Studying strong interaction and hadronic matter with hadron-hadron correlations

Romanenko G. (University and INFN Bologna)

Bologna, 23/01/2026

My research path

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Sep 2015
Jun 2019

Bachelor’s degree

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ALICE

CosmicAntiNuclei

erc

INFN BOLOGNA
Istituto Nazionale di Fisica Nucleare
Sezione di Bologna

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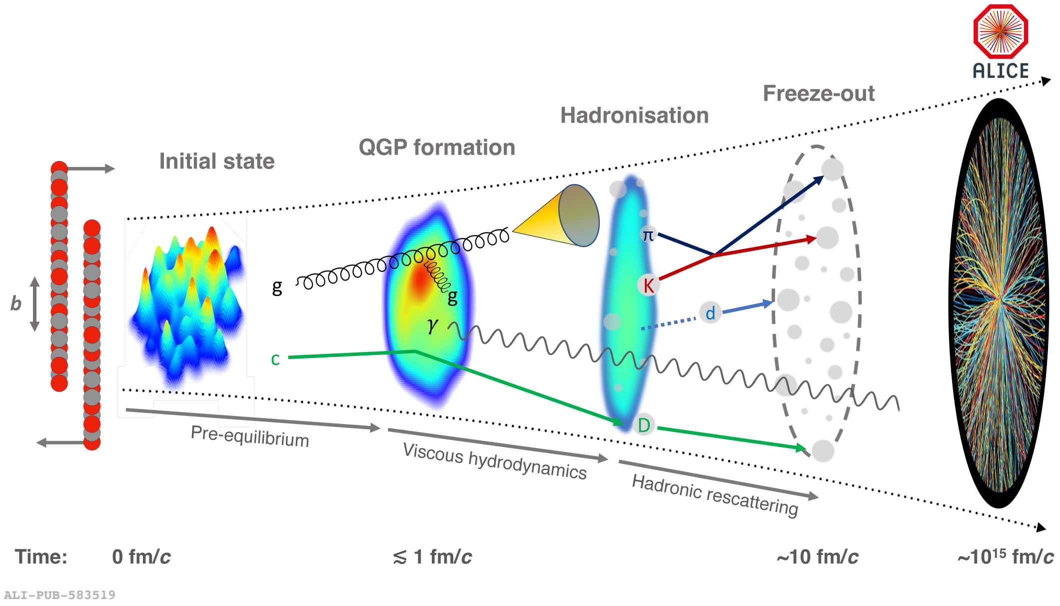
Heavy-Ion Collision (HIC) evolution

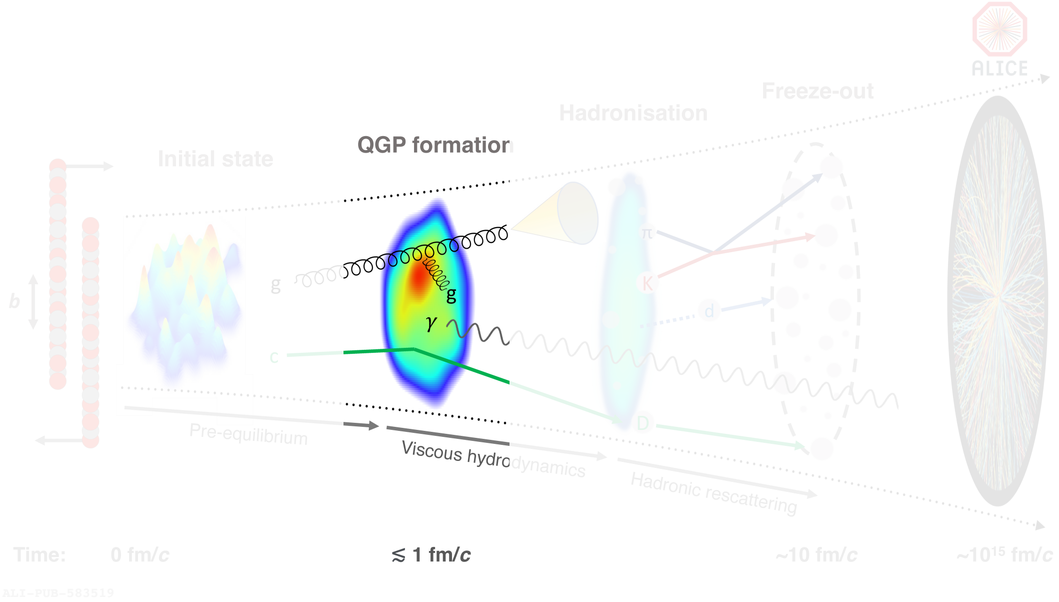


At the LHC the **Pb nuclei collide** at the energy of **5.36 TeV per nucleon (!)**

The matter undergoes through several stages in an **extremely short time**

1 fm/c ~= 3.3(3) * 10^(-24) s

Heavy-Ion Collision (HIC) evolution



- **Deconfined** partons (quarks and gluons)
- Extremely hot: ~ **300-400 MeV** (1 MeV = 11.6 * 10^9 K)
- Energy density > 1 GeV/fm^3 (1.6*10^{35} J/m^3)
- **Almost perfect fluid**: estimated $\eta/s = 0.12$ (lower bound for “most perfect fluid” is $1/(4\pi)$)
- **Expands** rapidly (“explosively”)
- **Extremely small**: few femtometers (!)

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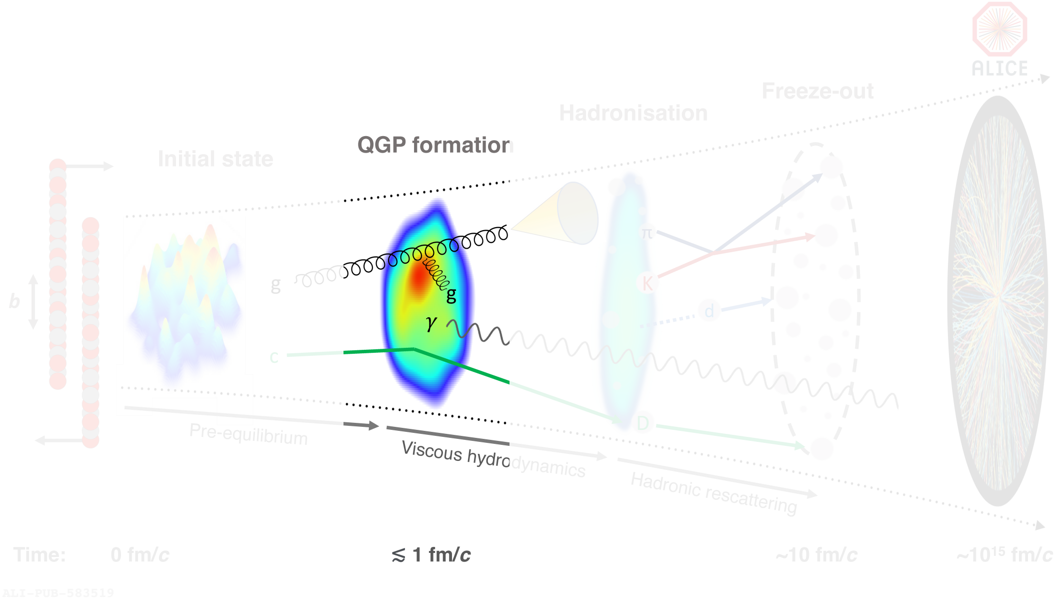

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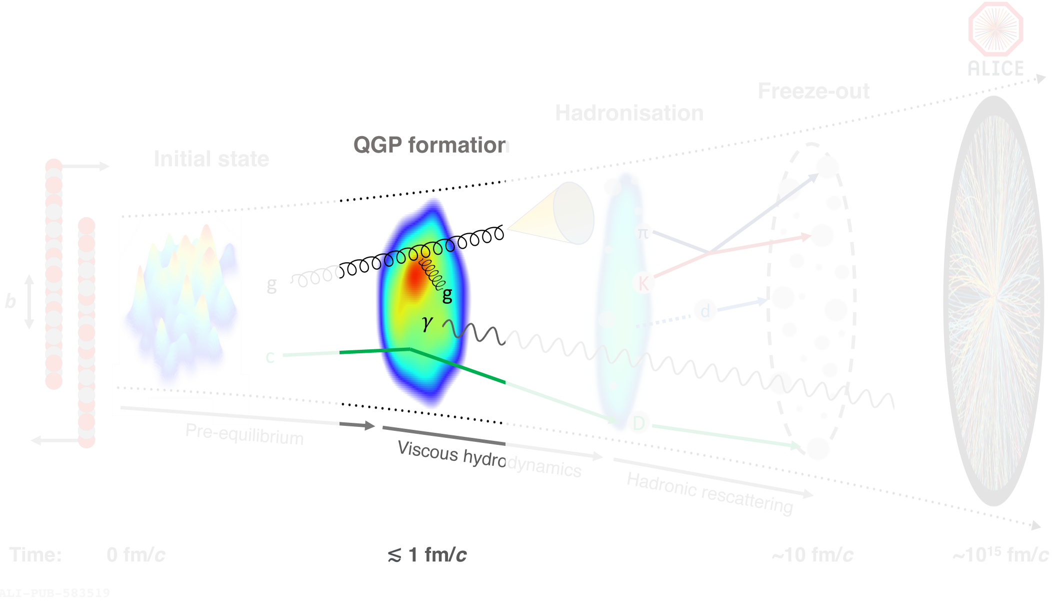

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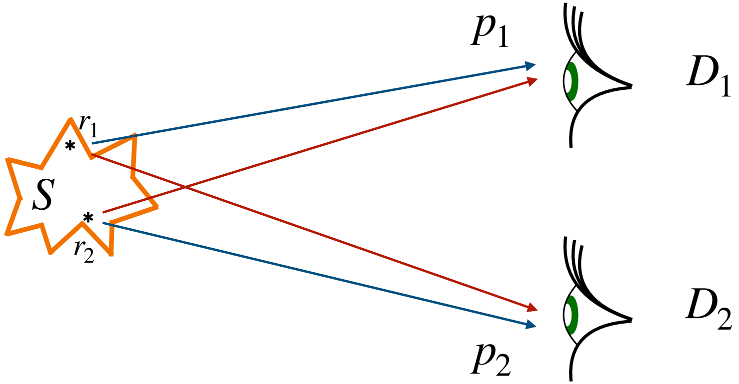
Femtoscopy!

Femtoscopy: a femtometer-scale ruler



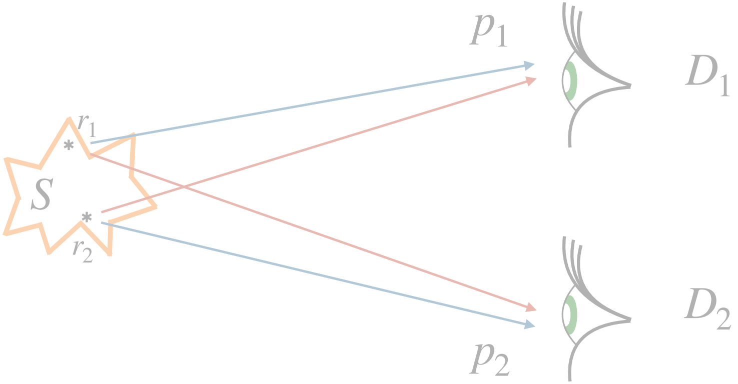
In the **simplest case** of two chargeless bosons

$$\Psi \propto e^{i(\vec{d}_1-\vec{r}_1)\vec{p}_1} e^{i(\vec{d}_2-\vec{r}_2)\vec{p}_2} + e^{i(\vec{d}_1-\vec{r}_2)\vec{p}_1} e^{i(\vec{d}_2-\vec{r}_1)\vec{p}_2}$$

$$P = \left|\Psi\right|^2 \propto 1 + cos\left(\Delta\vec{p}\Delta\vec{r}\right)$$

Method to **extract the distance** between emission points

Femtoscopy: a femtometer-scale ruler



In experiment we measure integrated pair spectra called correlation function (CF):

$$C(\Delta p) = \int d^3\Delta r\, S(\Delta r) \left| \Psi(\Delta p, \Delta r) \right|^2$$

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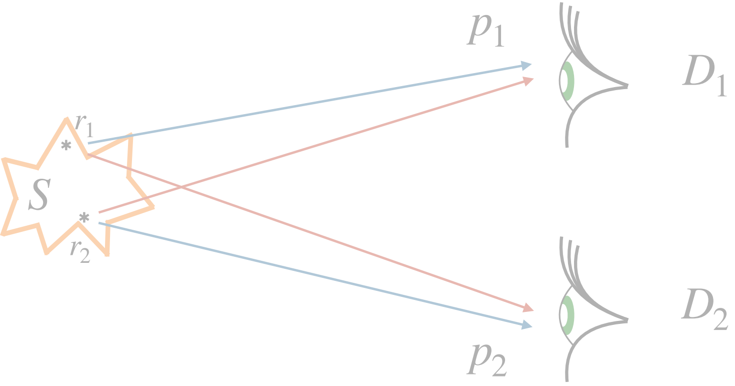

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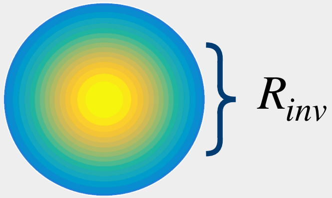
- Characterises the fireball with an **effective size R_{inv}**



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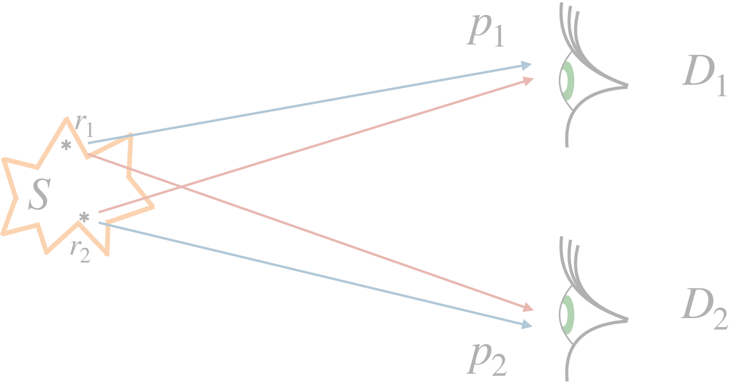

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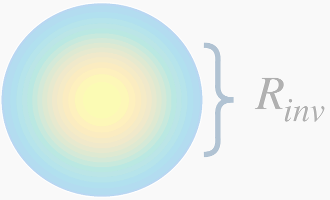
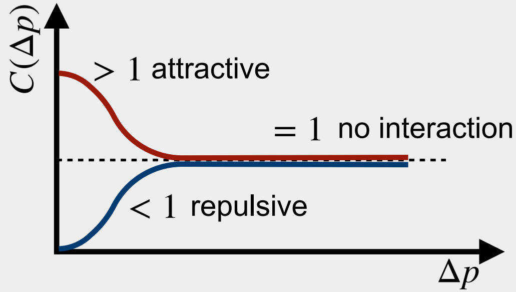


Wave function Ψ:

- Contains information on the **interaction**
- Defines the **profile of the CF**



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Method to **extract the distance** between emission points

Why proton femtoscopy

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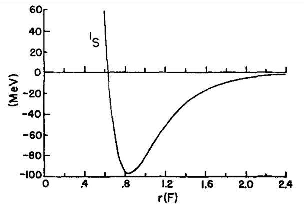
State-of-art strong potential models (Argonne v18, Nimj93 etc.):

- Realistic shape



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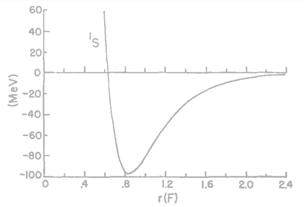

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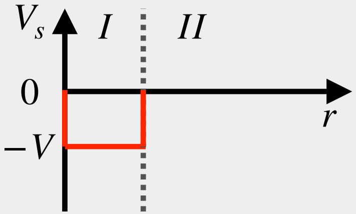
- Square-well shape



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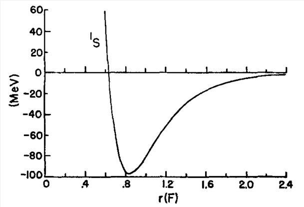

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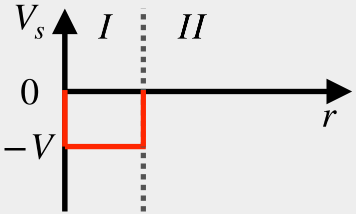


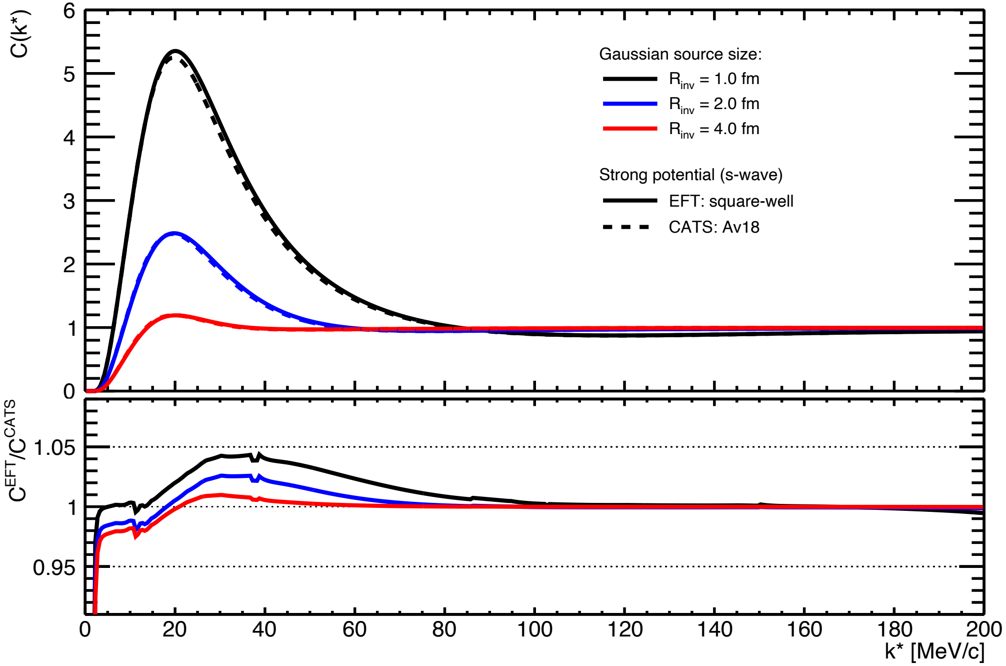
- **I managed to obtain an analytical WF**

numerical

fully analytical

Comparing my effective QM calculations with the numerical ones
I check the sensitivity of femtoscopy to the strong interaction

Comparing different approaches



CATS framework: solves the Schrodinger's equation numerically for a given strong potential shape (Av18 here)

versus

Effective approach: my analytical calculations with the square-well

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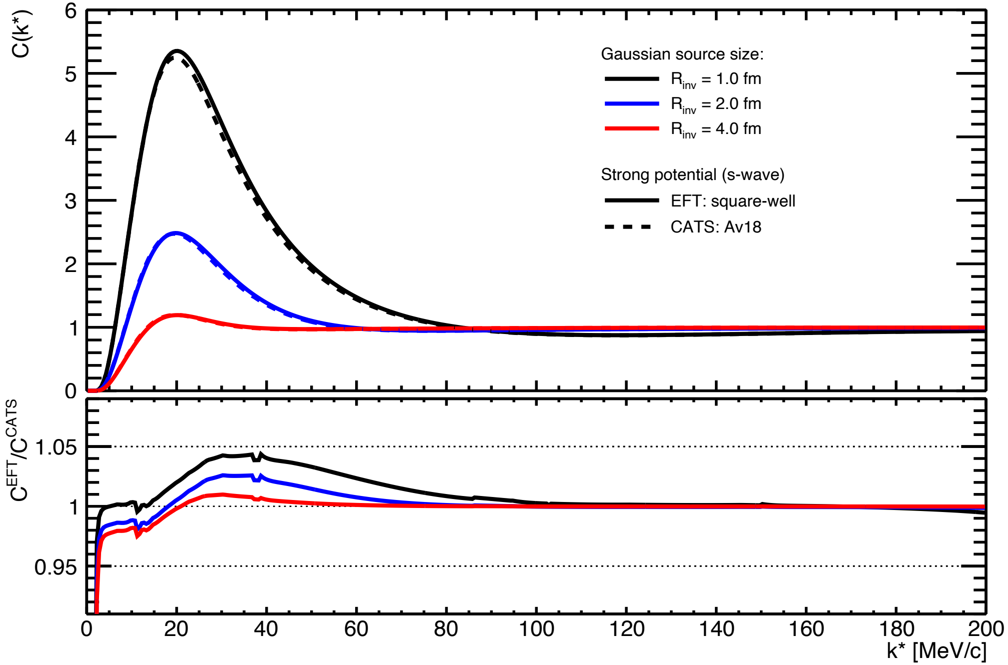

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The difference is very small ($\leq$ 4% for a 1fm source) and might be **too insignificant** in application to experimental data. **Not sensitive to the shape? Why?**

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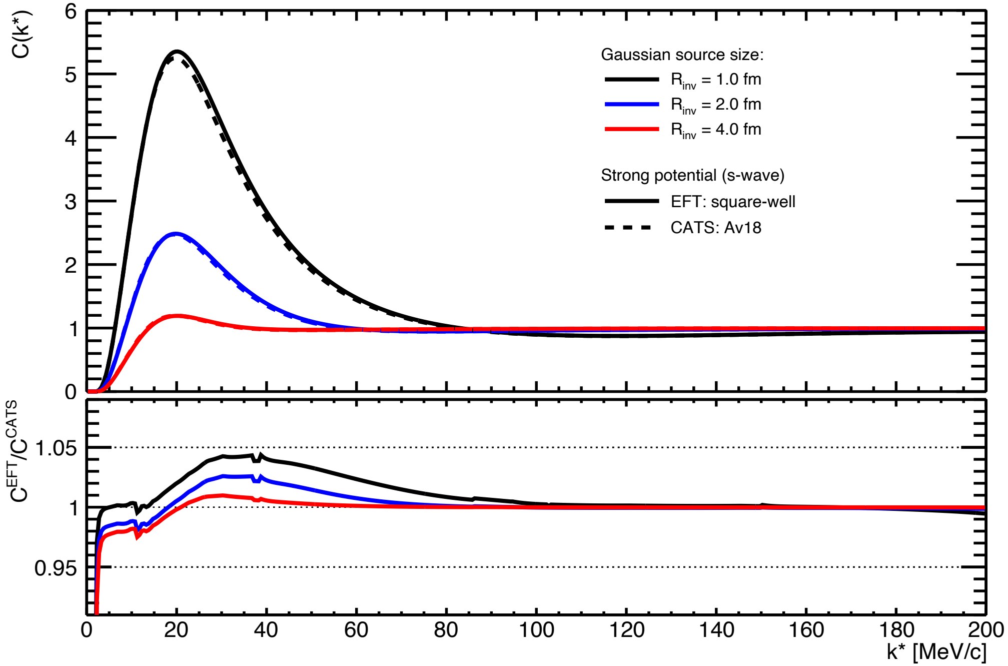

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For example, it's been well known for decades from phase shifts analyses that we are not sensitive to the shape of a short-range potential (as the strong one) at small energies (e.g. H. A. Bethe, Phys. Rev. 76, 38).

And we see our femtoscopic peak in small energy (momentum) region!

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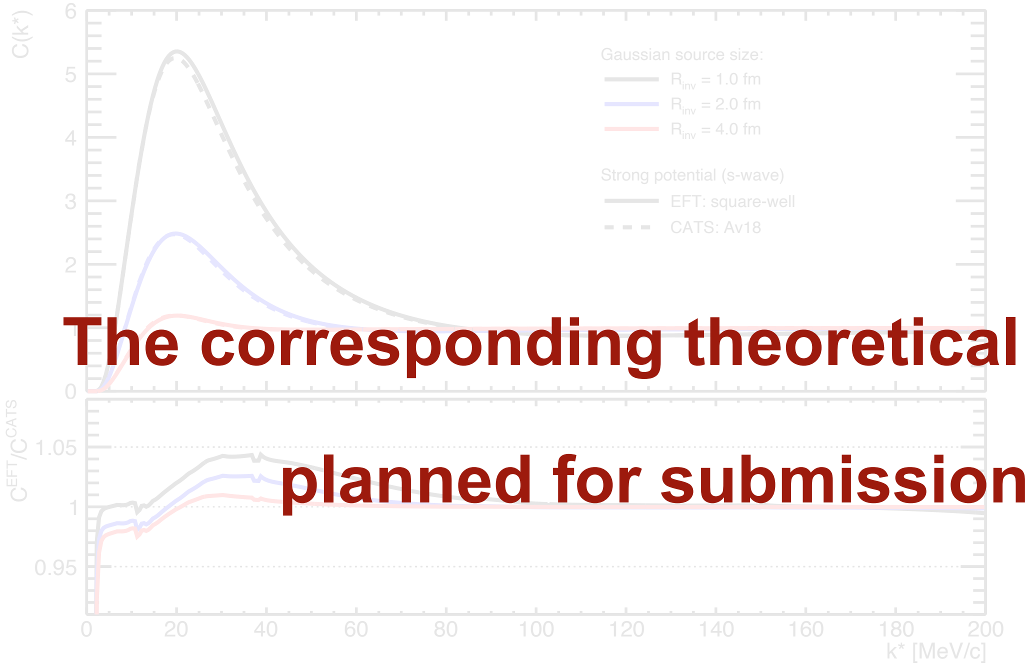

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The corresponding theoretical paper is being finalised and planned for submission by the end of winter!

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Proton source in Pb-Pb at 5.36 TeV Run3

Analysis recipe

- ☐ **Theoretical model**
- ☐ **Data**
- ☐ **Analysis framework**

Analysis recipe

- ☑ **Theoretical model** **My calculations**
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Analysis recipe

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- ☐ **Analysis framework**

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☐ **Analysis framework** **Femto3D package**

During my PhD we developed the **Femto3D package** inside ALICE's new **O2 framework** introduced for Run 3 to operate **in continuous readout mode.** Our package implements a **full analysis chain**:

- **Event and track selection;**
- **Particle Identification (PID);**
  Here I used the experience gained during my Async. Quality Control service work for the ALICE Time-Of-Flight (TOF) detector as a part of the INFN-TOF team
- **Event mixing procedure needed to construct femtoscopic CF**

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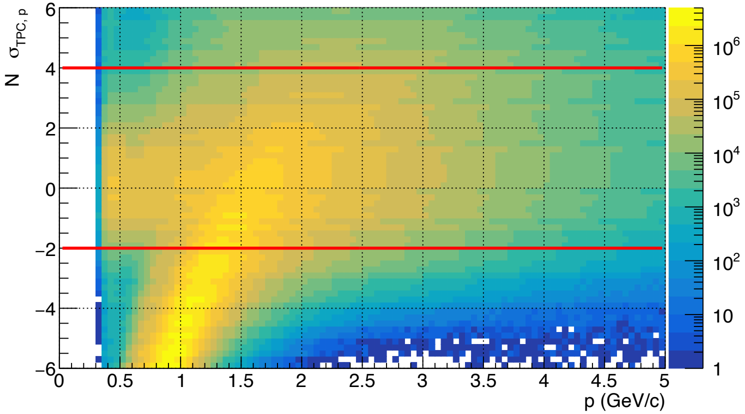
Proton signal with TPC



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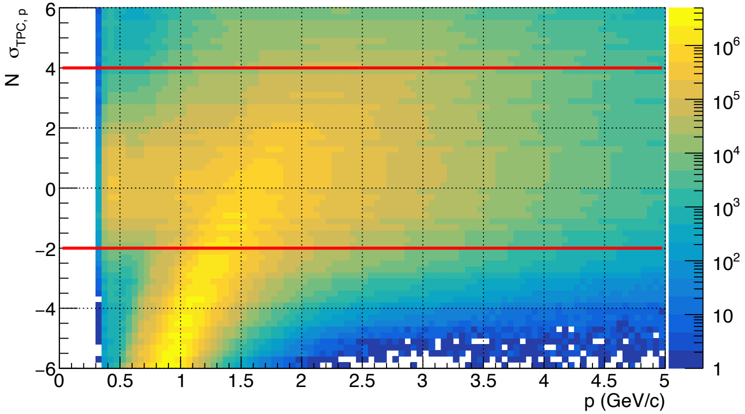

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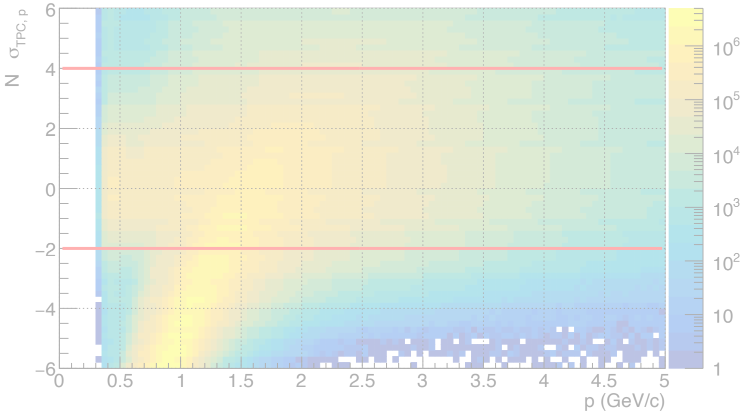
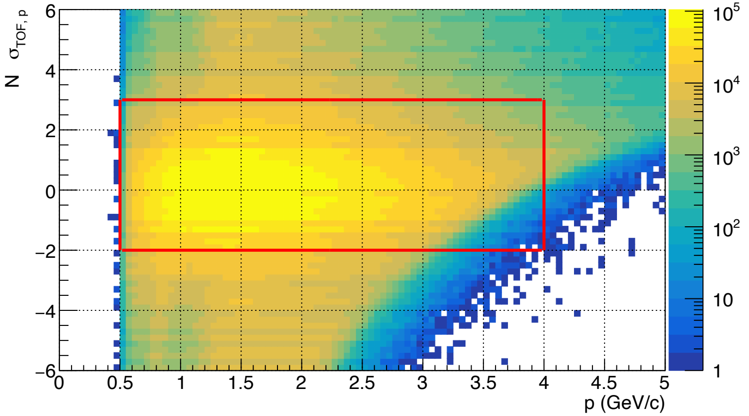


Proton signal with TPC+TOF



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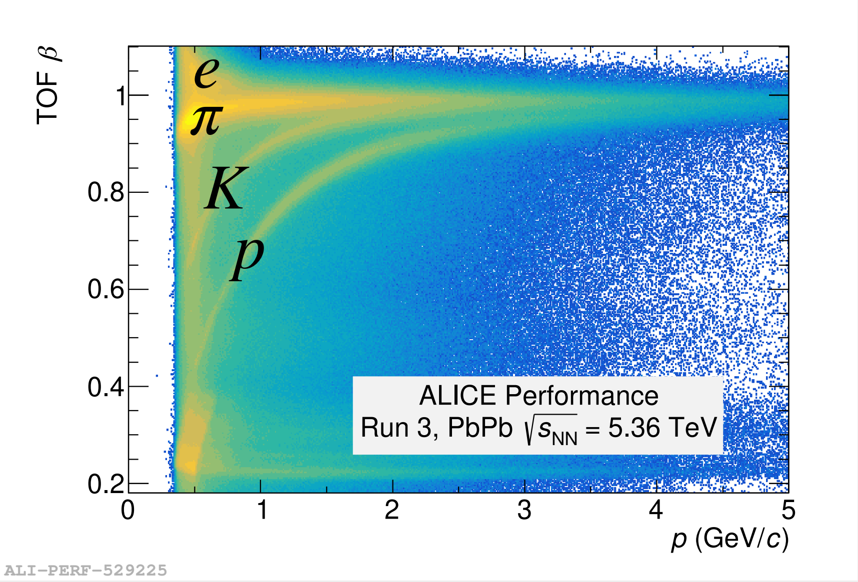
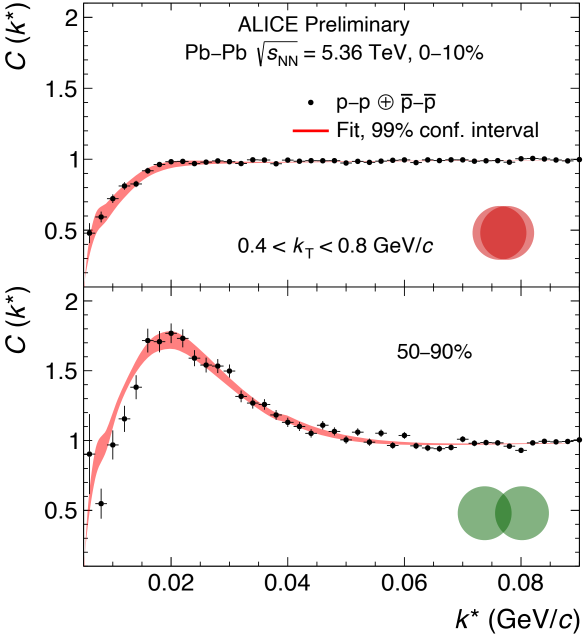
TOF shows excellent proton PID thankfully to the $3\sigma\ K - p$ separation up to 4 GeV/c!



Results


ALICE Preliminary
Pb–Pb $\sqrt{s_{\mathrm{NN}}}$ = 5.36 TeV, 0–10%
p–p ⊕ p̅–p̅
Fit, 99% conf. interval
0.4 < k_{T} < 0.8 GeV/c
50–90%
$C(k^*)$
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2
1.5
1
0.5
0.02
0.04
0.06
0.08
k^* (GeV/c)

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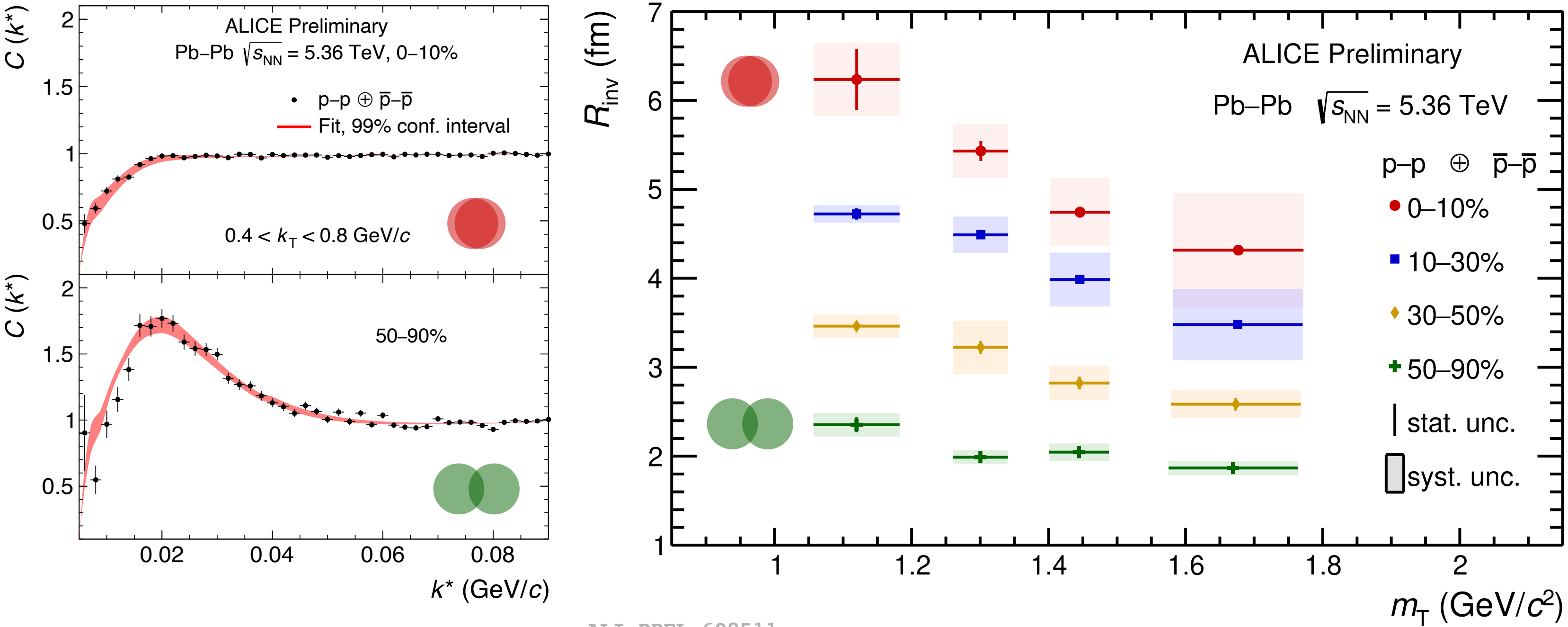
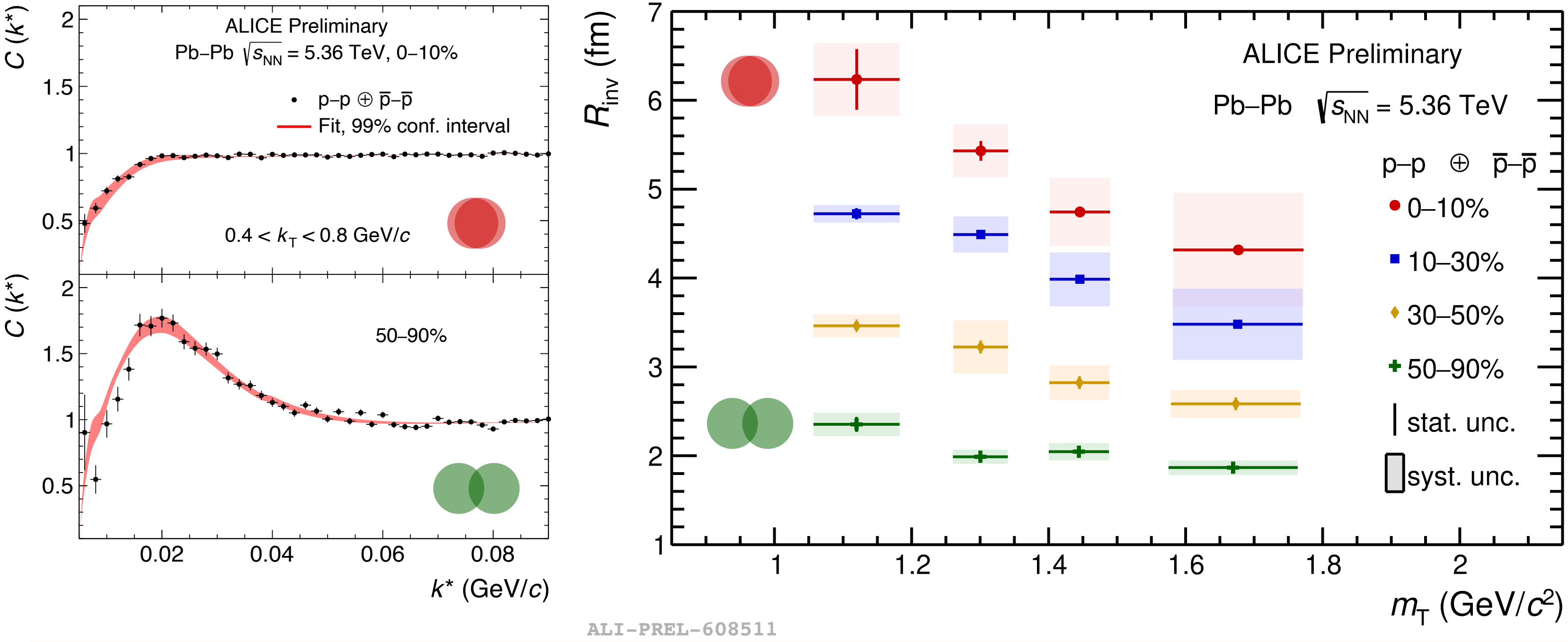


ALI-PREL-608511

Results



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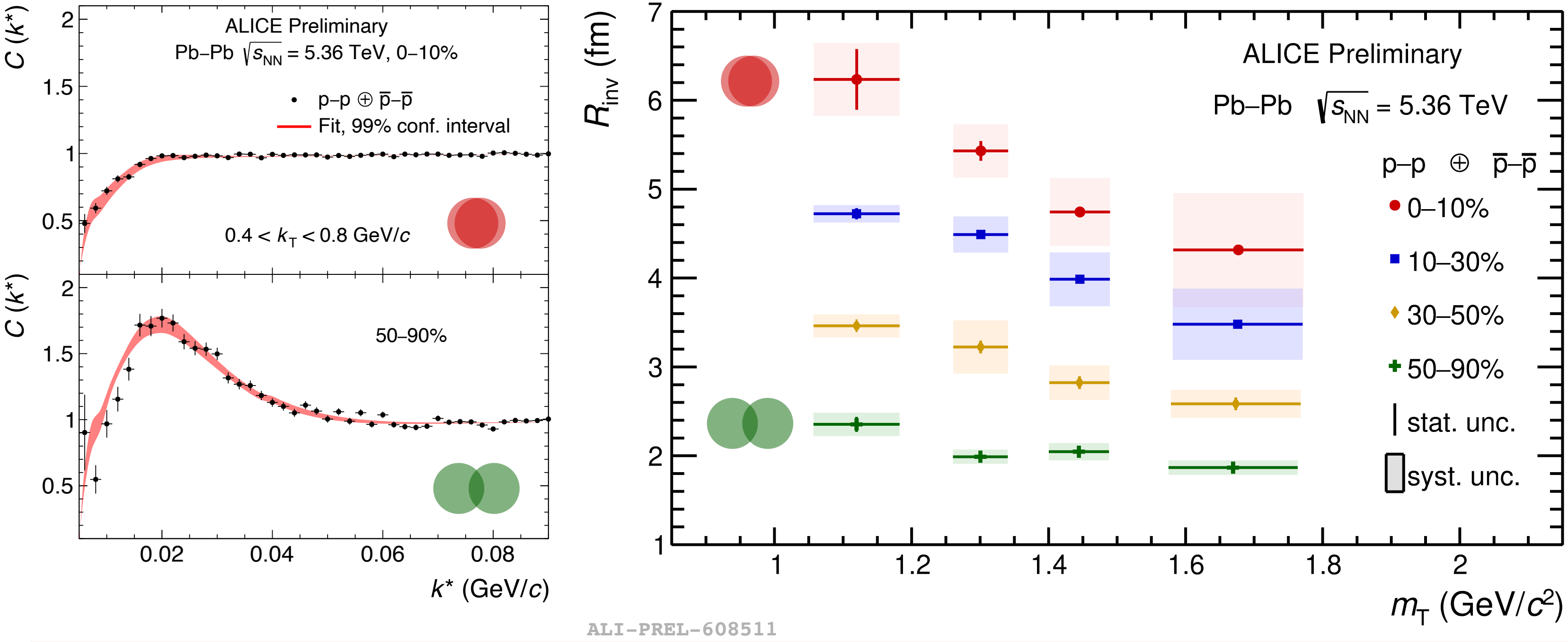

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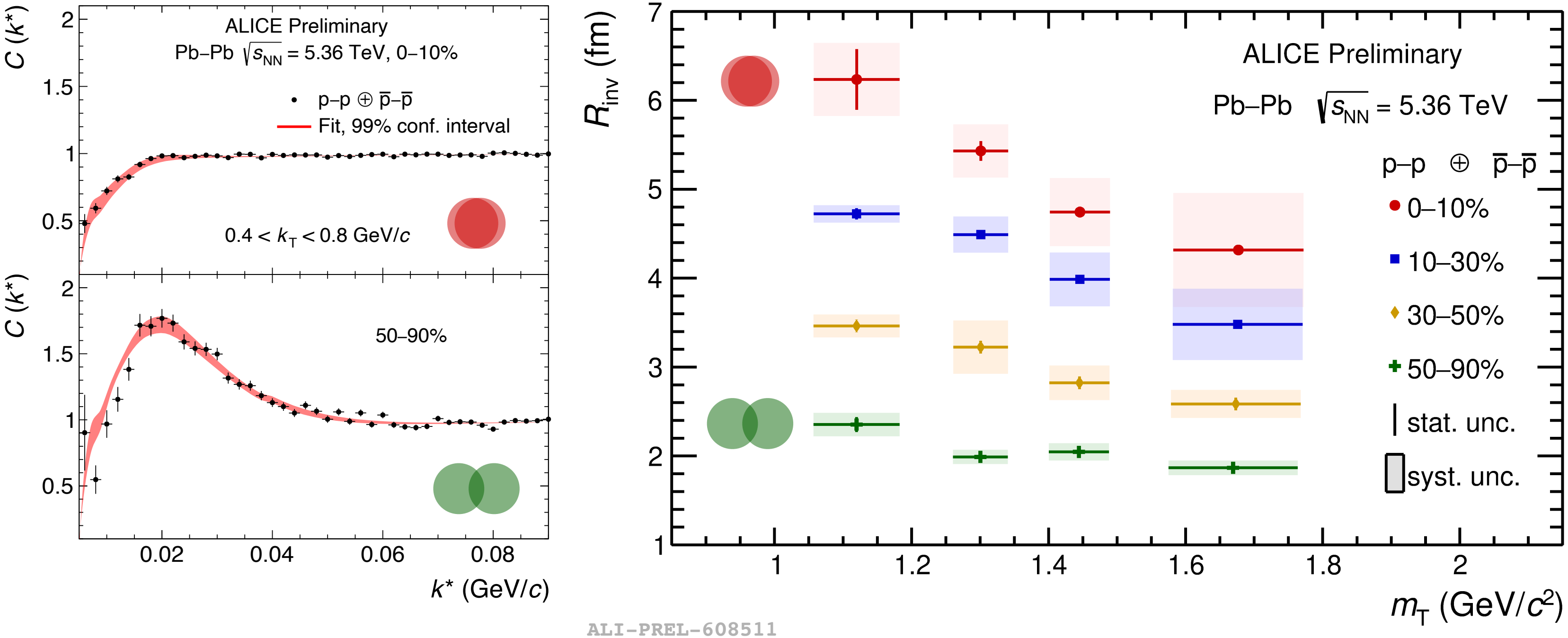

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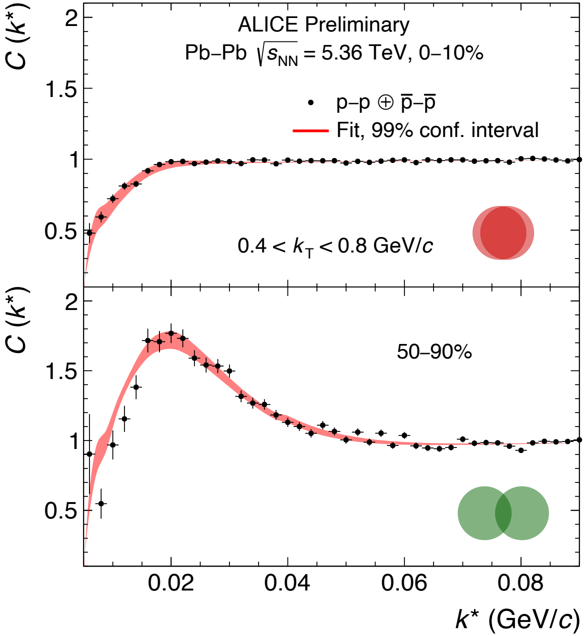

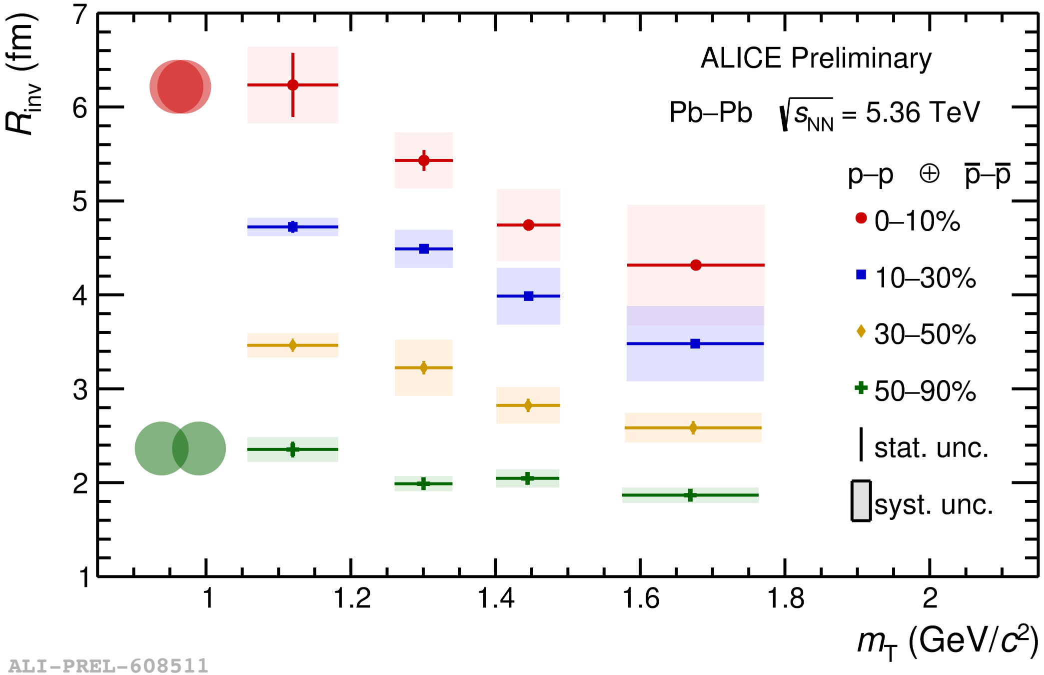

ALI-PREL-608511

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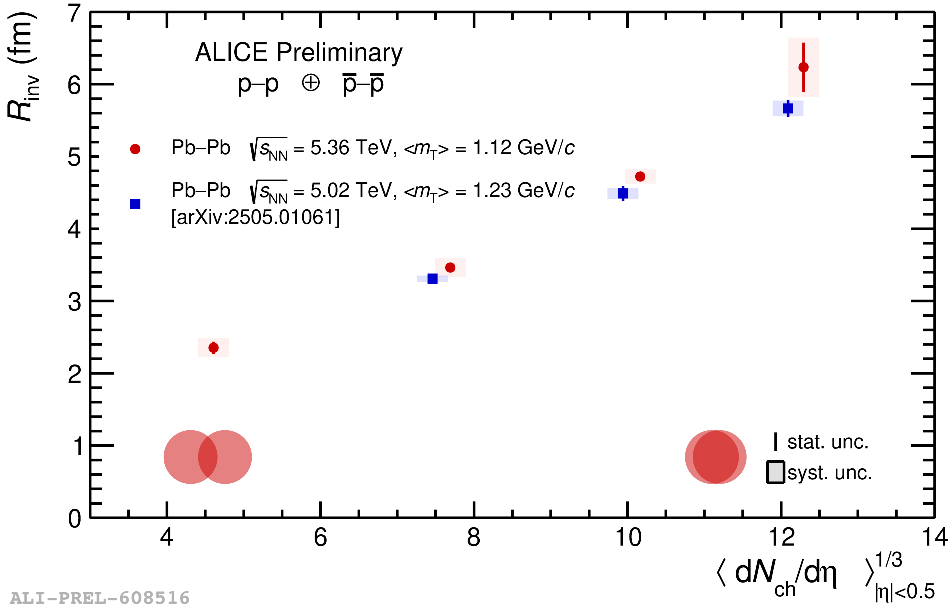
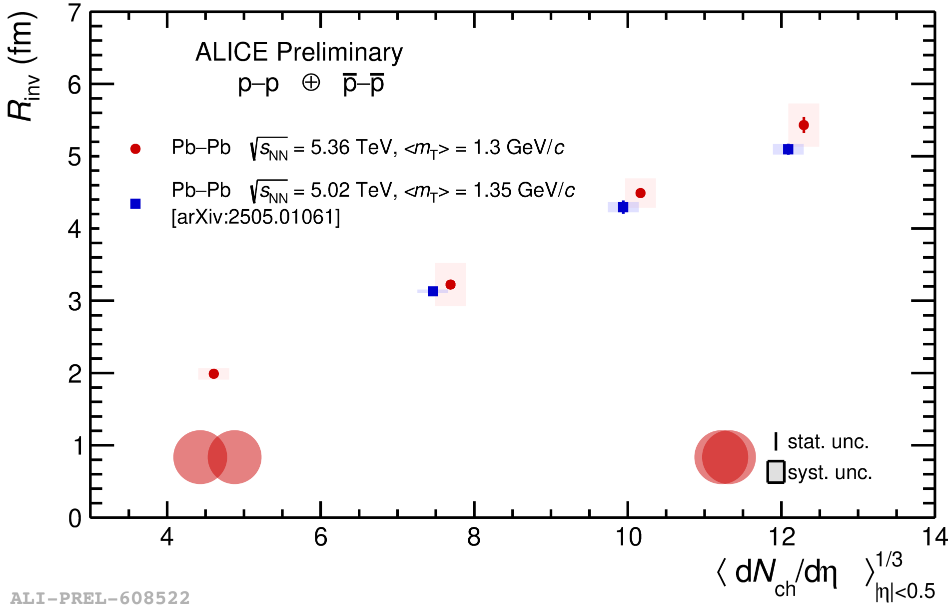
Results: comparison with Run 2



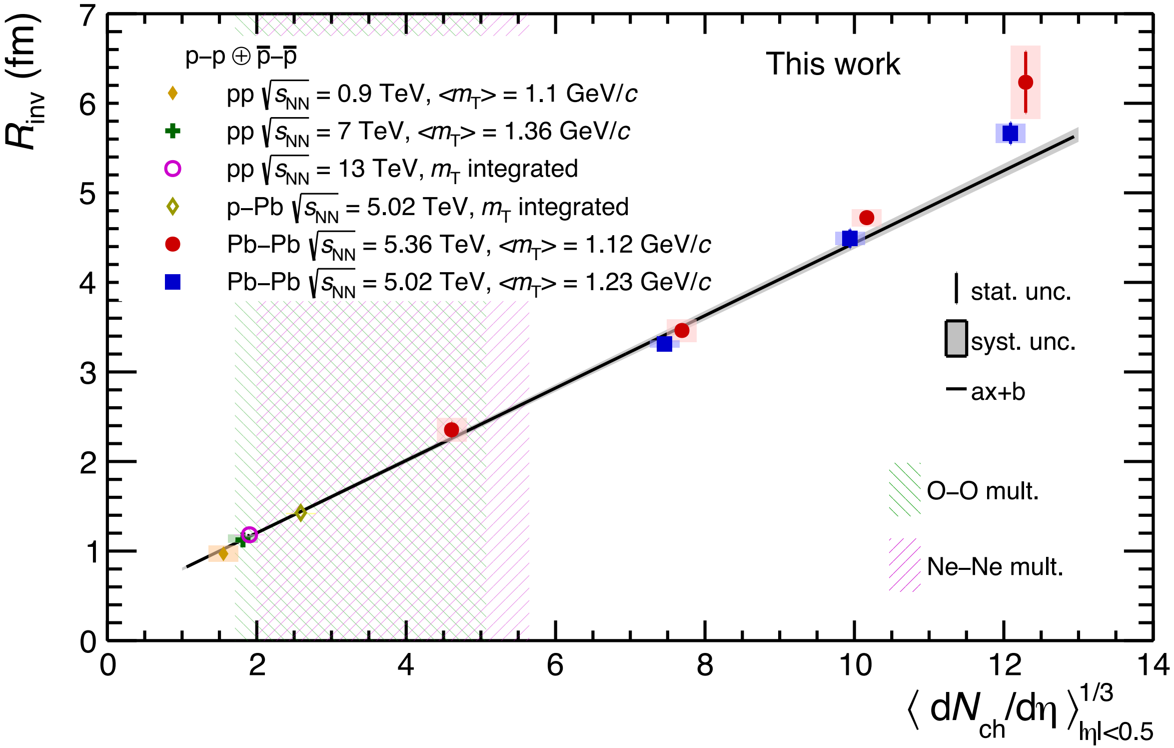
ALI-PREL-608516



ALI-PREL-608522

- **The new Run 3 results are consistent with Run 2 data (obtained using CATS) proving the reliability of the theoretical model and the analysis framework we developed**
- **More peripheral events are accessed w.r.t. Run 2 results**
- **My results are preliminary. Precision is expected to improve.**

More results



The analysis chain we developed is **successfully used** not only in my work but **in our Bologna group** providing a **detailed scan of proton-emitting source sizes** in various colliding systems used at the LHC.

Future prospects and plans

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- **ALICE3:** more precise measurements of proton-proton correlation — possible **sensitivity to the strong potential shape** (?) + **3D measurements** (!)
- **ePIC (EIC):** polarised colliding beams — possibility to **study strong potential for a chosen spin-orbital pair configuration** (?)

3D proton femtoscopy

Experimental measurement of proton-emitting source **has never been done with a 3D source parametrisation**.

Main challenges:

- Much **more statistics is needed** w.r.t. 1D (excited for the **ALICE3** !).
- **Complex fitting procedure** (need to fit a 3d hypersurface instead if 1d distribution).
- Center-of-Mass frame is used in 1D for both source parametrisation and WF calculation. 3D uses the Longitudinally Co-Moving System (LCMS) for the source. **Need to connect two frames.**

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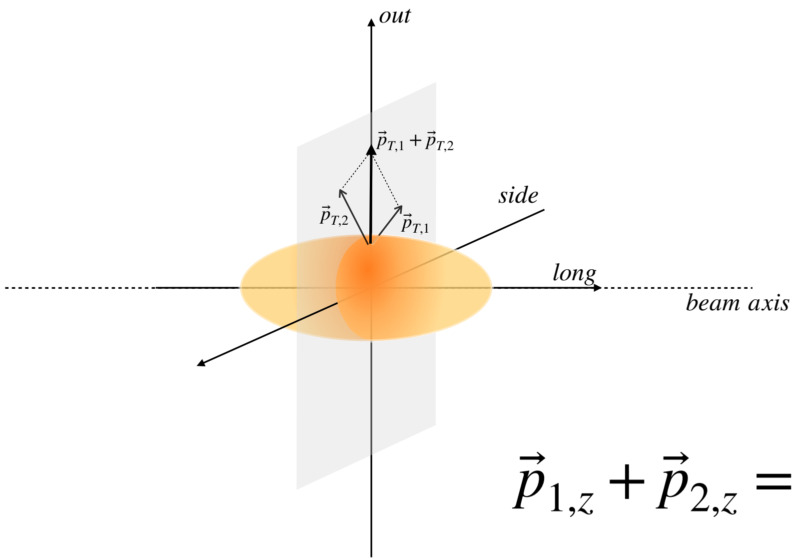
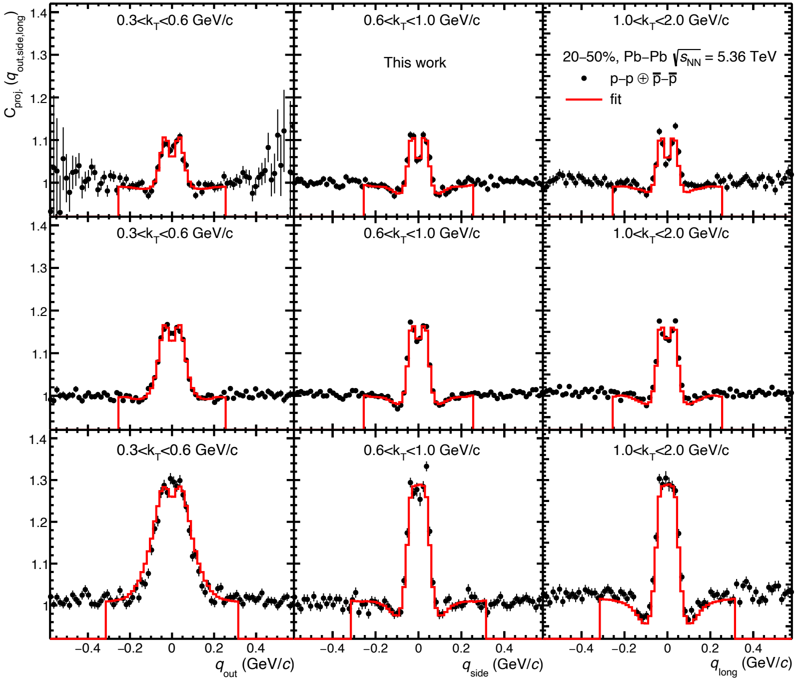
LCMS frame



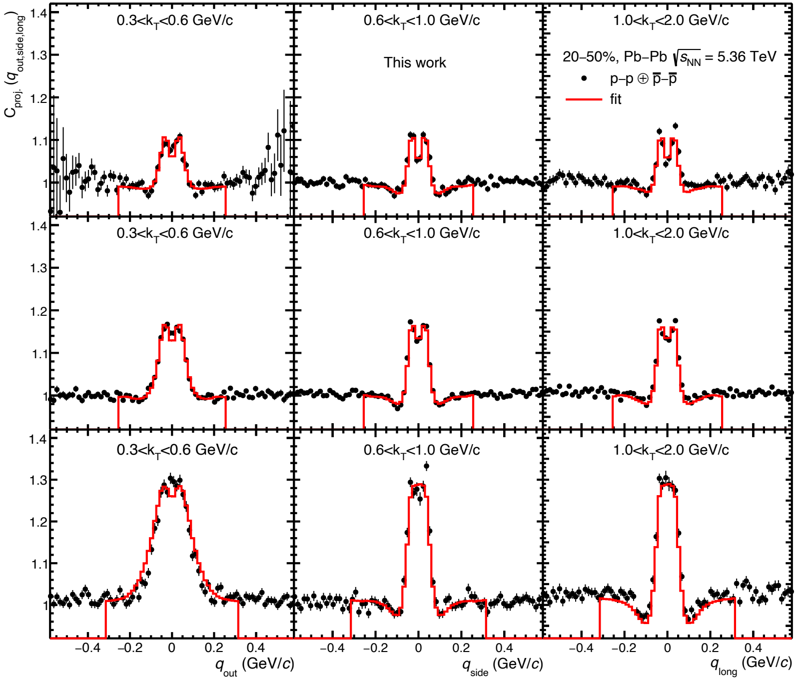
$$\vec{p}_{1,z} + \vec{p}_{2,z} = 0$$

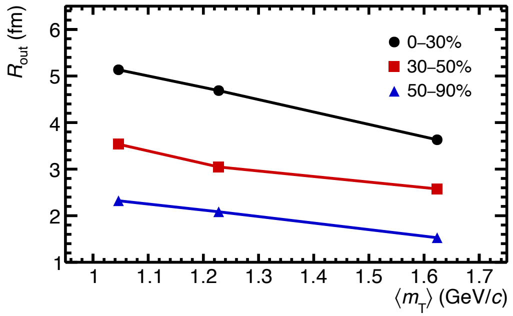
"long": along the beam
"out": along pair's total transverse momentum
"side": perpendicular to the two

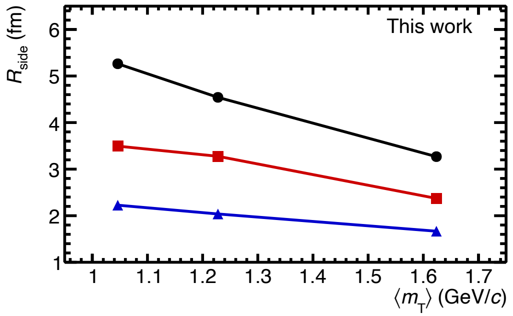
3D proton femtoscopy

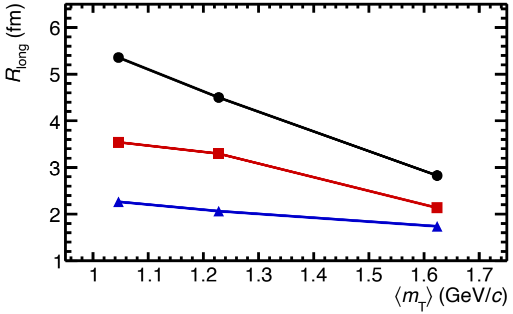


3D proton femtoscopy



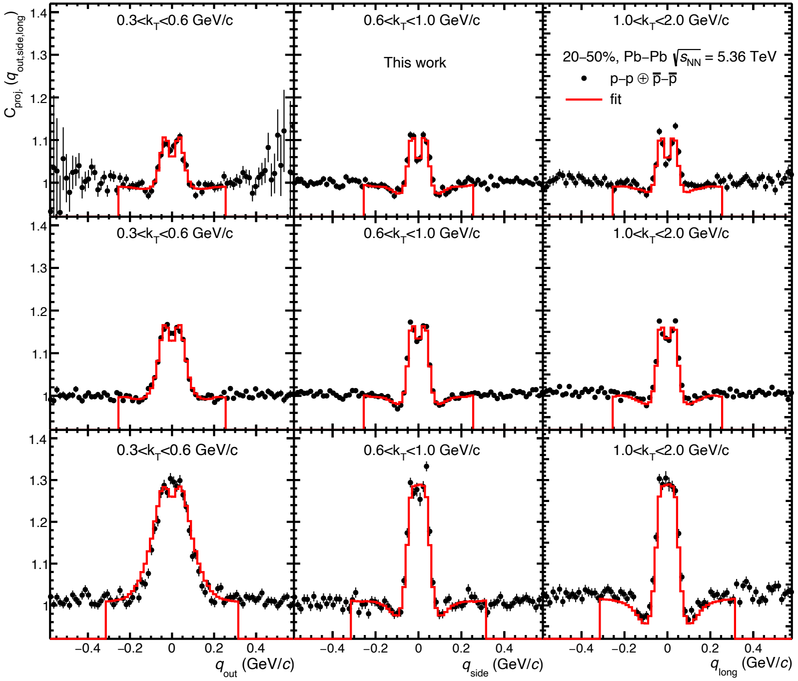


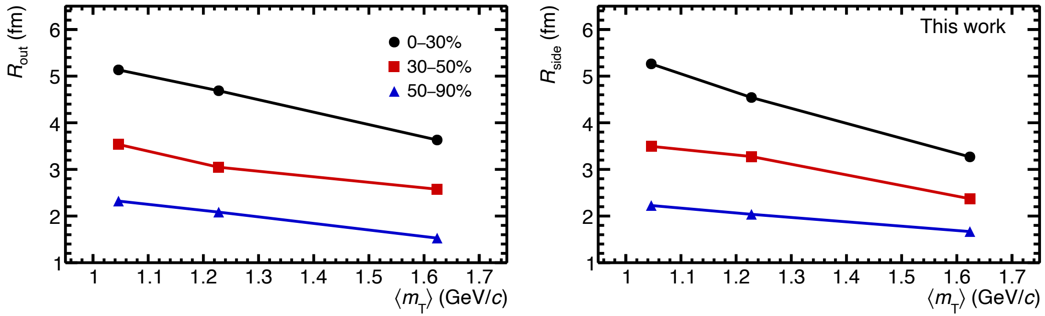


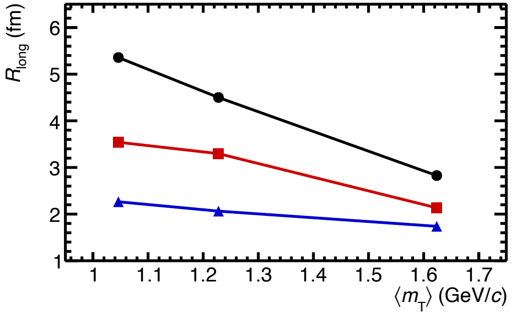


First ever for protons!

3D proton femtoscopy



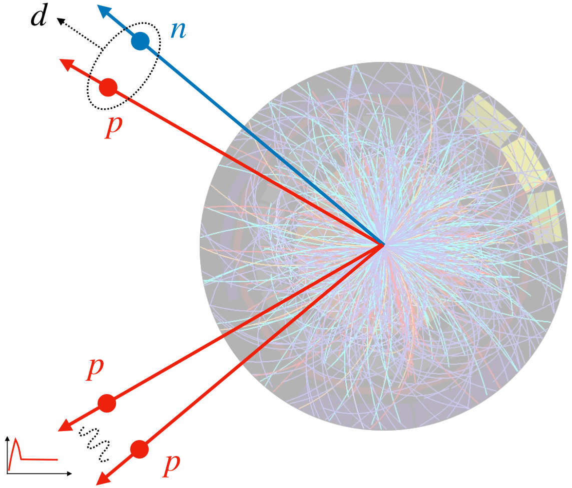




First ever for protons!

- Measurement of an **asymmetric proton-emitting source** in three dimensions
- Sizes scale with m_T and centrality — as it should be
- **I claim that such analysis is possible!**

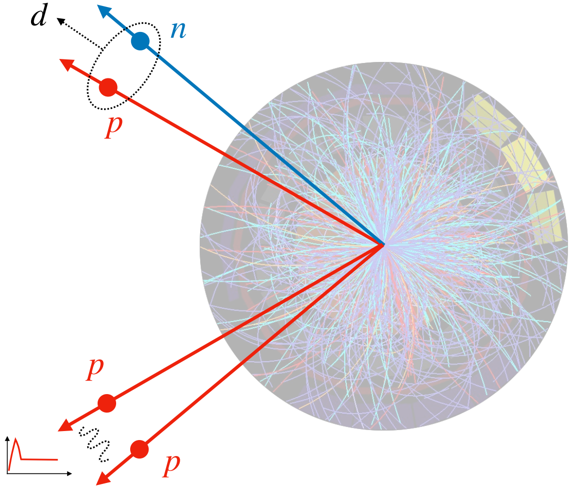
Coalescence



Coalescence models describe light (anti) nuclei production. They employ Quantum Mechanical formalism to estimate the probability of nuclei formation from unbound nucleons. Main inputs:

- Momentum distribution (given by event generators)
- Spatial distribution of nucleon emission (provided by **femtoscopy**)
- **WF calculation**

Coalescence



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Application of my research results:

- Extracted **proton-emitting source sizes** are adopted straightaway
- Adoption of **my theoretical calculations for QM description of nucleon-nucleon system** (already validated with femtosopy)

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- **Adoption of the results by coalescence models helps in understanding light (anti) nuclei production in HICs**
- **Exciting perspectives to study physics with the upcoming ALICE3 and ePIC (EIC) experiments!**

Backup slides

Starting with the Coulomb

The WF that satisfies the Schrodinger's equation for the Coulomb potential is well known (here we already anticipate the partial wave expansion):

$$\psi^{reg} = \sum_{l=0}^{\infty} (2l+1) i^l e^{i\sigma_l} \frac{F_l(\eta, \rho)}{\rho} P_l(cos\ \theta)$$

we need the solution to be regular at 0 so we close the regular Coulomb WF

or $$\psi^{reg} = \frac{1}{2\rho} \sum_{l=0}^{\infty} (2l+1)\ i^{l+1} \left(u_l^-(\eta,\rho) - e^{2i\sigma_l} u_l^+(\eta,\rho) \right) P_l(cos\ \theta)$$

Adding here an additional phase shift one can obtain the analytical solution for a short-range potential in the asymptotic region

$\rho_l = k_l \cdot r$

$\eta_l = \dfrac{1}{k_l a_B}$

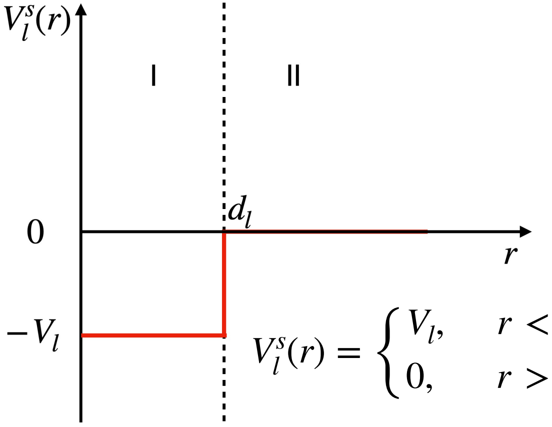
Albert Messiah, Quantum Mechanics, v1

Adding box potential

Let’s introduce a short-range potential as a square-well for each value of l.

Radial Schrodinger’s equation in (I) sector:

$$\frac{\mathrm{d}^2 R_l^{(I)}}{\mathrm{d}r^2} + \left[k^2 - \frac{l(l+1)}{r^2} - \frac{2}{a_B r} - \frac{2\mu}{\hbar^2} V_l\right] R_l^{(I)} = 0$$



$$V_l^s(r) = \begin{cases} V_l, & r < d_l \quad (I) \\ 0, & r > d_l \quad (II) \end{cases}$$

Substituting the sq.-well potential:

$$\frac{\mathrm{d}^2 R_l^{(I)}}{\mathrm{d}r^2} + \left[\tilde{k}^2 - \frac{l(l+1)}{r^2} - \frac{2}{a_B r}\right] R_l^{(I)} = 0$$

$$\tilde{k}_l = \sqrt{k^2 - \frac{2\mu}{\hbar^2} V_l}$$

But we already know the solution (here we put directly the total):

$$\psi^{reg} = \sum_{l=0}^{\infty} (2l+1) i^l e^{i\tilde{\sigma}_l} \frac{F_l(\tilde{\eta}, \tilde{\rho})}{\tilde{\rho}} P_l(cos\,\theta)$$

Final solution and CF

After matching the two WFs (within the box potential and the asymptotical one) one can get the final WF:

$$\psi_{c+s}(k,\, r) = \frac{1}{r} \sum_{l=0}^{\infty} (2l+1)\; i^{l} e^{i\sigma_l}\; u_l(k,\, r)\; P_l(cos\;\theta)$$

$$u_l(k,\, r) = \begin{cases} \frac{F_l(\tilde{\eta}_l,\, \tilde{k}_l r)}{F_l(\tilde{\eta}_l,\, \tilde{k}_l d)} \left(\frac{F_l(\eta, kd)}{k} + f_l(k)\; \left(G_l(\eta, kd) + i\; F_l(\eta, kd) \right) \right), & r < d \\ \left(\frac{F_l(\eta, \rho)}{k} + f_l(k)\; \left(G_l(\eta, \rho) + i\; F_l(\eta, \rho) \right) \right), & r \geq d \end{cases}$$

General expression for the CF:

$$C(k,\, R_{inv}) = \int d^3r \cdot S(r,\, R_{inv}) \cdot |\psi(\vec{k},\, \vec{r})|^2$$

$$S(r,\; R_{inv}) = \frac{1}{8\pi^{\frac{3}{2}} R_{inv}^3} exp(-\frac{r^2}{4R_{inv}^2})$$

— assuming Gaussian source

For a pair of protons with L=[0, 1]. Corresponding states:

$$C_{pp}(k*,\, R_{inv}) = \frac{1}{2} \sum_{S=0}^{1} \frac{2S+1}{(2s_p+1)^2} \sum_{L,\, J} \omega_{LJ} \int d^3r\; S(r,\, R_{inv})\, |\psi_{-\vec{k}}^{S}(\vec{r}) + (-1)^S \psi_{\vec{k}}^{S}(\vec{r})|^2$$

$$\omega_{LJ} = \frac{2J+1}{(2L+1)(2S+1)}$$

Defining the potential parameters

OK, we have an analytical WF for a square-well potential, but what are its parameters (depth and width)?

One can obtain them by fitting momenta(energy)-dependent phase shifts with our matching condition:

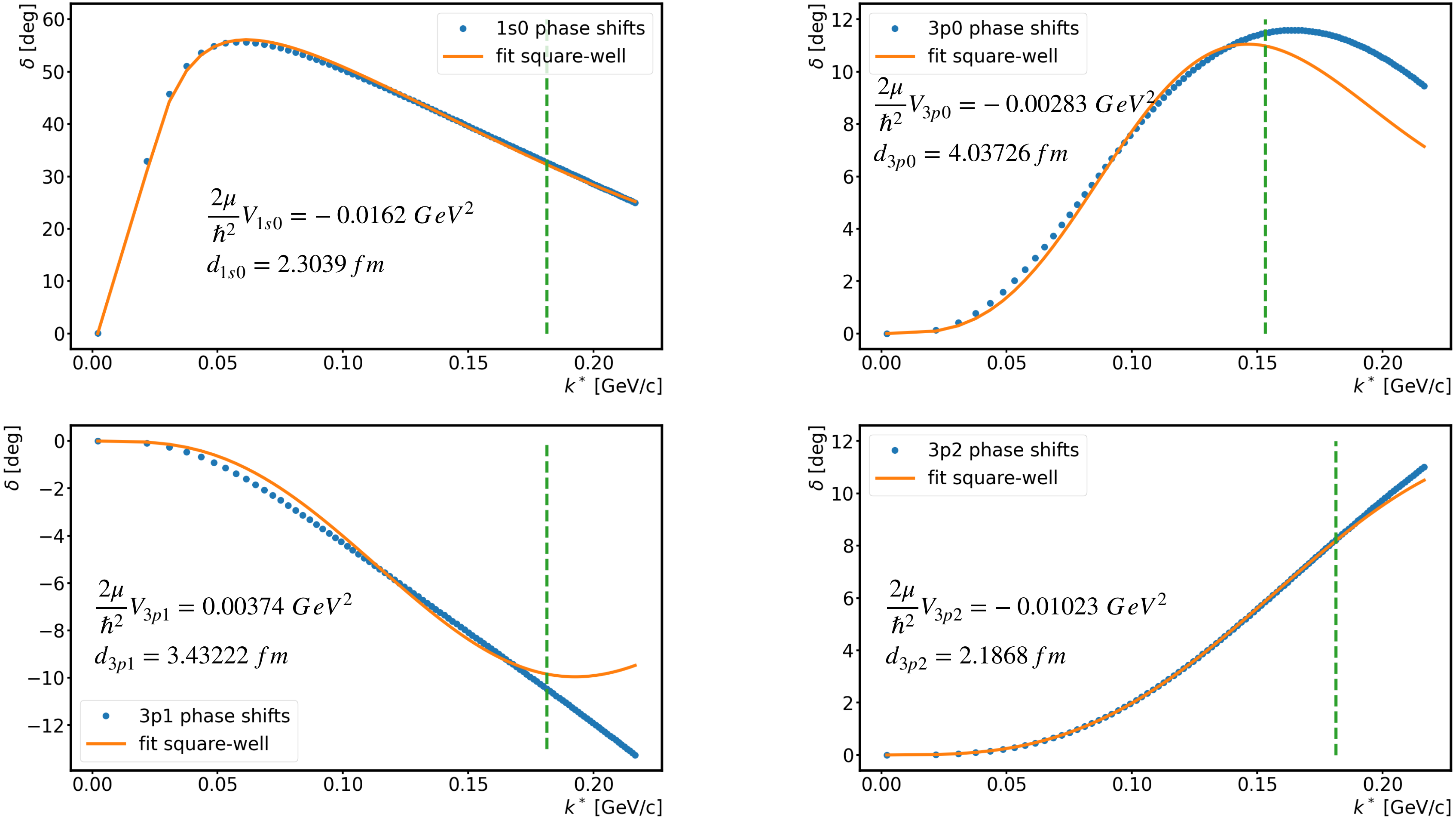
$$ctg\,\delta_l = \frac{G_l(\eta, kd)}{F_l(\eta, kd)}\,\frac{\tilde{k}\,f_l(\tilde{\eta}, \tilde{k}d) - k\,g_l(\eta, kd)}{k\,f_l(\eta, kd) - \tilde{k}\,f_l(\tilde{\eta}, \tilde{k}d)}$$

$$f_l(\eta,\rho) = \frac{\mathrm{d}}{\mathrm{d}r}\Big(ln\,(F_l(\eta,\rho))\Big)\Big|_{r=d}$$

$$g_l(\eta,\rho) = \frac{\mathrm{d}}{\mathrm{d}r}\Big(ln\,(G_l(\eta,\rho))\Big)\Big|_{r=d}$$

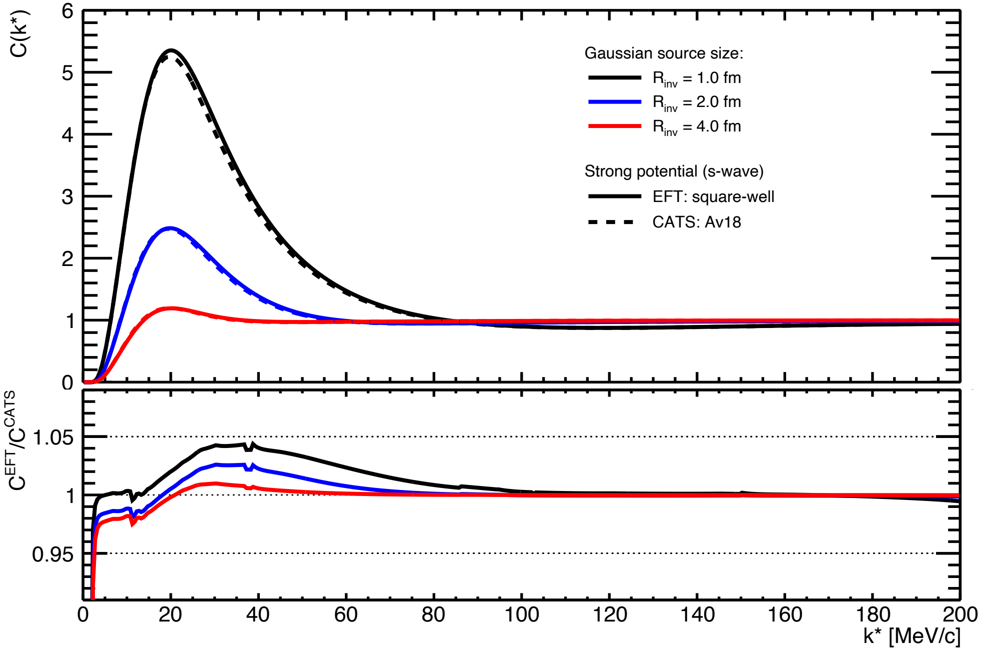
Defining the potential parameters

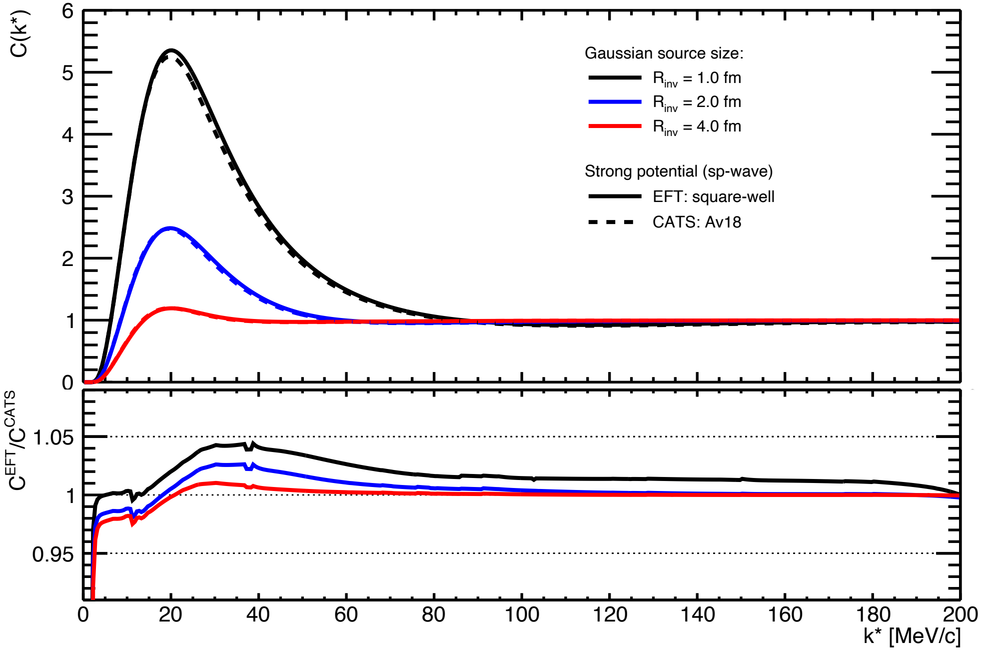
Phase shifts data are taken from: https://nn-online.org/



Green line — chosen range of the fit

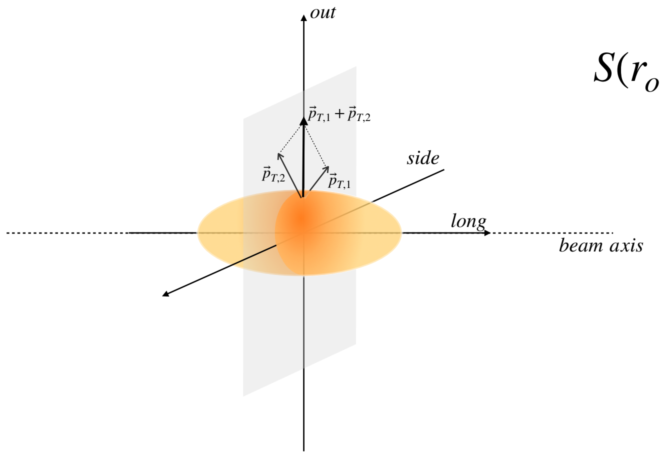
Comparing different approaches

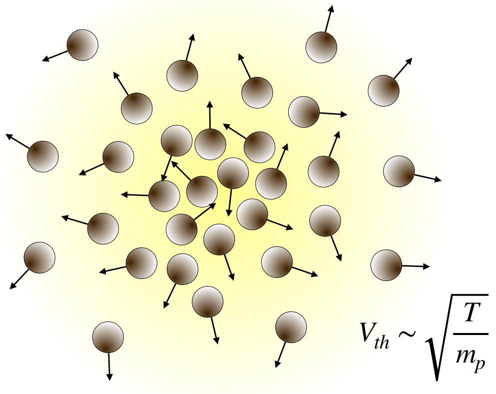




Longitudinally Co-Moving System (LCMS)

$$S(r_{out}, r_{side}, r_{long}) \propto e^{-\frac{r_{out}^2}{4R_{out}^2}} e^{-\frac{r_{side}^2}{4R_{side}^2}} e^{-\frac{r_{long}^2}{4R_{long}^2}}$$





$$V_{th} \sim \sqrt{\frac{T}{m_p}}$$

$$r_{out} = x_2 - x_1 = \gamma_T(x_1^* + v_T t_1^*) - \gamma_T(x_2^* + v_T t_2^*) = \gamma_T(r_{out}^* + v_T \Delta t^*)$$

3D sizes:

- $R_{out}^2 = \gamma_T^2\left({R_{out}^*}^2 + v_T^2\sigma_\tau^2\right)$ — size and emission duration
- $R_{side} = R_{side}^*$ — size
- $R_{long} = R_{long}^*$ — emission moment

* — corresponds to the PRF (at freeze-out)

- No pressure gradient along the beam -> expands thermally -> $R_{long} \propto \tau_B V_{th} = \tau_B \sqrt{\frac{T}{m_T}}$
- Explosive expansion in the transverse direction -> collectivity -> $V = V_{th} + V_{coll}(r)$
- **Radii scale with particle transverse mass** $m_T = \sqrt{E^2 - p_z^2} = \sqrt{m^2 + p_T^2}$

Bridge between LCMS and PRF

The definition of the PRF: $\vec{p}_1 = -\vec{p}_2$

The definition of the LCMS:

1. boost along Z =>
2. "out" axis is along pair's k$_T$ =>

$$p_{1\ long} = -p_{2\ long}$$
$$p_{1\ side} = -p_{2\ side}$$

2 out of 3 components are already in the PRF

Now if we remember that we deal with momenta differences we can connect the "out" component of the relative momentum in the LCMS with it's component in the PRF:

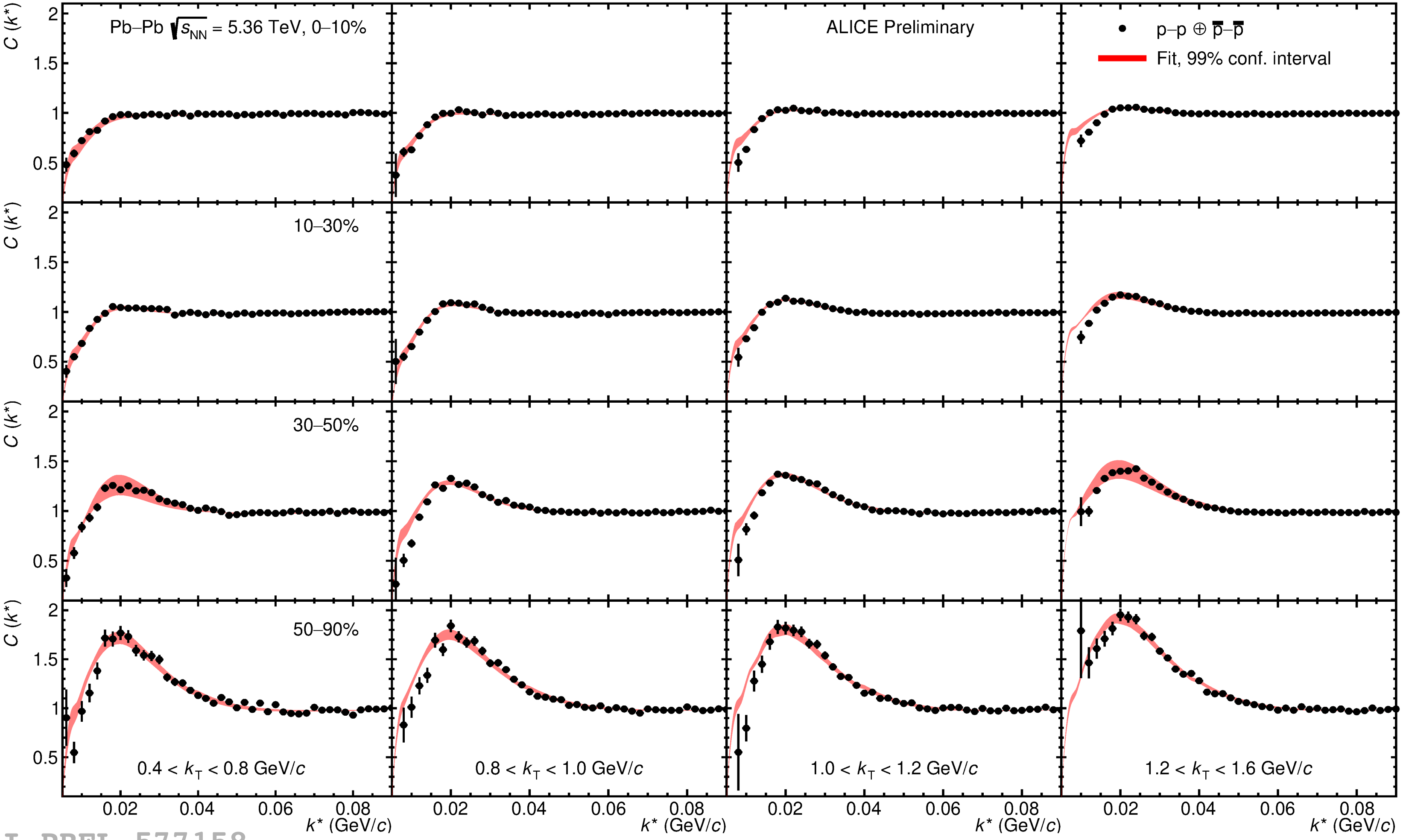
$$q_{out}^{PRF} = \frac{q_{out}^{LCMS}}{\gamma}$$

γ — Lorentz factor between PRF and LCMS

The same way we can connect the "out" component of the source size:

$$R_{out}^{PRF} = \gamma \cdot R_{out}^{LCMS}$$

Proton CFs in Pb–Pb at 5.36 TeV



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