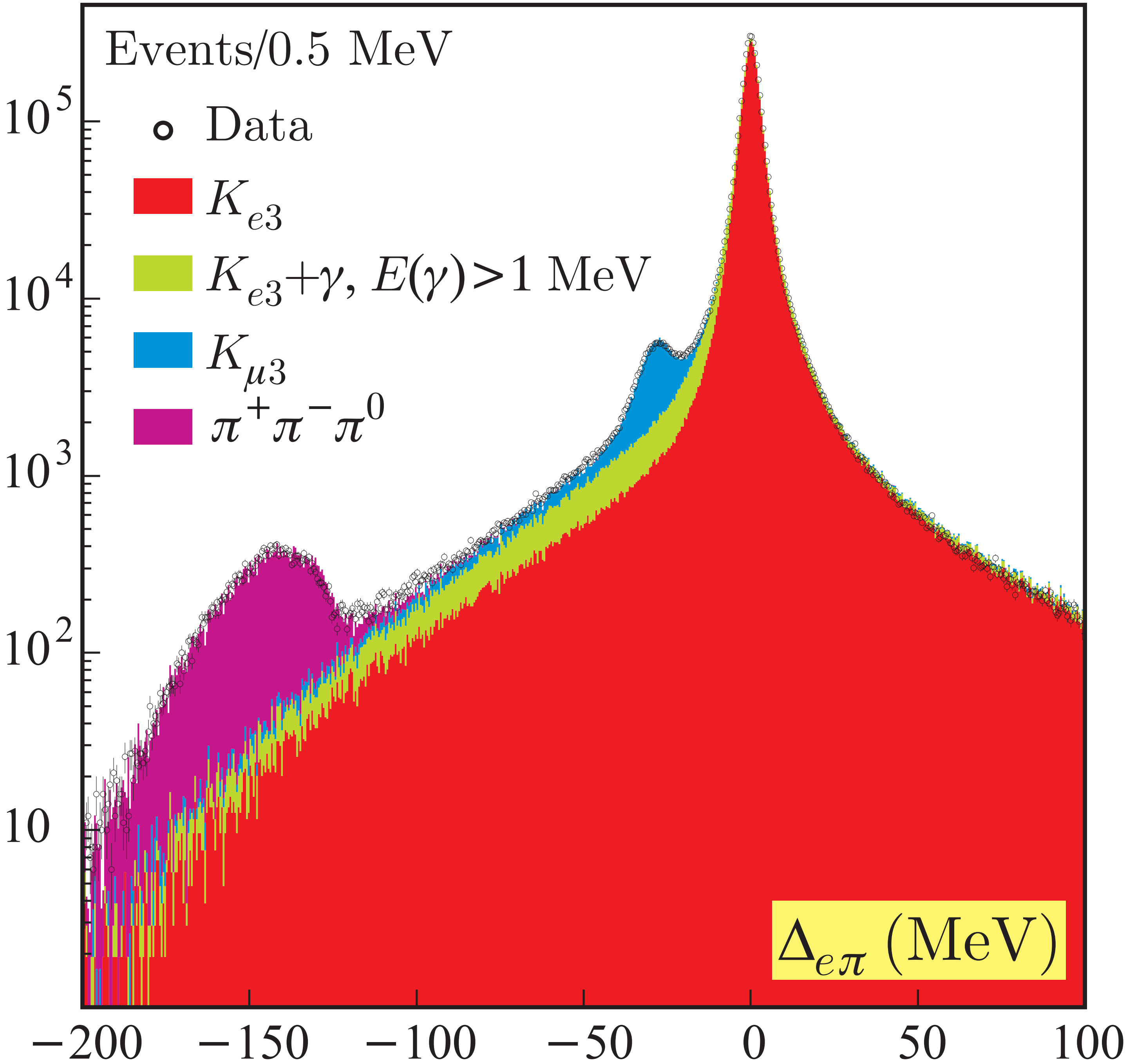
$\Delta_{\mu^+\pi^-}$, $\Delta_{\pi^+\mu^-}$

Fit to MC shapes

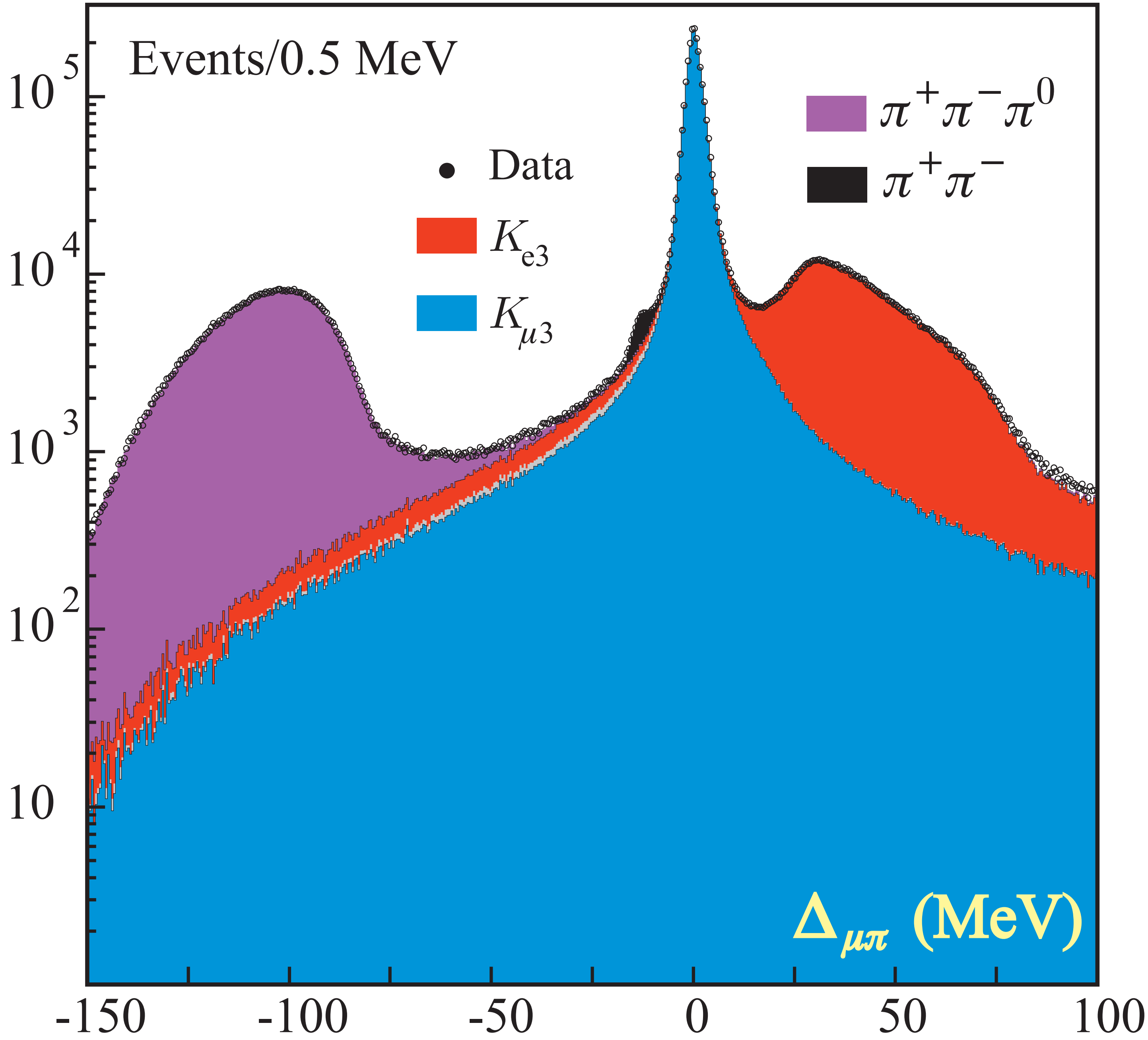
Count, BRs

14 sub-samples

Select electrons



Select muons



$K_L \to \pi^0\pi^0\pi^0$

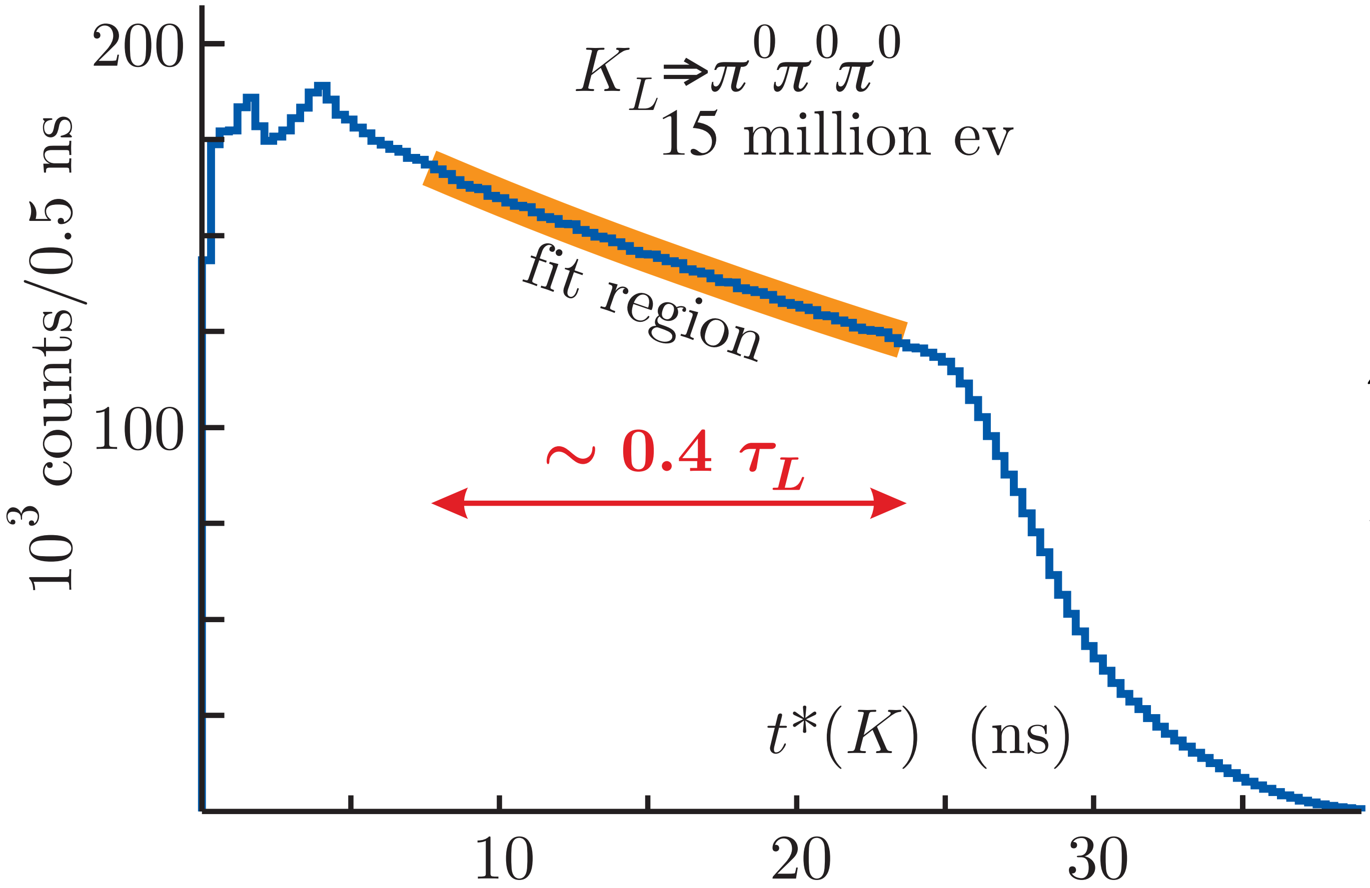
Count tagged K_L decaying to 3γ, 4γ, 5γ, 6γ,7γ

Efficiency $\mathcal{O}$(100%)

MC corrections

Background $\mathcal{O}$(0.001)

Time dependence of $K_L \to \pi^0\pi^0\pi^0$



$\tau(K_L) =$

50.92 ± 0.30 ns

All KLOE K_L results

Param.	Value	Correlation coefficients						
BR(K_{Le3})	0.4008(15)							
BR($K_{L\mu3}$)	0.2699(14)	−0.31						
BR($3\pi^0$)	0.1996(20)	−0.55	−0.41					
BR($+-0$)	0.1261(11)	−0.01	−0.14	−0.47				
BR($\pi^+\pi^-$)	$1.96(2) \times 10^{-3}$	−0.15	0.50	−0.21	−0.07			
BR($\pi^0\pi^0$)	$8.49(9) \times 10^{-4}$	−0.15	0.48	−0.20	−0.07	0.97		
BR($\gamma\gamma$)	$5.57(8) \times 10^{-4}$	−0.37	−0.28	0.68	−0.32	−0.14	−0.13	
τ_L	50.84(23) ns	0.16	0.22	−0.14	−0.26	0.11	0.11	−0.09

We can do it!

When PDG does it, combining inconsistent results from different experiments it can be a disaster!!! Till '06 they could not input correlations.

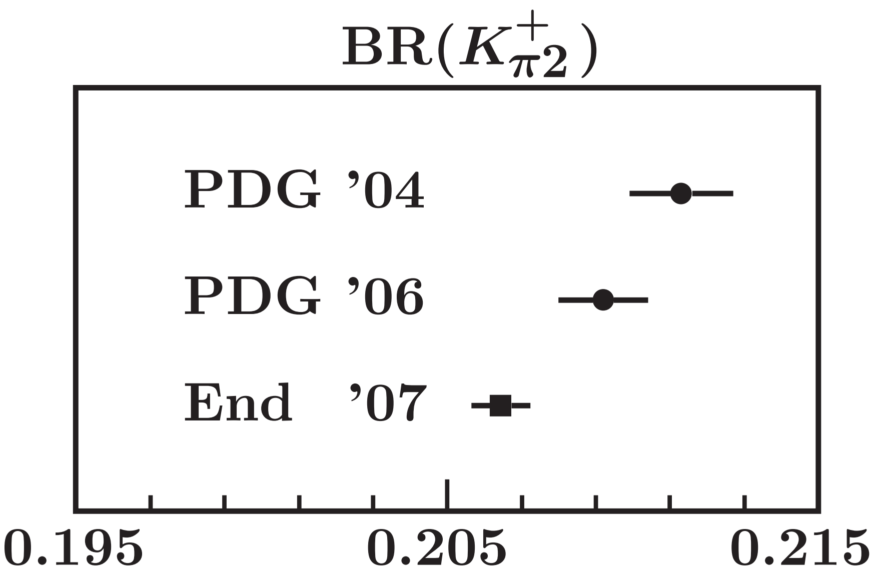
PDG Example

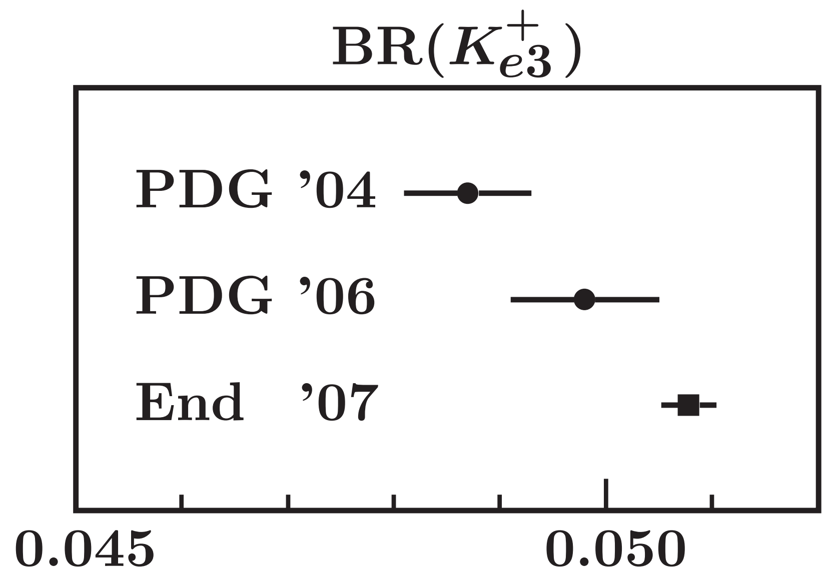
1. BR($K \to 3\pi^0$). NA31 '95: 0.2105±.0028 . PDG'04: 0.2105 ±0.0021. '08: 0.1951±0.0008. 5.3σ to 6.8σ off!

2. $3\pi^0/K_{e3}$ NA31 '95 and PDG '04 6.4σ to 7.6σ off!

3. BR($K^\pm \to \pi^+\pi^0$). Chiang '72: 0.2118±0.0028. PDG'04: 0.2113±0.0014. '08: .2064±.0008. 1.9σ to 3.2σ off!

4.





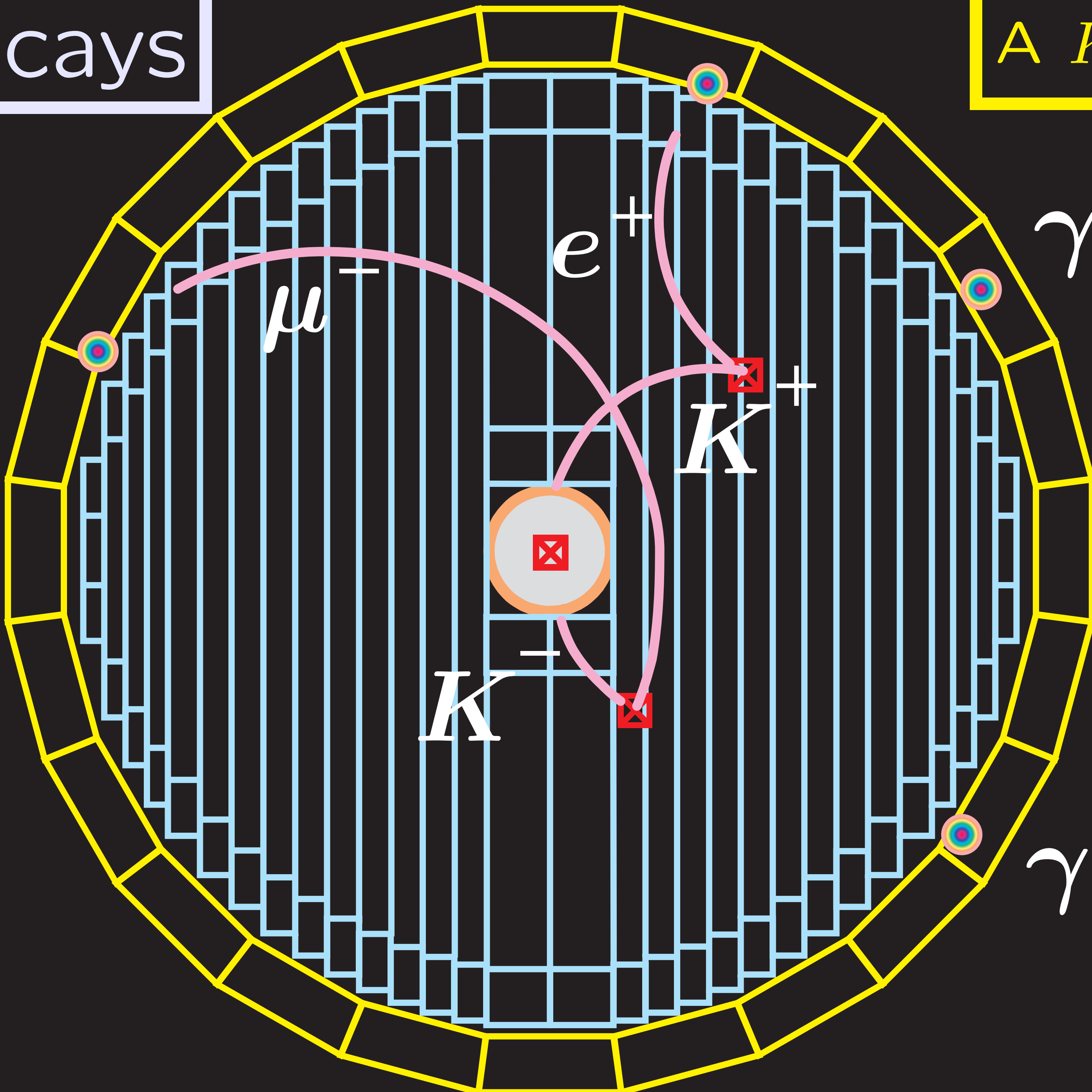
PDG masses

5. η mass. The PDG procedure should give $m(\eta)$=547.53±0.26 MeV, CL=10^{-15}. They include 3 old measurements which should give $m(\eta)$=547.51±0.13 MeV, CL=3×10^{-13}. They give $m(\eta)$= 547.51±0.18 MeV, CL=0.001???

6. K^+ mass. PDG procedure ⇒ $m(K^+)$=493.677±0.012 from 6 values. From only last two $m(K^+)$=493.686±0.019. They give 493.677±0.013. Also CL is 0.00035 (2.1×10^{-6}), they give CL=0.001.

$K^{\pm}$ decays

A $K^{\pm}_{e3}$ decay



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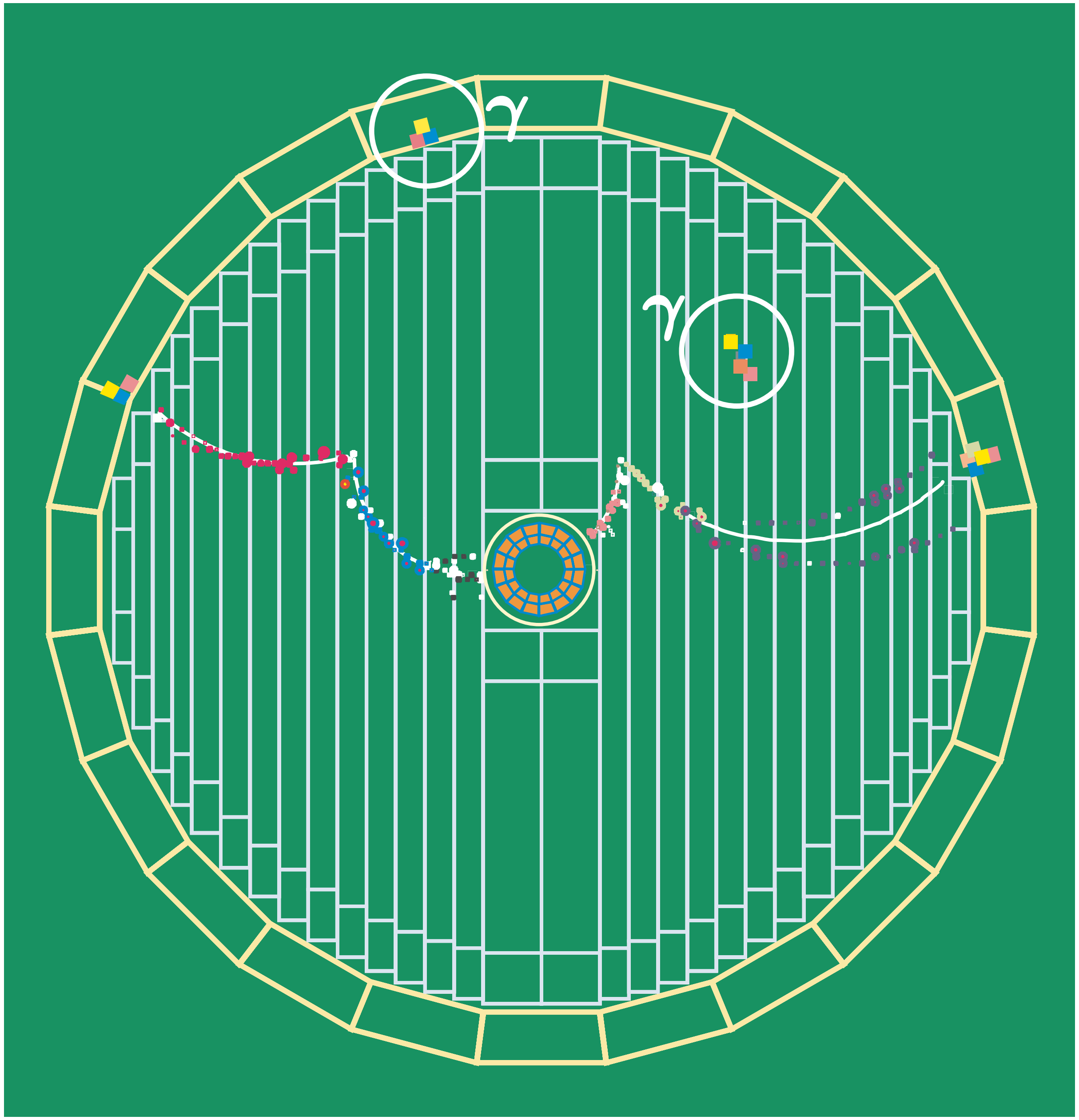
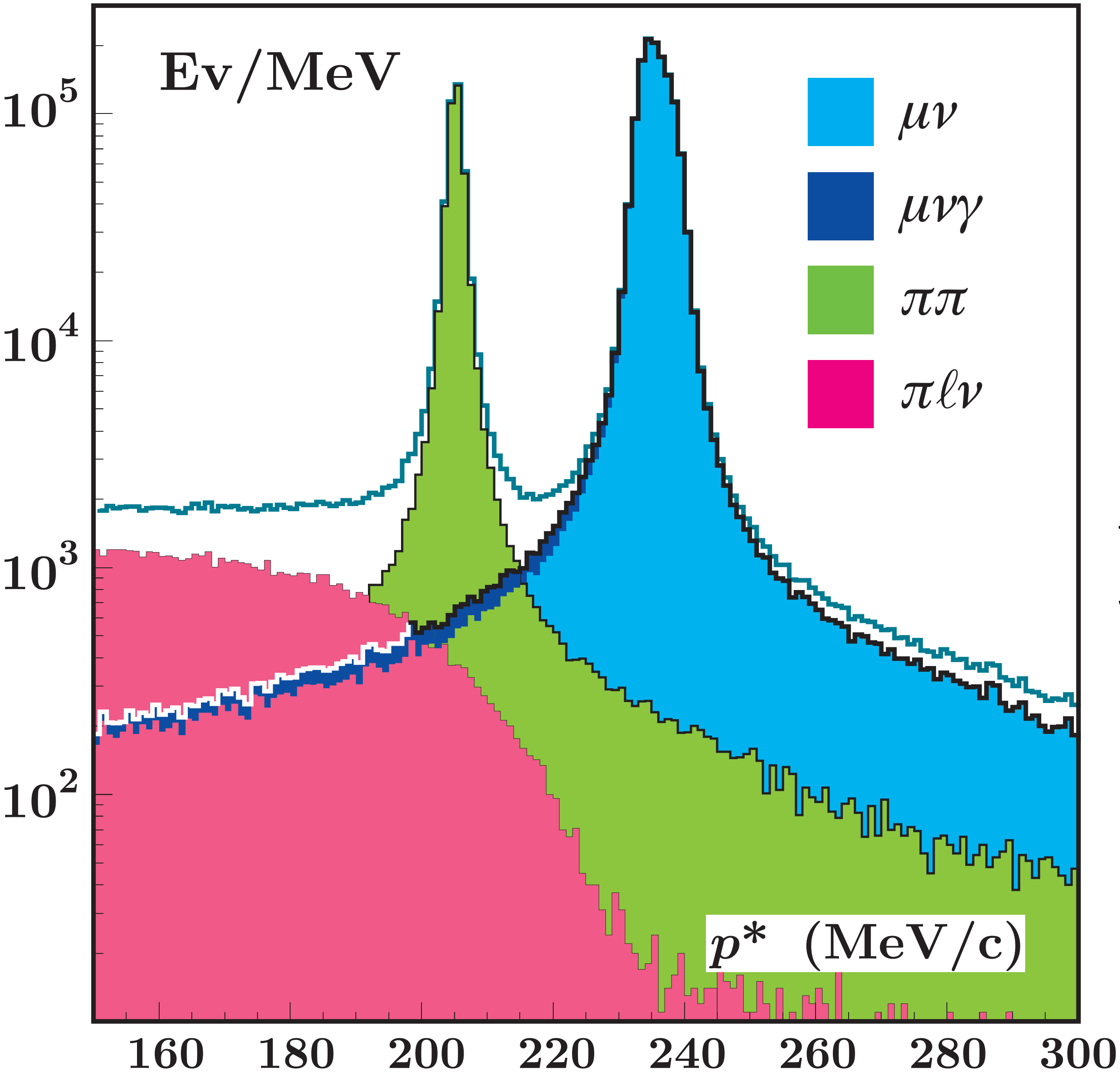
$K_{\ell 3}^{\pm}$ decays



Tag	–	Decay
$K^+ \to \mu^+\nu$	–	$K^- \to \pi^0 e^- \bar{\nu}$
$K^+ \to \pi^+\pi^0$	–	$K^- \to \pi^0 \mu^- \bar{\nu}$
$K^- \to \mu^-\bar{\nu}$	–	$K^+ \to \pi^0 e^+ \nu$
$K^- \to \pi^-\pi^0$	–	$K^+ \to \pi^0 \mu^+ \nu$

Tagging decays and $K^{\pm} \to \mu\nu$ decays



Compute CM momentum using $m(\pi)$. The $\mu\nu$ decay peak is distorted but quite recognizable for tagging.

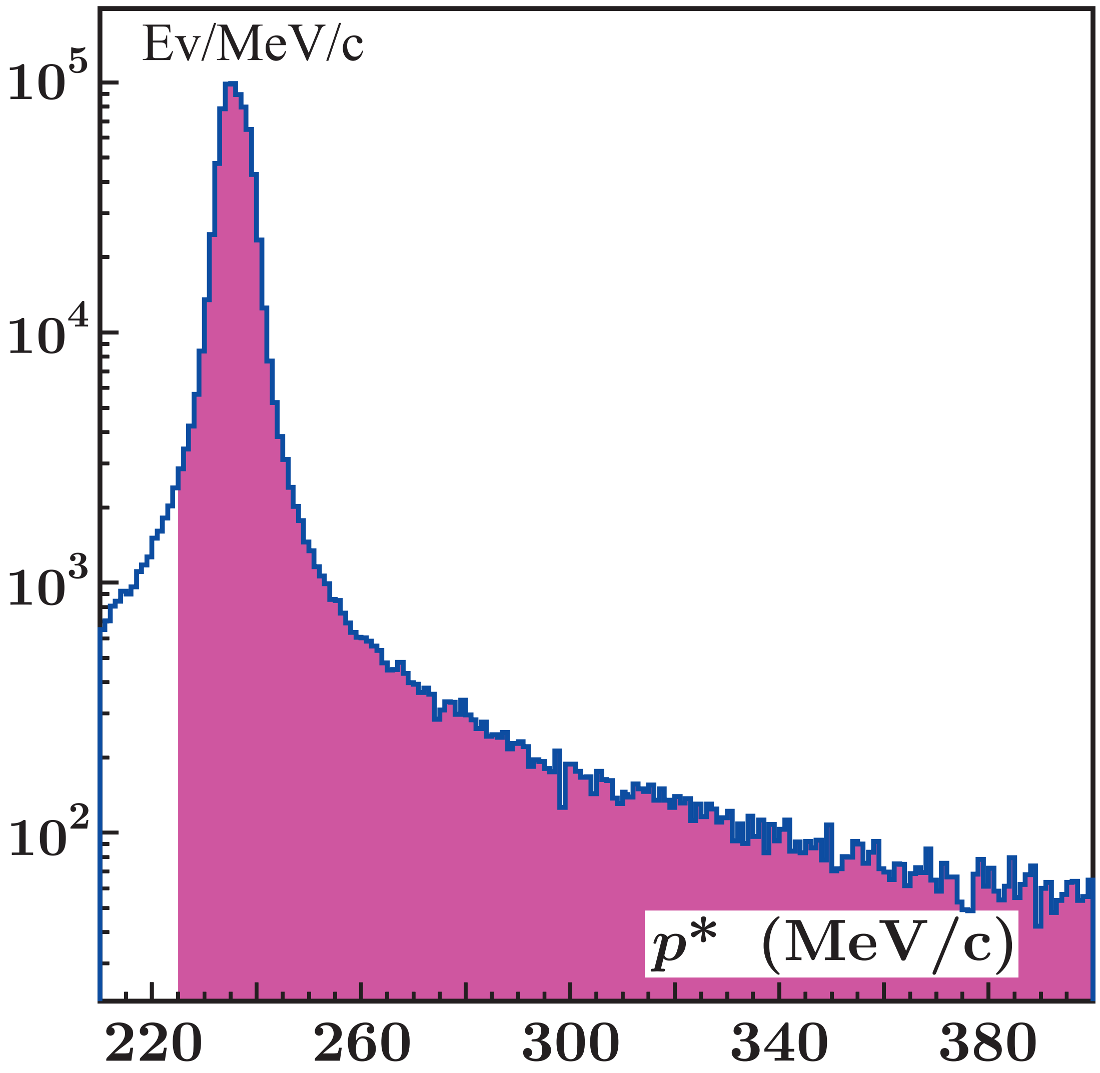
We also get

BR($K \to \mu\nu(\gamma)$) and

BR($K \to \pi\pi^0(\gamma)$)

Note the radiative tail!

$K \to \mu\nu(\gamma)$ BR

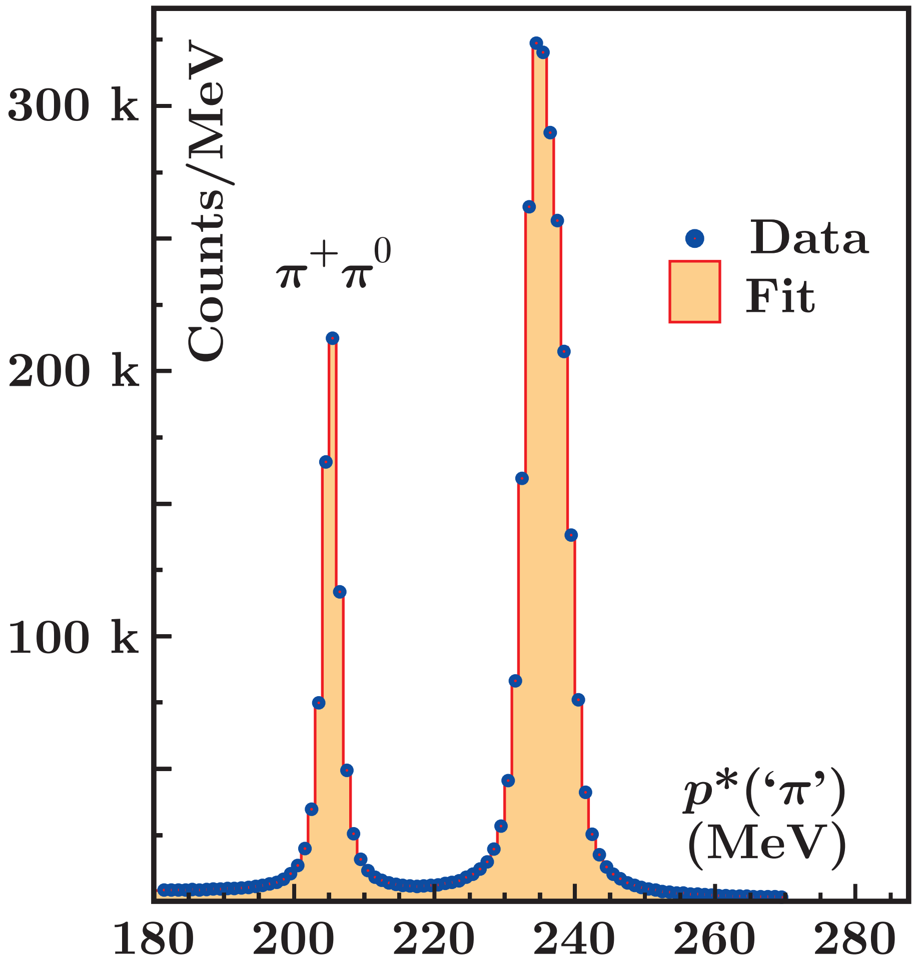


All background decays, contain a π^0. The background shape is obtained from a control sample

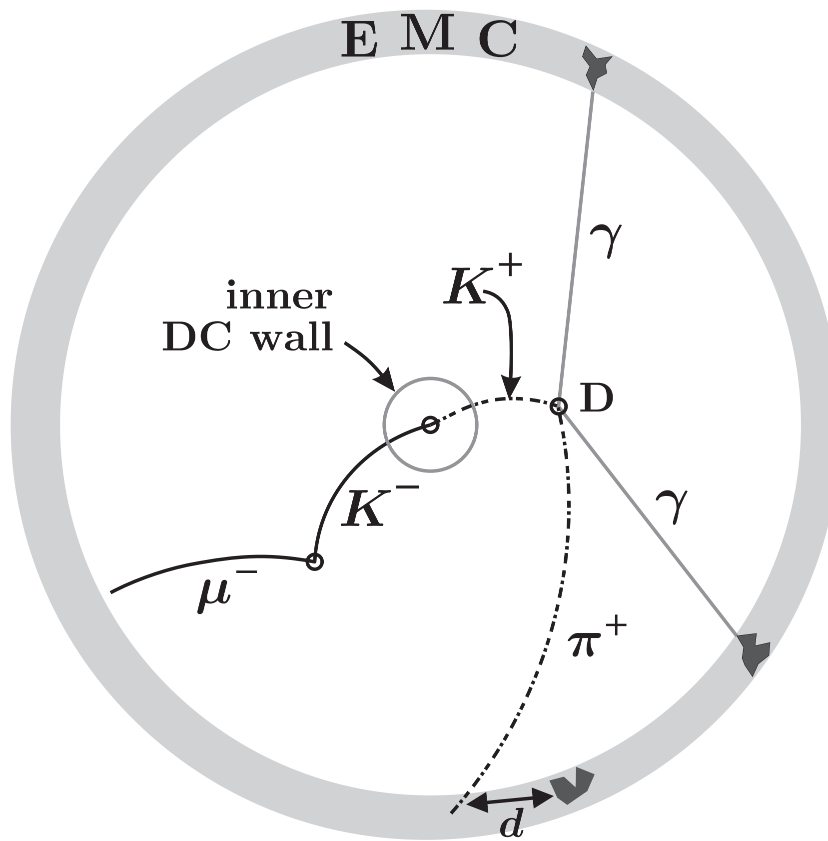
$K_{\mu 2}$ events are counted only in the shaded area, after background subtraction, ~2%. BR$(K^+ \to \mu^+\nu(\gamma)) =$ 0.6366±0.0009±0.0015.

$K^\pm \to \pi^+\pi^0$ decays

No π^0 requirement!



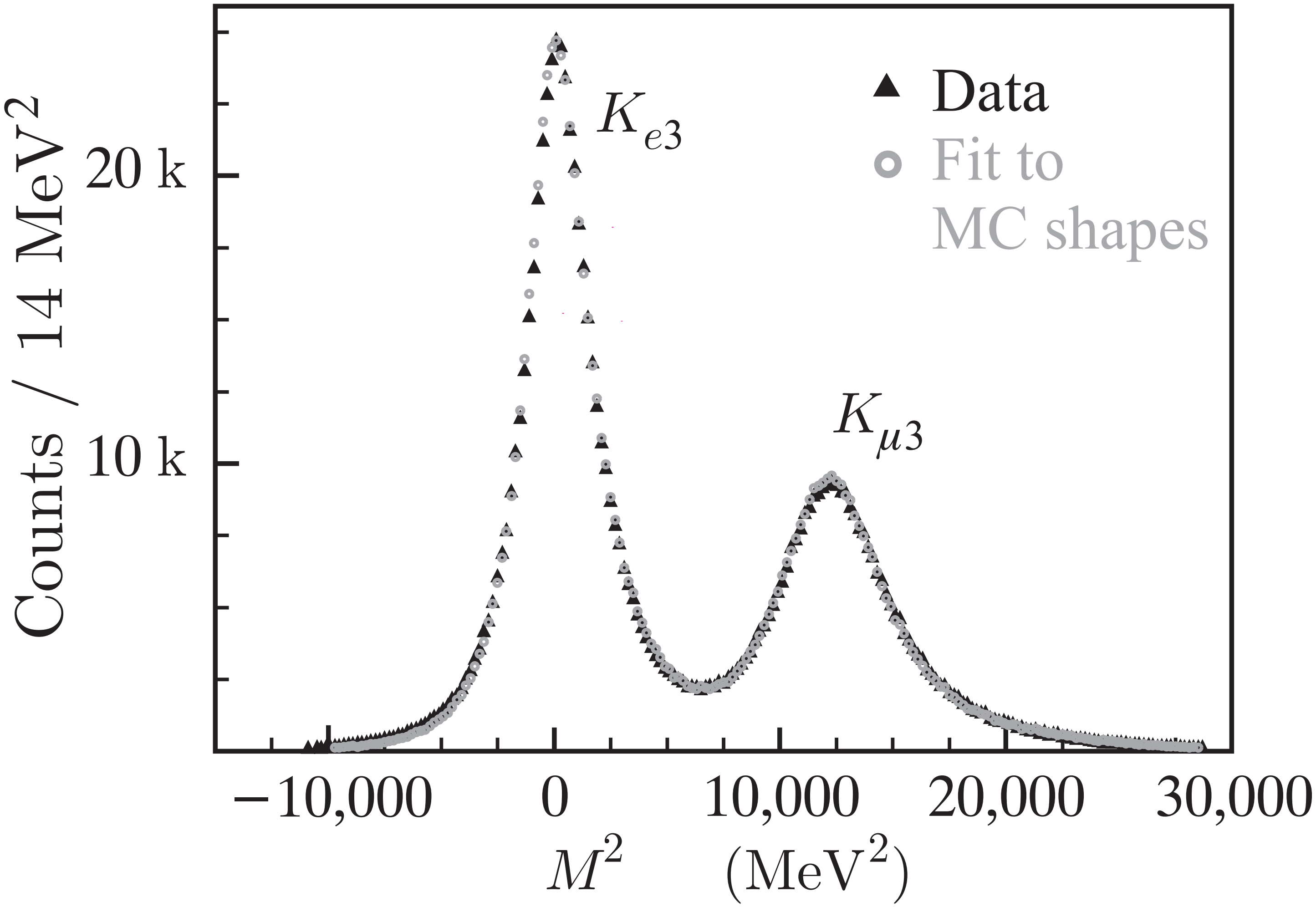
Measuring the $K \to \pi$ detection efficiency



$$\text{BR}(K^+ \to \pi^+\pi^0(\gamma)) = 0.2065 \pm 0.0005_{\text{stat}} \pm 0.0008_{\text{syst}}$$

Average of $K^-_{\mu 2}$ and $K^-_{\pi 2}$ tags. Normalization for many $K^\pm$ BRs

$K^{\pm}_{\ell 3}$ decays



$K \to \mu\nu$ and $K \to \pi^{\pm}\pi^{0}$ tag.

Verify $\pi^0 \to 2\gamma$.

Lepton mass from p and TOF.

$K^{\pm} \to \ell^{\pm}\pi^{0}\nu$ BRs

BR \ Tag	$K^{+}_{\mu 2}$	$K^{+}_{\pi 2}$	$K^{-}_{\mu 2}$	$K^{-}_{\pi 2}$
BR(K_{e3})	0.0495(7)	0.0493(10)	0.0497(8)	0.0502(10)
BR($K_{\mu 3}$)	0.0322(6)	0.0322(9)	0.0323(5)	0.0327(9)

8 measurements, very good consistency:

$$\mathrm{BR}(K^{\pm} \to e^{\pm}\pi^{0}\nu) = 0.0497 \pm 0.0005$$

$$\mathrm{BR}(K^{\pm} \to \mu^{\pm}\pi^{0}\nu) = 0.0323 \pm 0.0004$$

All KLOE $K^{\pm}$ results

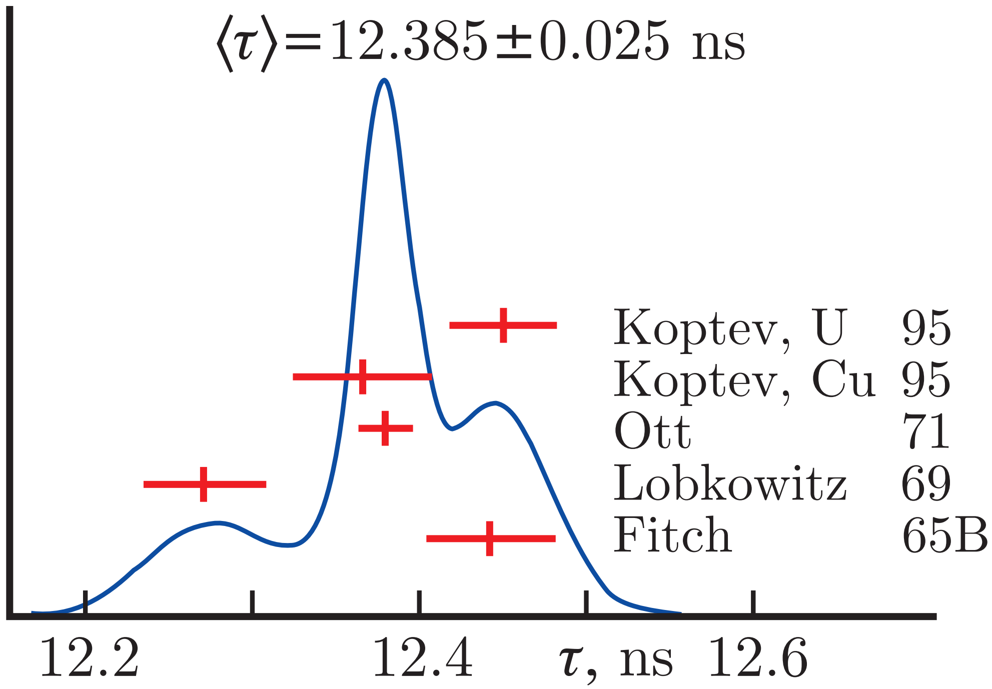
Constrained fit:

$$\mathrm{BR}(K^{\pm} \to \mu^{\pm}\nu) = 0.63765 \pm 0.0013$$
$$\mathrm{BR}(K^{\pm} \to \pi^{\pm}\pi^{0}) = 0.20680 \pm 0.0009$$
$$\mathrm{BR}(K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) = 0.05534 \pm 0.0009 \text{ (input PDG '04)}$$
$$\mathrm{BR}(K^{\pm} \to e^{\pm}\pi^{0}\nu) = 0.04978 \pm 0.0005$$
$$\mathrm{BR}(K^{\pm} \to \mu^{\pm}\pi^{0}\nu) = 0.03242 \pm 0.0004$$
$$\mathrm{BR}(K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}) = 0.01765 \pm 0.0002$$

sum = 1.0

$\chi^2/\mathrm{dof} = 0.78/1$, CL=38%

Must check $\tau(K^{\pm})$



Ott et al.: error is 0.032, not 0.016

Koptev: τ=12.422±0.048

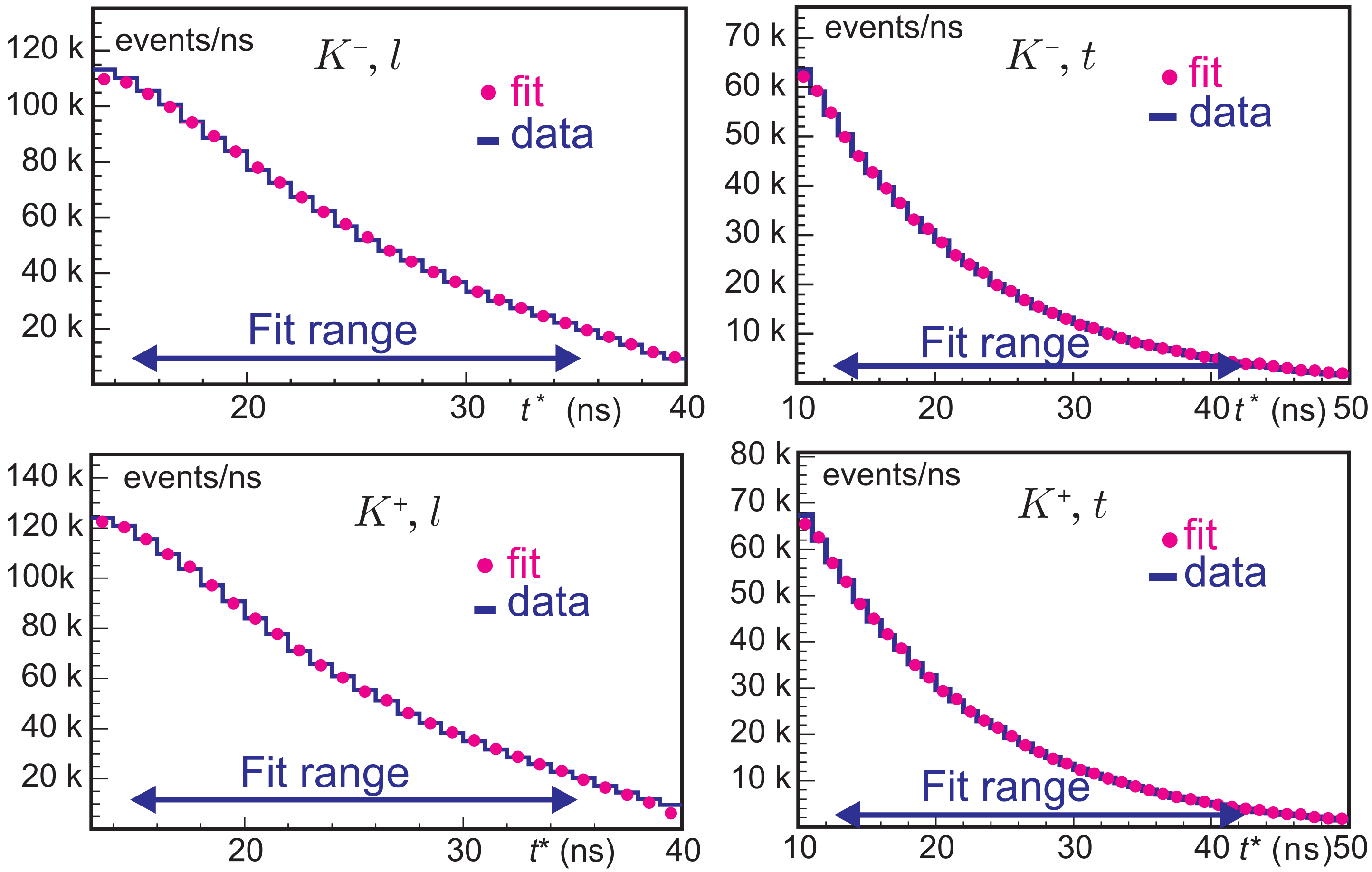
Lobkowicz is way off

$\delta\tau$=0.0119 ns ⇒ 0.025 ns

Better remeasure!

$\tau = 12.409 \pm 0.022$ ns

KLOE $\tau(K^{\pm})$



4 measurement of τ –

Consistent –

Can average

Use only our result

$$\tau(K^{\pm}) = 12.347 \pm 0.030, \quad 0.25\%$$

$$\tau(K^{+})/\tau(K^{\pm}) = 1.004 \pm 0.004$$

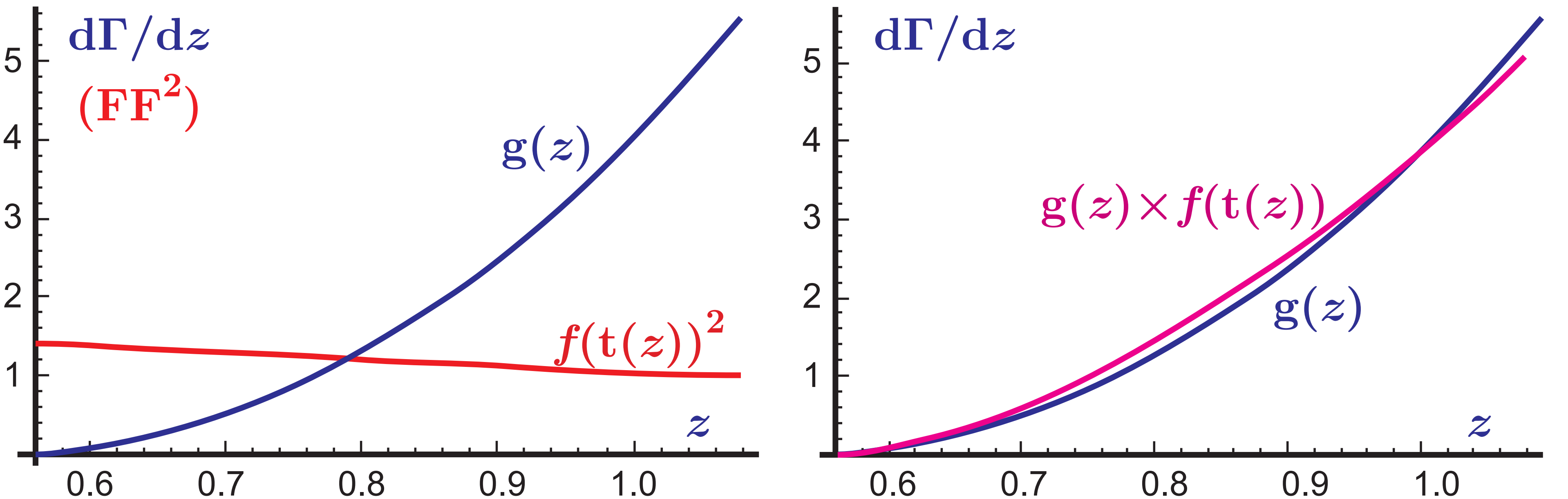
this is tex

The form factors

$\tilde{f}_{+,0}(t)$?? $\rightrightarrows$ Phase Space Integrals $I_{K\ell}$

$t = (P-p)^2 = M^2 + m^2 - 2M\,E_\pi$, $\quad 0 < t < M^2 + m^2$ for K_{e3}

No need to fit Dalitz Plot. All info in E_π spectrum.

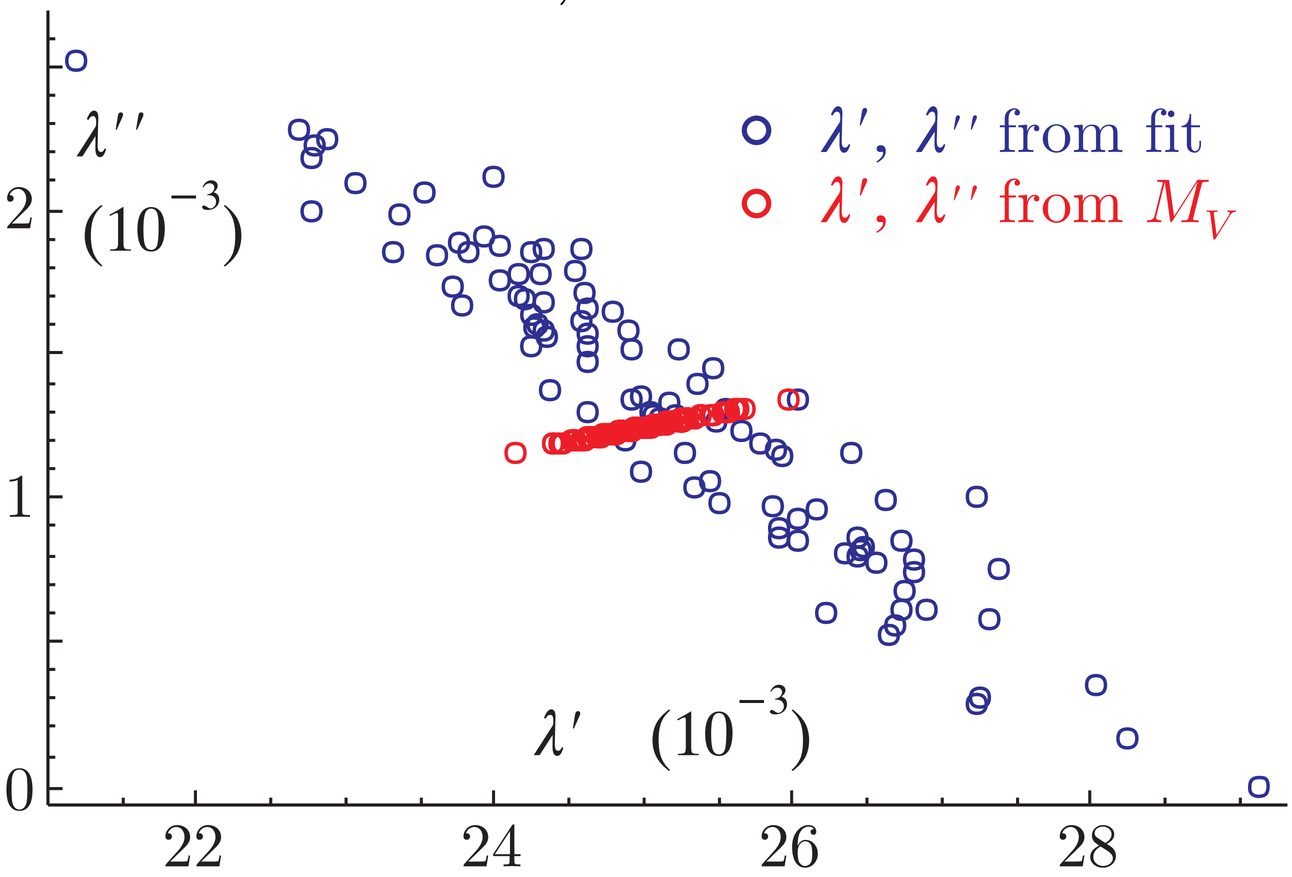


$$z = \frac{2\,E_\pi}{M}$$

FF: quadratic, pole, disp.

$$\tilde{f}_+(t) = 1 + \lambda' \frac{t}{m} + \frac{\lambda''}{2}\left(\frac{t}{m}\right)^2 \text{ or } \tilde{f}_+(t) = \frac{M_V^2}{t - M_V^2}$$

λ', λ'' are 95% correlated, errors $\times$3-4



$\lambda' = (m/M_V)^2, \lambda'' = 2\,\lambda'^2$

Pole and quad give different I, KTeV, '04

KTeV unlucky, but should have understood.

Pole was right!!

Common practice

Quadratic FF_+ and linear FF_0: three parameters λ'_+, λ''_+, λ_0

$$\begin{pmatrix} 1.79^2 & 3.51 & -1.98 \\ 3.51 & 3.15^2 & -4.04 \\ -1.98 & -4.04 & 1.37^2 \end{pmatrix} \times \frac{1}{N}$$

This systematically over-estimates λ_0 by $\sim$20%. Errors "look" acceptable.

$$\begin{matrix} \lambda'_0 & \lambda''_0 & \lambda'_+ & \lambda''_+ \end{matrix}$$

$$\begin{pmatrix} 63.9^2 & -1200 & -923 & 197 \\ -1200 & 18.8^2 & 272 & -59 \\ -923 & 272 & 14.8^2 & -49 \\ 197 & -59 & -48 & 3.4^2 \end{pmatrix} \times \frac{1}{N}$$

Errors are unacceptable, correlations reach -99.96%!!

NEED HELP!

Dispersive approach

Use a dispersion relation for $\log(\tilde{f}_+(t))$, twice subtracted at t=0. $\tilde{f}_+(t) = \exp\left[(t/m^2)(\lambda_+ + H(t))\right]$. λ_+ slope at t=0 to be measured. $H(t)$ is obtained from $K\pi$ p-wave phase, dominated by $K^*(892)$. $\tilde{f}_+$ expands to

$$\tilde{f}_+ = 1 + \lambda_+ \frac{t}{m^2} + \frac{\lambda_+^2 + 0.00058}{2}\left(\frac{t}{m^2}\right)^2 + \frac{\lambda_+^3 + 3 \times 0.00058 \times \lambda_+ + 0.00003}{6}\left(\frac{t}{m^2}\right)^3$$

The Callan-Treiman relation relates the scalar FF at $t = \Delta_{K\pi}$ to the ratio of the pseudoscalar decay constants f_K/f_π.

$$\tilde{f}_0(\Delta_{K\pi}) = \frac{f_K}{f_\pi}\frac{1}{f_+(0)} + \Delta_{\mathrm{CT}}$$

where $\Delta_{K\pi} = M^2 - m^2$, the so called Callan-Treiman point, and Δ_{CT} is a correction which at NLO in χPT is $\sim -3.5 \times 10^{-3}$.

Use dispersion relation subtracted at t=0 and $t = \Delta_{K\pi}$. Use s-wave $K\pi$ phases to compute the equivalent function $G(t)$. $\tilde{f}_0(t)$ expands to

$$\tilde{f}_+ = 1 + \lambda_+ \frac{t}{m^2} + \frac{\lambda_+^2 + 0.00042}{2}\left(\frac{t}{m^2}\right)^2 + \frac{\lambda_+^3 + 3 \times 0.00042 \times \lambda_+ + 0.000027}{6}\left(\frac{t}{m^2}\right)^3$$

(Use dispersion relations and χPT: Stern *et al.*. Also Pich *et al.*.)

FFs are power series in t but with only one free parameter each!

Fit for $\lambda_{+,0}$ and get "phase space integrals"

$I(K^0_{e3})$	$I(K^0_{\mu 3})$	$I(K^+_{e3})$	$I(K^+_{\mu 3})$
0.6191(14)	0.4105(18)	0.6365(14)	0.4224(18)

0.2 to 0.4 %. ∼1/3 of KTeV error.

[Γ]

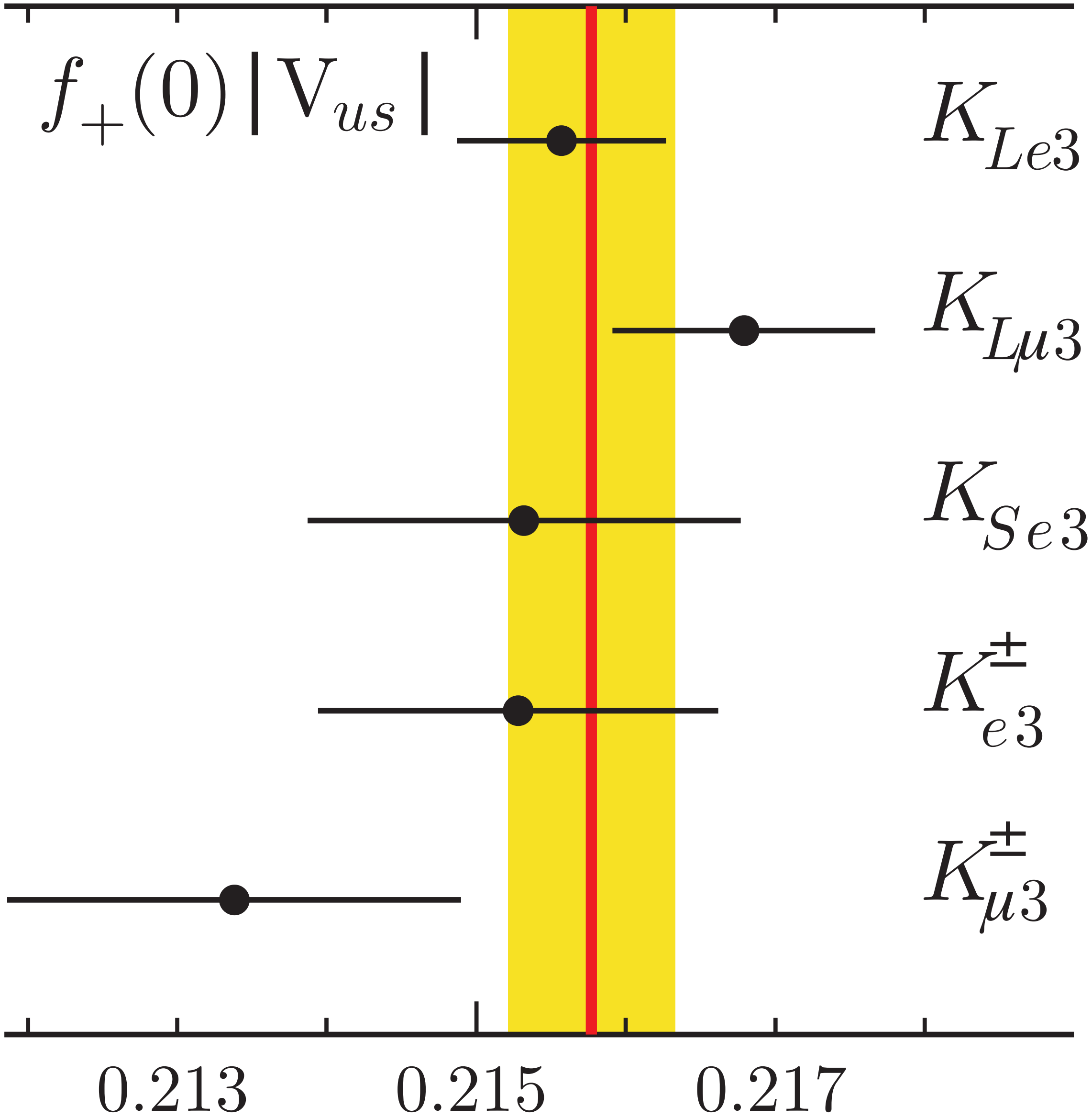
$f(0)|V_{us}|$

We are almost there. Let us begin with $f(0)|V_{us}|$. $f(0)$, by definition, belongs to $K^0 \to \pi^\pm$. If we know the I-spin corrections and the long distance EM corrections all decays must give the same value for $f(0)|V_{us}|$.

This is in fact a check of the $\delta\ldots$ estimates and of the experiment. We find

Channel	$f(0)\|V_{us}\|$	Correlation coefficients			
K_{Le3}	0.2155(7)				
$K_{L\mu3}$	0.2167(9)	0.28			
K_{Se3}	0.2153(14)	0.16	0.08		
$K^\pm_{e3}$	0.2152(13)	0.07	0.01	0.04	
$K^\pm_{\mu3}$	0.2132(15)	0.01	0.18	0.01	0.67

$f(0)|V_{us}|$



$\langle\rangle = -0.2157 \pm 0.0006$

χ^2/ndf $=7.0/4$
(CL=13%)

$SU(2)$ corrections are verified to 1.1 σ.

Lepton Universality

From:

$$R_{\mu e} \equiv \frac{[f(0)V_{us}]^2_{\mu 3}}{[f(0)V_{us}]^2_{e3}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \frac{I_{e3}\,(1+\delta_e)^2}{I_{\mu 3}\,(1+\delta_\mu)^2} \qquad [\Gamma]$$

$\Rrightarrow$

$$R_{\mu e} = \frac{g_e^2}{g_\mu^2} = 1.000 \pm 0.008 \qquad [W_\alpha J^\alpha]$$

Competitive with pure leptonic processes, τ decays

$(R_{\mu e})_\tau = 1.000 \pm 0.004$

Also with $\pi \to \ell\nu$ results

$(R_{\mu e})_\pi = 1.0042 \pm 0.0033$

$f(0)$ and $|V_{us}|$

The accepted value was $f(0)| = 0.961 \pm 0.008$, Roos & Leutwyler, 1984. Recently Lattice results have become convicing. We take $f(0) = 0.9644 \pm 0.0049$, RBC and UKQCD, 2007. Then

$$|V_{us}| = 0.2237 \pm 0.0013$$

$$|V_{us}|^2 = 0.05002 \pm 0.00057$$

$0^+ \to 0^+$ and $|V_{ud}|^2$

$$|V_{ud}|^2 = 0.9490 \pm 0.0005$$

$$1 - |V_{us}|^2 - |V_{ud}|^2 = 0.0009 \pm 0.0008 \quad (\sim 1.1\sigma)$$

But we can do better

$K \to \mu\nu / \pi \to \mu\nu$

$$\frac{\Gamma(K_{\mu 2(\gamma)})}{\Gamma(\pi_{\mu 2(\gamma)})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{m_K \left(1 - m_\mu^2/m_K^2\right)^2}{m_\pi \left(1 - m_\mu^2/m_\pi^2\right)^2} \times (0.9930 \pm 0.0035)$$

Cancellations in f_K^2/f_π^2. From lattice

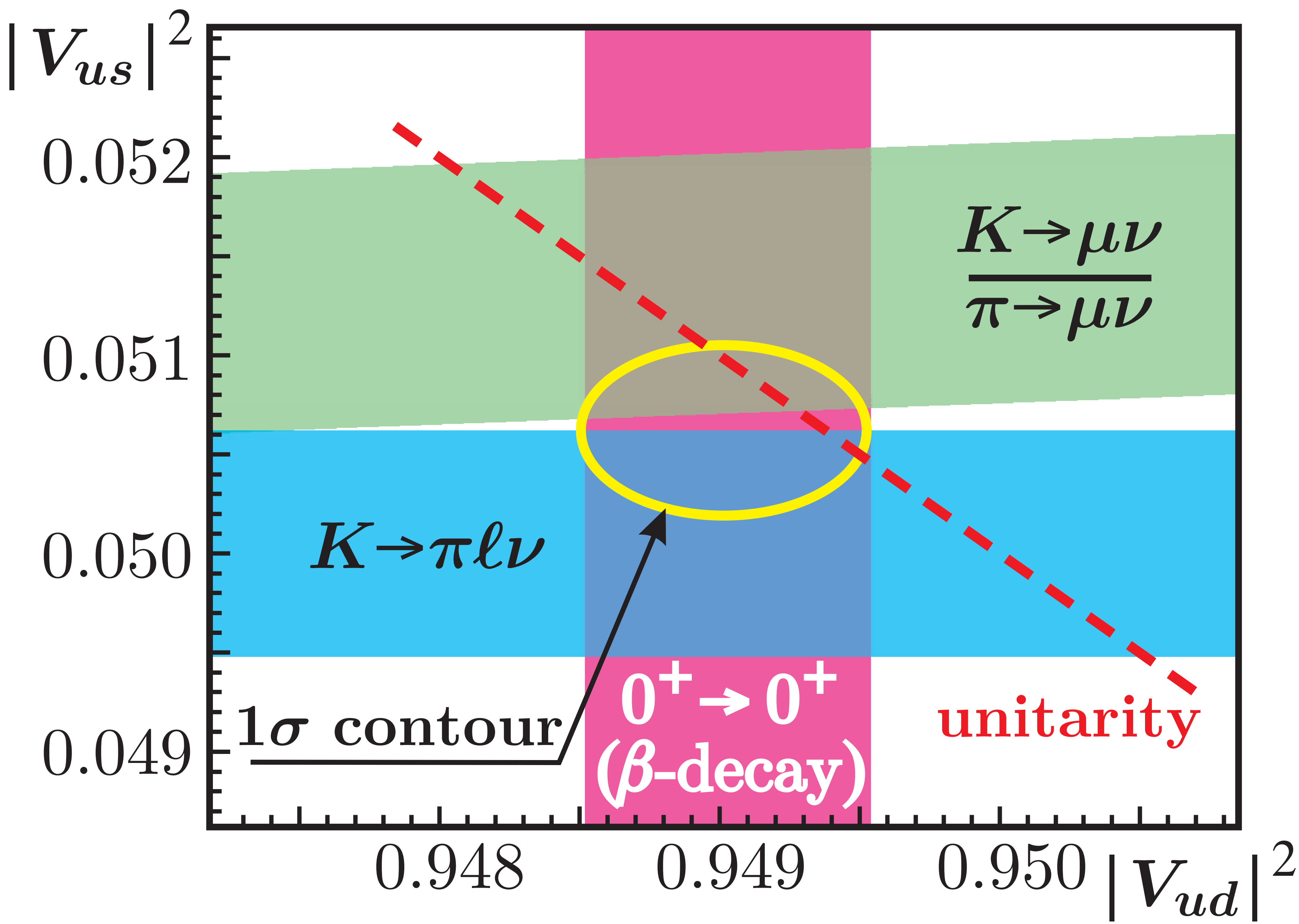
Cancellations in rad. cor. From Marciano.

KLOE $\Rrightarrow$ $|V_{us}/V_{ud} \times f_K/f_\pi|^2 = 0.0765 \pm 0.0005$

HPQCD/UKQCD $\Rrightarrow$ f_K/f_π=1.189±0.007

$$|V_{us}/V_{ud}|^2 = 0.0541 \pm 0.0007$$

CKM Unitarity



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From the fit

$$1 - |V_{us}|^2 - |V_{ud}|^2 = 0.0004 \pm 0.0007 \quad (\sim 0.6\sigma)$$

$$|V_{us}| = 0.2249 \pm 0.0010$$

$$|V_{ud}| = 0.97417 \pm 0.00026$$

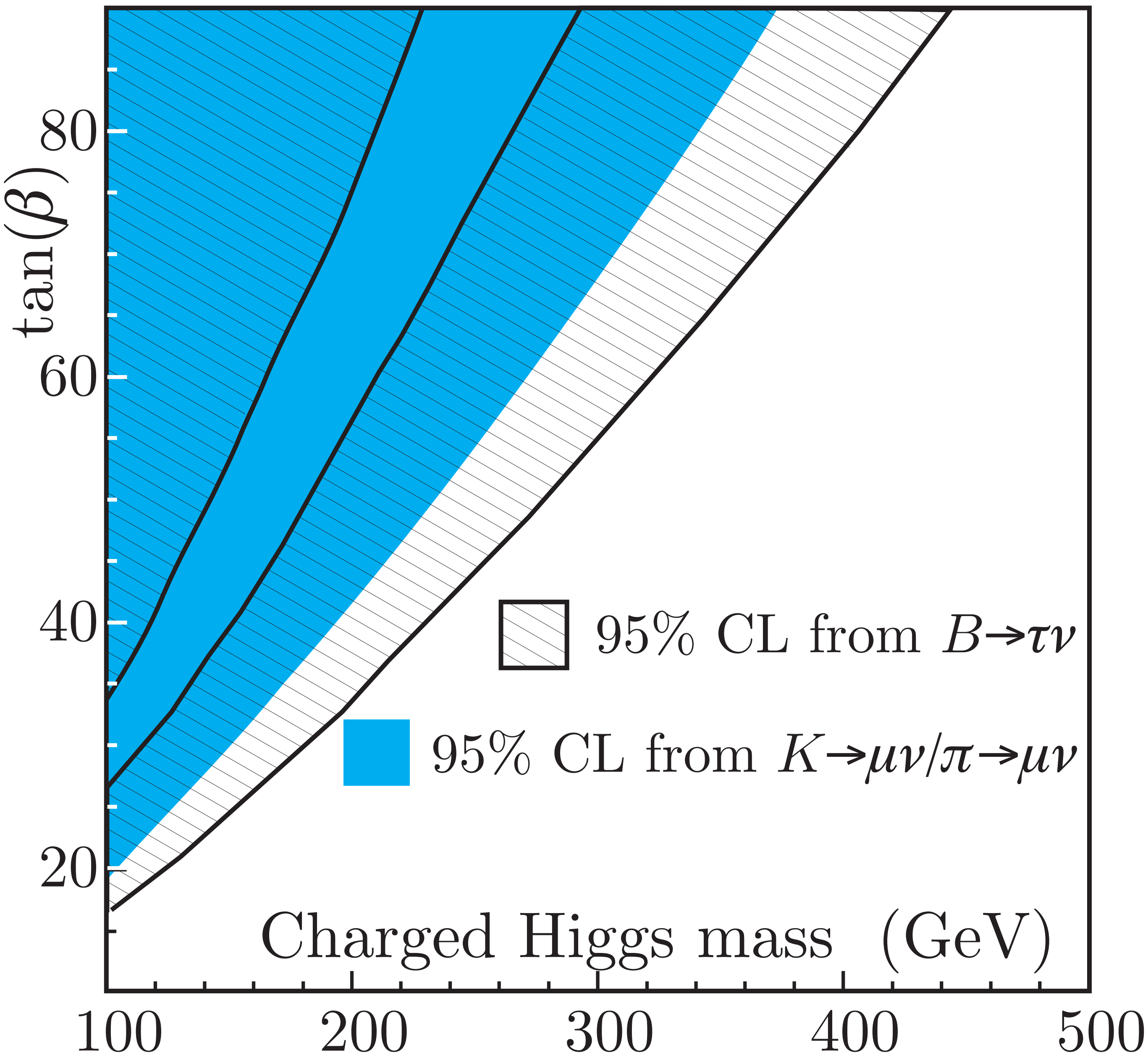
Fit with unitarity as constraint

$$|V_{us}| = 0.2253 \pm 0.0007$$

$$|V_{ud}| = \sqrt{1 - |V_{us}|^2} = 0.97429 \pm 0.00017.$$

$$\theta_C = 13.02^\circ \pm 0.06^\circ$$

$\tan\beta - M(\text{Higgs}^{\pm})$



$$R_{\ell 23} = \left| \frac{V_{us}(K_{\mu 2})}{V_{us}(K_{\ell 3})} \times \frac{V_{ud}(0^+ \to 0^+)}{V_{ud}(\pi_{\mu 2})} \right| = 1$$

in the SM. With charged Higgs exchange $R_{\ell 23}$=

$$\left| 1 - \frac{m^2_{K^+}}{m^2_{H^+}} \left(1 - \frac{m^2_{\pi^+}}{m^2_{K^+}}\right) \times \frac{\tan^2\beta}{1+\epsilon_0 \tan\beta} \right|$$

$$\epsilon_0 \approx 1/(16\pi^2)$$

Conclusions

Unitarity is verified at the 0.1% level

$$1 - |V_{us}|^2 - |V_{ud}|^2 = 0.0004 \pm 0.0007$$

Muons, electrons and quarks

carry the same weak charge to better than 0.5%

Kaon properties limit the parameter space

for some new physics models