Hyperon Form Factors in the Complex Plane

XXII LNF SPRING SCHOOL "BRUNO TOUSCHEK"
in Nuclear, Subnuclear and Astroparticle Physics

9th Young Researchers' Workshop

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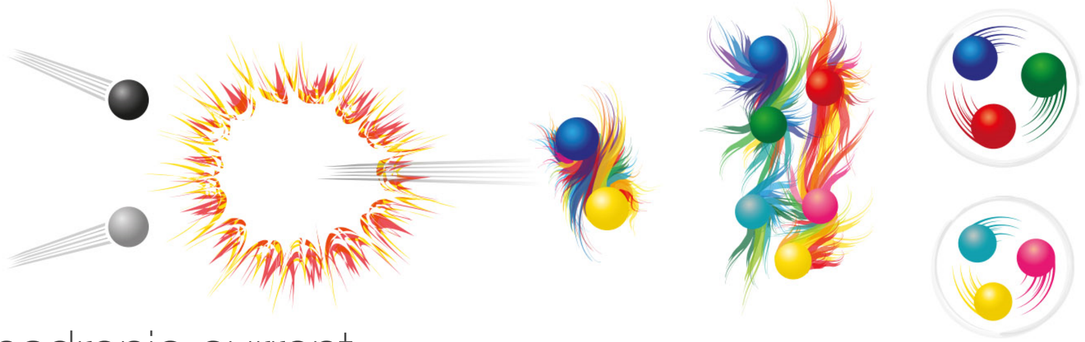
BESIII

Overview

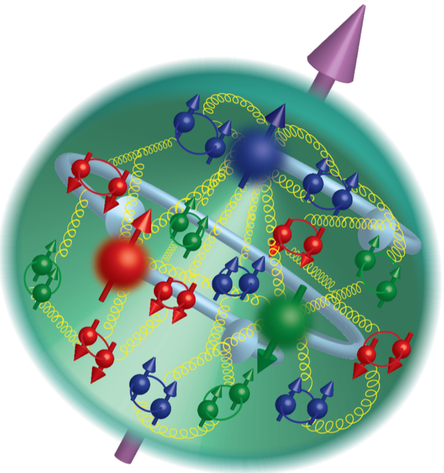
- Baryon form factor analyticity
- The Λ^0 and Σ^+ baryons special cases
- Dispersion relations for the form factor ratio
- Parametrization and χ^2 definition
- Results and discussions

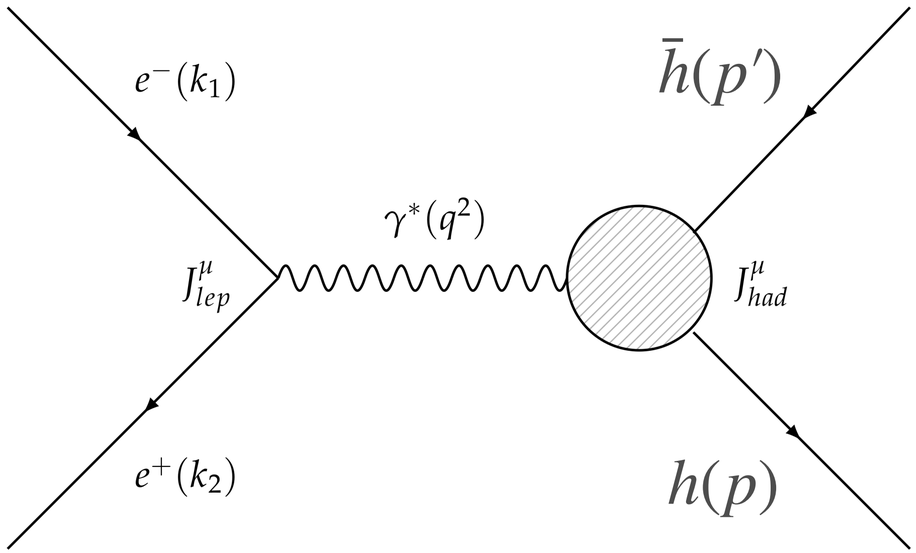
Hadronic Form Factors



The hadron h electromagnetic vertex $hh\gamma$ describes the hadronic current

$$\langle h(p',\lambda')\,|\,J^{\mu}_{had}(0)\,|\,h(p,\lambda)\rangle = \bar{u}(p',\lambda')\left[\sum_j F_j(q^2)C^{\mu}_j\right]u(p,\lambda)$$





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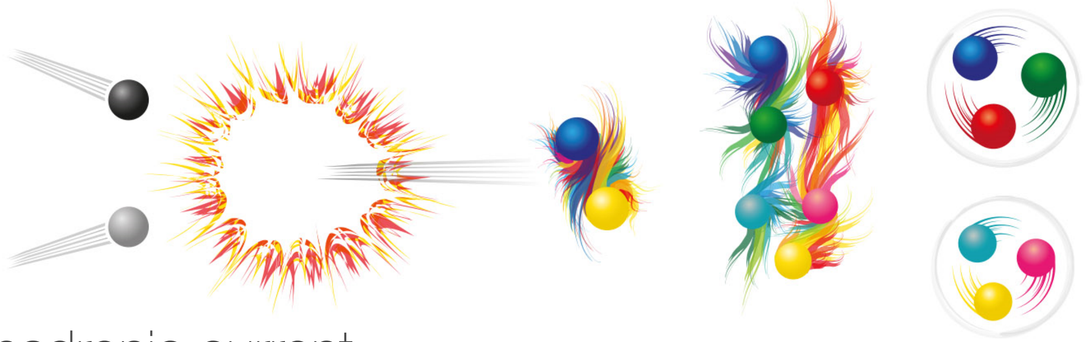

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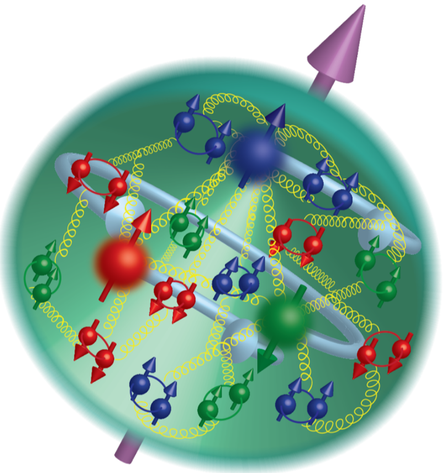

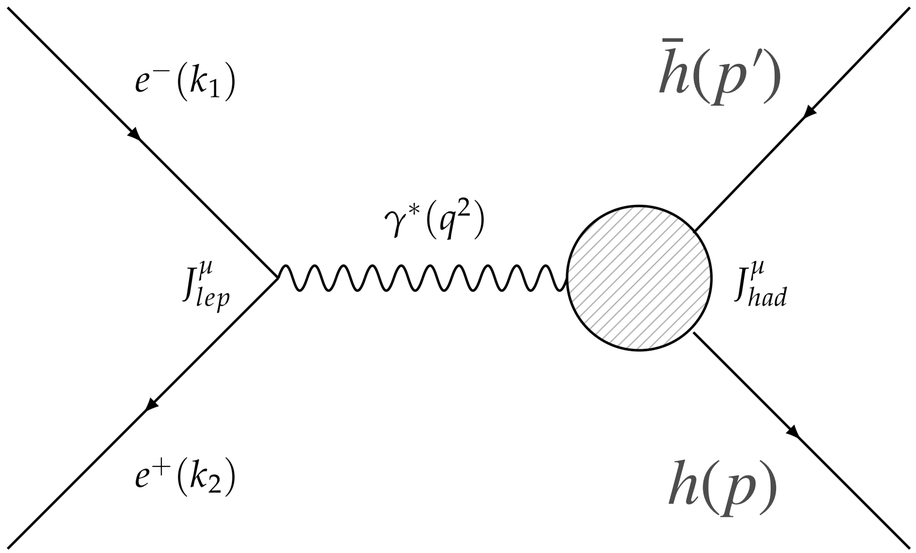

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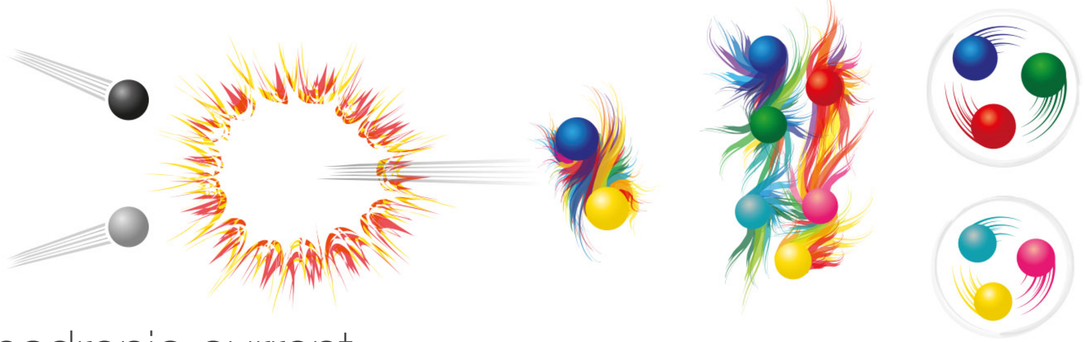

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Clifford Algebra Structures

$$\langle h(p',\lambda')\,|\,J^{\mu}_{had}(0)\,|\,h(p,\lambda)\rangle = \bar{u}(p',\lambda')\left[\sum_j F_j(q^2)\,C^{\mu}_j\right]u(p,\lambda)$$

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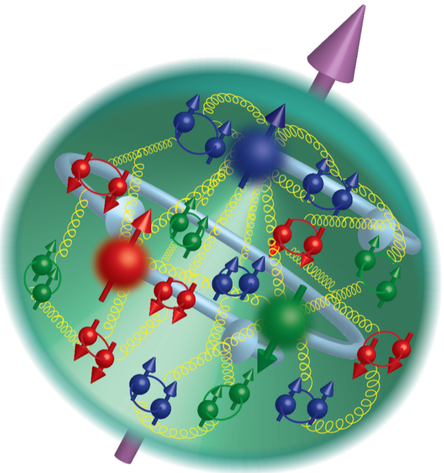

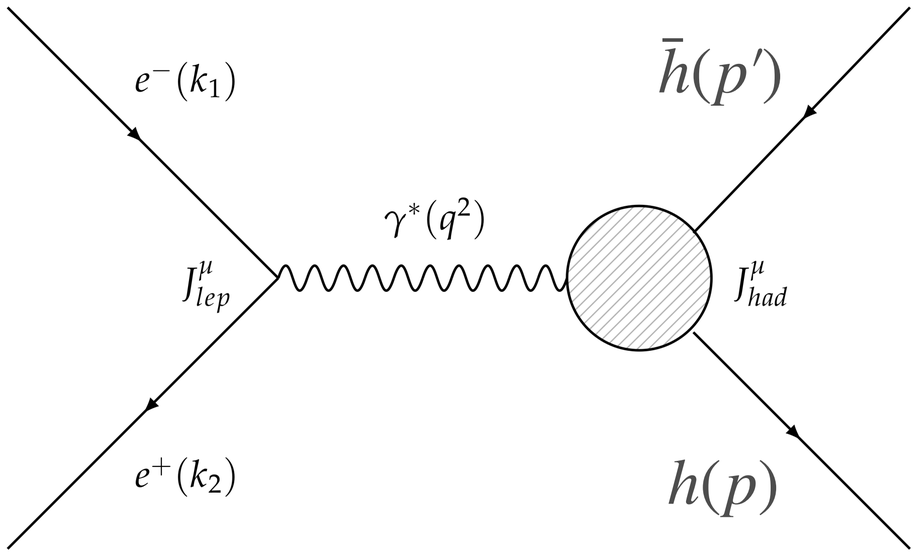

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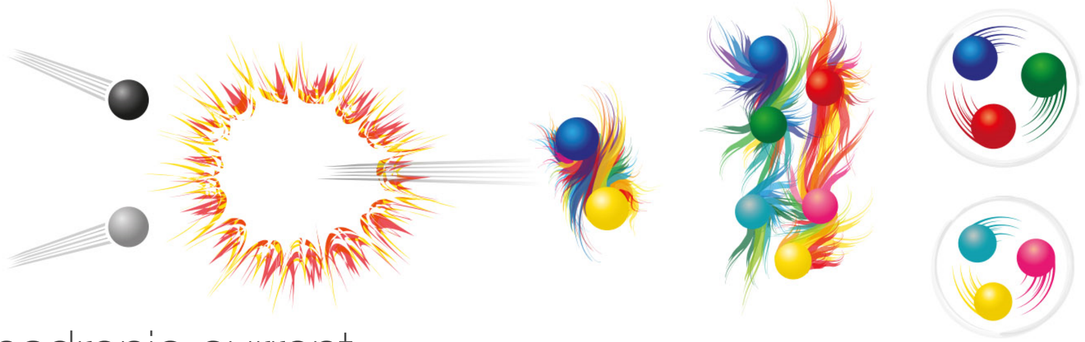

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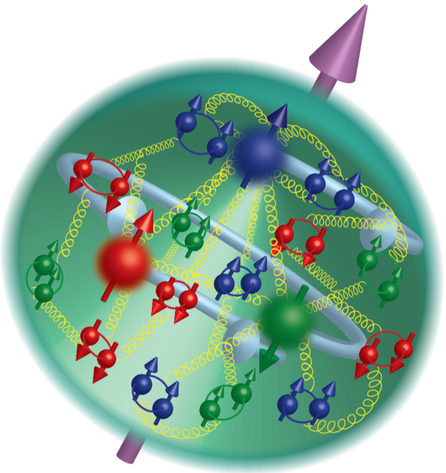
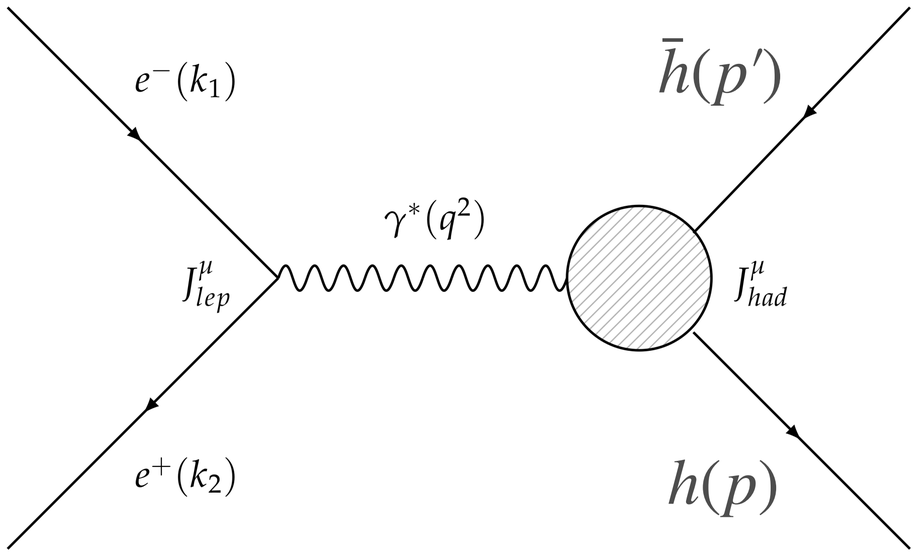
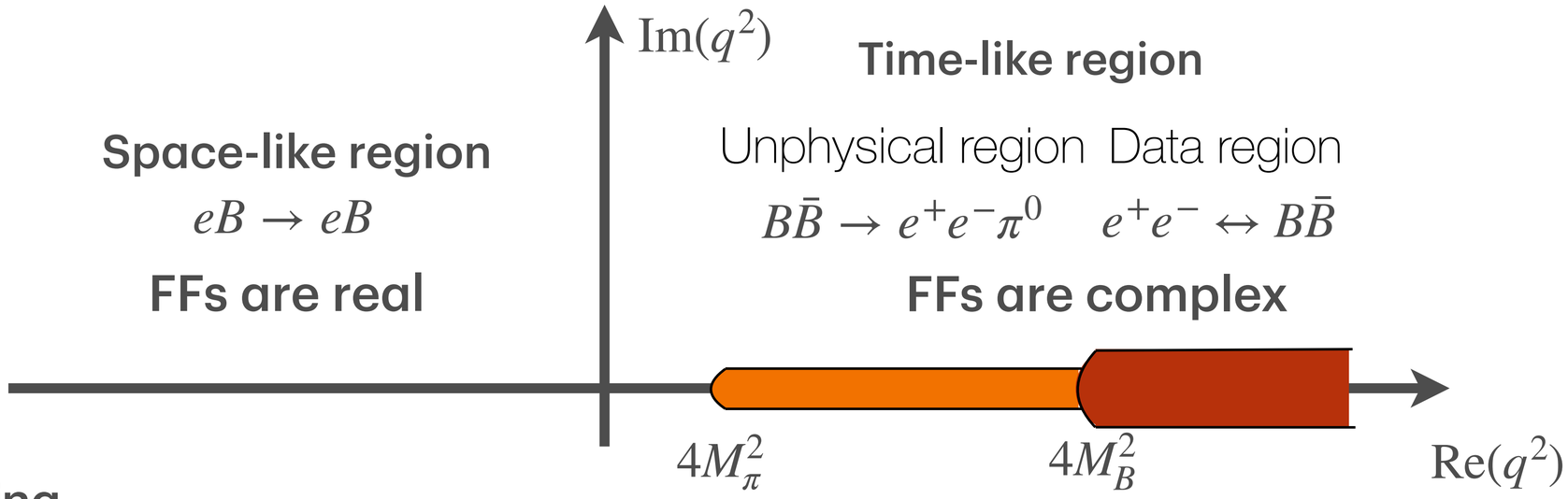




$2S+1$ independent FFs for a spin S hadron, e.g. $\mathscr{B}$

$$J^{\mu}_{\mathscr{B}}(p,p') = \gamma^{\mu}F_1^{\mathscr{B}}(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M_{\mathscr{B}}}F_2^{\mathscr{B}}(q^2)$$

Baryon Form Factors



Elastic Scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\omega_2\cos^2\frac{\theta_e}{2}}{4\omega_1^3\sin^4\frac{\theta_2}{2}}\left[(G_E^B)^2 - \tau\left(1+2(1-\tau)\tan^2\frac{\theta_e}{2}\right)(G_M^B)^2\right]\frac{1}{1-\tau} \qquad \tau = \frac{q^2}{4M_B^2}$$

Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha\beta\mathscr{C}}{4q^2}\left[(1+\cos^2\theta)\,|\,G_M^B\,|^2 + \frac{1}{\tau}\sin^2\theta\,|\,G_E^B\,|^2\right] \qquad \beta = \sqrt{1-\frac{1}{\tau}}$$

Form factors in the time-like region

In the time-like region, $G_E^{\mathscr{B}}(q^2)$ and $G_M^{\mathscr{B}}(q^2)$ are complex functions

Crossing symmetry: $\left\langle P(p')\middle|J^\mu\middle|P(p)\right\rangle \to \left\langle \bar{P}(p')P(p)\middle|J^\mu\middle|0\right\rangle$

Optical theorem

$$\mathrm{Im}\left(\left\langle \bar{P}(p')P(p)\middle|J^\mu\middle|0\right\rangle\right) \approx \sum_n \left\langle \bar{P}(p')P(p)\middle|J^\mu\middle|n\right\rangle\left\langle n\middle|J^\mu\middle|0\right\rangle \Rightarrow \begin{cases} \mathrm{Im}\left(F_{1,2}^{\mathscr{B}}\right) \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

Where $|n\rangle$ are intermediate states, i.e. $|n\rangle = 2\pi, 3\pi, \ldots$

Phragmén Lindelöf theorem

If $f(z) \to f_1$ as $z \to \infty$ along the straight line L_1 and $f(z) \to f_2$ as $z \to \infty$ along the straight line L_2 and $f(z)$ is regular and bounded in the angle between the lines, then $f_1 = f_2 \equiv f_{12}$ and $f(z) \to f_{12}$ in the region between L_1 and L_2.

Asymptotic behaviour in the time-like region

$$\lim_{q^2\to+\infty} G_{E,M}^{\mathscr{B}}(q^2) = \lim_{q^2\to-\infty} G_{E,M}^{\mathscr{B}}(q^2)$$

Λ^0 and Σ^+ cases

Λ^0 and Σ^+ cases

Theoretical threshold

Λ^0 and Σ^+ cases

Λ^0

$$q_{\text{th}}^2 = \left(2M_\pi + M_{\pi^0}\right)^2$$
$$\approx 0.171\,\text{GeV}^2$$

Theoretical threshold

$\Lambda^0\bar{\Lambda}^0$ state is pure isoscalar

Λ^0 and Σ^+ cases

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$\Sigma^+\bar{\Sigma}^-$ state is isoscalar + isovector (like the proton)

Σ^+

$$q_{\text{th}}^2 = \left(2M_\pi\right)^2$$
$$\approx 0.078\,\text{GeV}^2$$

Λ^0 and Σ^+ cases

Λ^0

$$q^2_{\text{th}} = \left(2M_\pi + M_{\pi^0}\right)^2$$
$$\approx 0.171\,\text{GeV}^2$$

$$q^2_{\text{phy}} = \left(2M_\Lambda\right)^2$$
$$\approx 4.979\,\text{GeV}^2$$

Theoretical threshold

$\Lambda^0\bar{\Lambda}^0$ state is pure isoscalar

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Physical threshold

Minimum energy required to produce the $\mathscr{B}\bar{\mathscr{B}}$ couple

Σ^+

$$q^2_{\text{th}} = \left(2M_\pi\right)^2$$
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$$q^2_{\text{phy}} = \left(2M_{\Sigma^+}\right)^2$$
$$\approx 5.658\,\text{GeV}^2$$

Λ^0 and Σ^+ cases

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Physical threshold

Minimum energy required to produce the $\mathscr{B}\bar{\mathscr{B}}$ couple

- Form factors have nonzero imaginary parts for $q^2 \geq q^2_{\text{th}}$
- Unphysical and space-like regions have no data

Dispersion relations

The form factors $G_{E,M}^{\mathscr{B}}$ are analytic functions on the q^2-complex plane with the cut $\left(q_{\text{th}}^2, \infty\right)$ on the real axis.

Dispersion relations are based only on unitarity and analyticity ⇒ **model independent approach**.

Dispersion relation for the imaginary part ($q^2 < 0$):

$$G(q^2) = \frac{1}{\pi}\int_{q_{\text{th}}^2}^{\infty} \frac{\text{Im}(G(s))}{s - q^2} ds$$

Experimental Inputs

- Time-like data for form factor's moduli from $e^+e^- \leftrightarrow \mathscr{B}\bar{\mathscr{B}}$
- Time-like data for the relative phase from $e^+e^- \leftrightarrow \mathscr{B}^{\uparrow}\bar{\mathscr{B}}$

Dispersion relation for the real part ($q^2 \geq q_{\text{th}}^2$):

$$\text{Re}\left(G(q^2)\right) = \frac{\text{Pr}}{\pi}\int_{q_{\text{th}}^2}^{+\infty} \frac{\text{Im}(G(s))}{s - q^2} ds$$

Theoretical Inputs

- Analyticity
- Threshold values
- Asymptotic behaviour

Dispersive procedure

We define the ratio $R(q^2) = \dfrac{G_E^{\mathscr{B}}(q^2)}{G_M^{\mathscr{B}}(q^2)} \Rightarrow \begin{cases} G_E^{\mathscr{B}}(0) = Q_{\mathscr{B}} \\ G_M^{\mathscr{B}}(0) = \mu_{\mathscr{B}} \\ G_E^{\mathscr{B}}(q^2_{\text{phy}}) = G_M^{\mathscr{B}}(q^2_{\text{phy}}) \end{cases} \Rightarrow \begin{cases} R(0) = Q_{\mathscr{B}}/\mu_{\mathscr{B}} \\ R(q^2_{\text{phy}}) = 1 \end{cases}$

The asymptotic behaviour

$$\lim_{q^2 \to \pm\infty} R(q^2) = \frac{G_E^{\mathscr{B}}(q^2)}{G_M^{\mathscr{B}}(q^2)} = \mathcal{O}(1)$$

Subtracted dispersion relations for real and imaginary part

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{q^2_{\text{th}}}^{\infty} \frac{\text{Im}(R(s))}{s(s-q^2)} ds, \quad \forall q^2 \notin \left[q^2_{\text{th}}, \infty\right)$$

$$\text{Re}\left(R(q^2)\right) = R(0) + \frac{q^2}{\pi} \text{Pr} \int_{q^2_{\text{th}}}^{\infty} \frac{\text{Im}(R(s))}{s(s-q^2)} ds, \quad \forall q^2 \in \left[q^2_{\text{th}}, \infty\right)^+$$

The subtracted dispersion relations ensure the convergence of the integral and the normalization at $q^2 = 0$

Parametrization for the form factors' ratio

Parametrization through the set of **Chebyshev polynomials** $\left\{ T_j(x) \right\}_{j=0}^{N}$.

$$\mathrm{Im}(R(q^2)) \equiv Y\left(q^2; \overrightarrow{C}, q^2_{\mathrm{asy}}\right) = \begin{cases} \sum_{j=0}^{N} C_j T_j(x(q^2)), & q^2_{\mathrm{th}} < q^2 < q^2_{\mathrm{asy}} \\ 0, & q^2 \geq q^2_{\mathrm{asy}} \end{cases}$$

$$x(q^2) = 2\frac{q^2 - q^2_{\mathrm{th}}}{q^2_{\mathrm{asy}} - q^2_{\mathrm{th}}} - 1$$

$$q^2 \in [q^2_{\mathrm{th}}, q^2_{\mathrm{asy}}] \Rightarrow x(q^2) \in [-1,1]$$

Theoretical constraints on $Y(q^2; \overrightarrow{C}, q^2_{\mathrm{asy}})$

$R(q^2_{\mathrm{th}})$ is real $\Rightarrow Y(q^2_{\mathrm{th}}; \overrightarrow{C}, q^2_{\mathrm{asy}}) = 0$

$R(q^2_{\mathrm{phy}})$ is real $\Rightarrow Y(q^2_{\mathrm{phy}}; \overrightarrow{C}, q^2_{\mathrm{asy}}) = 0$

$R(q^2 \geq q^2_{\mathrm{asy}})$ is real $\Rightarrow Y(q^2 \geq q^2_{\mathrm{asy}}; \overrightarrow{C}, q^2_{\mathrm{asy}}) = 0$

Theoretical constraints on $\mathrm{Re}(R(q^2))$

$$\mathrm{Re}\left(R(q^2_{\mathrm{phy}})\right) = \frac{q^2_{\mathrm{phy}}}{\pi} \mathrm{Pr} \int_{q^2_{\mathrm{th}}}^{q^2_{\mathrm{asy}}} \frac{Y(s; \overrightarrow{C}, q^2_{\mathrm{asy}})}{s(s - q^2_{\mathrm{phy}})} ds = 1$$

$$\left|\mathrm{Re}\left(R(q^2_{\mathrm{asy}})\right)\right| = \frac{q^2_{\mathrm{asy}}}{\pi} \left| \mathrm{Pr} \int_{q^2_{\mathrm{th}}}^{q^2_{\mathrm{asy}}} \frac{Y(s; \overrightarrow{C}, q^2_{\mathrm{asy}})}{s(s - q^2_{\mathrm{asy}})} ds \right| = 1$$

χ^2 definition

$$\chi^2\left(\vec{C}, q^2_{\text{asy}}\right) = \chi^2_{|R|} + \chi^2_{\phi} + \tau_{\text{phy}}\chi^2_{\text{phys}} + \tau_{\text{asy}}\chi^2_{\text{asy}} + \tau_{\text{curv}}\chi^2_{\text{curv}}$$

$$\chi^2_{|R|} = \sum_{j=1}^{8}\left(\frac{\sqrt{X^2(q^2_j) + Y^2(q^2_j)} - \left|R_j\right|}{\delta\left|R_j\right|}\right)^2 \qquad X(q^2) \equiv \text{Re}(R(q^2))$$

$$\chi^2_{\phi} = \sum_{k=1}^{7}\left(\frac{\sin\left(\arctan(Y(q^2_k)/X(q^2_k)\right) - \sin(\phi_k)}{\delta\sin(\phi_k)}\right)^2$$

Constraint at $q^2 = q^2_{\text{phy}}$ $\qquad \chi^2_{\text{phy}} = \left(1 - X(q^2_{\text{phy}})\right)^2$

Constraint at $q^2 = q^2_{\text{asy}}$ $\qquad \chi^2_{\text{asy}} = \left(1 - X^2(q^2_{\text{asy}})\right)^2$

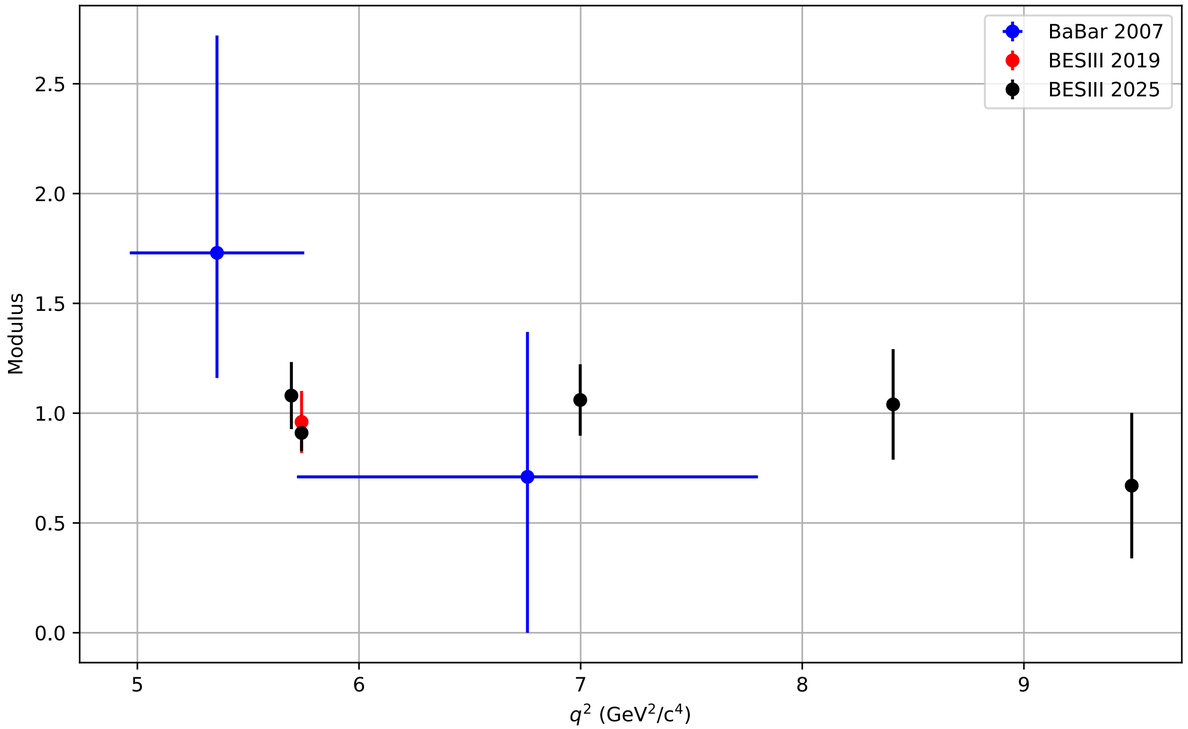
Oscillation damping $\qquad \chi^2_{\text{curv}} = \int_{q^2_{\text{th}}}^{q^2_{\text{asy}}}\left(\frac{d^2Y(s)}{ds^2}\right)^2 ds$

The values of τ_{phys} and τ_{asy} are chosen so that the theoretical conditions are exactly verified.

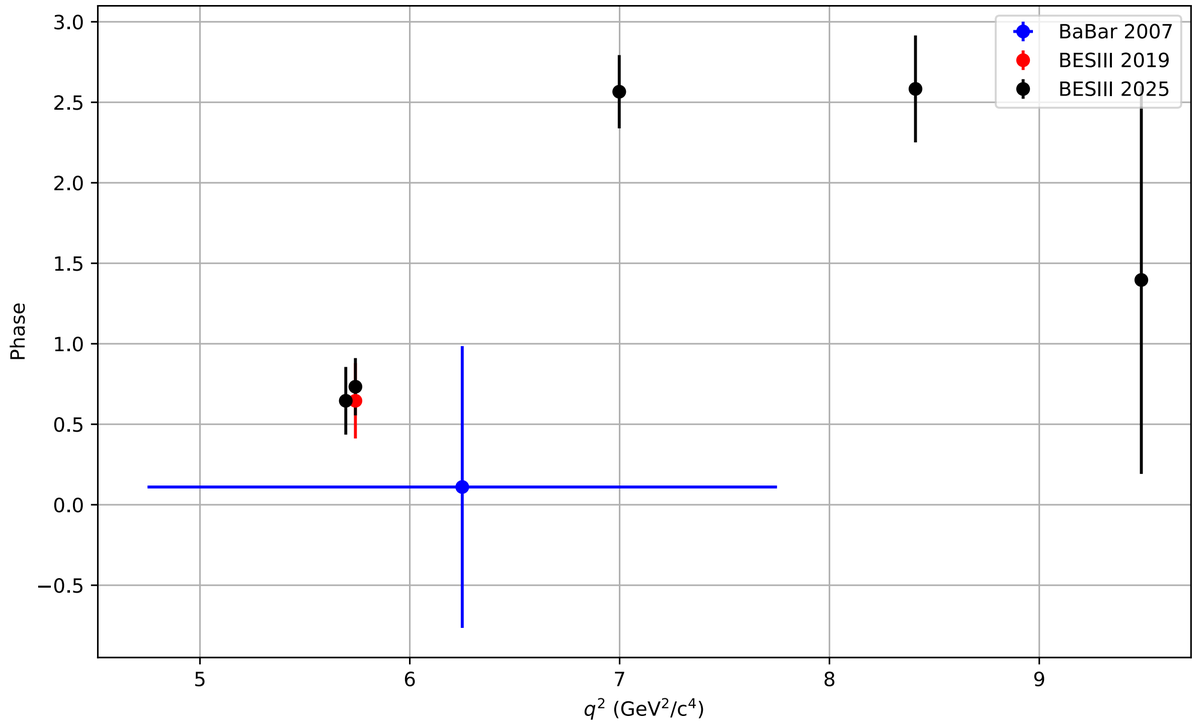
The minimization procedure implies the solution of an ill-posed problem which has to be regularized.

Λ^0 case

Data for Λ^0



BaBar collaboration, Study of $e^+e^- \to \Lambda\bar{\Lambda}, \Lambda\bar{\Sigma}^0, \Sigma^0\bar{\Sigma}^0$ using initial state radiation with BaBar, Phys. Rev. D 76 (2007) 092006 [0709.1988]



M. Ablikim et al. (BESIII Collaboration), Complete Measurement of the Λ Electromagnetic Form Factors, Phys. Rev. Lett. 123, 122003

- Sine of the relative phase accessible through polarization
- No hints on the determination of the relative phase

$$\mathscr{P}_y = -\frac{2M_\Lambda\sqrt{q^2}\sin(2\theta)\left|G_E^\Lambda/G_M^\Lambda\right|\sin\left(\arg(G_E^\Lambda/G_M^\Lambda)\right)}{q^2\left(1+\cos^2(\theta)\right)+4M_\Lambda^2\left|G_E^\Lambda/G_M^\Lambda\right|\sin^2(\theta)}$$

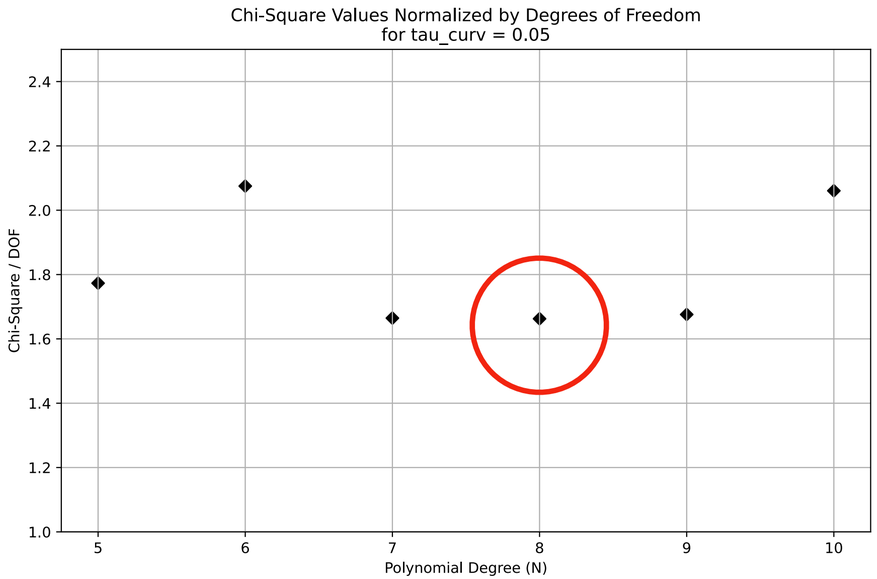
Mutual correlation between $\tau_{\rm curv}$ and N

The theoretical constraints $Y(q^2_{\rm th}; \overrightarrow{C}, q^2_{\rm asy}) = Y(q^2_{\rm phy}; \overrightarrow{C}, q^2_{\rm asy}) = Y(q^2_{\rm asy}; \overrightarrow{C}, q^2_{\rm asy}) = 0$
remove three degrees of freedom, allowing to determine three coefficients, i.e. C_0, C_1, C_2.

The asymptotic threshold $q^2_{\rm asy}$ is used as a free parameter.

If we consider $(N+1)$ Chebyshev polynomials, we are left with $(N-2)$ free coefficients.

We have 13 independent data points $\Rightarrow$ The number of *d.o.f.* is $\mathrm{d.o.f.} = 13 - (N-2) - 1 = 14 - N$.



- $\tau_{\rm phy} = 10^4 \Rightarrow$ The real part of the ratio is forced to the unity at $q^2 = q^2_{\rm phy}$
- $\tau_{\rm asy} = 0 \Rightarrow$ No constraint for the real part at $q^2 = q^2_{\rm asy}$
- $\tau_{\rm curv} = 0.05 \Rightarrow$ Dumping relevant only for high degree polynomials

Results and discussion

At the thresholds q^2_{th} and q^2_{asy} the values of the ratio are real, so the relative phases are integer multiples of of π radians.

$$N_{\text{th,asy}} = \frac{1}{\pi} \arg\left(\frac{G_E^{\Lambda}(q^2_{\text{th,asy}})}{G_M^{\Lambda}(q^2_{\text{th,asy}})} \right) \in \mathbb{Z}$$

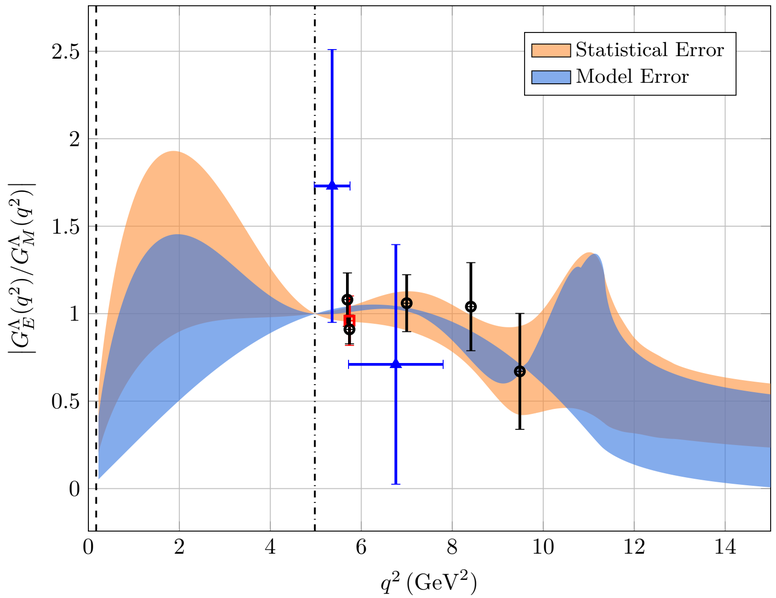
The χ^2 minimization alongside with the theoretical constraints allows to produce the $(N_{\text{th}}, N_{\text{asy}})$ pairs compatible with the data points.

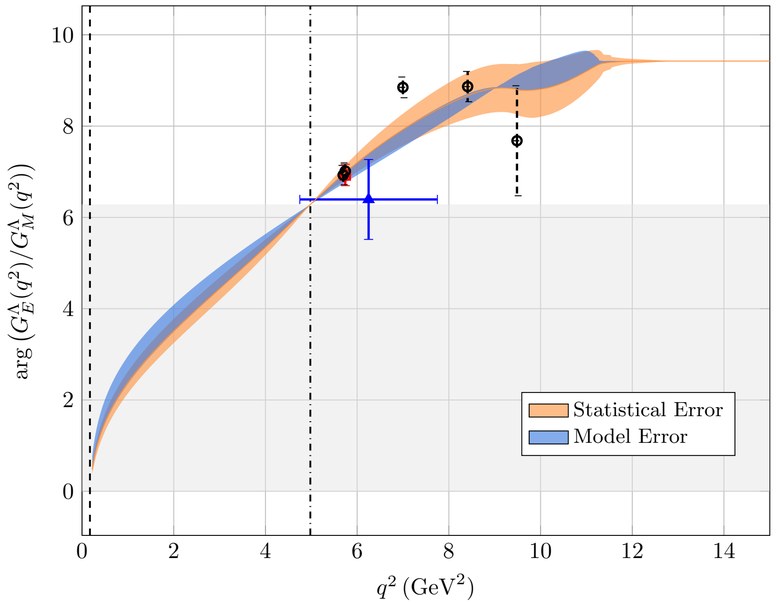
A Monte Carlo procedure determines the probability of occurrence for each pair $(N_{\text{th}}, N_{\text{asy}})$.

N_{th}	N_{asy}	%
0	3	85%
0	4	7.6%
1	3	3.8%
-1	1	1%

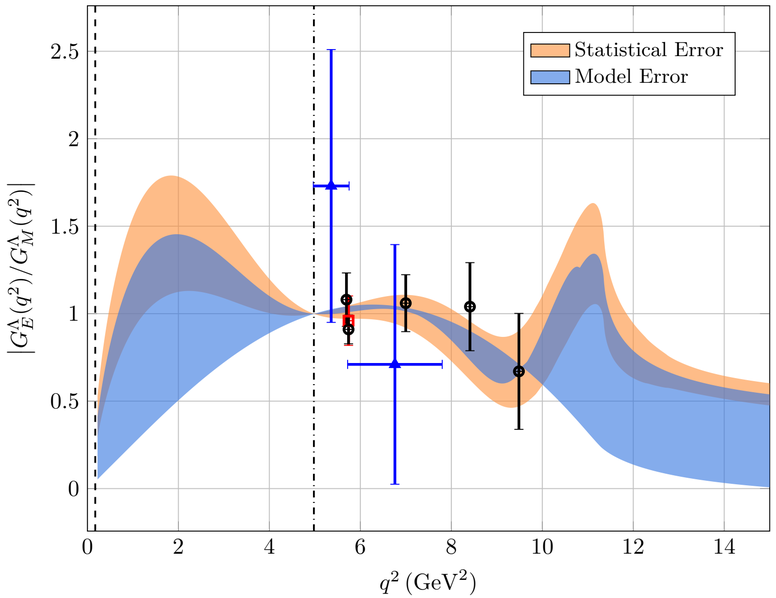
Results and discussion

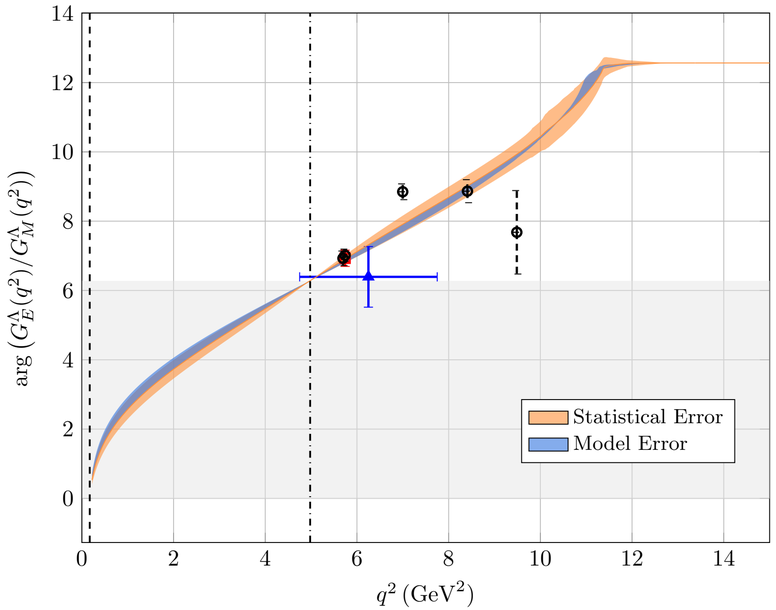
$(N_{\text{th}}, N_{\text{asy}}) = (0,3)$





$(N_{\text{th}}, N_{\text{asy}}) = (0,4)$





Charge radius of a neutral baryon

In terms of the dispersion relations for the imaginary part, the first derivative of the ratio $R(q^2)$ at $q^2 = 0$ is computed as

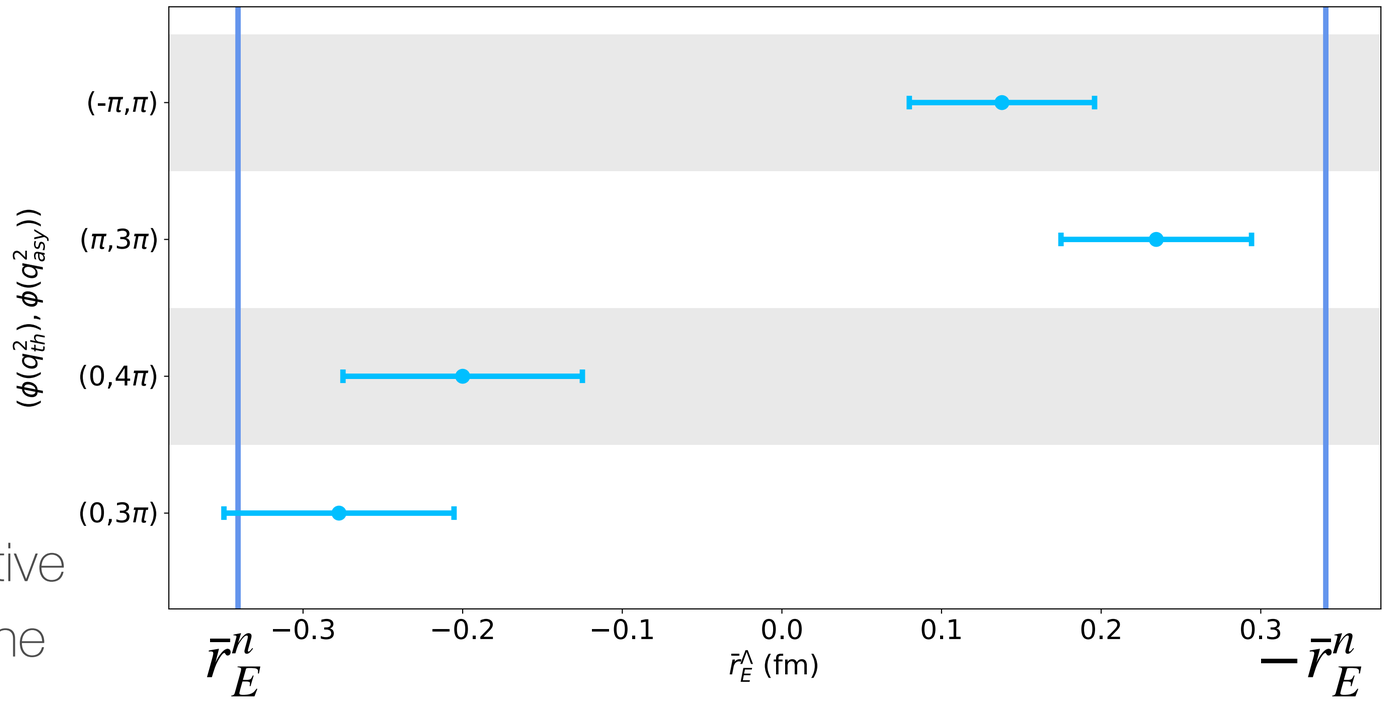
$$\langle r_E \rangle^2 = 6\mu_\Lambda \frac{dR(q^2)}{dq^2}\bigg|_{q^2=0} = \frac{6\mu_\Lambda}{\pi}\int_{q_{\rm th}^2}^{\infty} \frac{\mathrm{Im}\left(R(s)\right)}{s^2} ds = \frac{6\mu_\Lambda}{\pi\Delta q^2}\sum_{j=0}^{N} C_j \int_{-1}^{1} \frac{T_j(x)dx}{(x+1+q_{\rm th}^2/\Delta q^2)^2},$$

$$\Delta q^2 = (q_{\rm asy}^2 - q_{\rm th}^2)/2$$

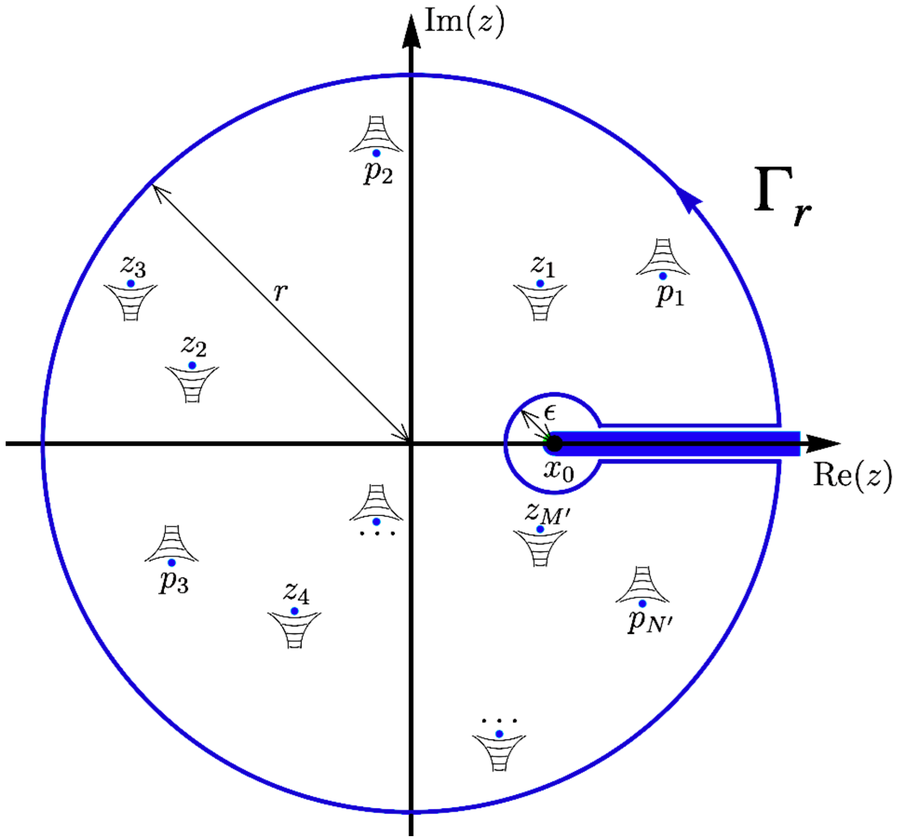
$$\bar{r}_E^{\mathscr{B}} \equiv \mathrm{Sgn}\left(\langle r_E^{\mathscr{B}} \rangle^2\right)\sqrt{\left|\langle r_E^{\mathscr{B}} \rangle^2\right|}$$

The Λ baryon charge radius is comparable with the neutron one $\bar{r}_E^n$.

The $\bar{r}_E^{\Lambda}$ value for $(0,3\pi)$ suggests that the negative charge of the Λ baryon's s quark lies closer to the center than the d quark of the neutron.



The meaning of the phase: locating the zeroes



- Consider the complex function $R(z)$ with N poles $\{p_j\}_{j=1}^{N}$ and M zeroes $\{z_k\}_{k=1}^{M}$ and a branch cut (x_0, ∞)

- Taking the integral over the contour Γ_r gives the Cauchy's argument principle

$$\lim_{r\to\infty} \frac{1}{2i\pi} \oint_{\Gamma_r} \frac{d\ln\left(R(z)\right)}{dz} dz = M - N$$

- By taking each contribution into account

$$\lim_{r\to\infty} \frac{1}{2i\pi} \oint_{\Gamma_r} \frac{d\ln\left(R(z)\right)}{dz} dz = \frac{1}{\pi}\left(\arg(R(\infty)) - \arg(R(x_0))\right)$$

$$\arg(R(\infty)) - \arg(R(x_0)) = \pi\,(M - N) = \arg(R(q_{\text{asy}}^2)) - \arg(R(q_{\text{th}}^2))$$

Levinson's Theorem

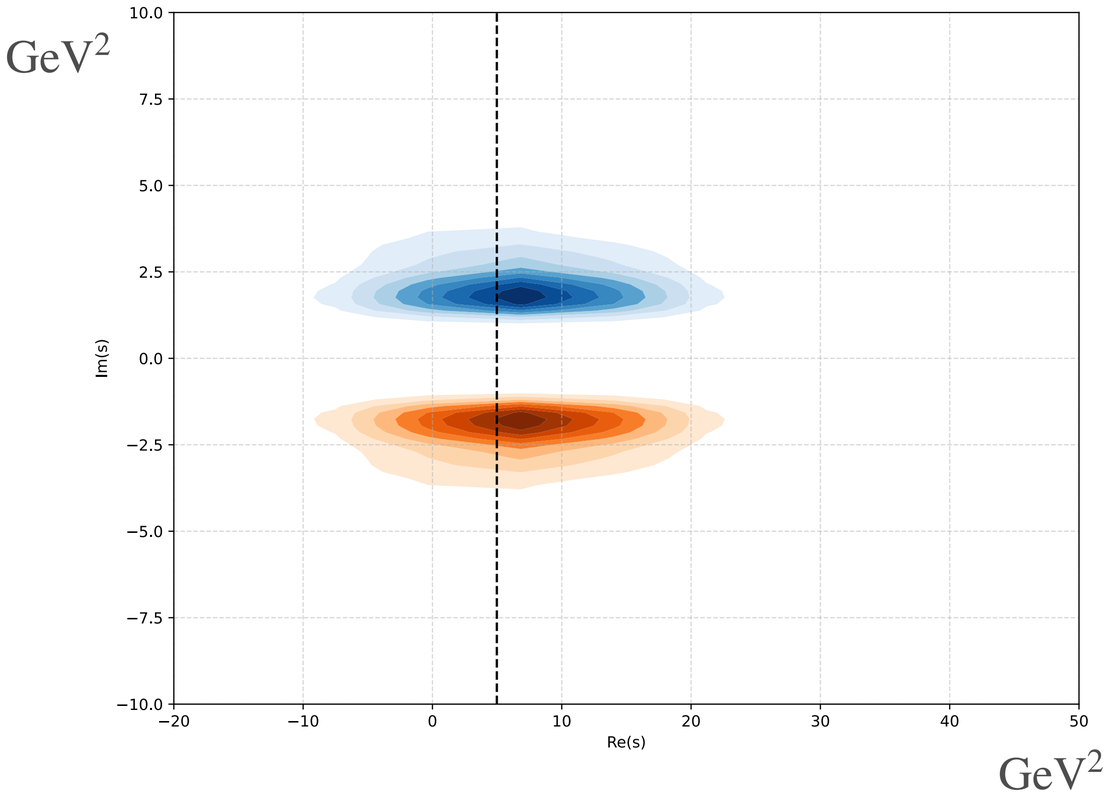
Location of the zeroes

$(N_{\mathrm{th}}, N_{\mathrm{asy}}) = (0,3)$

The presence of a zero of the electric FF is quite plausible for the proton form pQCD calculations in the space-like, what can we say for the hyperons?

The modulus $|R(s)|$ can be plotted for values of $s = q^2$ in the complex plane.

By Schwarz reflection principle $f(z^*) = f^*(z)$ for analytic functions, zeroes of the ratio come in pairs of complex conjugates.



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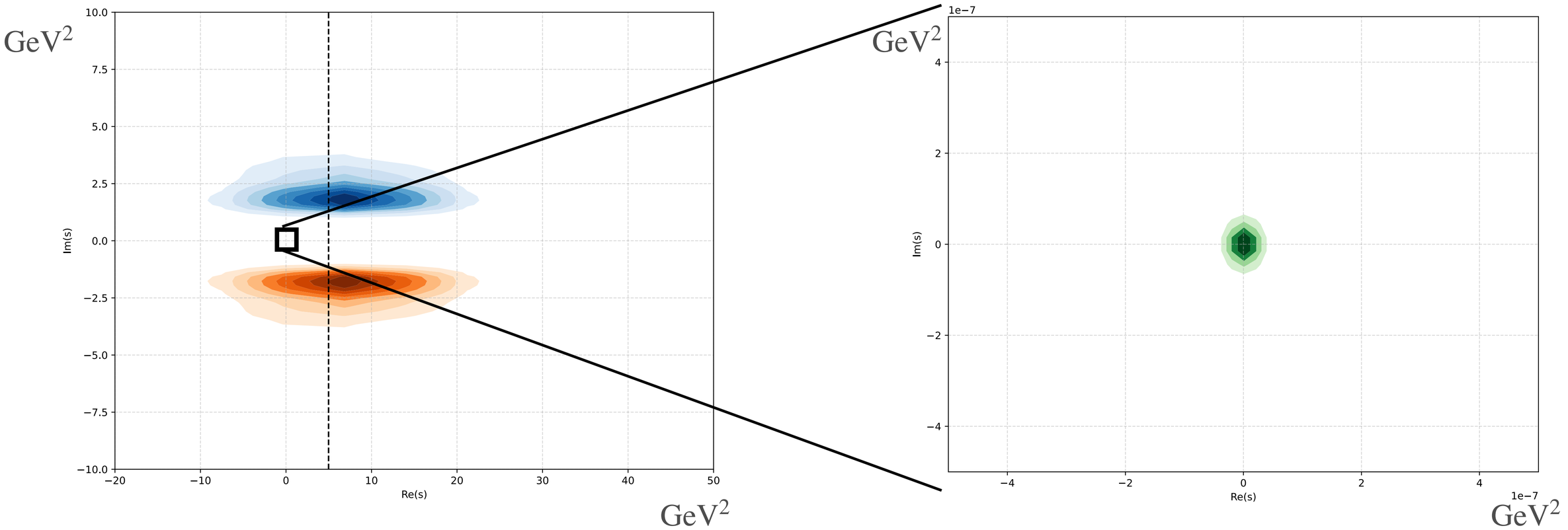
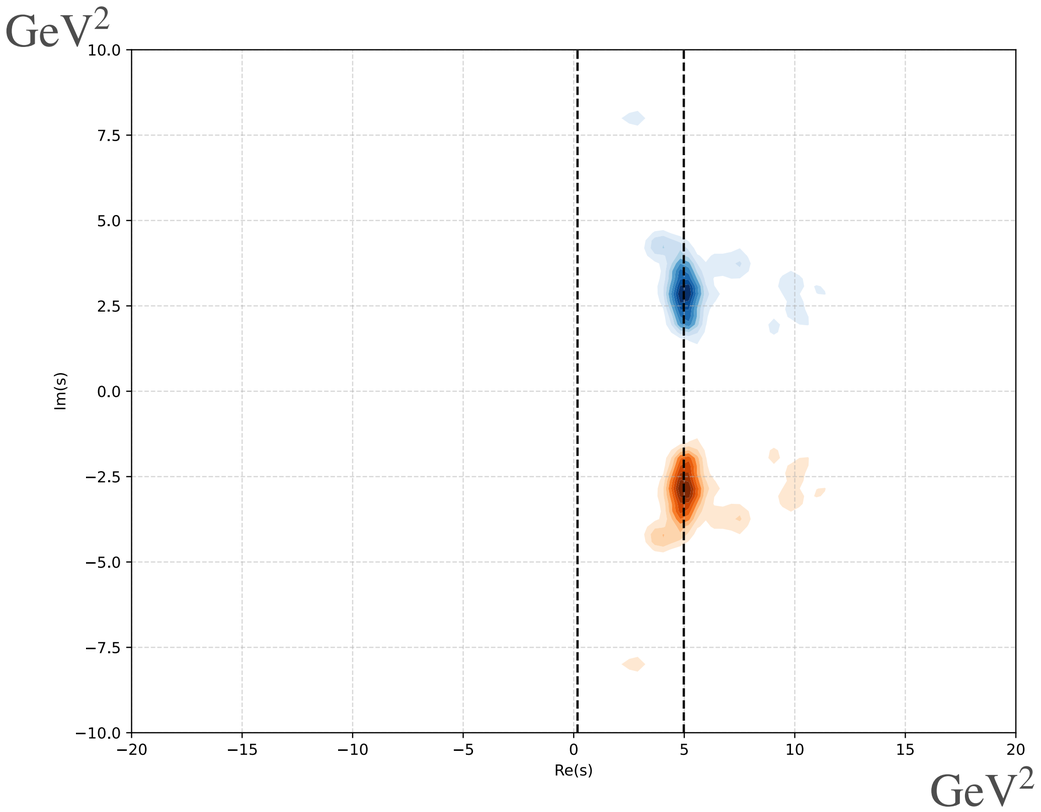


Location of the zeroes

$(N_{\text{th}}, N_{\text{asy}}) = (0,4)$



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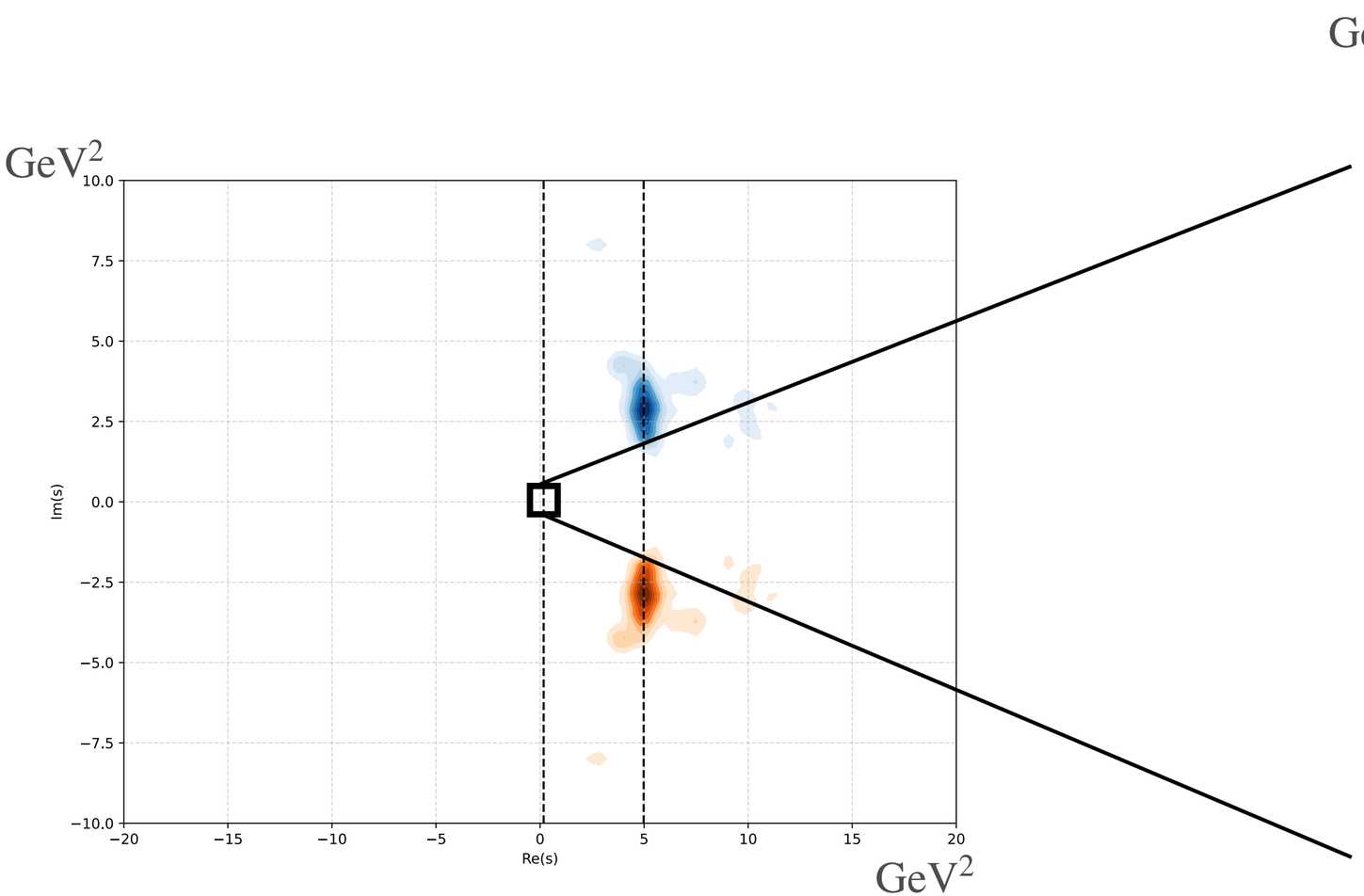
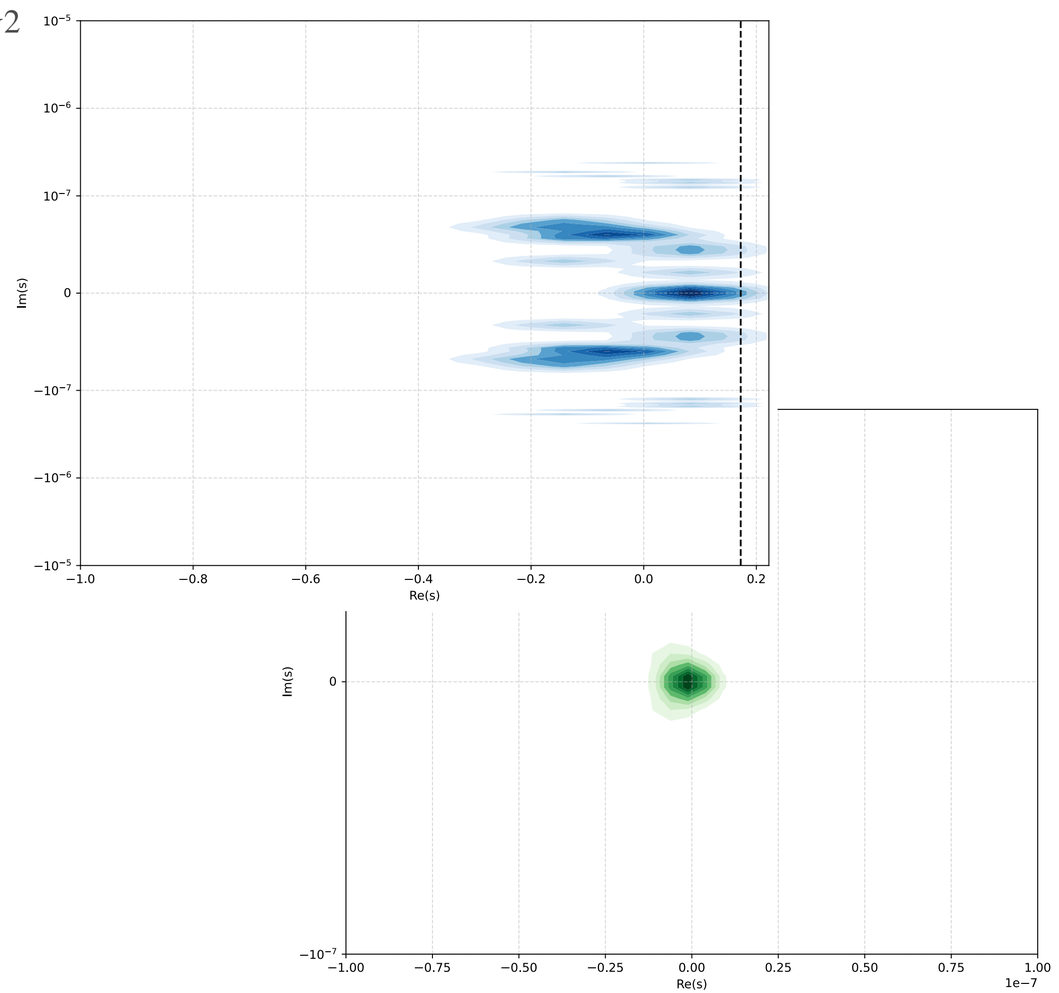
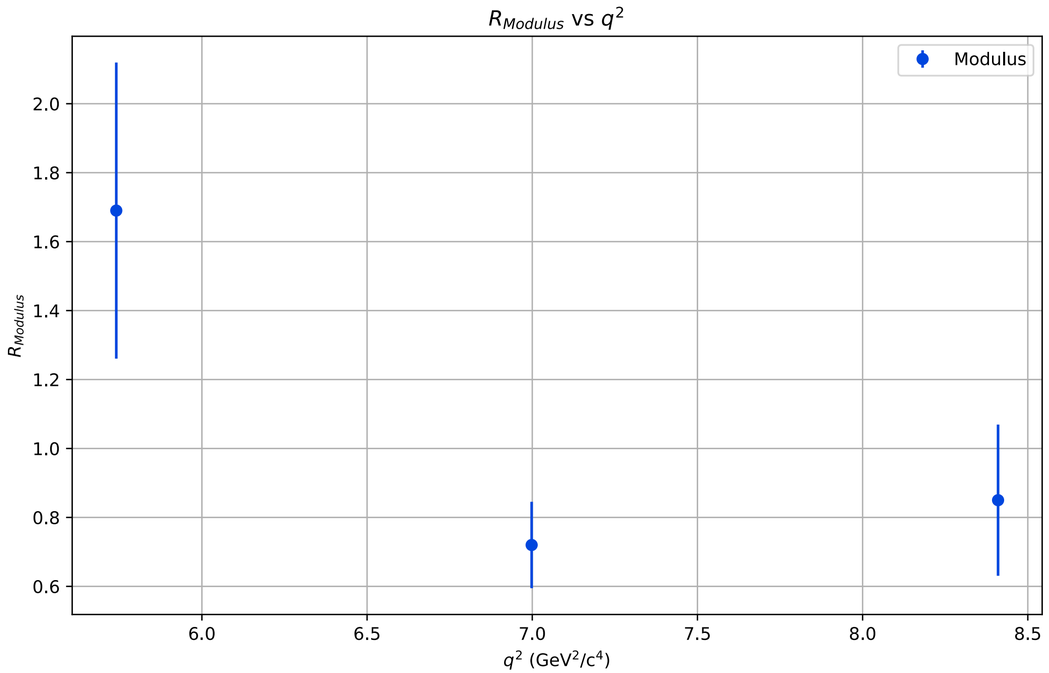




Σ^+ case

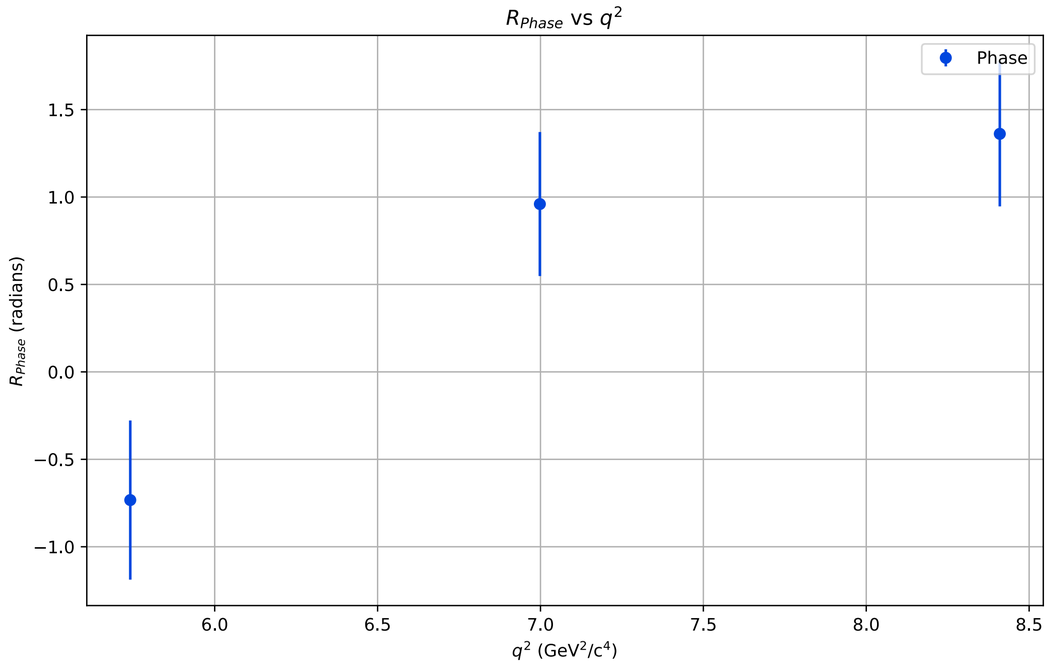
Data for Σ^+



M. Ablikim et al. (BESIII Collaboration), Determination of the Σ^+ Timelike Electromagnetic Form Factors, Phys.Rev.Lett. 132 (2024) 8, 081904

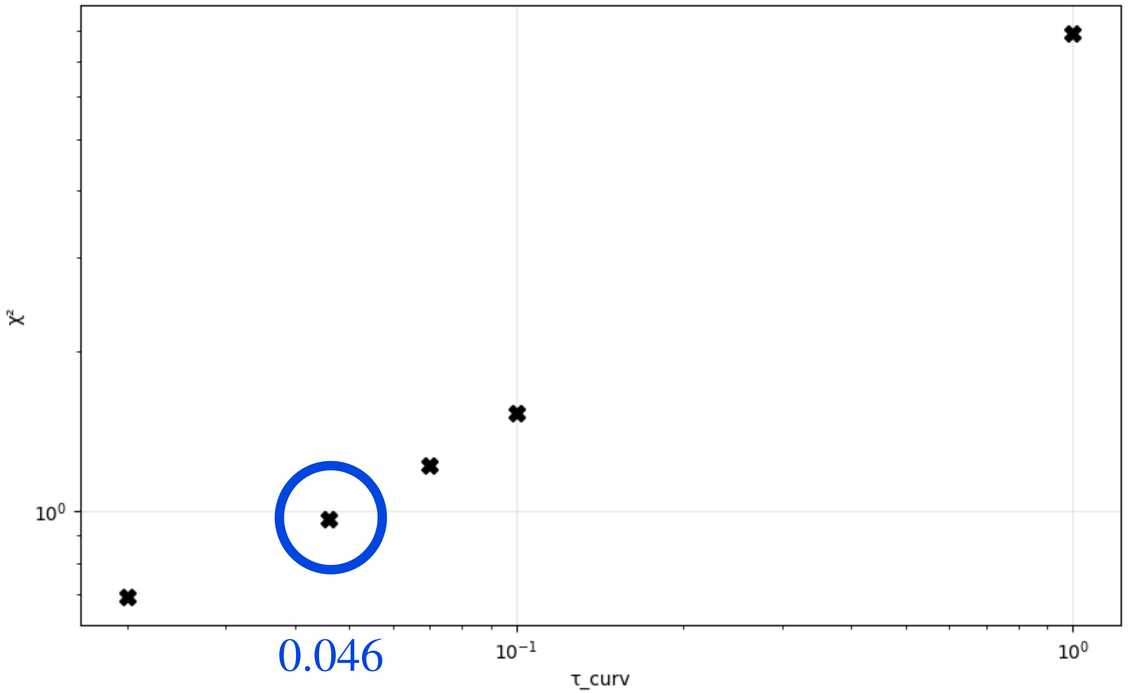
$\sqrt{s}$ [GeV]	2.3960	2.6454	2.9000
$\lvert G_E^{\Sigma^+}/G_M^{\Sigma^+} \rvert$	1.68 ± 0.38 ± 0.20	0.72 ± 0.11 ± 0.06	0.85 ± 0.16 ± 0.15
$\Delta\phi$ (°)	-42 ± 22 ± 14	55 ± 19 ± 14	78 ± 22 ± 19

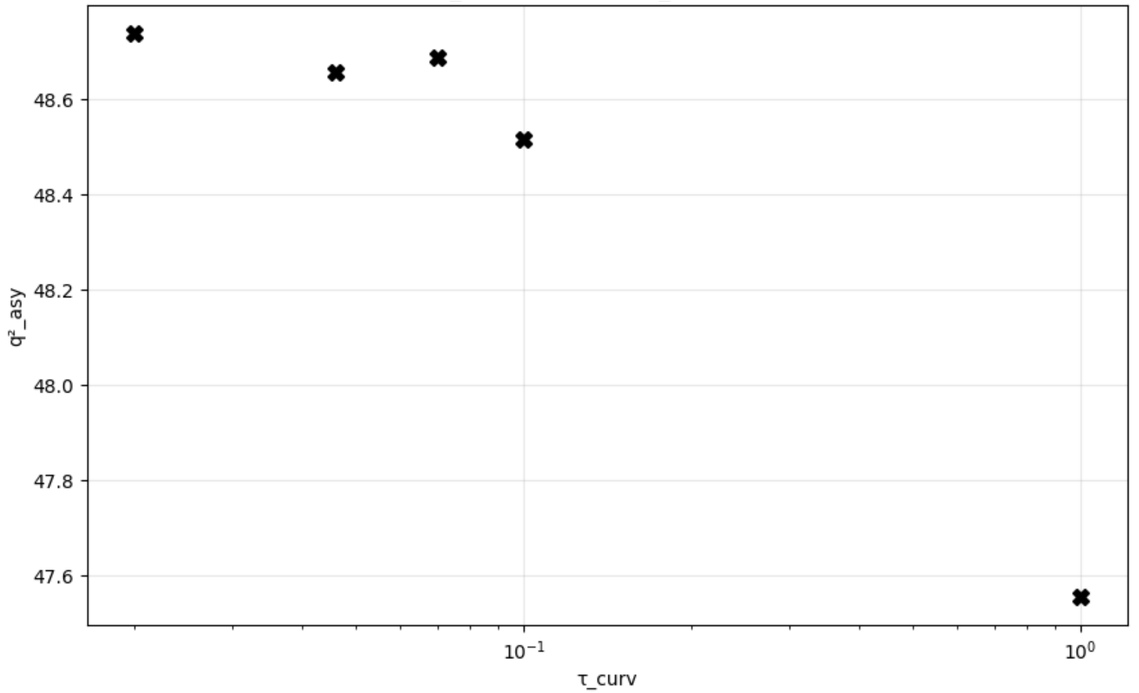
M. Ablikim et al. (BESIII Collaboration), Determination of the Σ^+ Timelike Electromagnetic Form Factors, Phys.Rev.Lett. 132 (2024) 8, 081904



- No hints on the determination of the relative phase
- Near-threshold data differ from theoretical predictions

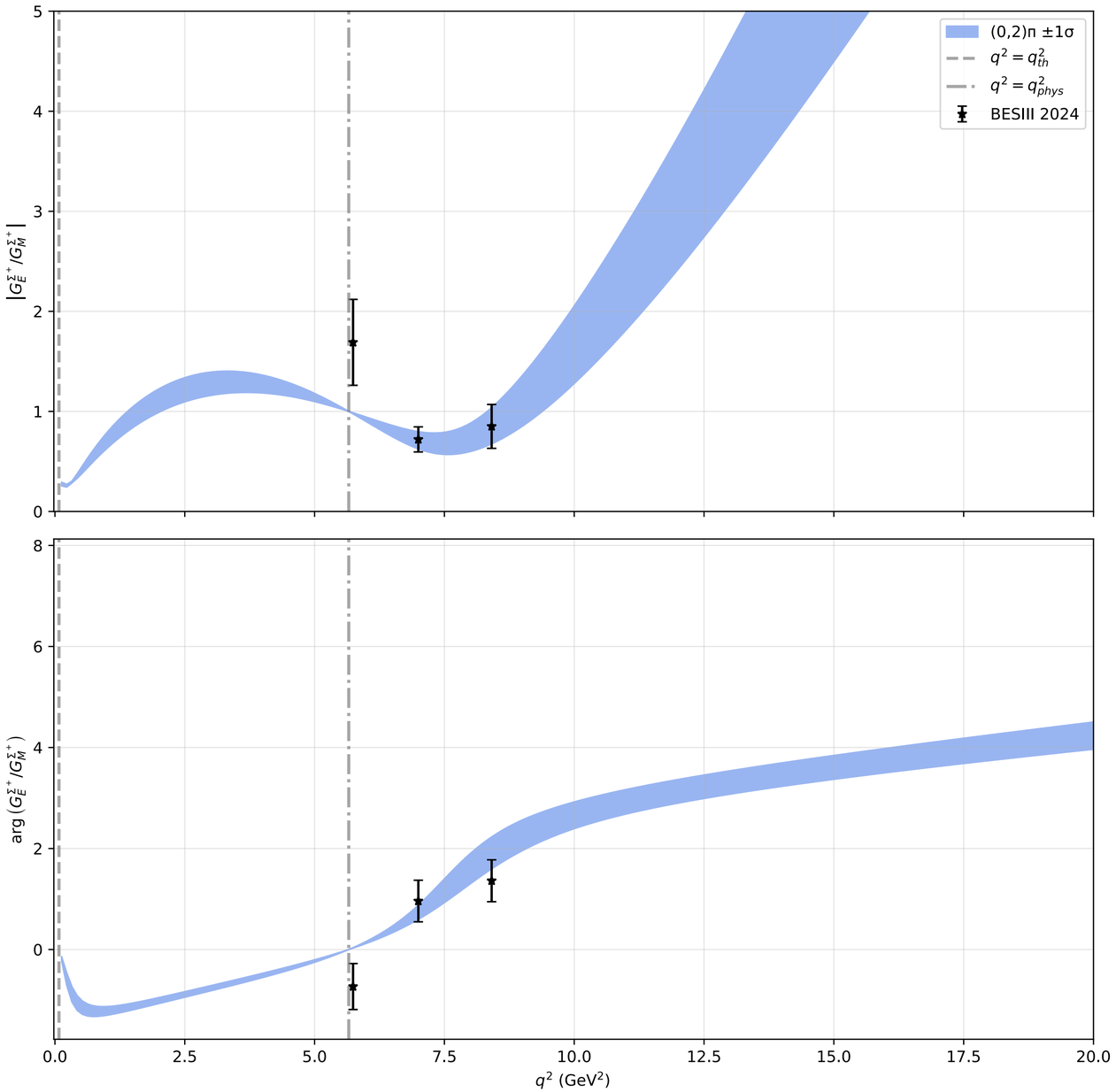
Results and discussion





- Choice on the curvature parameter given by the value of reduced χ^2.

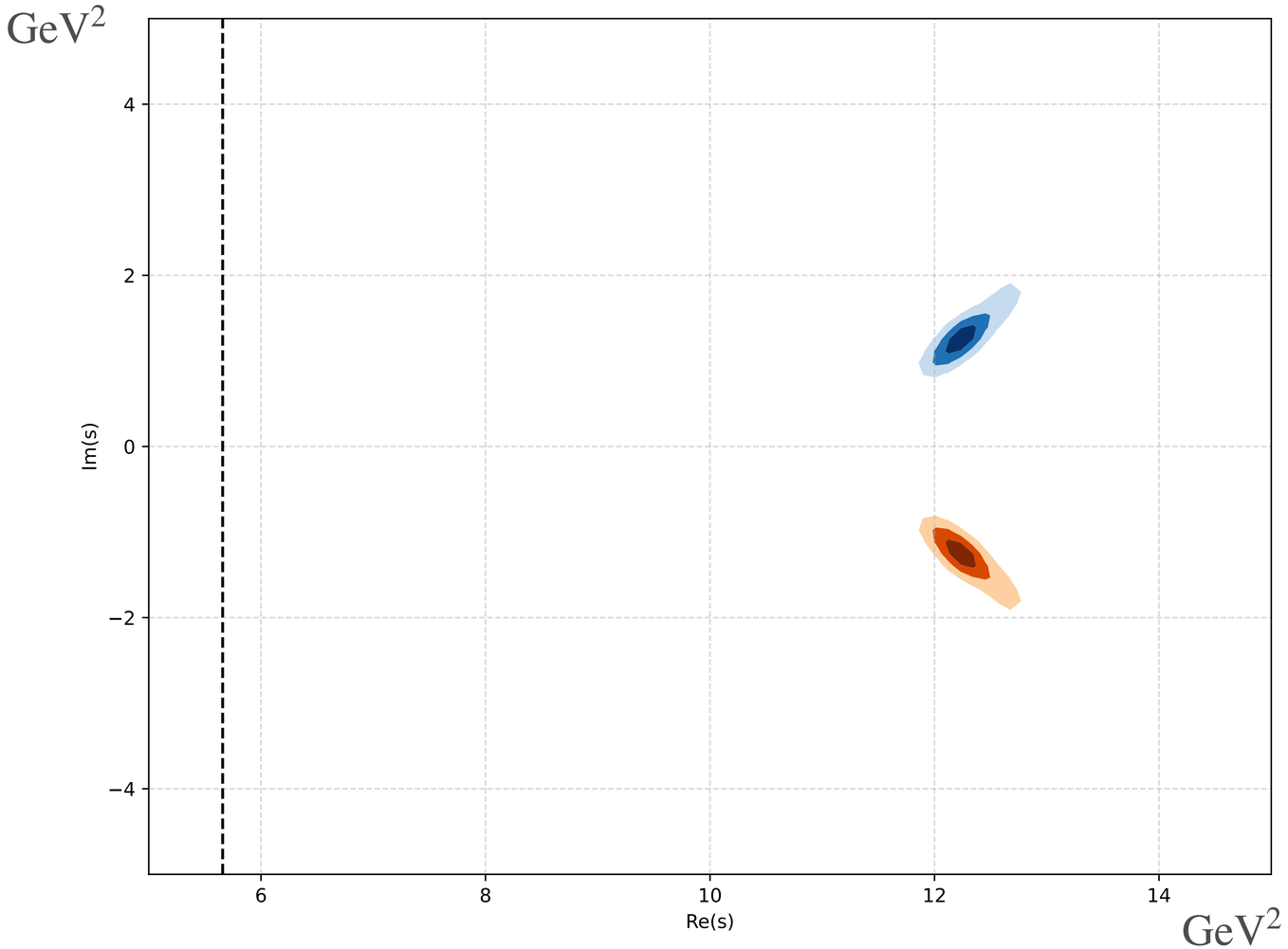
Results and discussion



$P(0,2) \approx 100\,\%$

$(N_{\text{th}}, N_{\text{asy}}) = (0,2)$

Location of the zeroes



$(N_{\text{th}}, N_{\text{asy}}) = (0,2)$

First derivative of the $G_E^{\Sigma^+}/G_M^{\Sigma^+}$ ratio

Let us consider the logarithmic derivative of $R(q^2)$ in $q^2 = 0$:

$$\left.\frac{d\ln\left(R(q^2)\right)}{dq^2}\right|_{q^2=0} = \left.\frac{1}{R(q^2)}\frac{dR}{dq^2}\right|_{q^2=0} = \left.\left(\frac{1}{G_E^{\Sigma^+}(q^2))}\frac{dG_E^{\Sigma^+}}{dq^2} - \frac{1}{G_M^{\Sigma^+}(q^2)}\frac{dG_M^{\Sigma^+}}{dq^2}\right)\right|_{q^2=0} = \frac{1}{6}\left(\left\langle r_E^2\right\rangle - \left\langle r_M^2\right\rangle\right)$$

$$\Delta\left\langle r^2\right\rangle = \left\langle r_E^2\right\rangle - \left\langle r_M^2\right\rangle = 6\mu_{\Sigma^+}\left.\frac{dR}{dq^2}\right|_{q^2=0}$$

$(N_{\text{th}}, N_{\text{asy}})$	$\Delta\left\langle r^2\right\rangle$ [fm^2]
(0 , 2)	-0.43 ± 0.06

Final considerations

The dispersive procedure, connecting time-like experimental values and theoretical constraints, allows to assign different determinations to the phase, and hence to the measured values of the phase. This gives information about the space-like behaviour of the form factor ratio.

Assuming no zeroes for the magnetic form factor, the Levinson's Theorem allows to count the number of zeroes of the electric form factor.

For the Λ^0 the most probable values for $(N_{\text{th}}, N_{\text{asy}})$ suggest the presence of two non-trivial zeroes for the function G_E^{Λ}.

The Λ^0 charge radius $\bar{r}_E^{\Lambda}$ seems to indicate a neutron-like behaviour, with the s-quark orbiting closer to the particle center than the neutron's d-quark.

The Σ^+ analysis determines a generally significant difference between the charge and magnetic radii of the particle. Further investigations are needed. Moreover, the most probable solution $(0,2)$ suggests the presence of two zeroes for the electric form factor in the complex plane.

Thanks for Your Attention!

谢谢您们的关注！

Backup slides

Asymptotic behaviour

$$G_E^{\mathscr{B}}(q^2) = F_1^{\mathscr{B}}(q^2) + \frac{q^2}{4M_{\mathscr{B}}^2} F_2^{\mathscr{B}}(q^2)$$

$$G_M^{\mathscr{B}}(q^2) = F_1^{\mathscr{B}}(q^2) + F_2^{\mathscr{B}}(q^2)$$

The space-like asymptotic form factors behaviour is given in pQCD by counting rules as $q^2 \to -\infty$

Helicity conservation

- $J^{\lambda,\lambda}(q^2) \propto G_M^{\mathscr{B}}(q^2)$
- 2 gluon propagators distributing the momentum transfer of the virtual photon
- $G_M^{\mathscr{B}}(q^2) \sim (q^2)^{-2}$

Helicity flip

- $J^{\lambda,-\lambda}(q^2) \propto G_E^{\mathscr{B}}(q^2)/\sqrt{-q^2}$
- [2 gluon propagators] / $\sqrt{-q^2}$
- $G_E^{\mathscr{B}}(q^2) \sim (q^2)^{-2}$

Dirac and Pauli Form Factors

$$F_1^{\mathscr{B}}(q^2) \underset{q^2\to-\infty}{\sim} (q^2)^{-2}$$

$$F_2^{\mathscr{B}}(q^2) \underset{q^2\to-\infty}{\sim} (q^2)^{-3}$$

Sachs Form Factor Ratio

$$\frac{G_E^{\mathscr{B}}(q^2)}{G_M^{\mathscr{B}}(q^2)} \underset{q^2\to-\infty}{\sim} \text{constant}$$

Analyticity of form factors

Spacelike region	Unphysical region	Timelike region
$q^2 < 0$	$q^2_{\text{th}} < q^2 \leq q^2_{\text{phys}}$	$q^2 > q^2_{\text{phys}}$
$e\mathscr{B} \to e\mathscr{B}$	$\mathscr{B}\bar{\mathscr{B}} \to e^+e^-\mathscr{M}_0$	$e^+e^- \leftrightarrow \mathscr{B}\bar{\mathscr{B}}$
$G_E^{\mathscr{B}}(q^2), G_M^{\mathscr{B}}(q^2)$	$\left\lvert G_E^{\mathscr{B}}(q^2)\right\rvert, \left\lvert G_M^{\mathscr{B}}(q^2)\right\rvert$	$\begin{cases} \left\lvert G_E^{\mathscr{B}}(q^2)\right\rvert, \left\lvert G_M^{\mathscr{B}}(q^2)\right\rvert \\ \arg\left(G_E^{\mathscr{B}}/G_M^{\mathscr{B}}\right)^*, \end{cases}$

* Sine of the argument measurable in polarized cross section only

Cross section

Scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_e \cos^2(\theta/2)}{4E_e^3 \sin^4(\theta/2)} \left[\left(G_E^{\mathscr{B}}\right)^2 - \tau \left(1 + 2\left(1-\tau\right)\tan^2(\theta/2)\right) \left(G_M^{\mathscr{B}}\right)^2 \right] \frac{1}{1-\tau}$$

Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathscr{C}}{16E^2} \left[\left(1 + cos^2(\theta)\right) \left| G_E^{\mathscr{B}} \right|^2 + \frac{1}{\tau} \sin^2(\theta) \left| G_M^{\mathscr{B}} \right|^2 \right]$$

Coulomb correction factor

$$\mathscr{C} = \frac{\pi\alpha}{\beta} \frac{1}{1 - e^{-\pi\alpha/\beta}}$$

Locating the zeroes: the proton predictions

For the proton, the presence of a zero crossing is predicted from pQCD:

$$\frac{G_E^p(-Q^2)}{G_M^p(-Q^2)} = \frac{1 - \frac{Q^2}{4m_p^2}\frac{F_2^p(-Q^2)}{F_1^p(-Q^2)}}{1 + \frac{F_2^p(-Q^2)}{F_1^p(-Q^2)}} \to -\frac{1}{4m_p^2}[\alpha_S(Q)]^{8/(9\beta)}\,\log^2(Q^2/\Lambda^2) \to -\infty$$

If that holds, then the relative phase in the space-like region becomes:

$$\lim_{q^2\to-\infty} \arg\left(G_E^p(q^2)/G_M^p(q^2)\right) = \pm\,\pi \qquad \text{but} \qquad G_E^p(0)/G_M^p(0)$$

Hence the natural presence of a zero crossing.