TENSOR NETWORK SIMULATION OF MULTI-EMITTER WAVEGUIDE QED

DIPARTIMENTO INTERATENEO DI FISICA
UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO
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Bari Theory Xmas Workshop
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Introduction

- *Challenging problem*: simulating quantum many-body systems

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$$\sim 2^n$$

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  - Available RAM in a common laptop: $\sim 16\ GB$

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- Quantum states *exponentially large* in the size of the system:

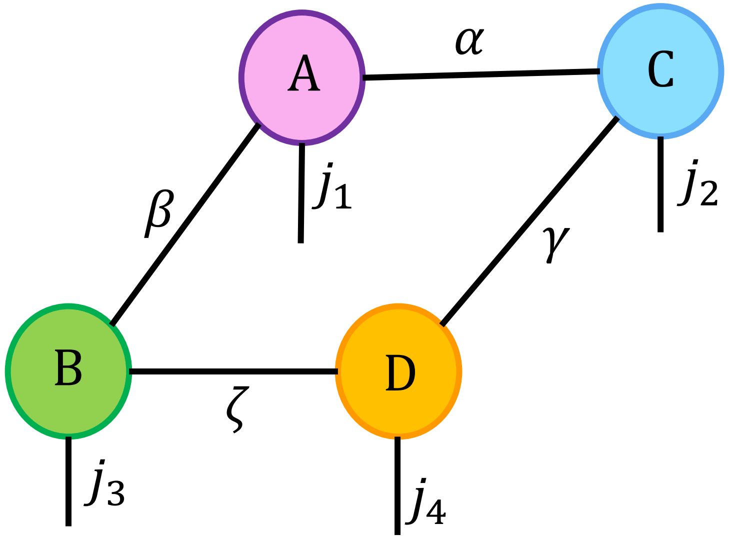
$$\sim 2^n$$

- *Concrete idea*:
  - 40 spins $\longrightarrow$ $4\ TB$ of memory
  - Available RAM in a common laptop: $\sim 16\ GB$
- New *quantum-inspired simulation techniques*

$\Longrightarrow$ **Tensor networks (TNs)**

Tensor network (TN)

- Simulation of *large* many-body quantum systems with *good approximation*



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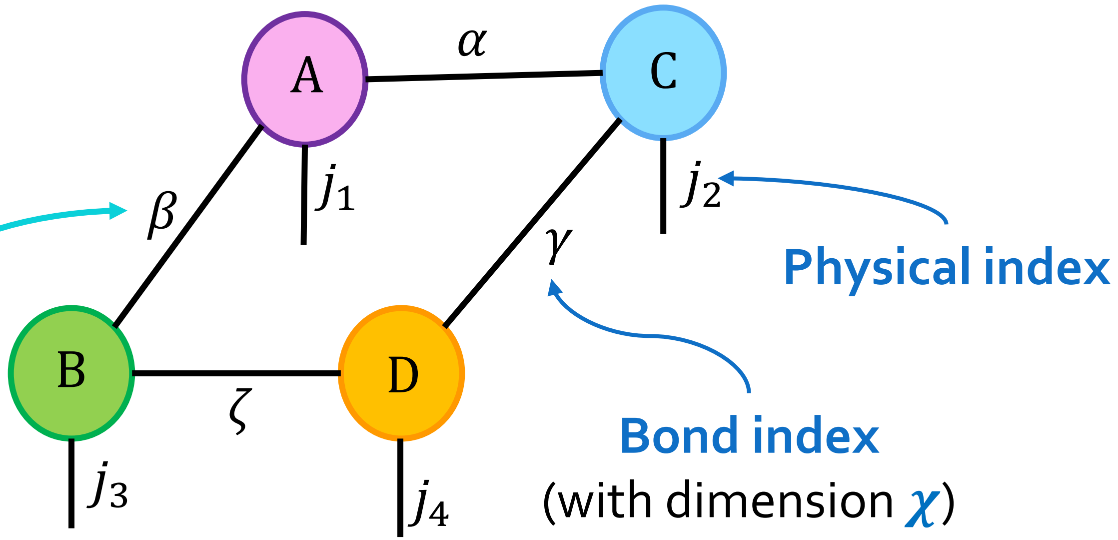
- $|\psi\rangle$ modelled as a TN



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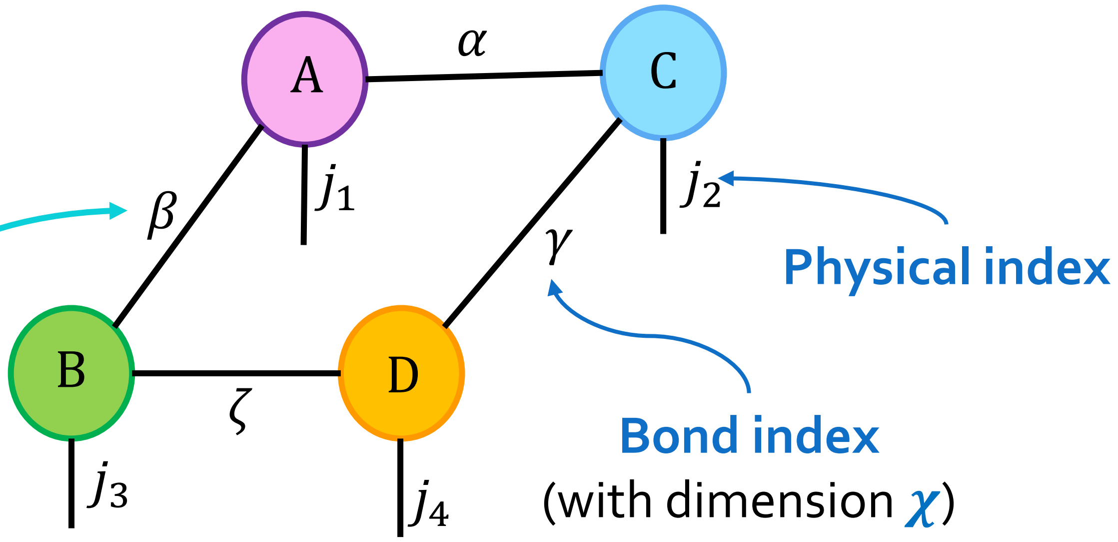
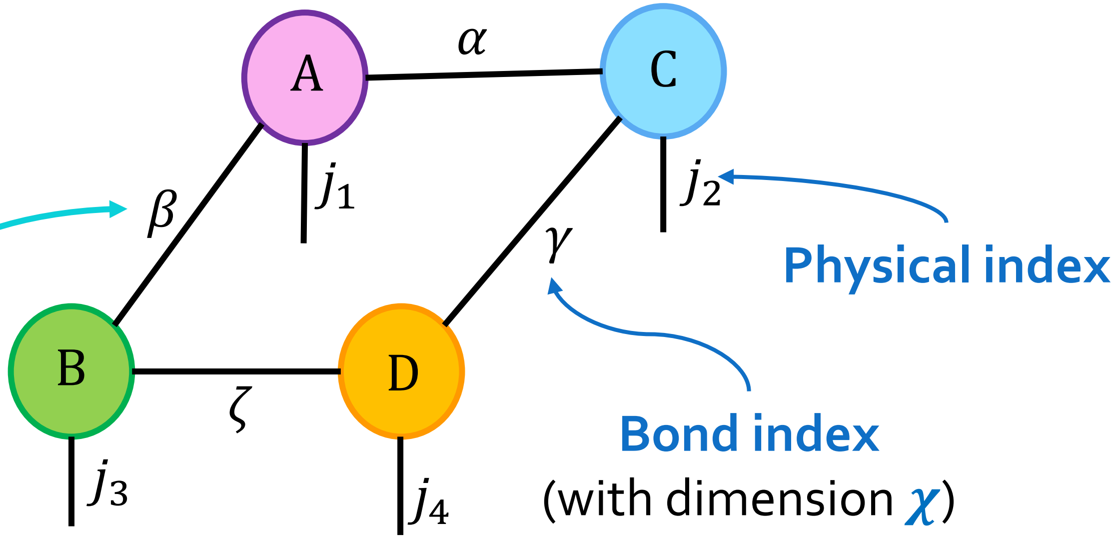
- *Entanglement* described by the *network structure*



Tensor network (TN)

- Simulation of *large* many-body quantum systems with *good approximation*
- $|\psi\rangle$ modelled as a TN
- *Entanglement* described by the *network structure*
- Reduction of *computational complexity*

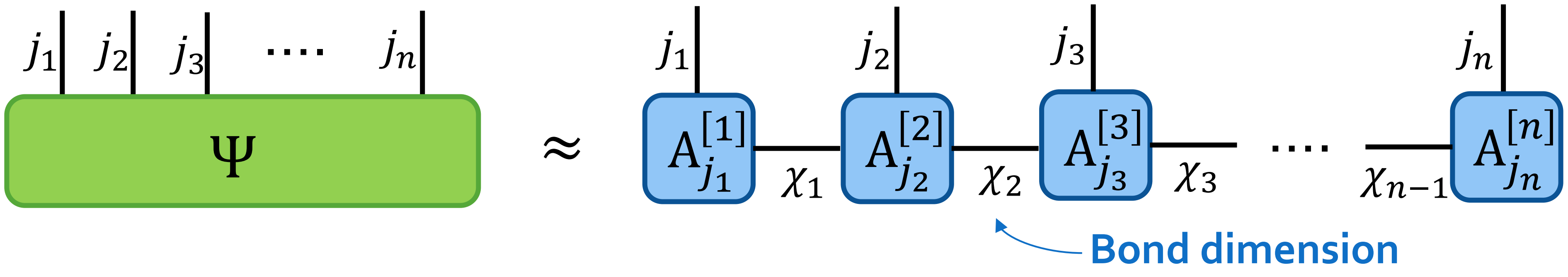


#parameters: $O(poly(n)poly(\chi))$

$\Longrightarrow$ ***Efficient***!

Matrix Product State (MPS)

- *Factorization* of a rank-n tensor into a chain-like product of lower-rank tensors:



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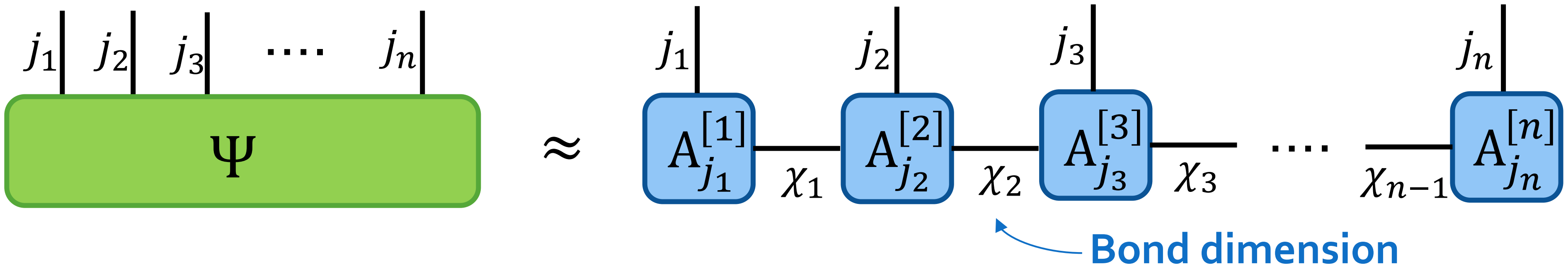

- Overall size: $\begin{cases} \psi: \sim 2^n \implies \textit{exponential} \text{ in } n \\ MPS: \sim 2n\chi_{max}^2 \implies \textit{linear} \text{ in } n \end{cases}$

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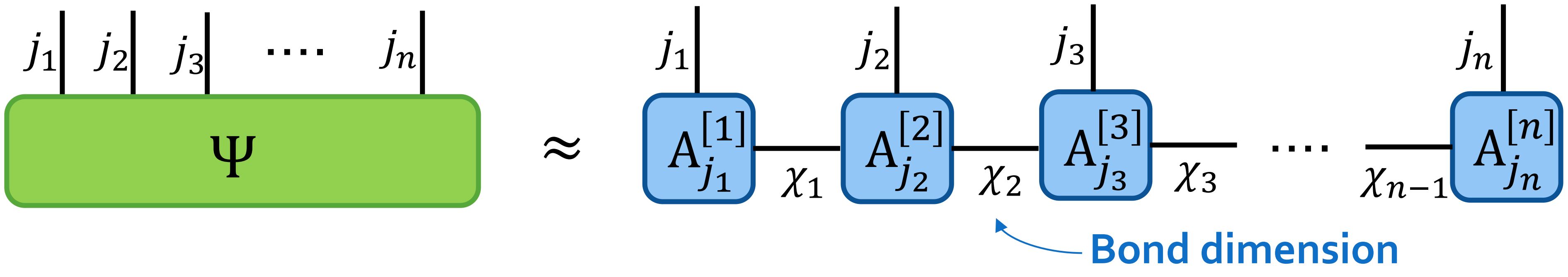

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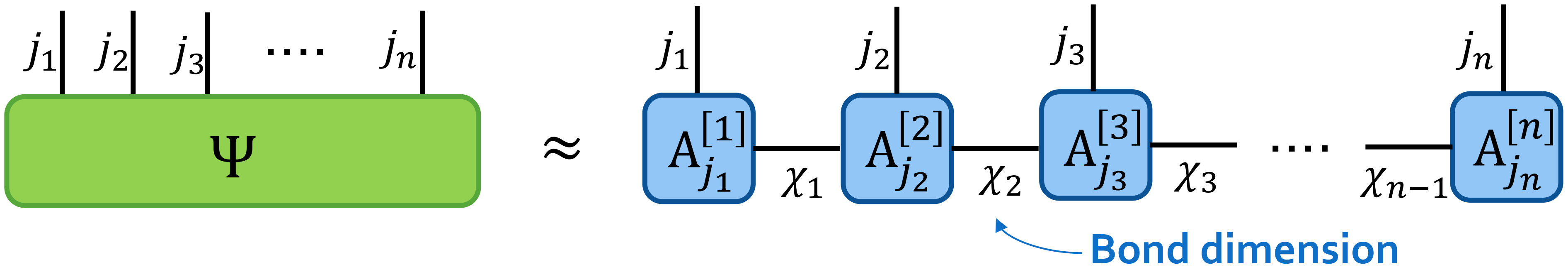

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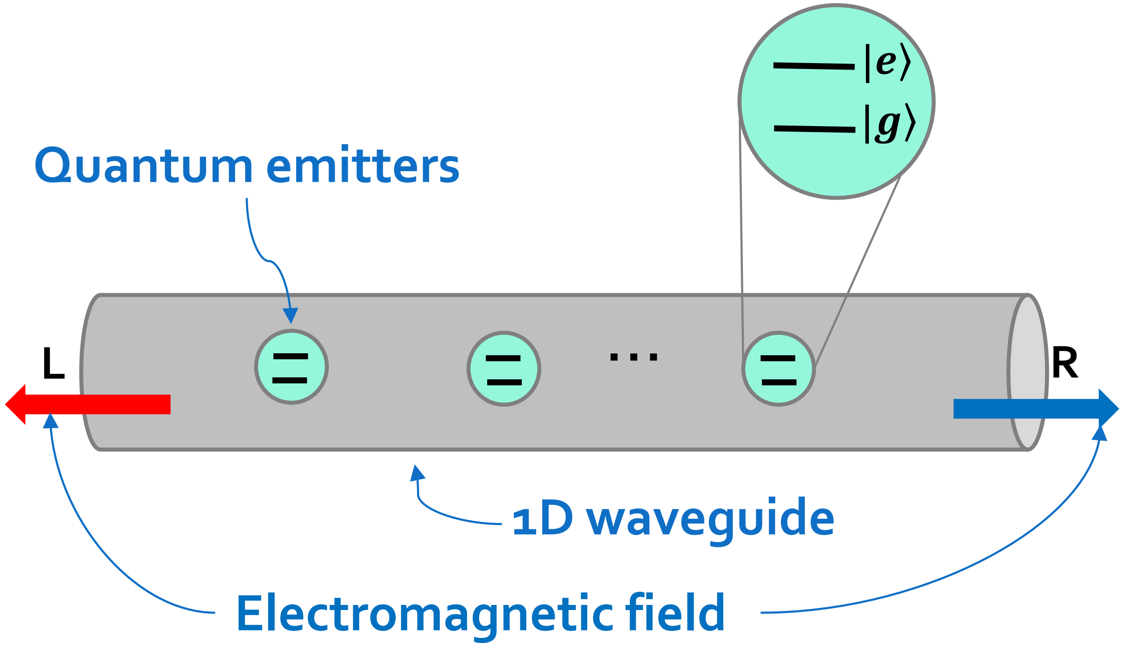
- $\chi_{max} \sim 2^{\lfloor n/2 \rfloor} \implies$ Introducing some *approximation:* **truncation** of the *bond dimension values*

Waveguide QED

- *TN application*: **waveguide QED**
  - quantum technology implementations
  - *quantum light-matter interactions* and other interesting *physical phenomena*



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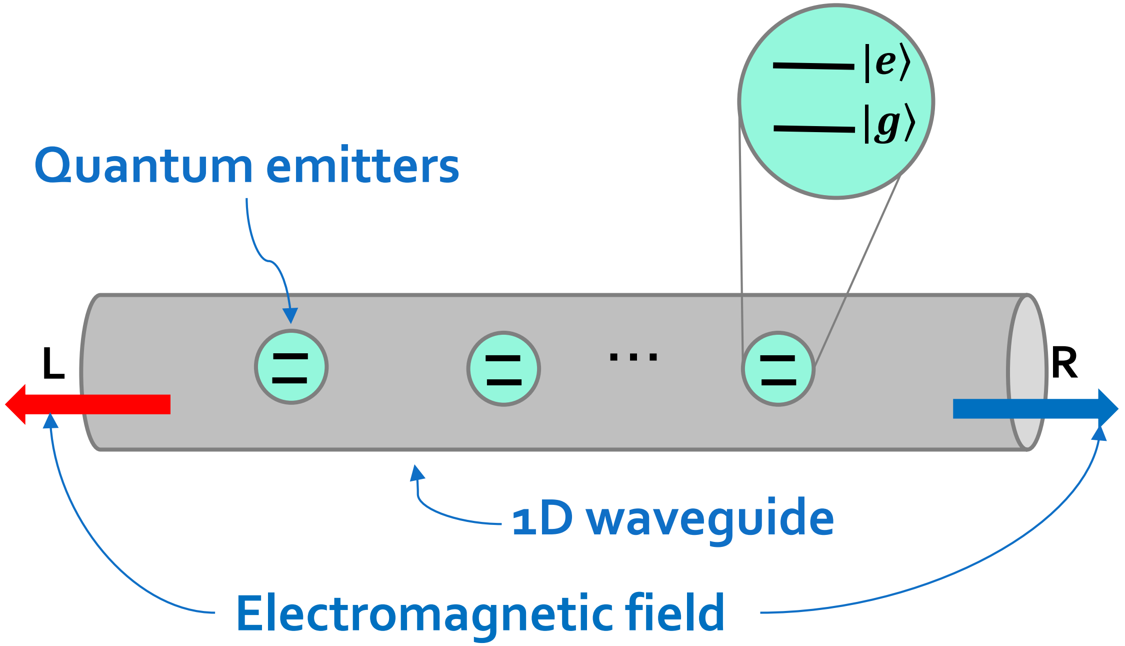
$\Longrightarrow$ well-understood in the *Markovian setting (Lindblad Master Equation)*



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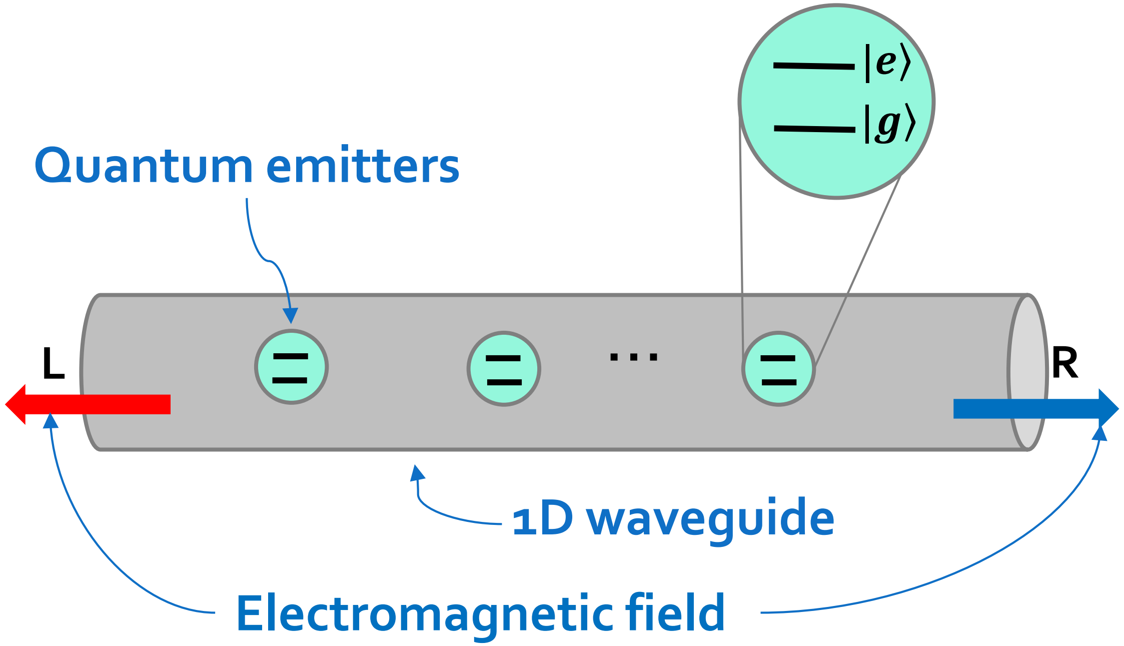

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$$\tau \gtrsim \gamma^{-1}$$

  - *Time delay*: $\tau = d/v_g$
  - Spontaneous emission *characteristic time* : γ^{-1}

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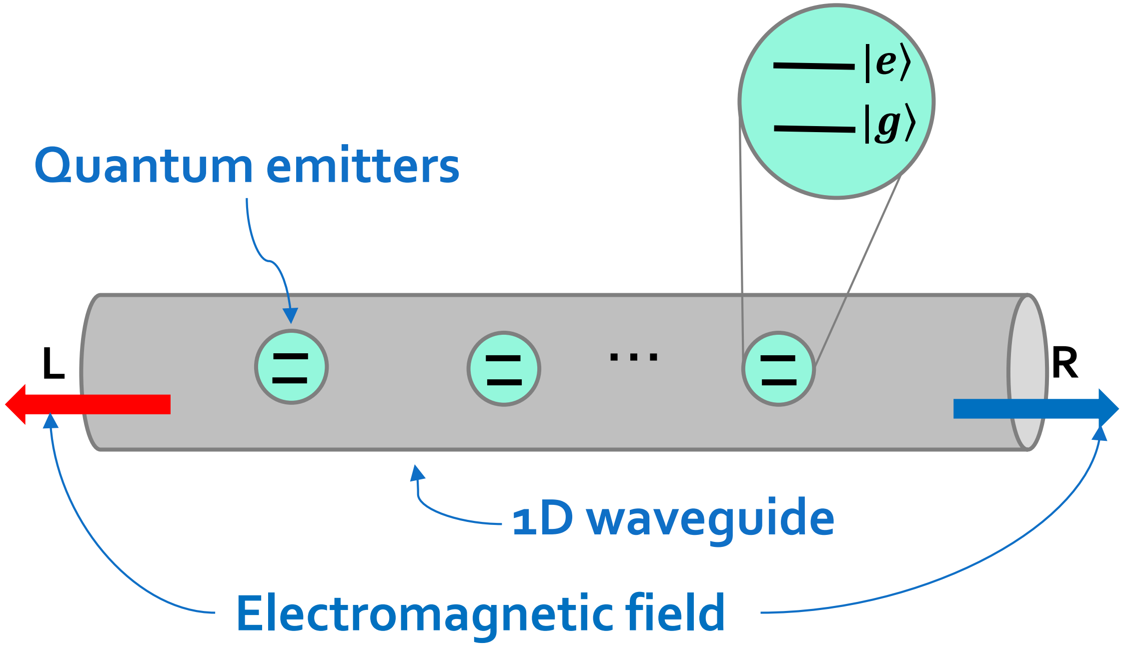

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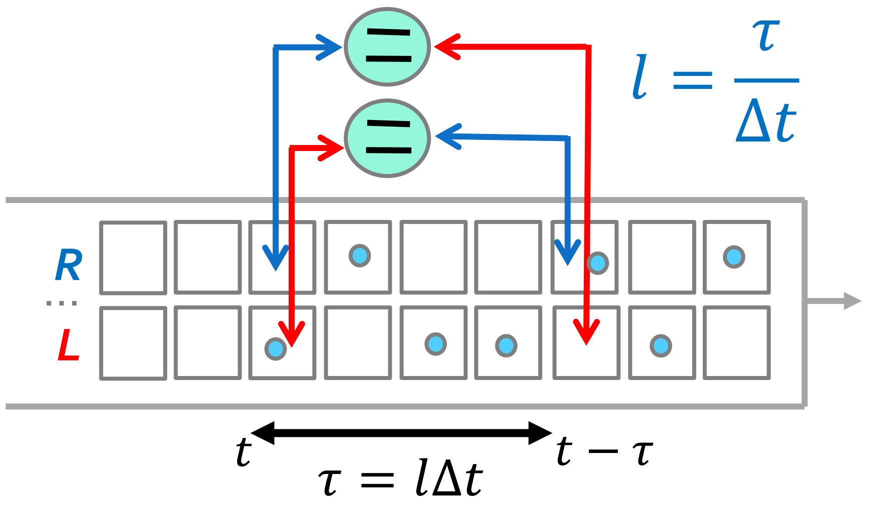
How to address wQED non-Markovian dynamics ?

wQED: Non-Markovian dynamics

- *Collision model*:
  - Set a *coarse-graining time interval*: $\Delta t \ll \gamma^{-1}$
  - Decompose the evolution: $\hat{U} = \prod_k \hat{U}_k = \prod_k e^{-i\hat{O}_j}$
- *Dynamical map*:

$$|\psi(t_{k+1})\rangle = \exp\left\{-i\left[\sum_{j=1}^{N_e} \hat{O}_j(t_k)\right]\right\}|\psi(t_k)\rangle$$



where:

- $\hat{O}_j(t_k) = \sqrt{\frac{\gamma\Delta t}{2}}\hat{\sigma}_j\left[e^{-i(j-1)\phi}\hat{B}^{\dagger}_{k-(j-1)l,R} + e^{i(j-1)\phi}\hat{B}^{\dagger}_{k+(j-1)l,L}\right] + \text{H.c.}$
- $\phi = k_0 d = \omega_0 \tau$

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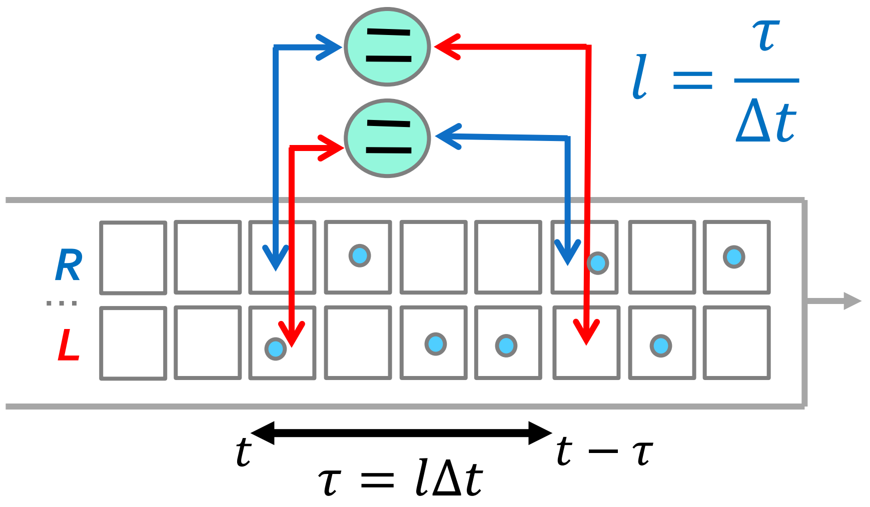
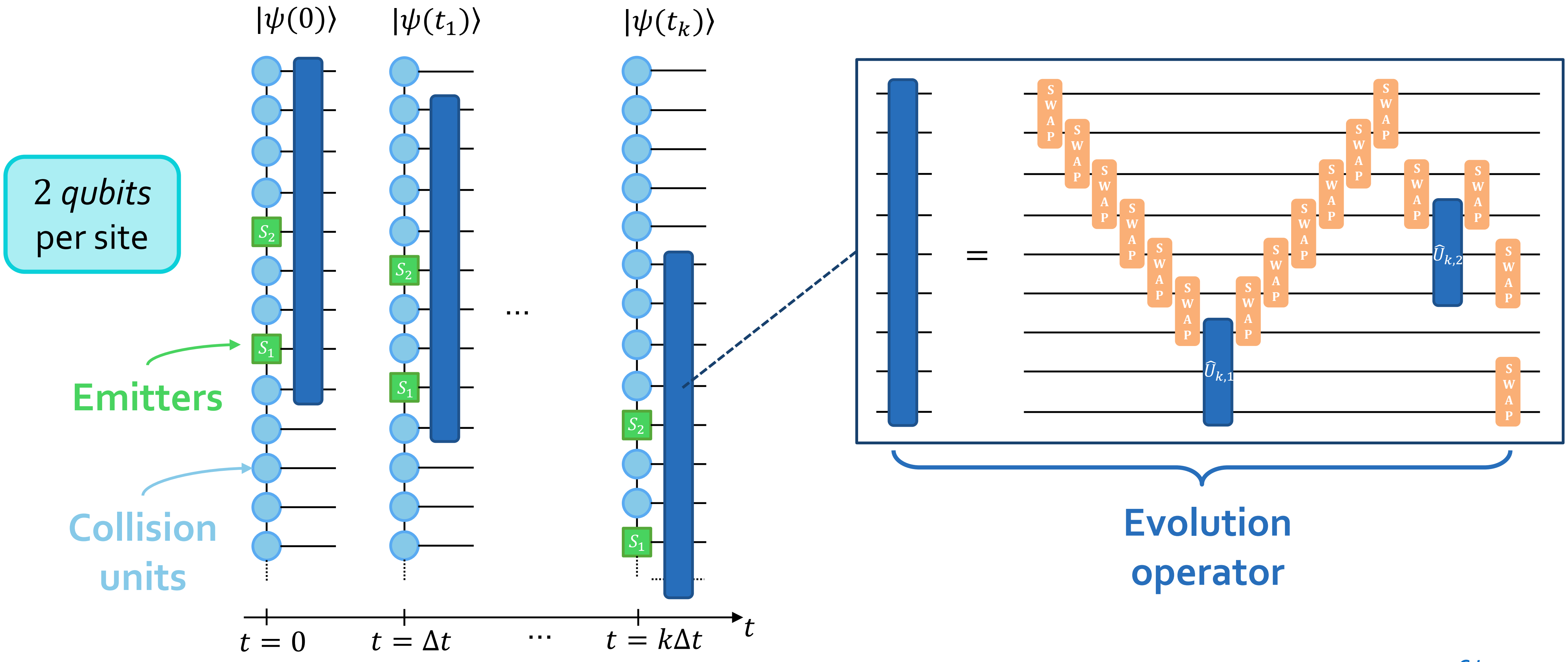
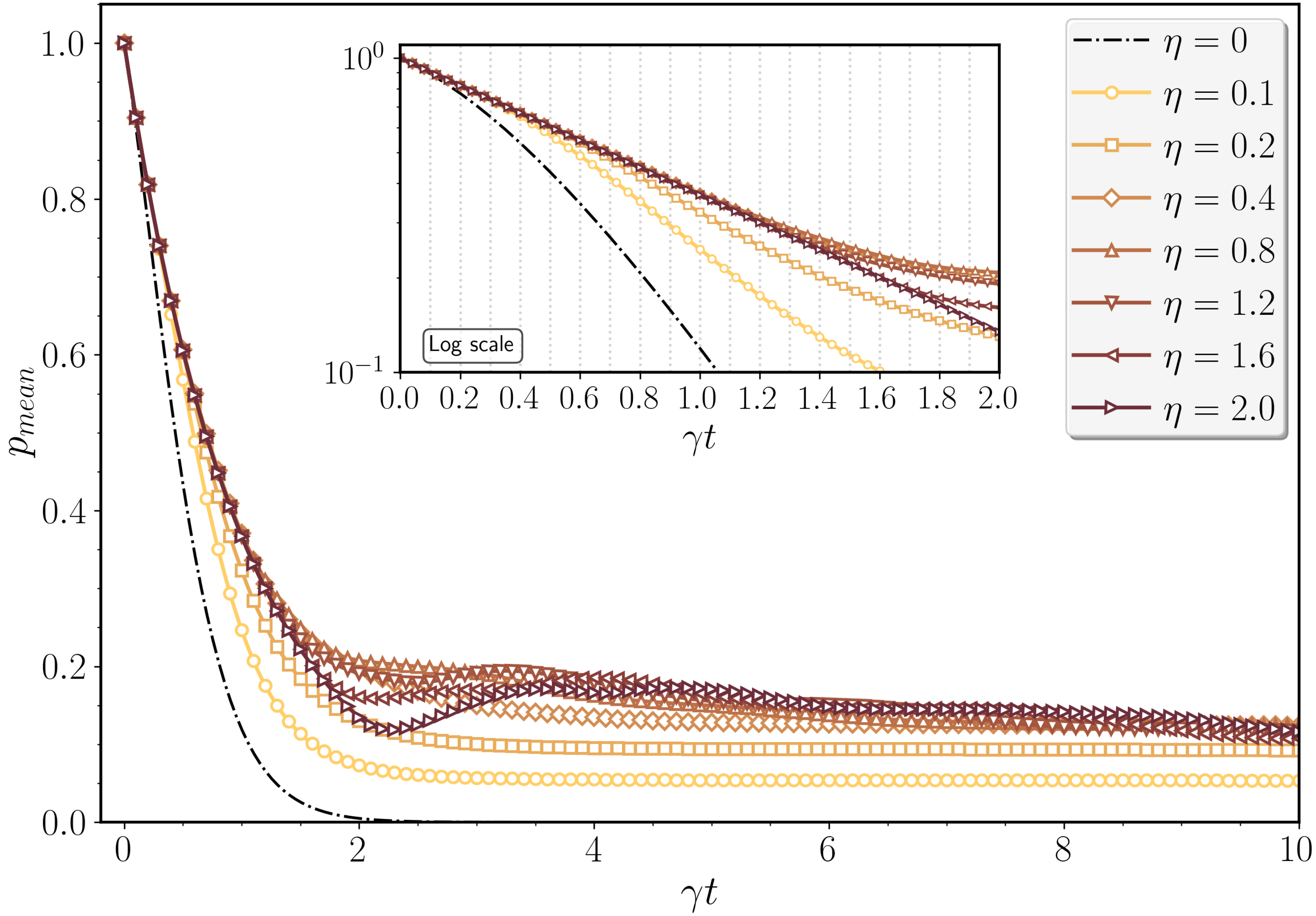


At each time step, *each* emitter pair interacts with **4** different *collision units*

MPS model for Waveguide QED

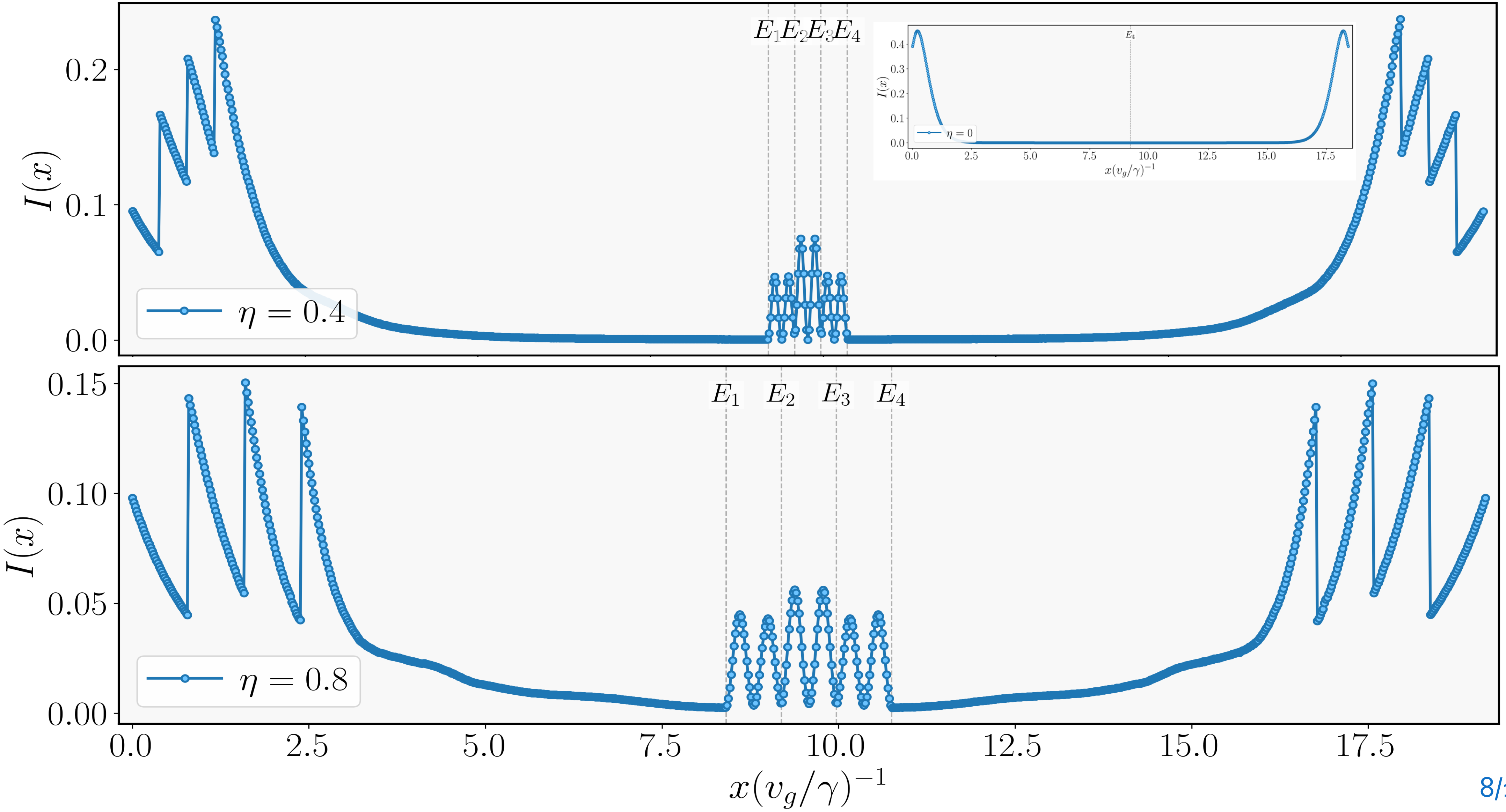


Results: Mean excitation population



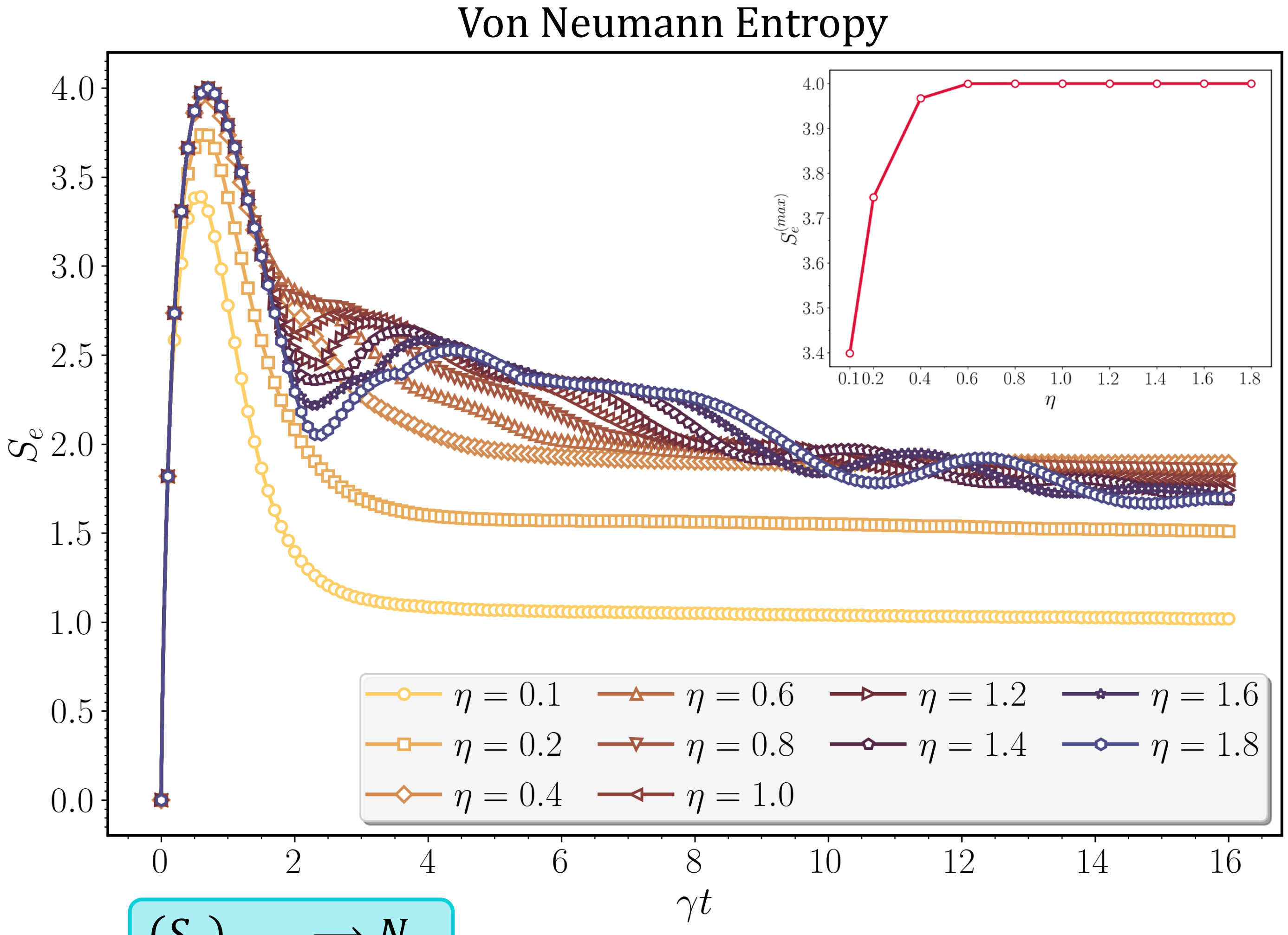
- $N_e = 4$
- *Initial emitter state:* $|\uparrow,\uparrow,\uparrow,\uparrow\rangle$
- $\eta = \gamma\tau = \gamma l \Delta t$
  - $\gamma = 0.2$
  - $\Delta t = 0.1$

Results: Field intensity profile



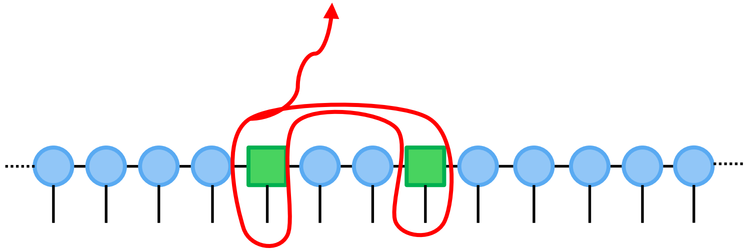
$\phi = 2\pi$

Results: Entanglement Entropy

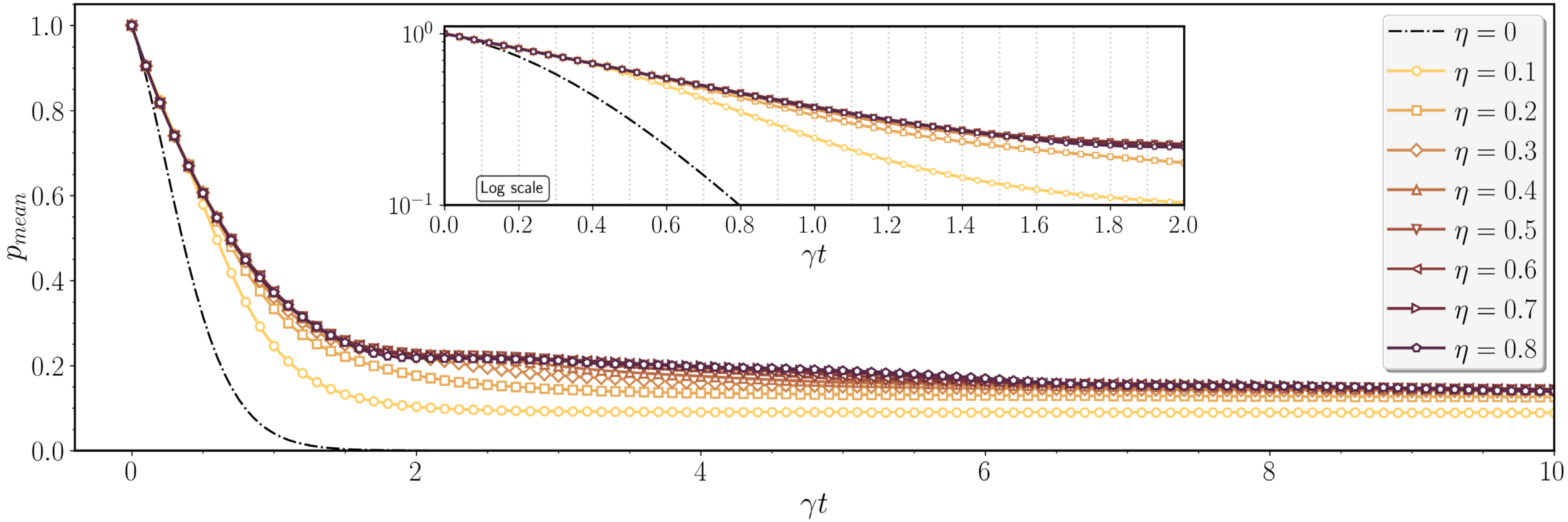


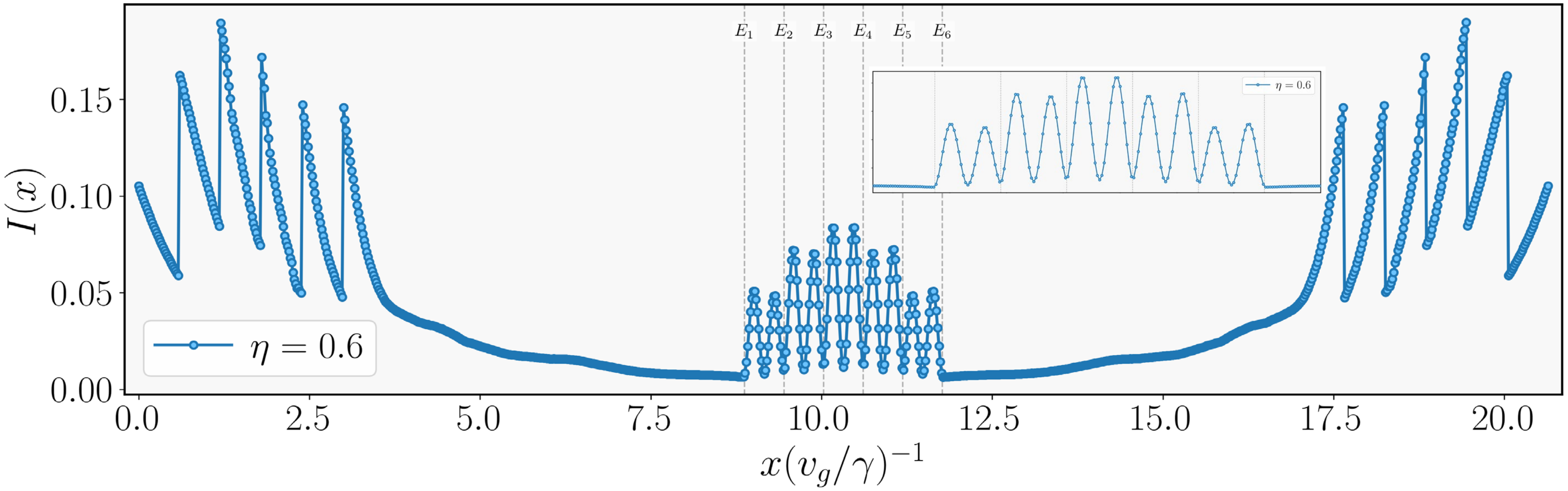
$(S_e)_{max} \longrightarrow N_e$

- $S_e = -Tr(\rho_e log_2 \rho_e)$



Results: 6 emitters



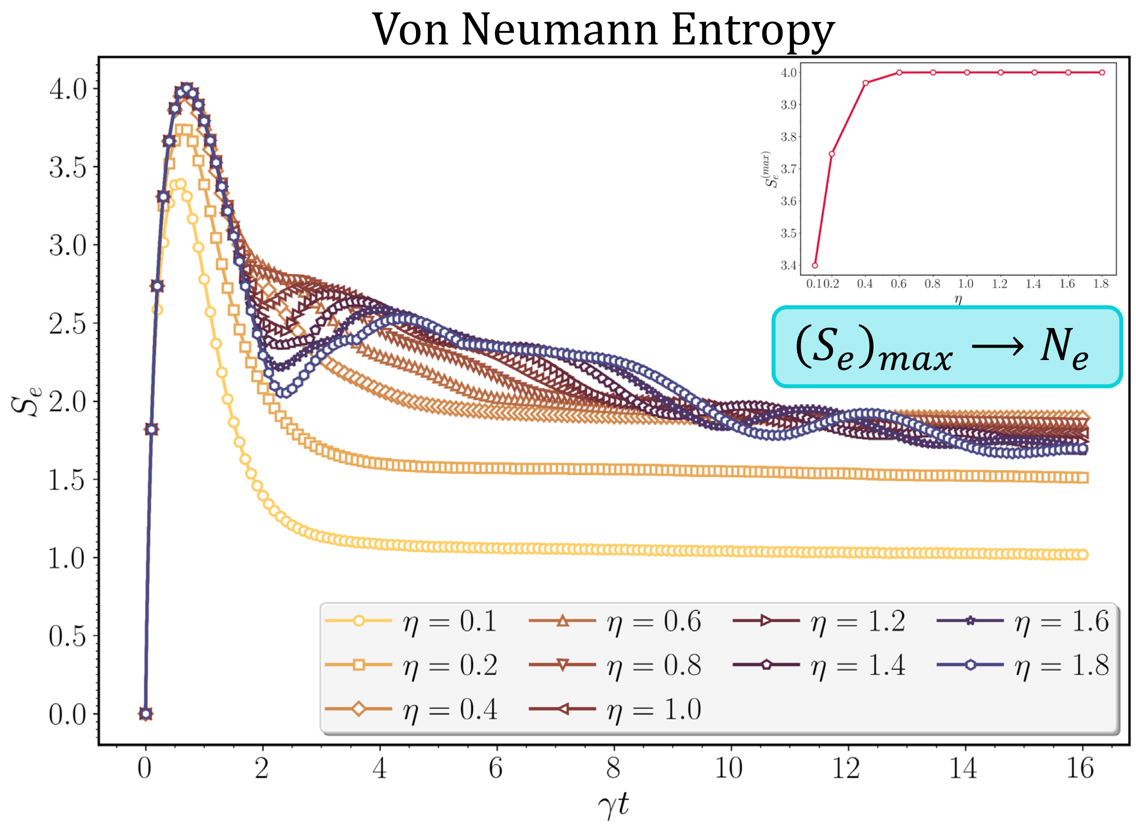


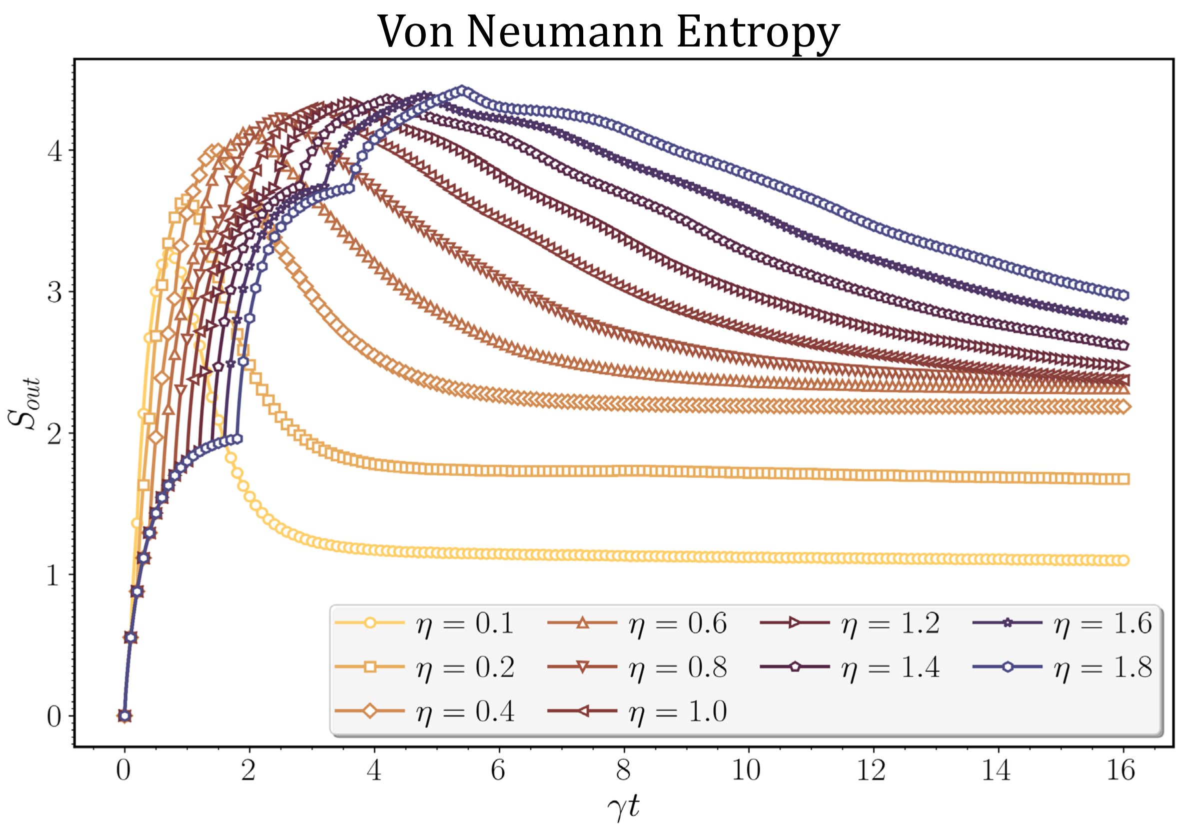
Conclusions

- Tensor Network methods *efficiently* address many-body quantum system time evolution
- Simulation of the non-Markovian dynamics of a *1D waveguide QED platform* with *4 emitters* via Tensor Network, varying η
- Formation of a *Bound State in the continuum (BIC)*:
  - *Entanglement generation*
  - *Photon trapping*
- *Entanglement entropy* analysis

Thanks for your attention!

Results: Entanglement Entropy





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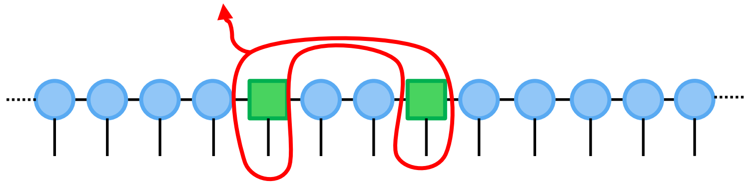
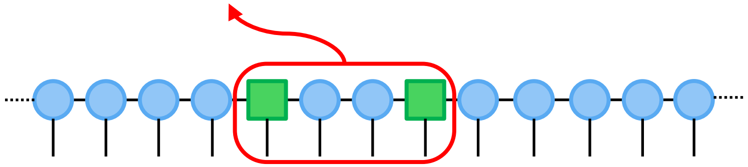
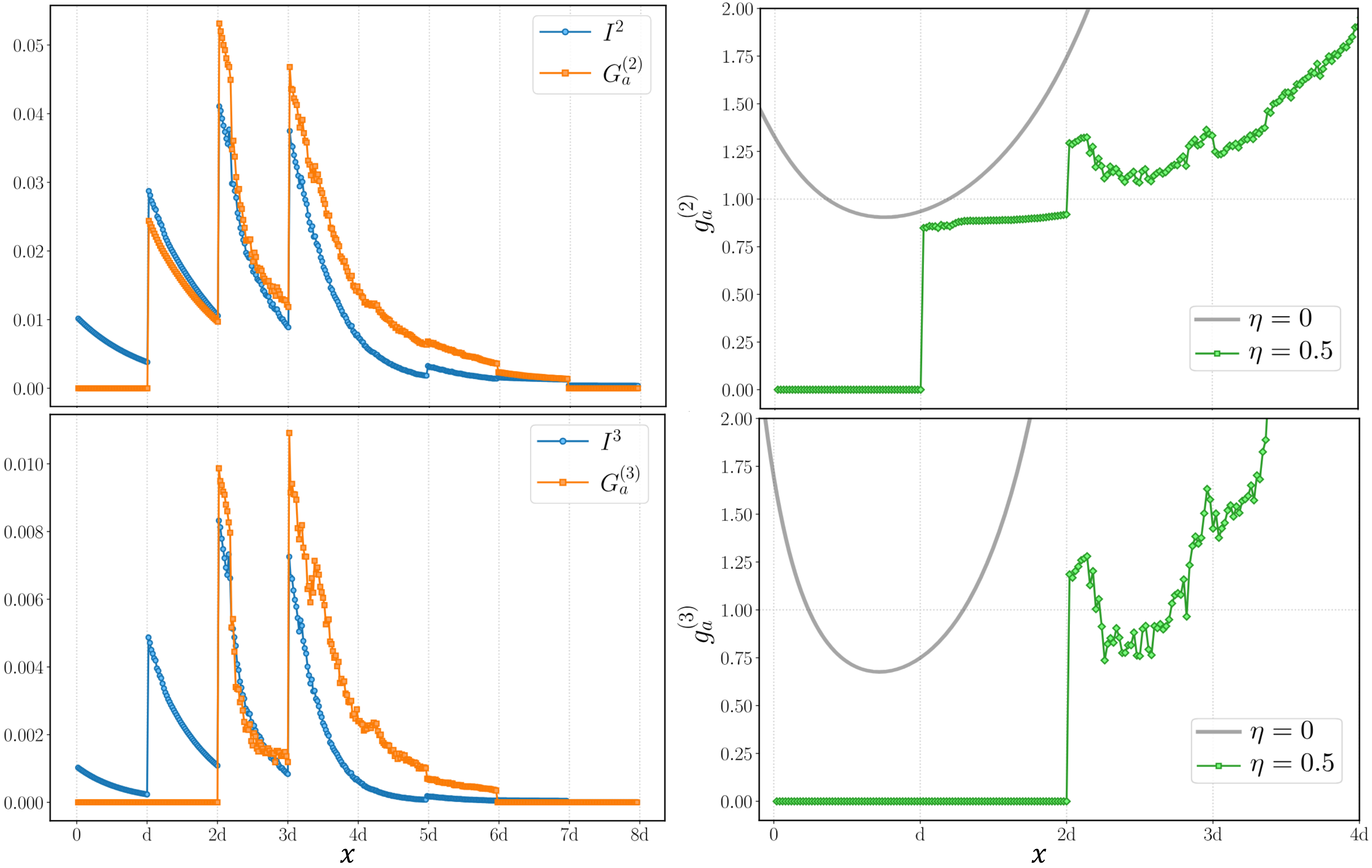


- $S_{out} = -Tr(\rho_{out} log_2 \rho_{out})$



Results: Field auto-correlation functions



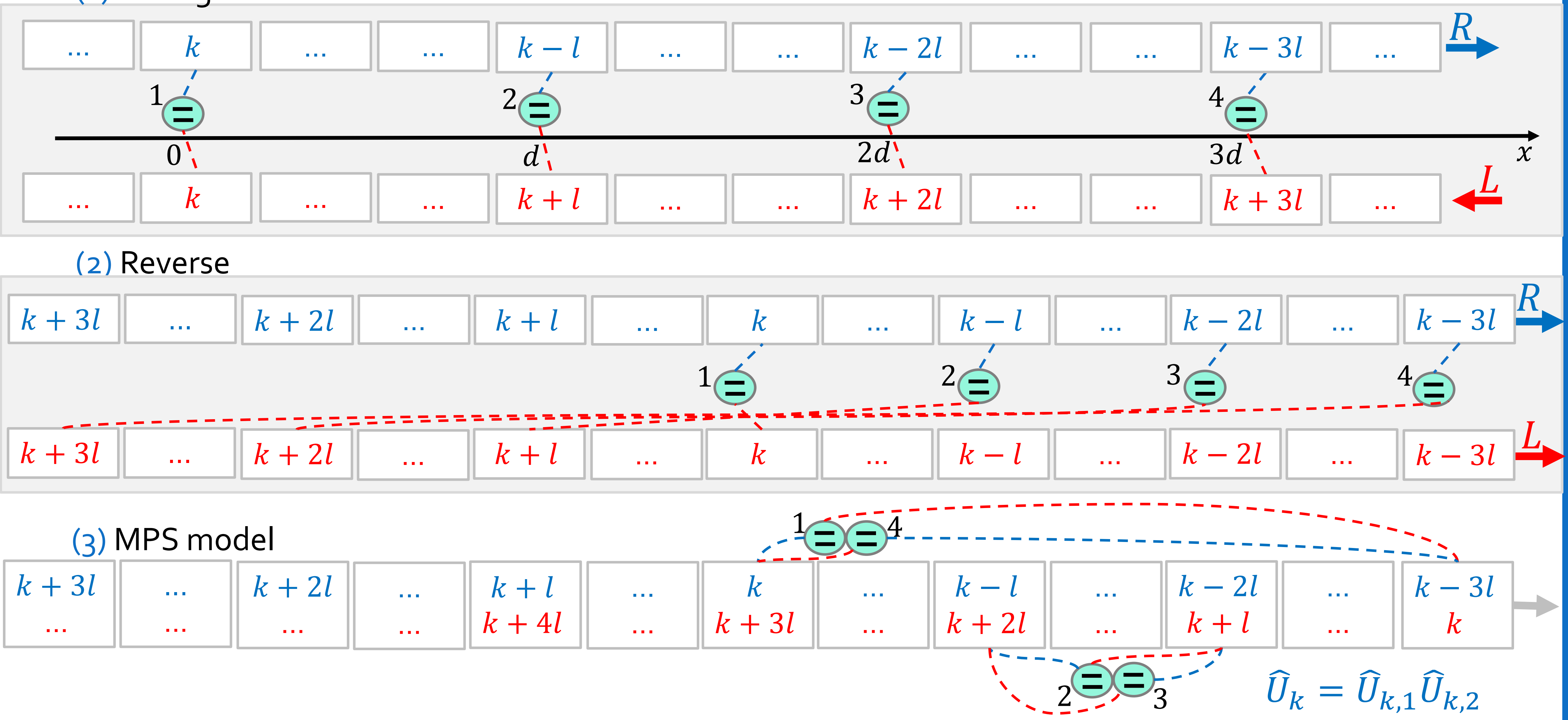
$$g_a^{(m)} = \frac{G_a^{(m)}}{I^m}$$

MPS model for Waveguide QED

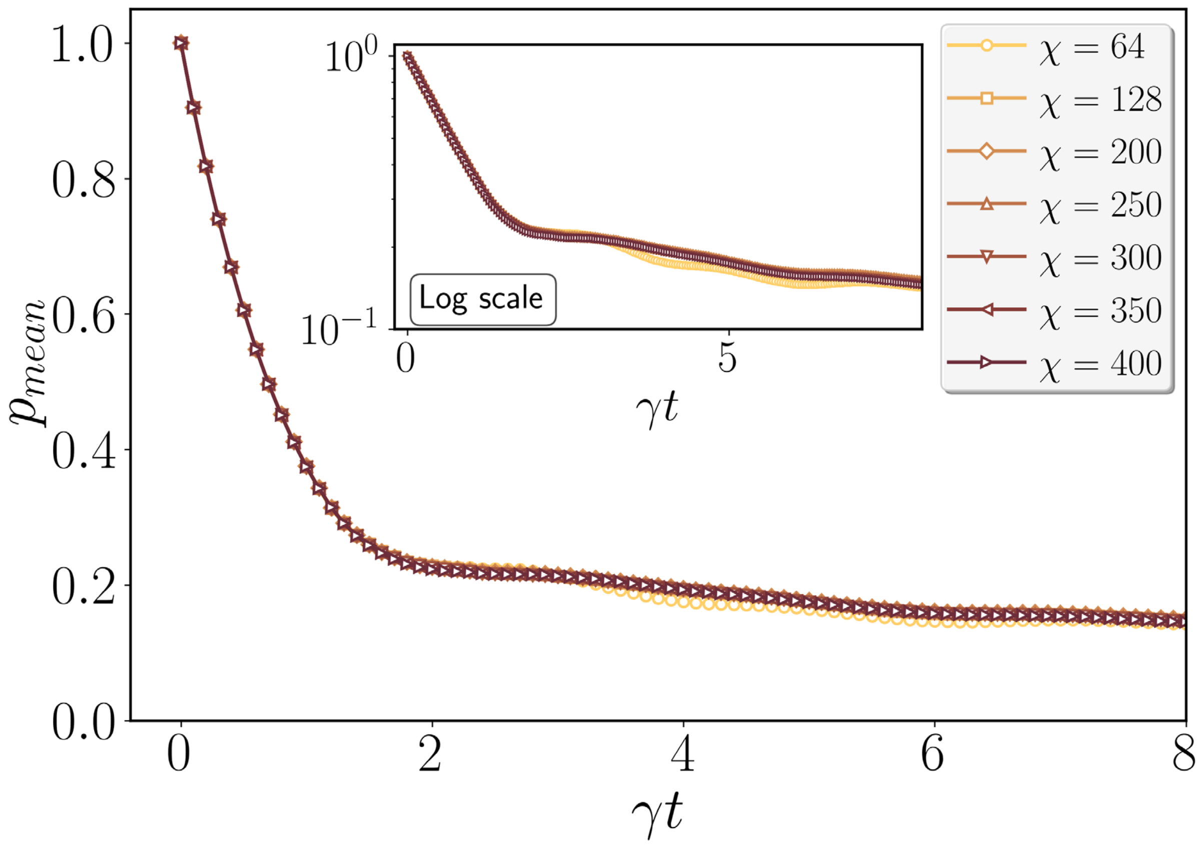
(1) Waveguide QED

(2) Reverse

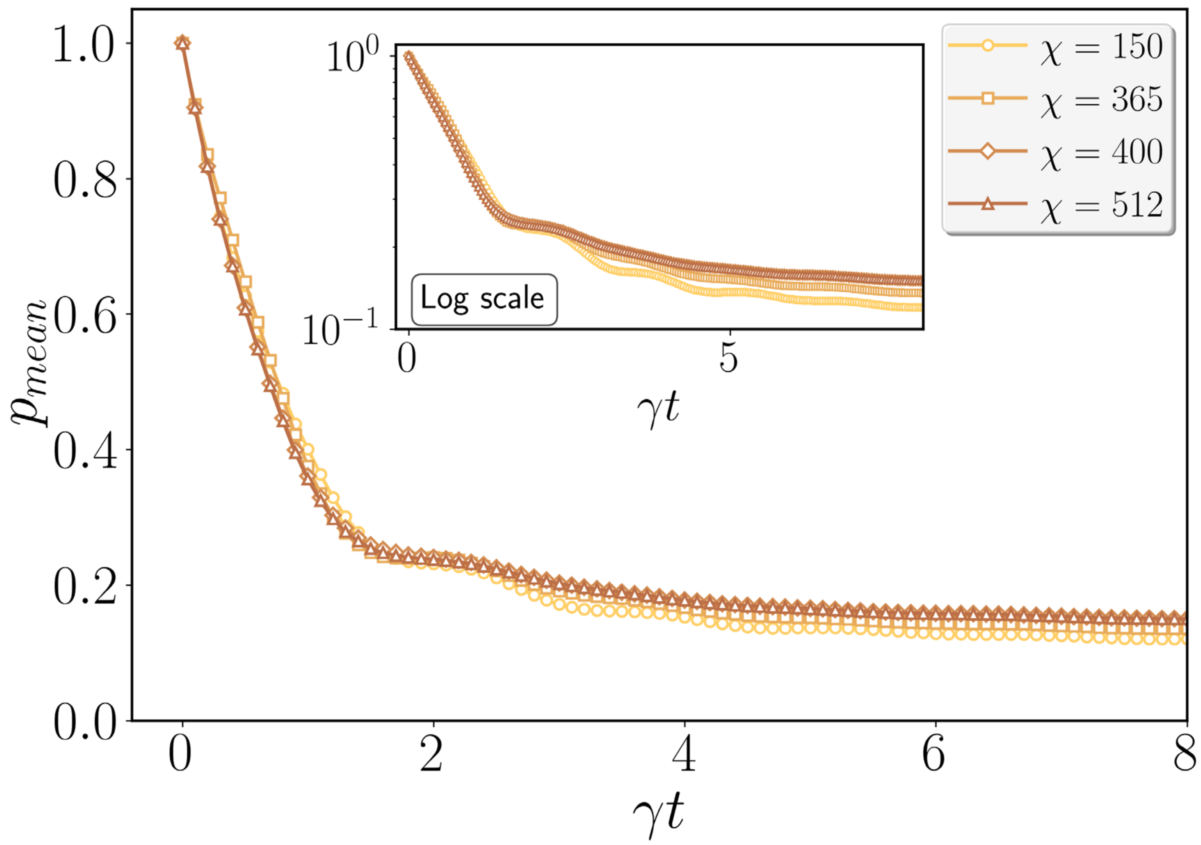
(3) MPS model



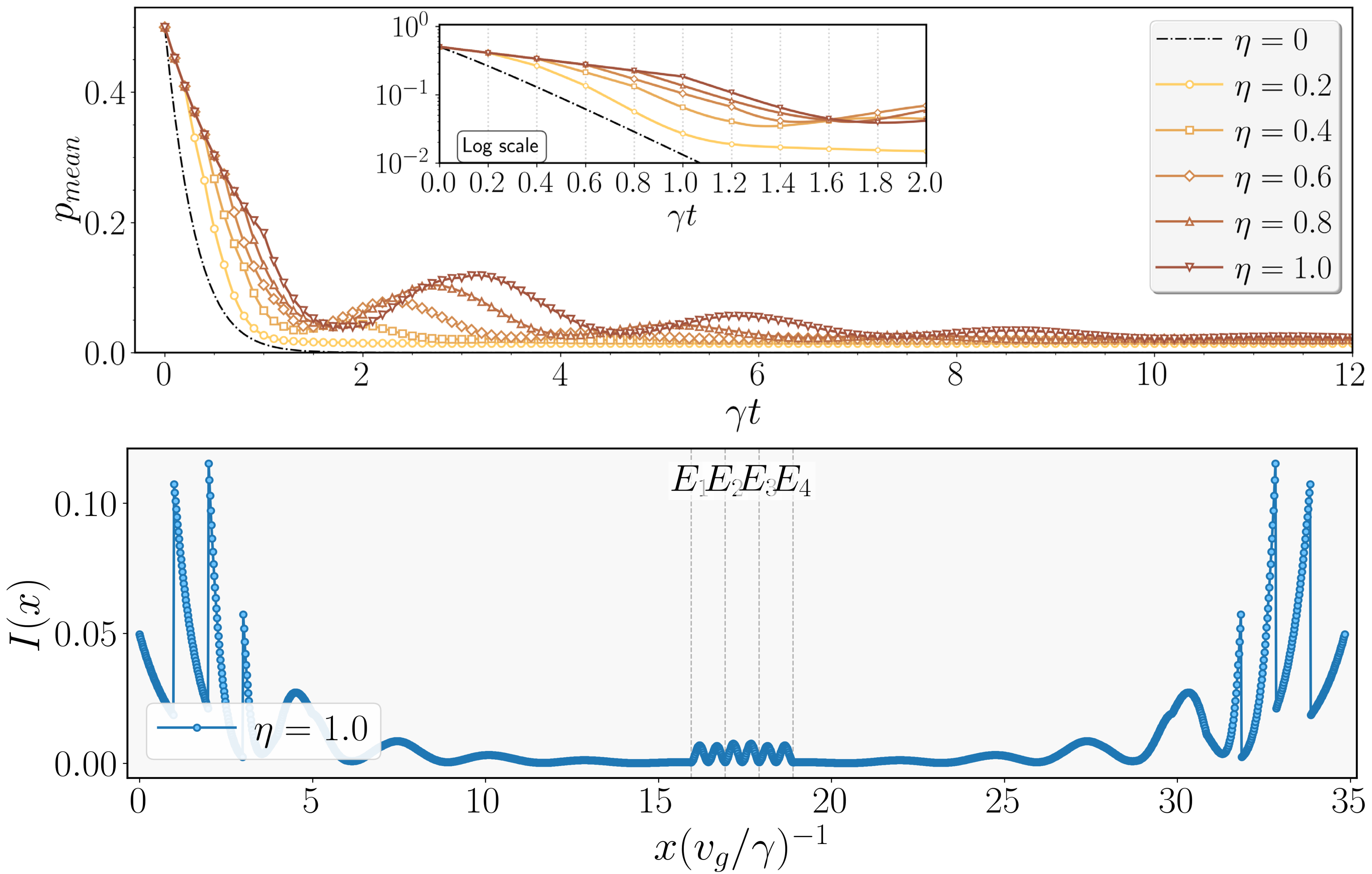
Numerical Convergence



$N_e = 6, \eta = 0.6$



$N_e = 8, \eta = 0.3$



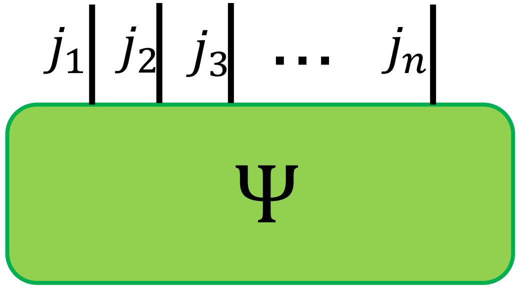
- $N_e = 4$
- *Initial state:* $\frac{1}{\sqrt{6}}(|\uparrow\uparrow\downarrow\downarrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle)$
- $\eta = \gamma\tau = \gamma l \Delta_t$
  - $\gamma = 0.2$
  - $\Delta_t = 0.1$

TN diagram for a Quantum State

$$|\psi\rangle = \sum_{j_1, j_2, \ldots, j_n} C_{j_1 j_2 \ldots j_n} |j_1\rangle \otimes |j_2\rangle \otimes \ldots \otimes |j_n\rangle$$

d-level quantum system

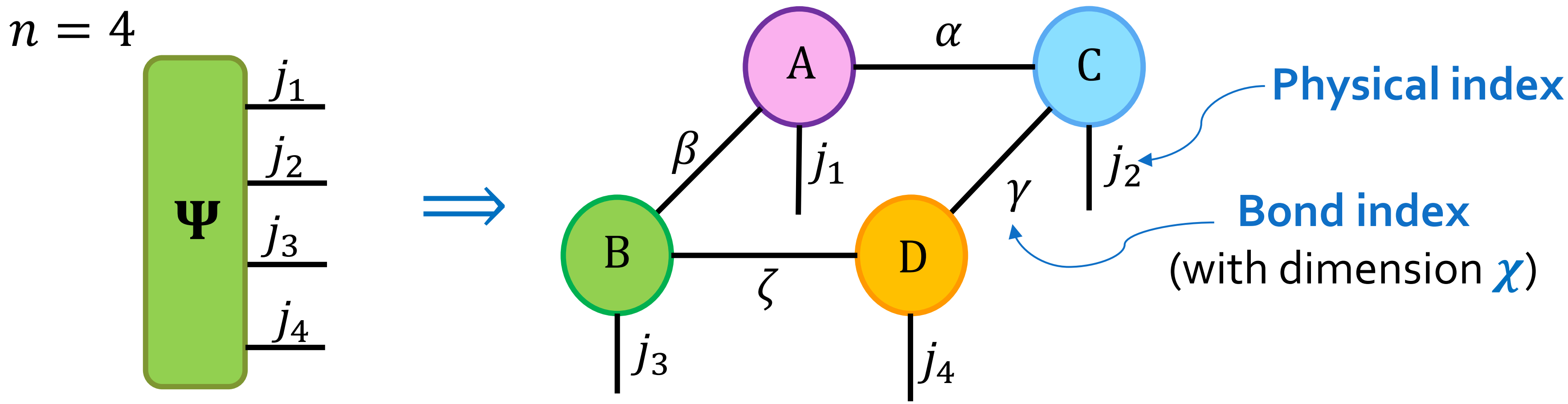
- $\{C_{j_1 j_2 \ldots j_n}\}_{j_1, j_2, \ldots, j_n} \rightarrow d^n$ complex numbers
- Coefficients of a **tensor** Ψ with n indices taking d different values $\Longrightarrow$ Overall size: d^n
- The indices j_i are called **physical indices** (*physical dimension* of the *subsystem* i)



\# parameters: $O(d^n)$
$\Longrightarrow$ ***Inefficient!***

TN diagram for a Quantum State

- *Aim* $\Longrightarrow$ reduction of computational complexity
- **How?** $\Longrightarrow$ replacing the «big» tensor $\mathbf{\Psi}$ with a TN of *smaller-rank tensors*



parameters: $O(d^n)$

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parameters: $O\big(poly(n)poly(\chi)\big)$

$\Longrightarrow$ ***Efficient***!

TN algorithm for Quantum Simulations

- **TEBD algorithm** $\Longrightarrow$ simulation of the *time evolution* of quantum systems with *short-ranged* Hamiltonian
- *Key idea* $\Longrightarrow$ decomposing the evolution operator into *local gates*[1]

$$U(t) = e^{-i\mathcal{H}t}$$

Hamiltonian:

$$\mathcal{H} = \sum_{even\ i} h_{i,i+1} + \sum_{odd\ i} h_{i,i+1}$$

Suzuki-Trotter formula:

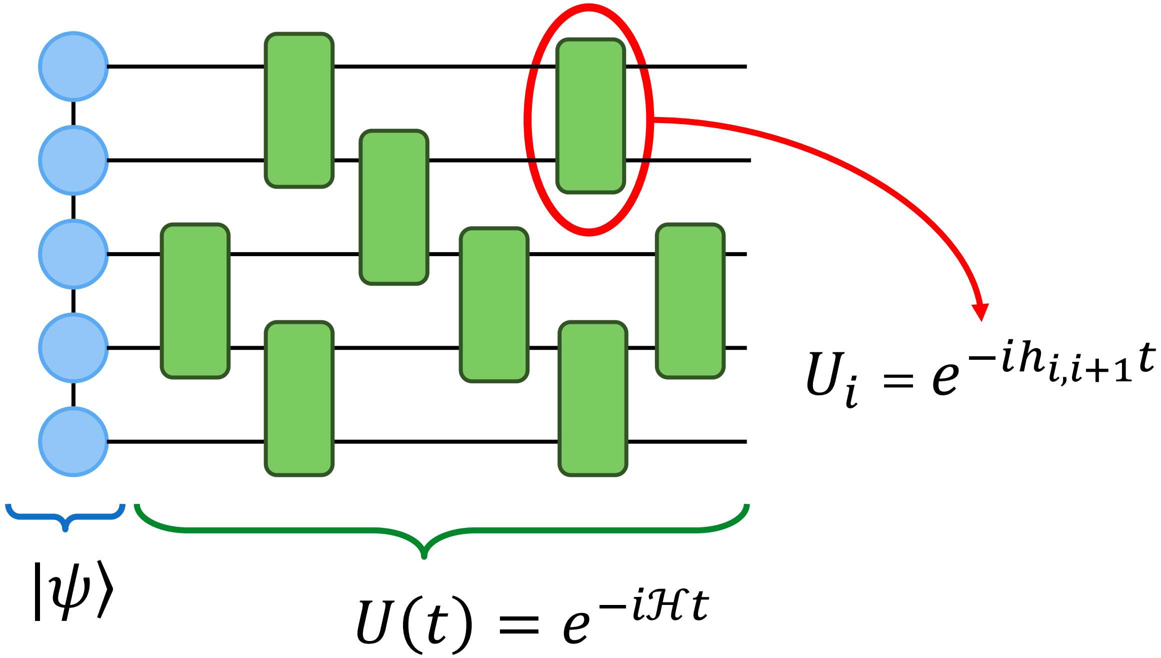
$$U(t) \approx \left(\prod_{even\ i} e^{-ih_{i,i+1}\Delta t} \prod_{odd\ i} e^{-ih_{i,i+1}\Delta t} \right)^n$$

with $\boldsymbol{\Delta t} = \boldsymbol{t/n}$

[1] S. Paeckel, T. Köhler, A. Swoboda, S. Manmana, U. Schollwöck, C. Hubig; 10.48550/arXiv.1901.05824 (2019)

TN algorithm for Quantum Simulations

- *Time evolution* decomposed into «*blocks*» $\Longrightarrow$ *quantum circuit representation*
- Every *quantum circuit* is in **one-to-one correspondance** with a *tensor network*



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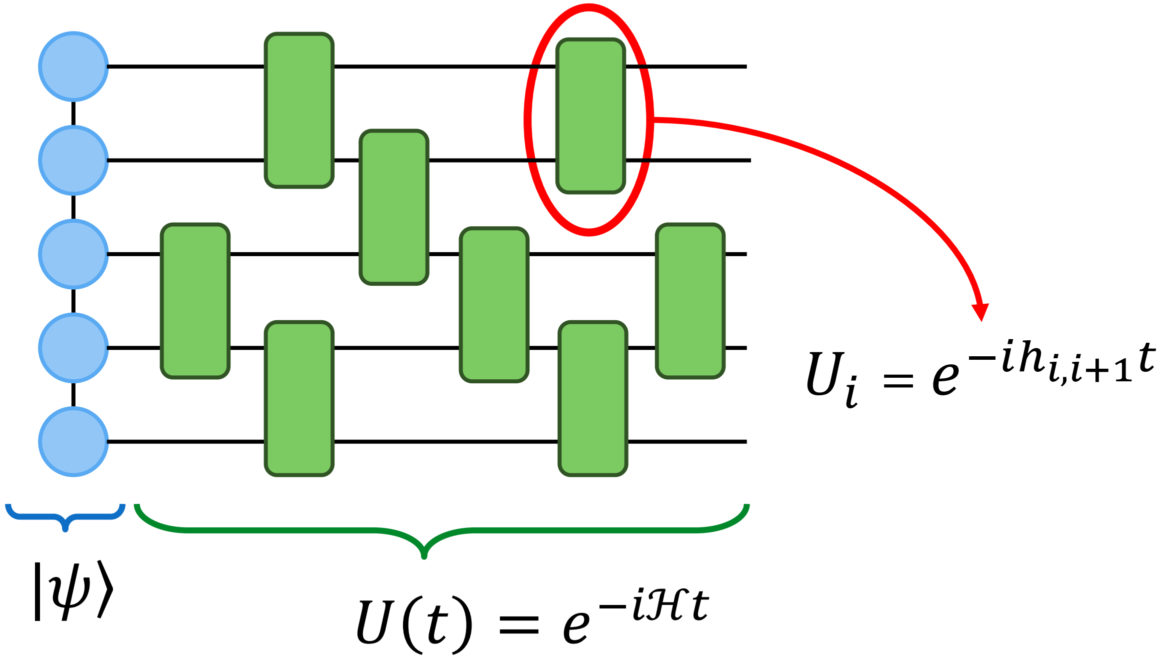
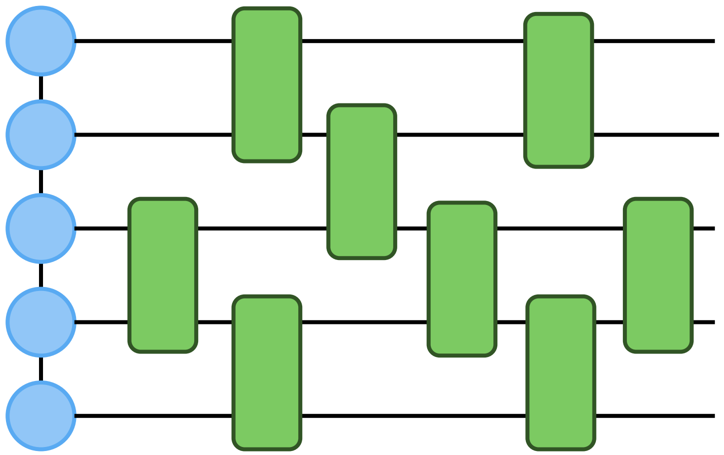


Any n-qubit gate F can be reshaped into a rank-$2n$ tensor $\mathbf{F}$:

$$F \in \mathbb{C}^{2^n \times 2^n} \Longrightarrow$$
$$\mathbf{F} \in \mathbb{C}^{2\times 2\times \cdots \times 2}$$

Time-evolving block decimation (TEBD)

- TEBD algorithm $\Longrightarrow$ *series of contraction operations*



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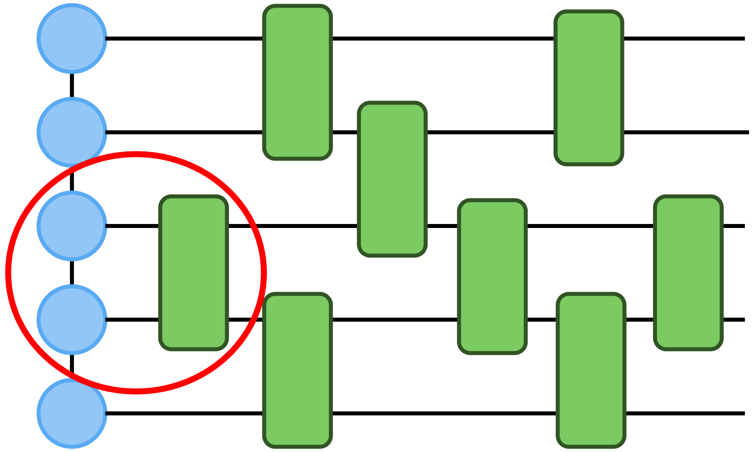

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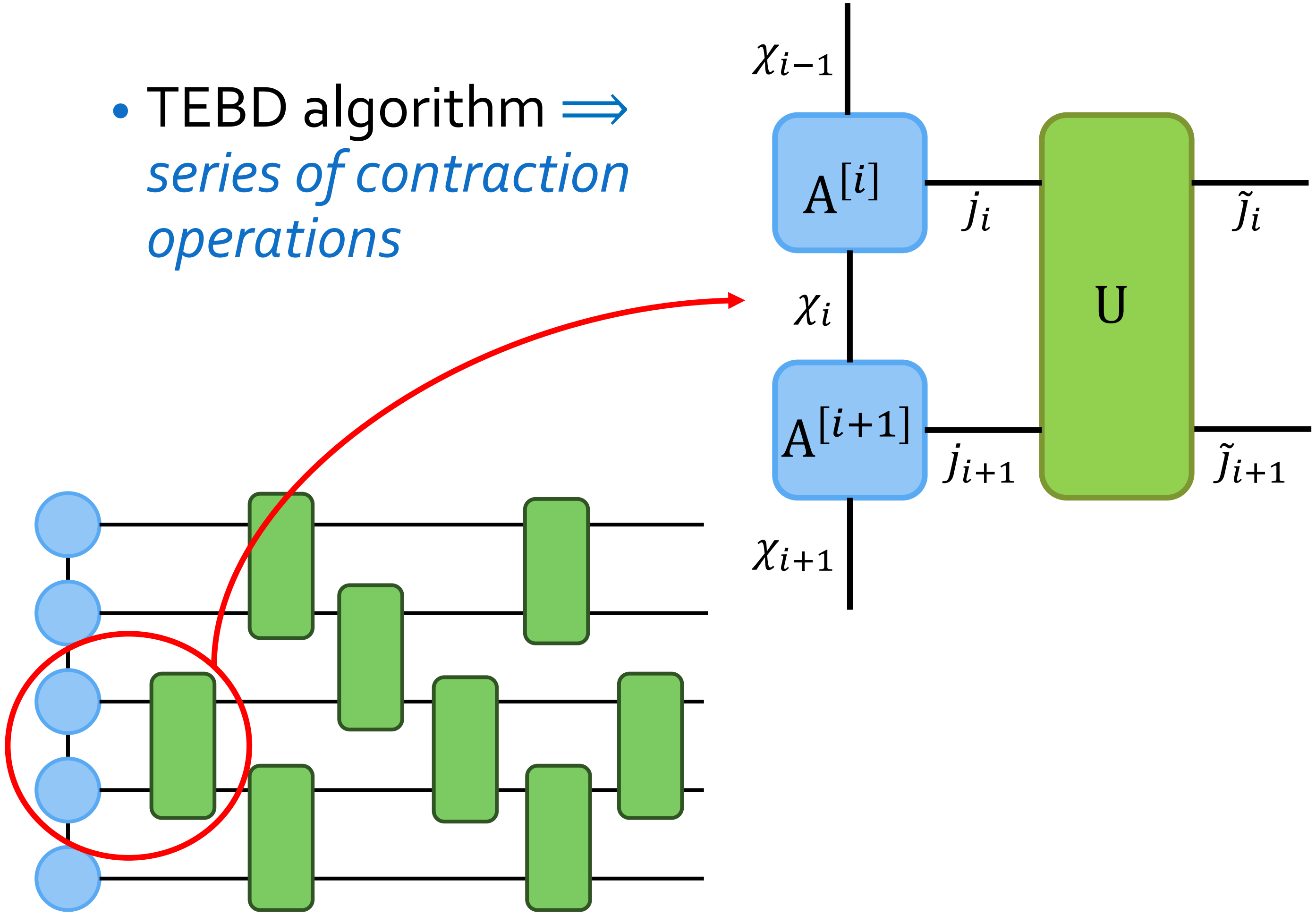

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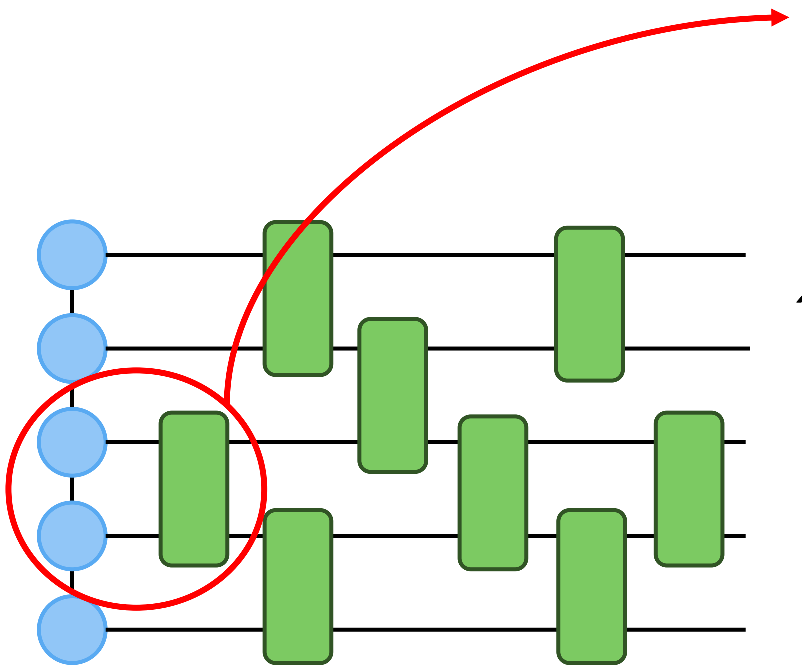

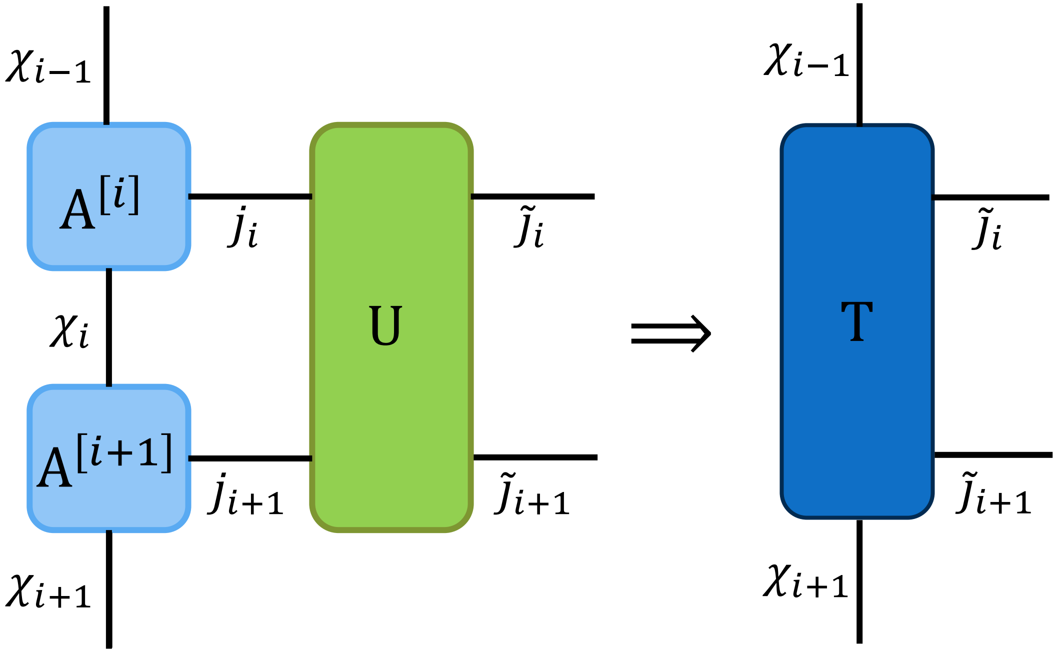

MPS contracted with *one layer* of the circuit at each step

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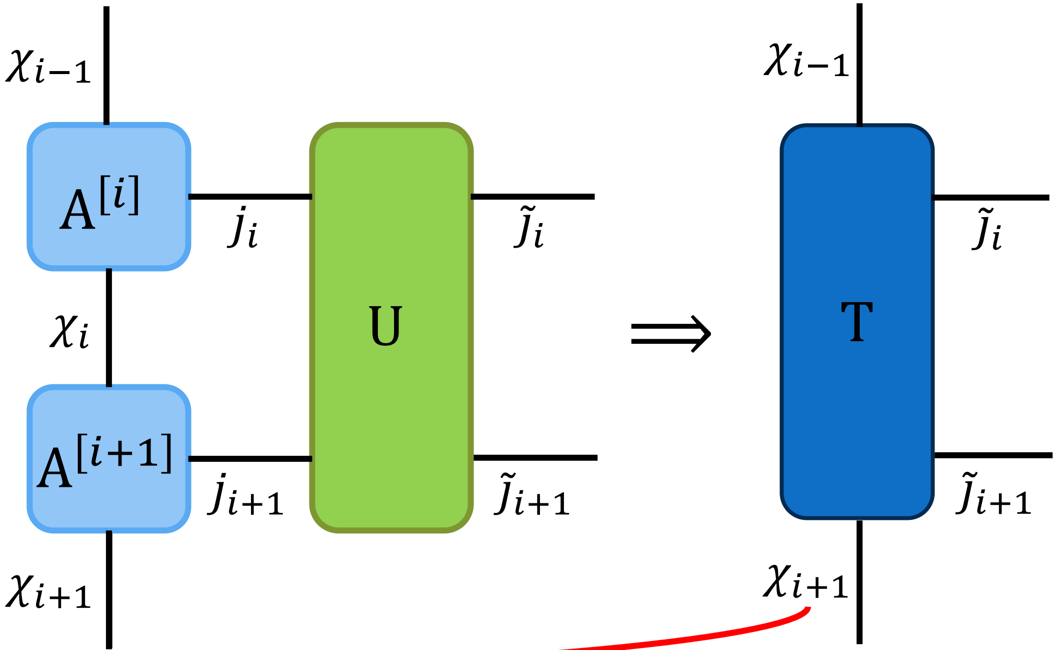

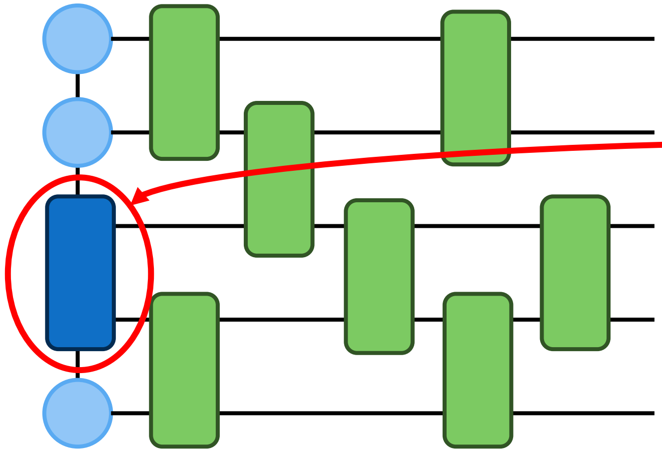

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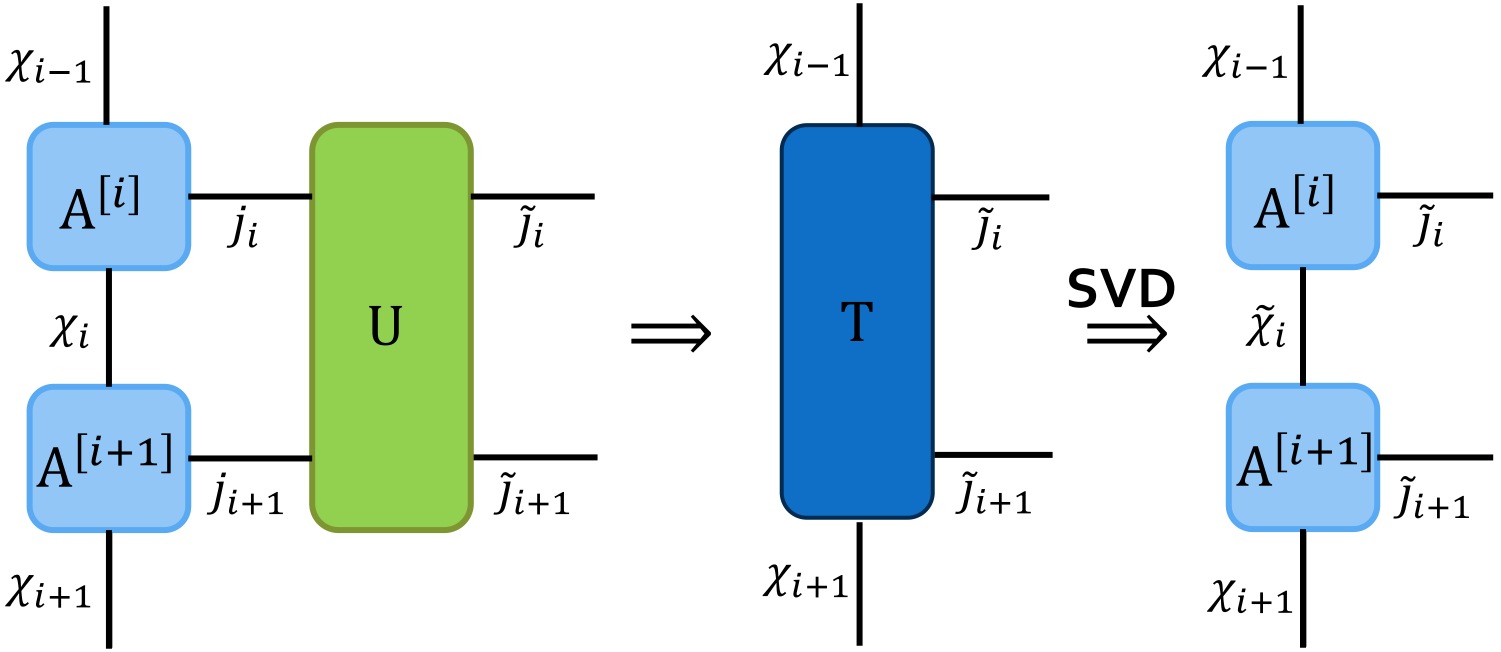
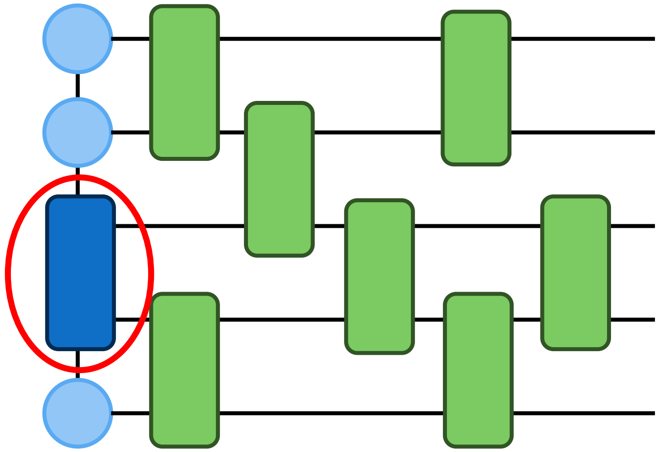
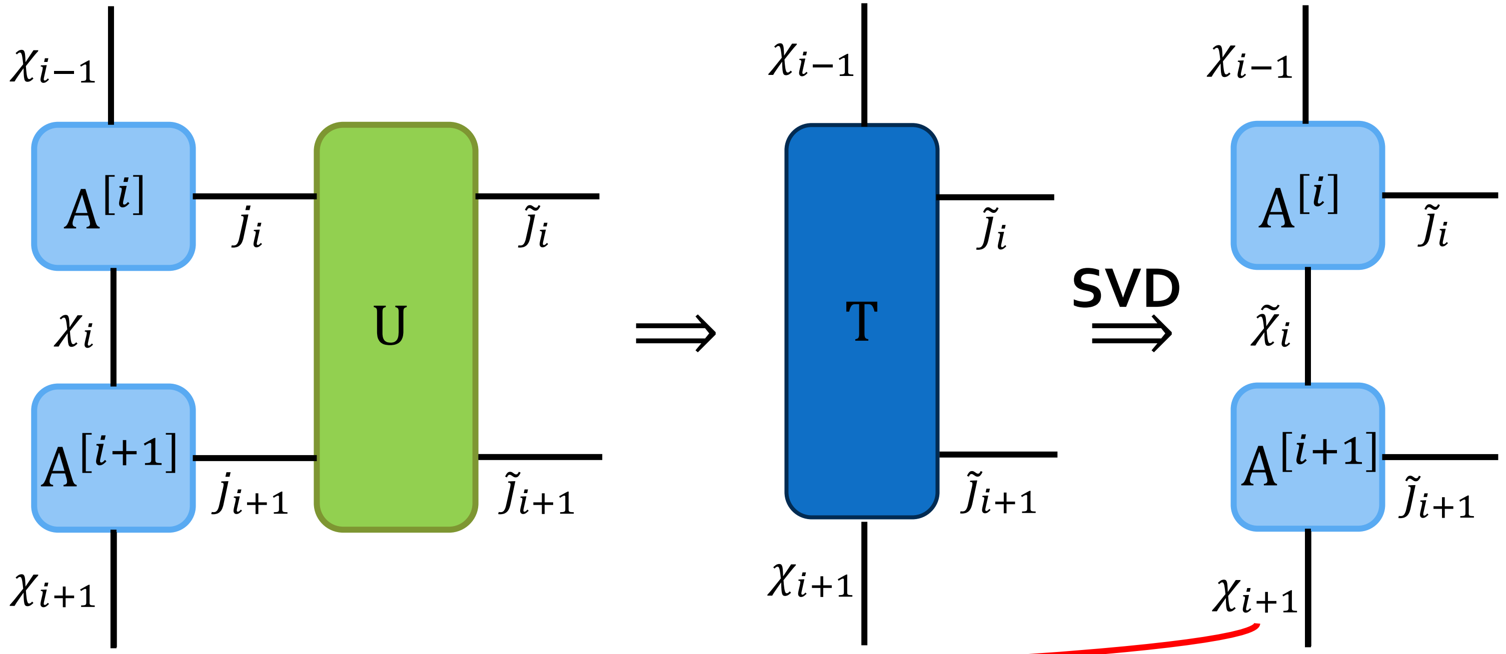


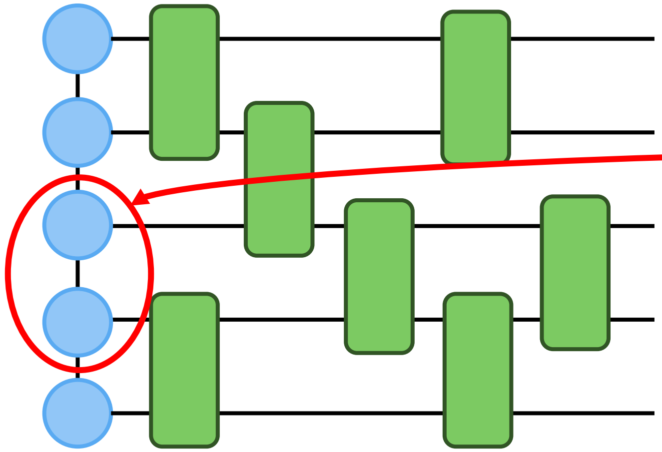


After each contraction step, MPS form of the evolving state *restored*

Time-evolving block decimation (TEBD)

- TEBD algorithm $\Longrightarrow$ *series of contraction operations*





After each contraction step, MPS form of the evolving state *restored*

Singular value decomposition (SVD)

- **SVD** of a matrix $\mathbf{A} \in \mathbb{C}^{m \times n} \Longrightarrow$ *factorization* of $\mathbf{A}$ into the product of 3 matrices

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^{\dagger}$$

such that:

1. $\mathbf{U} \in \mathbb{C}^{m \times m}$ *unitary*
2. $\mathbf{V} \in \mathbb{C}^{n \times n}$ *unitary*
3. $\mathbf{S} \in \mathbb{R}^{m \times n}$, $\mathbf{S} \geq \mathbf{0}$ *diagonal* in the computational basis:

$$\mathbf{S} = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\min(m,n)} \end{pmatrix}, \quad \text{with } \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{\min(m,n)}$$

→ *singular values* of $\mathbf{A}$