

Optimal detection

The background features a series of overlapping, wavy shapes in two shades of blue. The top layer is a dark, navy blue, while the bottom layer is a slightly lighter, vibrant blue. These shapes create a layered, mountain-like or wave-like effect across the entire width of the image.

Detection problem

frequentist approach

$$\text{data } x(t) = h(t) + n(t)$$

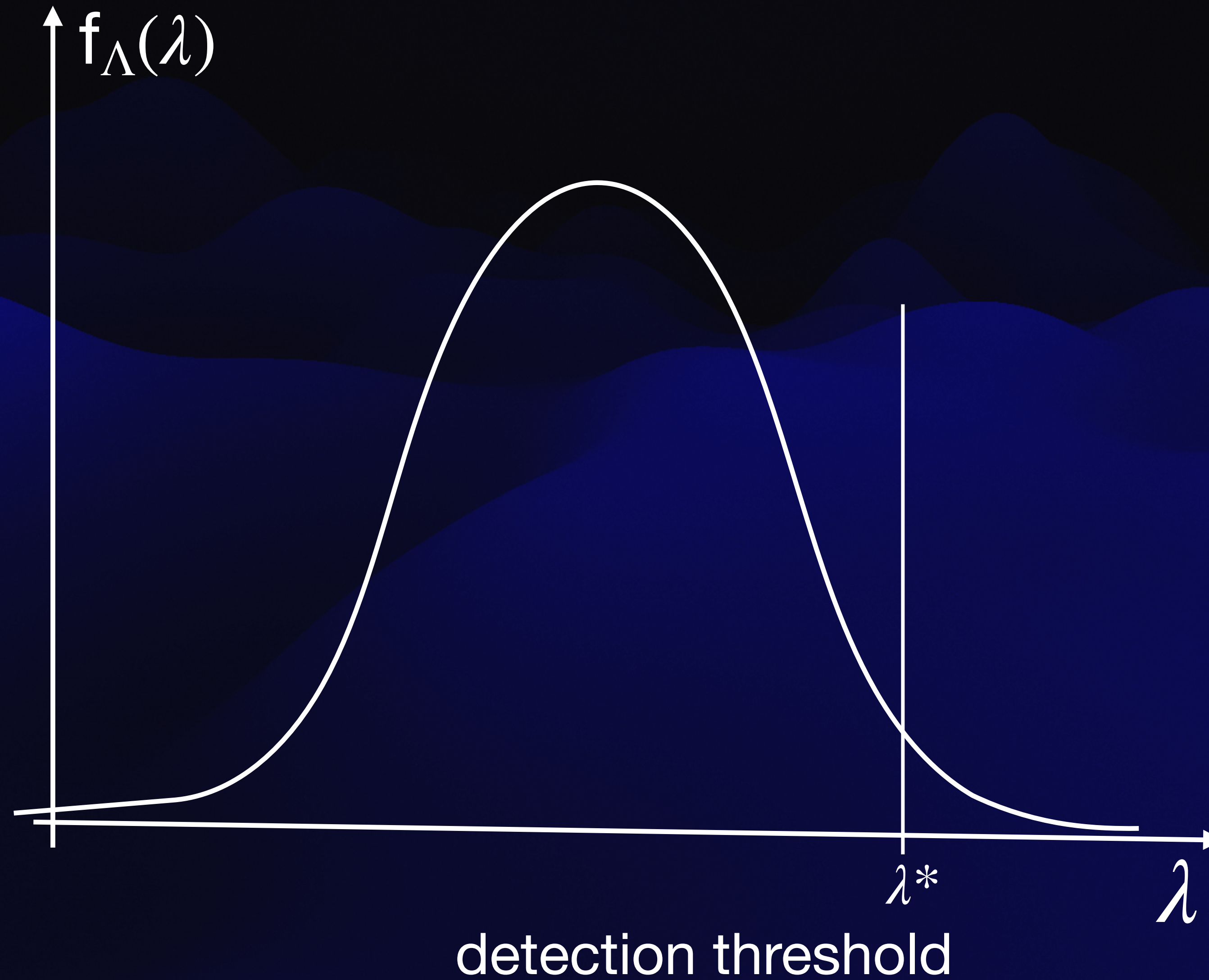
Detection statistic: is a rule that allows to construct a score based on which you can decide if a signal is present or not.

The likelihood Λ (and any monotonic function of it) is an optimal detection statistic

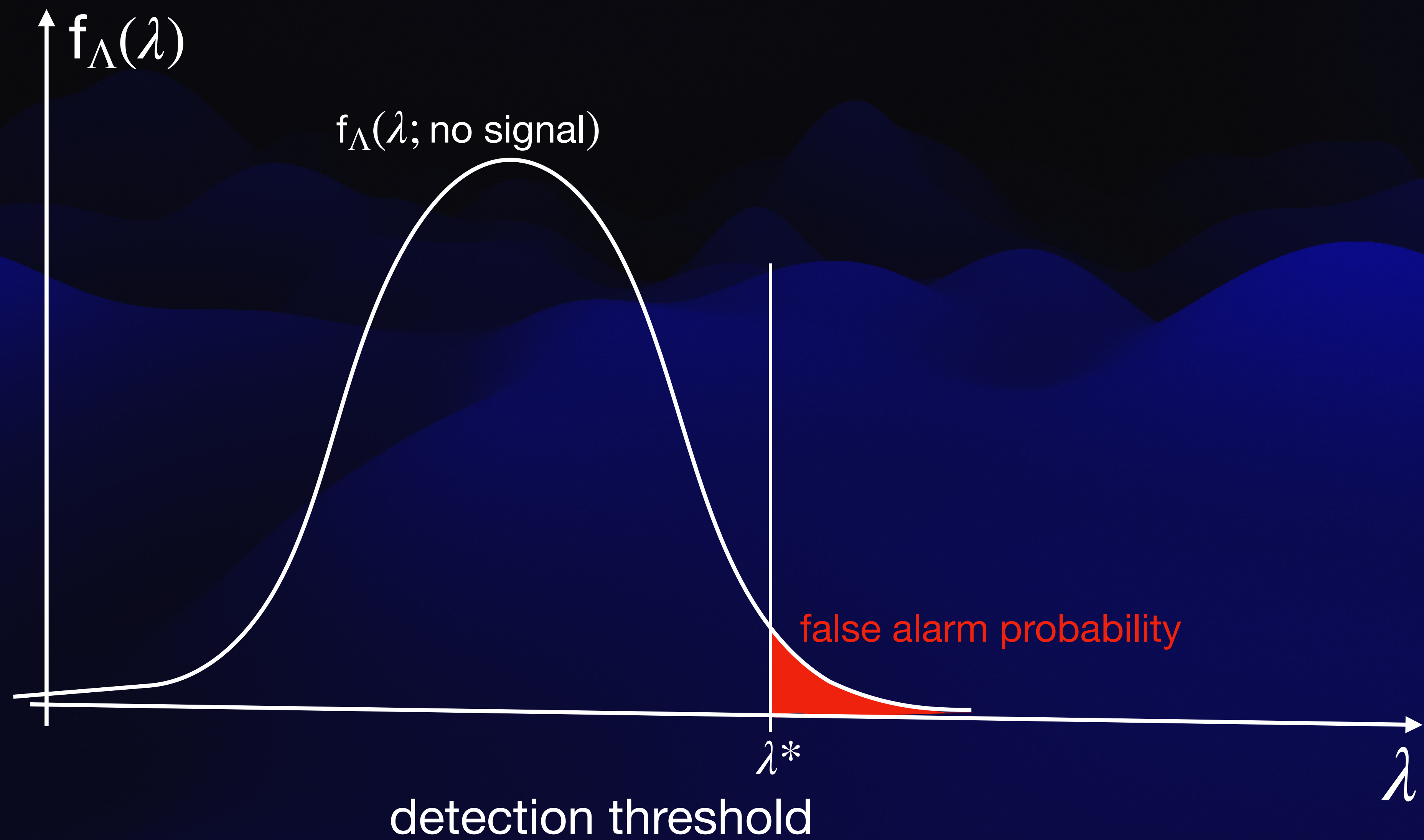
$$\Lambda(x; \text{signal}) \equiv \frac{\text{pdf}(x | \text{signal})}{\text{pdf}(x | \text{noise})}$$

so also optimal $\log \Lambda$ is an optimal statistic.

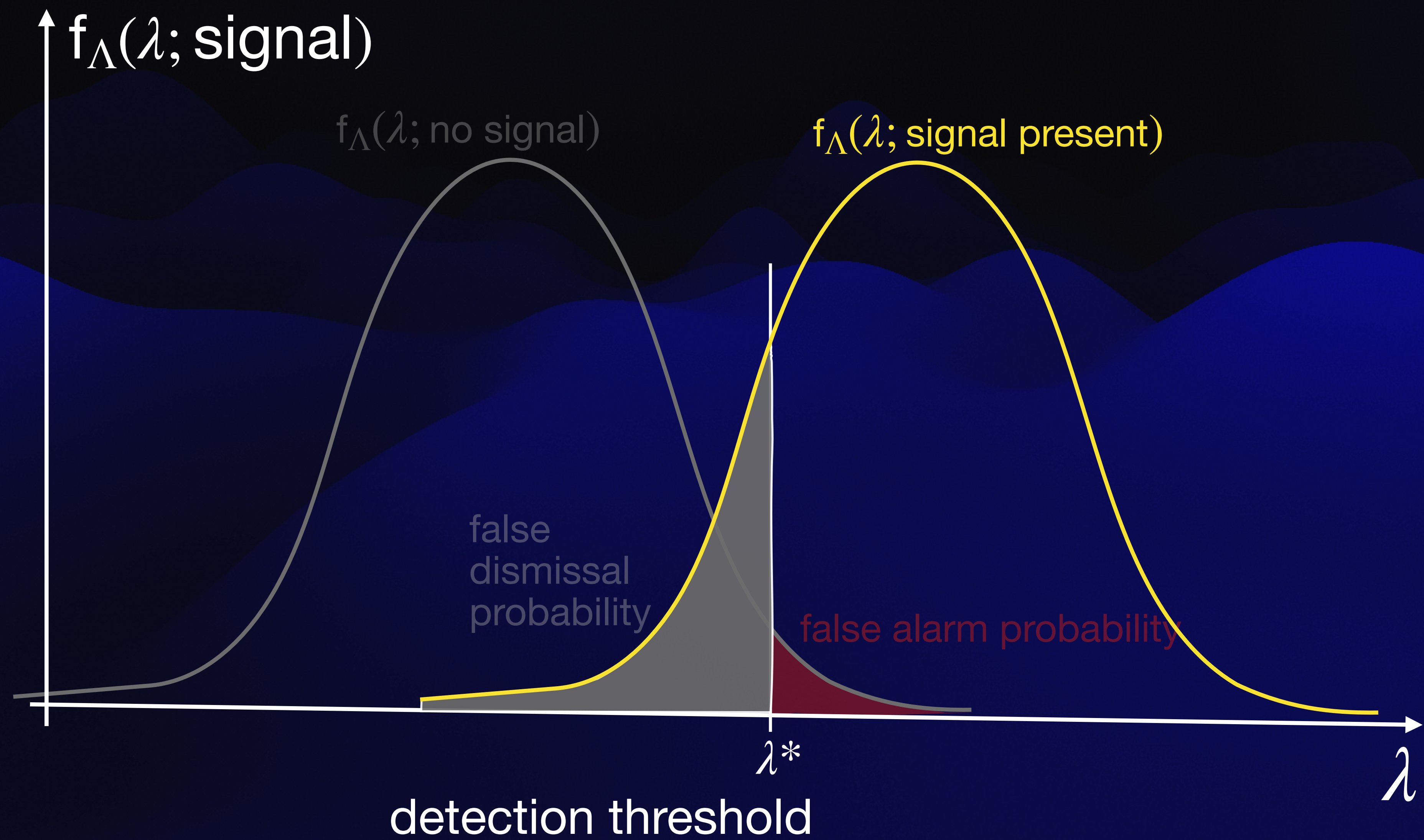
How do I decide if there is a signal based on the value of Λ ? I pick a threshold λ^* :
If $\Lambda \geq \lambda^*$ \rightarrow signal present. Else, signal absent.



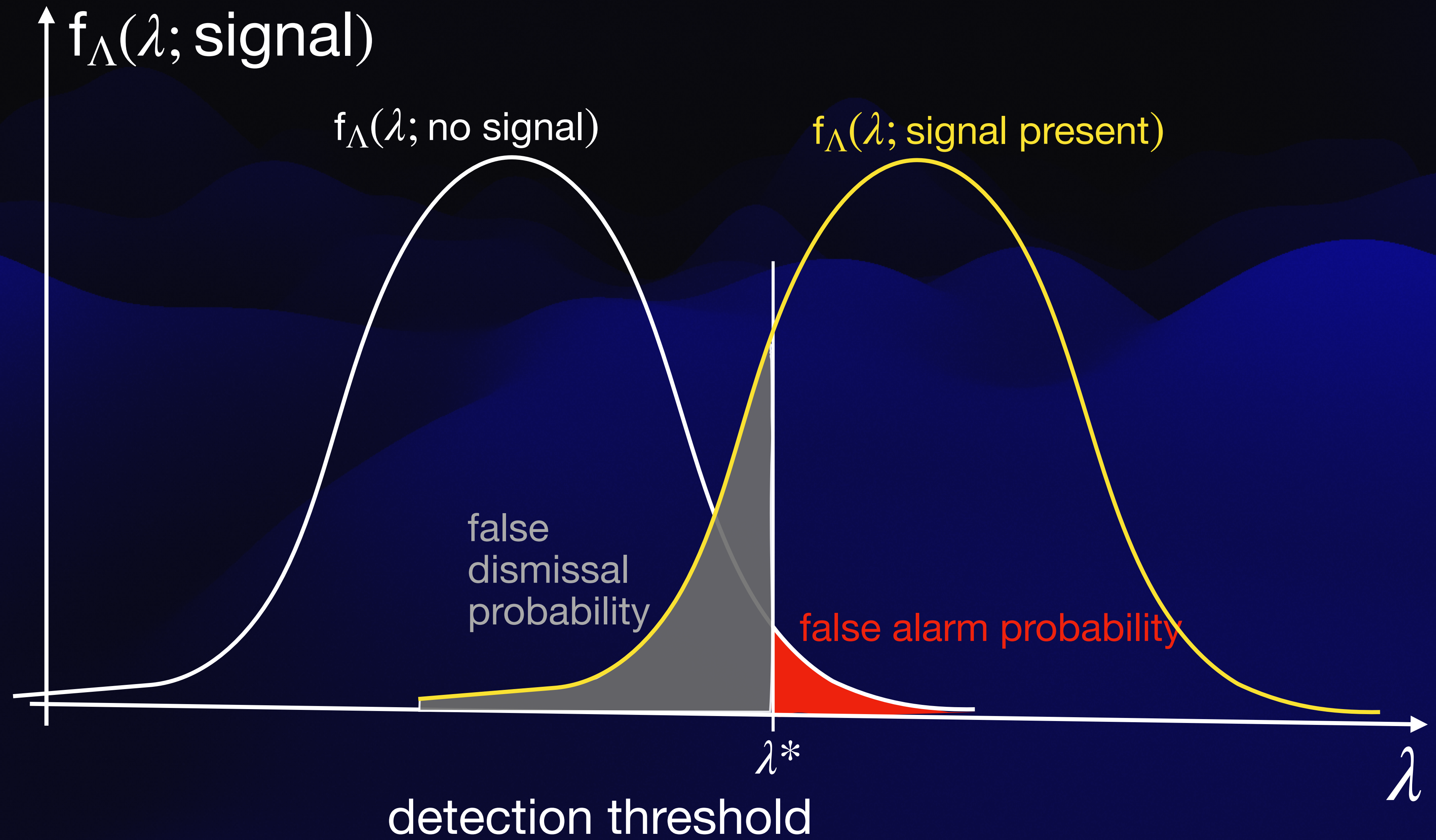
Set detection threshold λ^* \rightarrow false alarm rate



Set detection threshold λ^* \rightarrow false dismissal for any given signal



Neymann-Pearson optimal: smallest false dismissal at fixed false alarm



The log-likelihood

For stationary, zero-mean Gaussian noise

$$\log \Lambda(x; h) = (x | h) - \frac{1}{2}(h | h)$$

For observation T , data containing noise with one-sided noise spectral density $S_n(f)$ and a narrow-band signal with frequency $\approx f_0$

$$(x | h) \simeq \frac{2}{S_n(f_0)} \int_T dt x(t)h(t)$$

The log-likelihood $\log \Lambda(x; h) = (x | h) - \frac{1}{2}(h | h)$

with

$$h(t) = \sum_{i=1 \dots 4} A_i h_i(t)$$

$$\log \Lambda = A^i x_i - \frac{1}{2} A^i M_{ij} A^j$$

in Gaussian stationary noise

matched filters output $x_i(\lambda) := (x | h_i)$

antenna-pattern matrix $M_{ij}(\vec{n}_{sky}) := (h_i | h_j) = \gamma \begin{pmatrix} A & C & 0 & 0 \\ C & B & 0 & 0 \\ 0 & 0 & A & C \\ 0 & 0 & C & B \end{pmatrix}$
 $\gamma = \frac{\text{T}}{S_n}, A = \langle a^2 \rangle, B = \langle b^2 \rangle, C = \langle ab \rangle$

The maximum log-likelihood

With respect to the amplitude parameters

$$\log \Lambda(x; A_i, \lambda) = A^i x_i(\lambda) - \frac{1}{2} A^i M_{ij}(\vec{n}_{sky}) A^j$$

maximising wrt A_i

$$\mathcal{F}(x; \lambda) := \max_{\{A_i\}} [\log \Lambda(x; A_i, \lambda)]$$


re-plugging in the maximum likelihood estimators $\hat{A}^i = M^{ij} x_j$:

$$\mathcal{F} = \frac{1}{2} x_i M^{ij} x_j$$


The max likelihood ratio

$$2\mathcal{F} = 2D^{-1}[B\mathcal{F}_A + A\mathcal{F}_B - 2C\mathcal{F}_C]$$

The max likelihood ratio

$$2\mathcal{F} = 2D^{-1} \left[\mathcal{F}_A \mathcal{F}_B - 2C^2 \mathcal{F}_C \right]$$


The max likelihood ratio

$$2\mathcal{F} = 2D^{-1} \left[\mathcal{F}_A \mathcal{F}_B - 2C^2 \mathcal{F}_C \right]$$


$$\mathcal{F}_a = |F_A|^2 \sim |x_1 - ix_3|^2$$

$$\begin{cases} x_1 = (x | h_1) & h_1(t) = a(t) \cos 2\Phi(t) \\ x_3 = (x | h_3) & h_3(t) = a(t) \sin 2\Phi(t) \end{cases}$$

$$\longrightarrow F_A \approx \left| \int_T x(t) a(t) e^{-i2\Phi(t)} dt \right|^2$$

remember:

$$x_i \equiv \frac{2}{S_h} \int_T dt x(t) h_i(t)$$

The max likelihood ratio

$$2\mathcal{F} = 2D^{-1}[B\mathcal{F}_A + A\mathcal{F}_B - 2C\mathcal{F}_C]$$

with

$$\left\{ \begin{array}{l} \mathcal{F}_A = |F_a|^2 = |(x_1 - ix_3)/\gamma|^2 \\ \mathcal{F}_B = |F_b|^2 = |(x_2 - ix_4)/\gamma|^2 \\ \mathcal{F}_C = \Re(F_a^* F_b) \\ D = AB - C^2 \end{array} \right.$$

In practice

- Data (also from various detectors)
- Signal model (template):
 $\alpha, \delta, f_0, f_1, \dots$, binary parameters

lalapps_ComputeFstat



$2\mathcal{F}(data; signal)$

In practice

- Data (Short-Fourier-Transforms, SFTs)
- Signal model (template):
 $\alpha, \delta, f_0, f_1, \dots$, binary parameters

lalapps_ComputeFstat



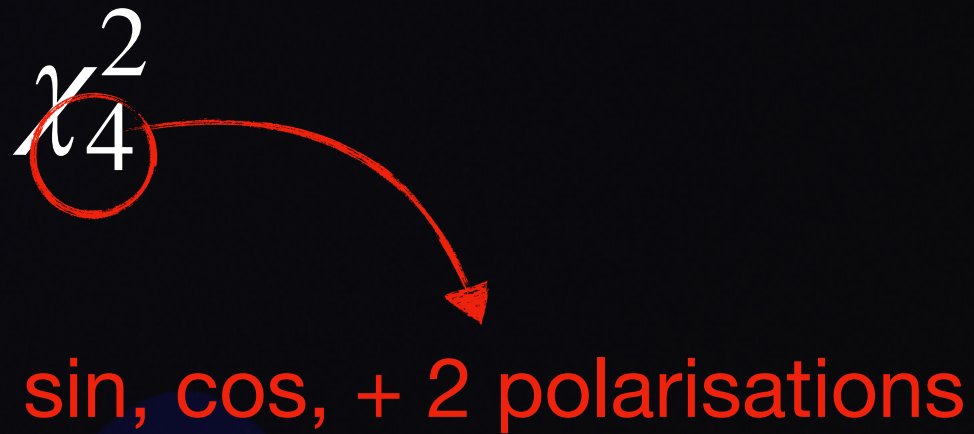
$2\mathcal{F}(data; signal)$

You get “for free” the search and maximisation over the amplitude parameters. You just have to search explicitly over phase-evolution parameters.

lalapps_ComputeFstat

- Can combine data for more detectors [Cutler&Schutz, PRD72 (2005)]
- Can do more than a template. Various ways to specify the template bank.
- Input data, SFTs: specific format that code recognises [SFT specs]. Baseline: 60s-1800s, depending on searched signal.
- Need Ephemerides data
- Assumptions of stationary Gaussian noise :
 - Relaxed to noise stationary on SFT time baseline
 - Heart is computation of matched filters $x_i = (x | h_i)$ that feature $1/S_n(f_0)$ that is estimated per-SFT from the data itself in a way that is insensitive to signals.

Distribution of $2\mathcal{F}$

- Chi-square distribution with 4 degrees of freedom, χ_4^2


sin, cos, + 2 polarisations
- When a signal at the searched parameters is present, a non-centrality parameter arises: $\rho_{opt}^2 = (h^{sig} | h^{sig})$, that JKS call the optimal SNR²

depends on amplitude parameters, on relative source-detector position, on detector noise, on amount of data

$$\bullet E[2\mathcal{F}] = 4 + \rho^2$$

$$VAR[2\mathcal{F}] = 8 + 4\rho^2$$

The optimal ρ^2

relation with h_0 and T

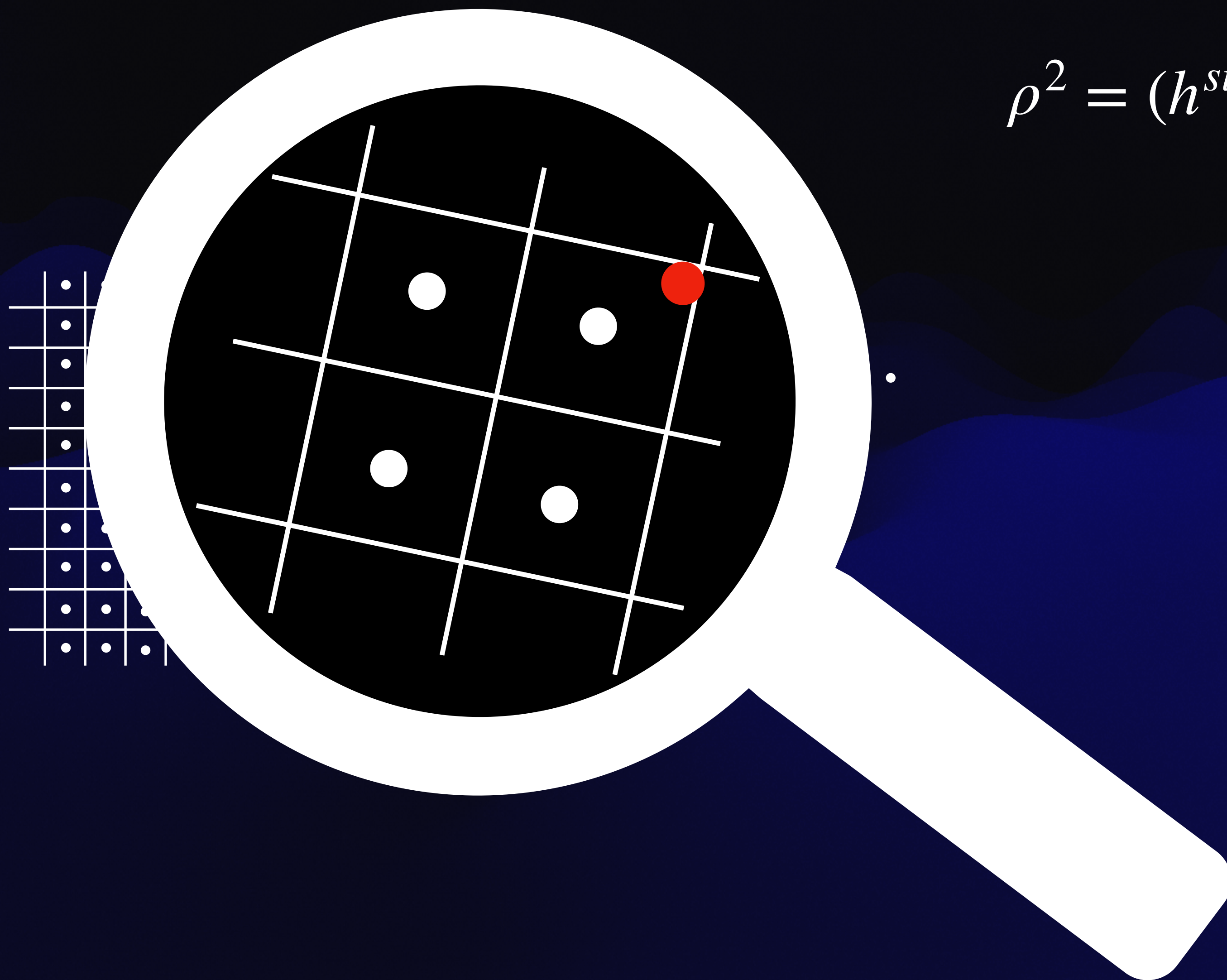
$$\rho_{opt}^2 = \frac{h_0^2}{S_n} \left[\frac{1}{4} (1 + \cos^2 \iota)^2 \int_T F_+^2 dt + \cos^2 \iota \int_T F_\times^2 dt \right]$$
$$\simeq \frac{h_0^2}{S_n} [G_1(\delta, \psi, \iota) T + G_2(\alpha, \delta, \psi, \iota; T)]$$

$$\langle \rho_{opt}^2 \rangle_{\alpha, \delta, \psi, \iota} \simeq \frac{4}{25} \frac{h_0^2 T}{S_n}$$

With 1% false alarm the $h_0^{detectable}$ with 10% false dismissal is $\approx 11.4 \sqrt{\frac{S_n}{T}}$ in a 1-template search.

Mismatch

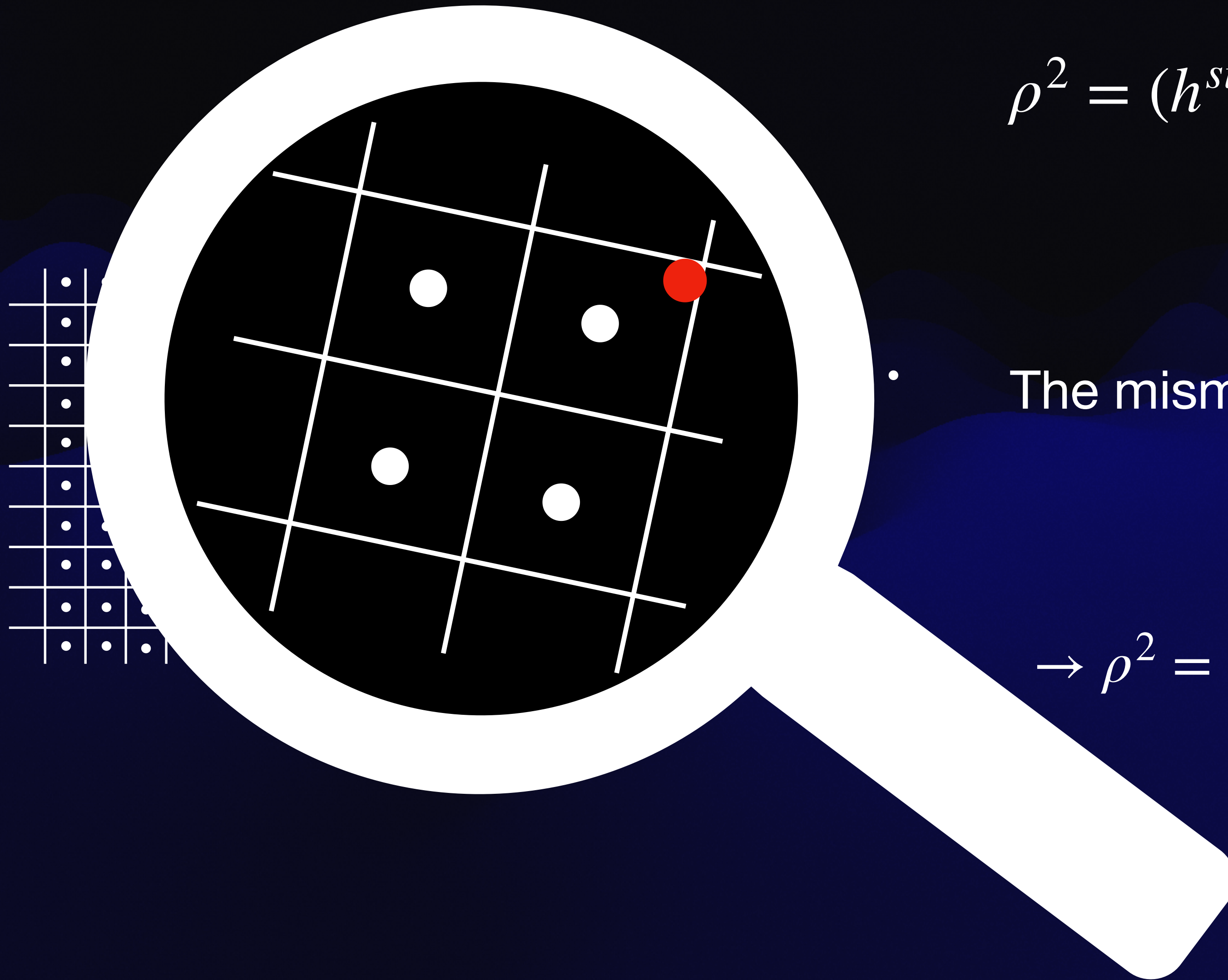
signal



$$\rho^2 = (h^{sig} | h^{templ}) \leq \rho_{opt}^2 \equiv (h^{sig} | h^{sig})$$

Mismatch

signal



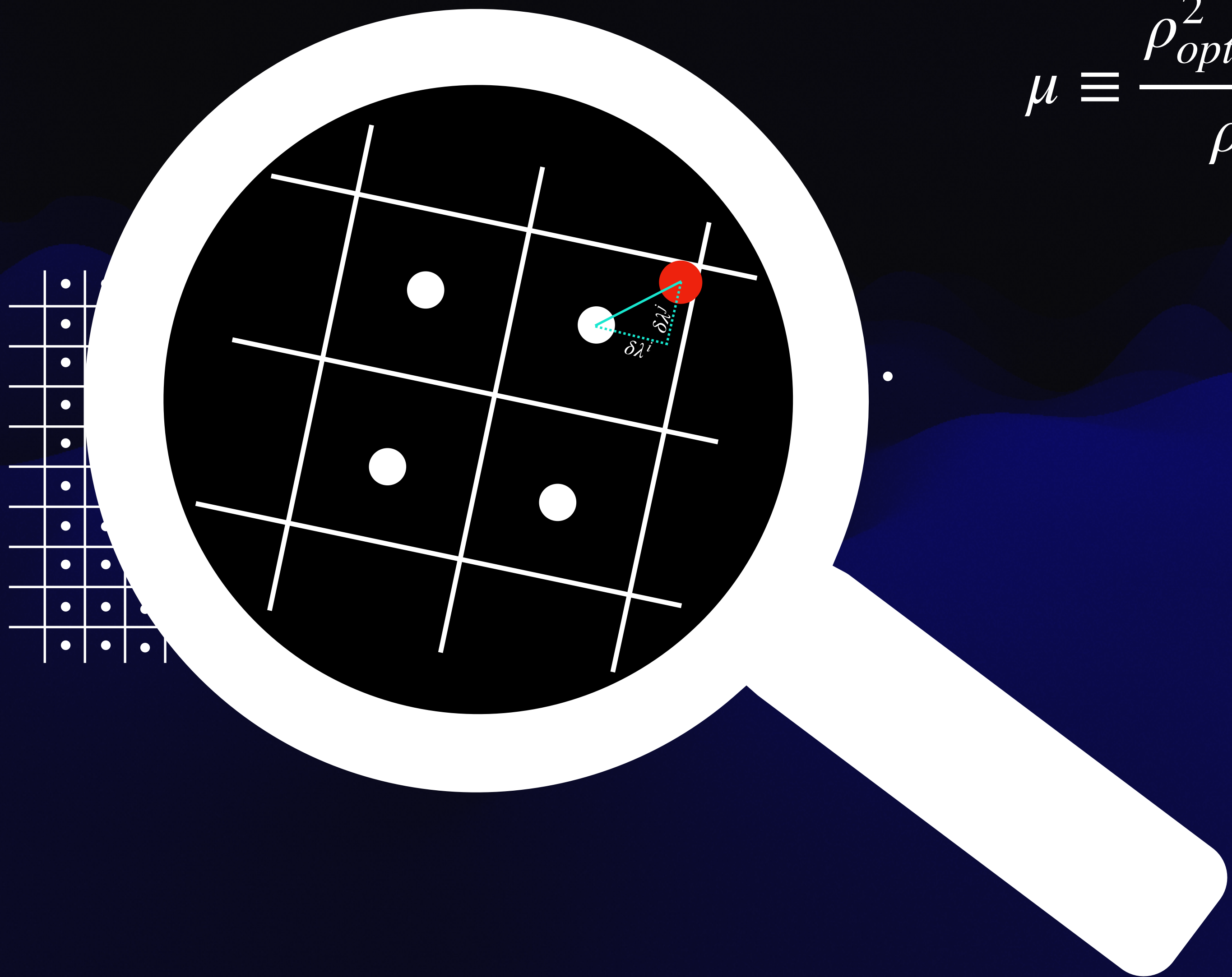
$$\rho^2 = (h^{sig} | h^{templ}) \leq \rho_{opt}^2 \equiv (h^{sig} | h^{sig}) .$$

The mismatch $\mu \equiv \frac{\rho_{opt}^2 - \rho^2}{\rho_{opt}^2}$

$$\rightarrow \rho^2 = (1 - \mu) \rho_{opt}^2$$

Mismatch

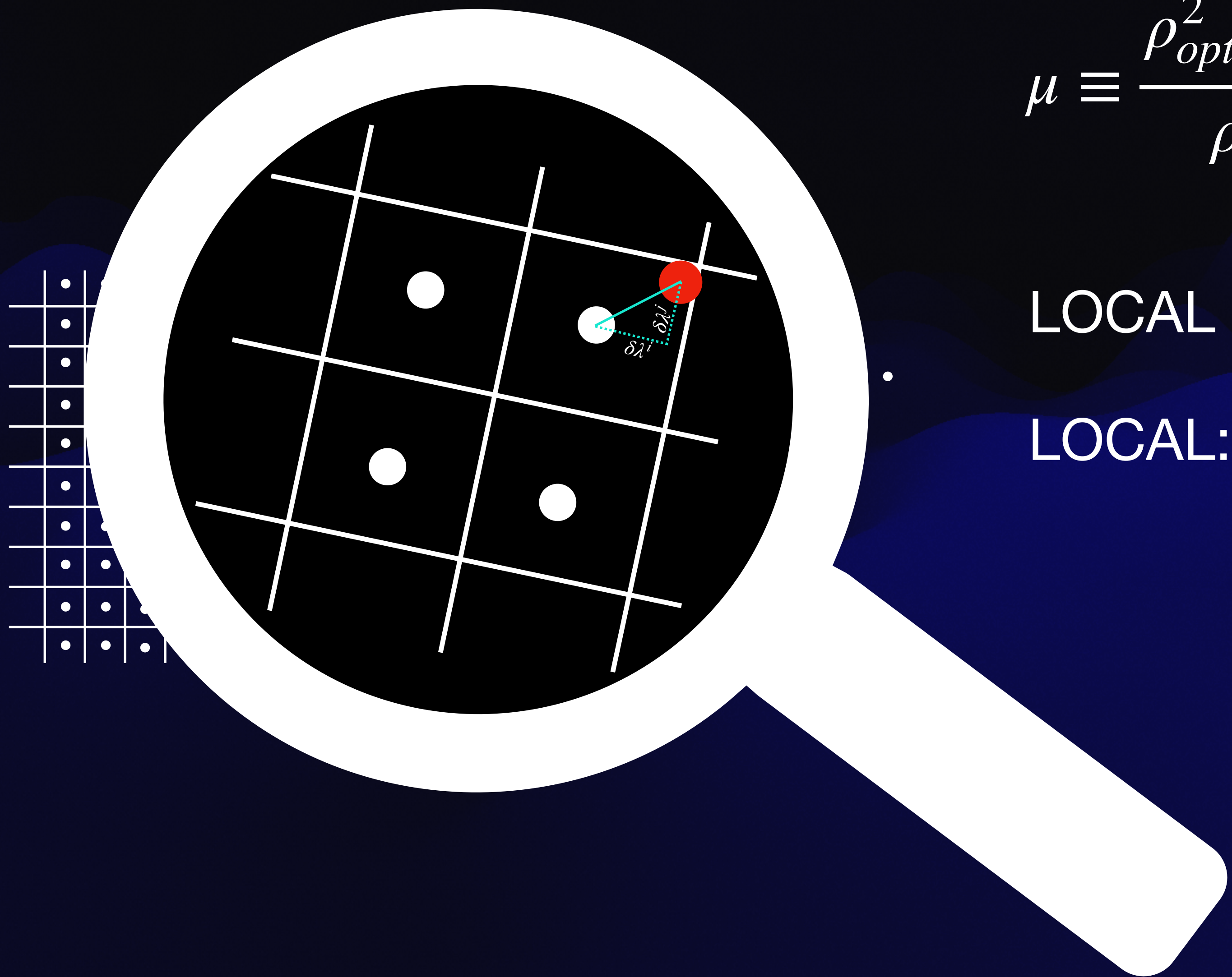
signal



$$\mu \equiv \frac{\rho_{opt}^2 - \rho^2}{\rho_{opt}^2} = g_{ij} \delta\lambda^i \delta\lambda^j + \mathcal{O}(\delta\lambda^3)$$

Mismatch

signal



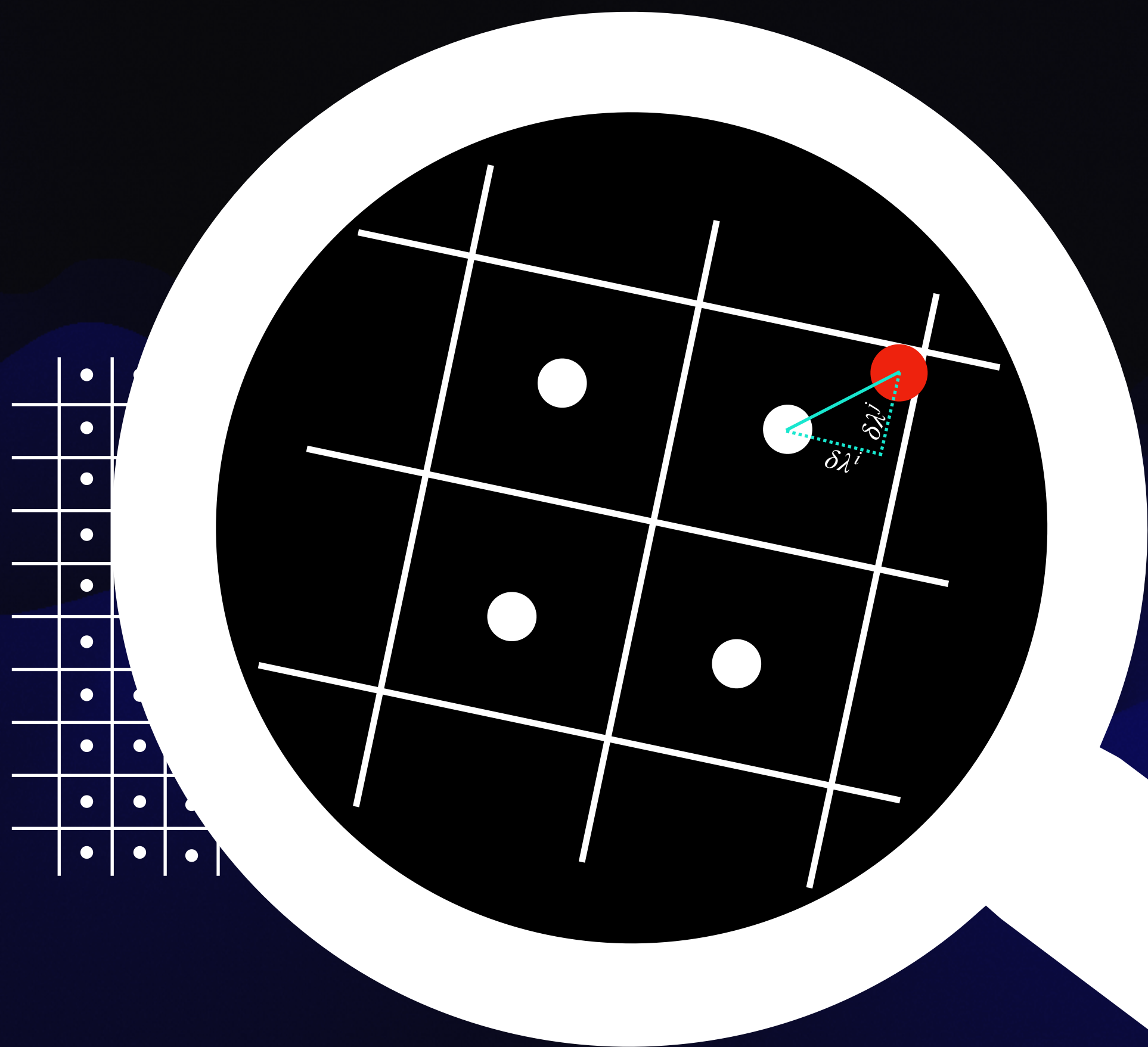
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LOCAL DESCRIPTION OF SNR² LOSS

LOCAL: $\mu \lesssim 5\%$

Mismatch

signal



$$\mu \equiv \frac{\rho_{opt}^2 - \rho^2}{\rho_{opt}^2} = g_{ij} \delta\lambda^i \delta\lambda^j + \mathcal{O}(\delta\lambda^3)$$

LOCAL DESCRIPTION OF SNR² LOSS

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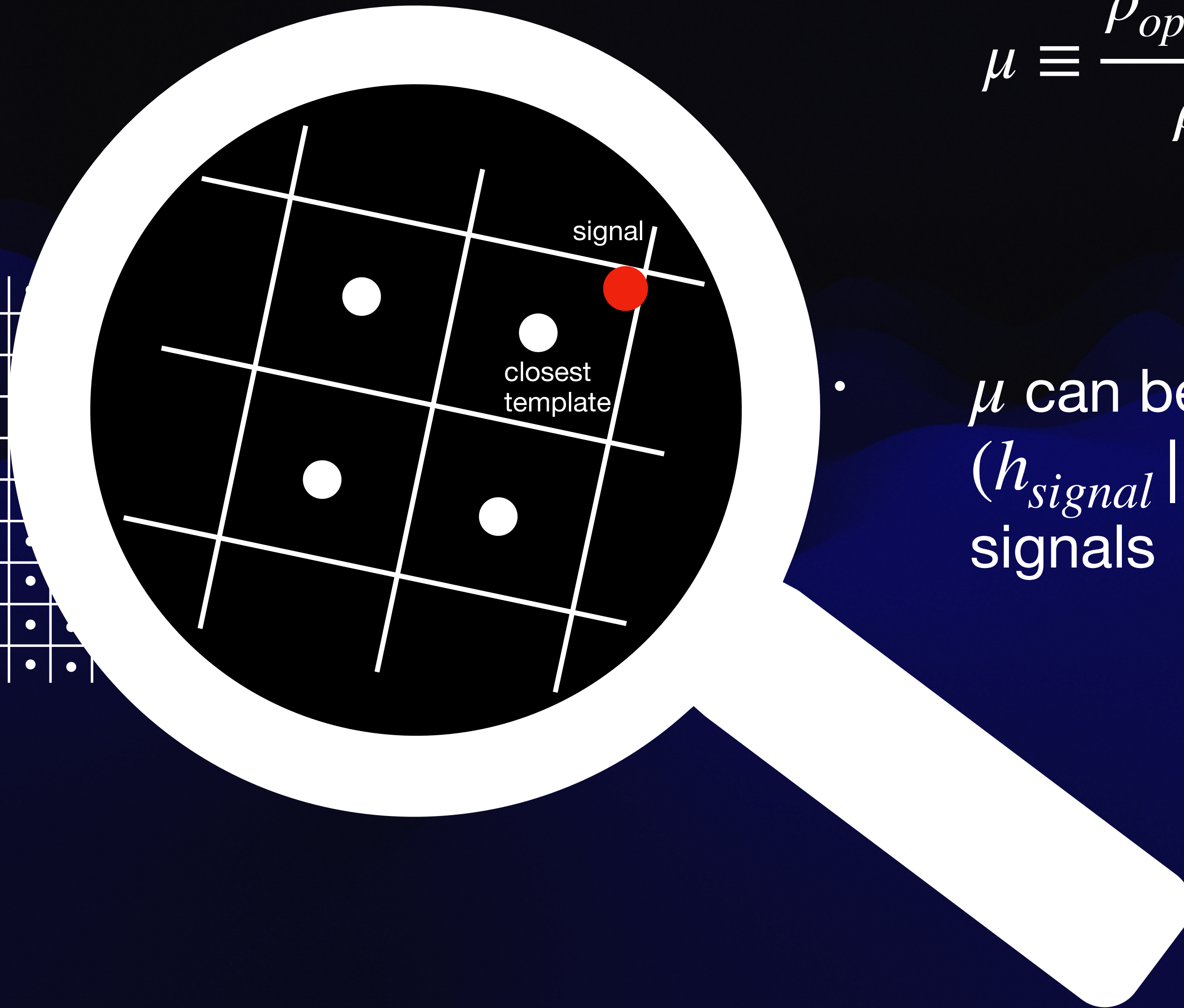
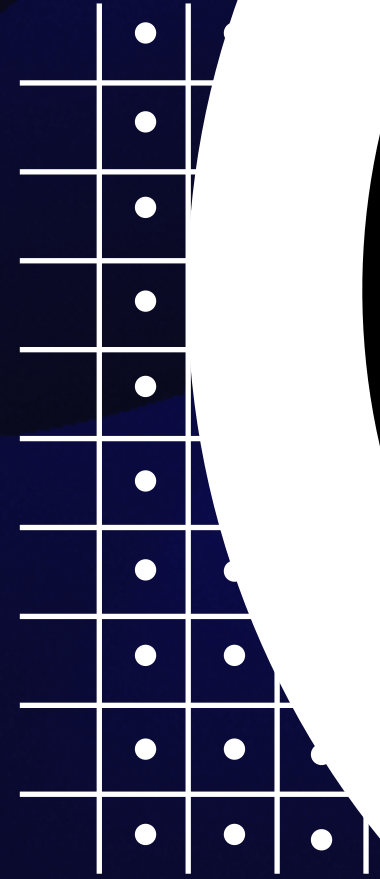
GENERAL CLOSED-FORM
EXPRESSION FOR g_{ij} DOES NOT
EXIST

APPROXIMATION: PHASE METRIC

Mismatch distribution

$$\mu \equiv \frac{\rho_{opt}^2 - \rho^2}{\rho_{opt}^2}$$

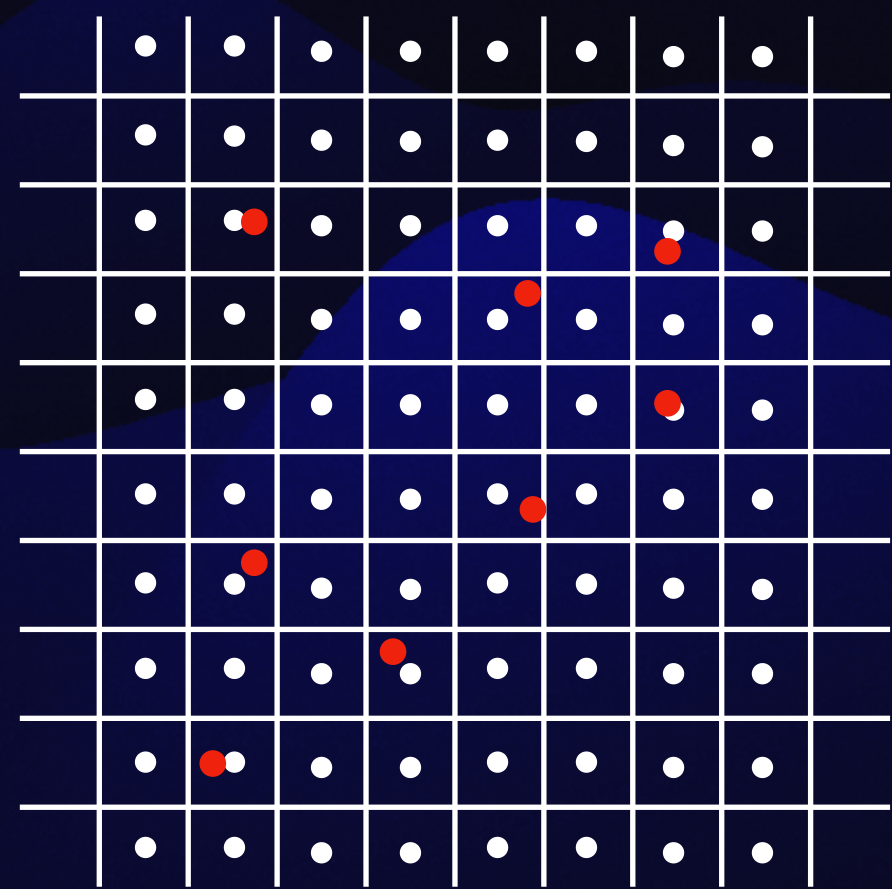
signal



• μ can be measured by computing $(h_{signal} | h_{closest-template})$ for different signals

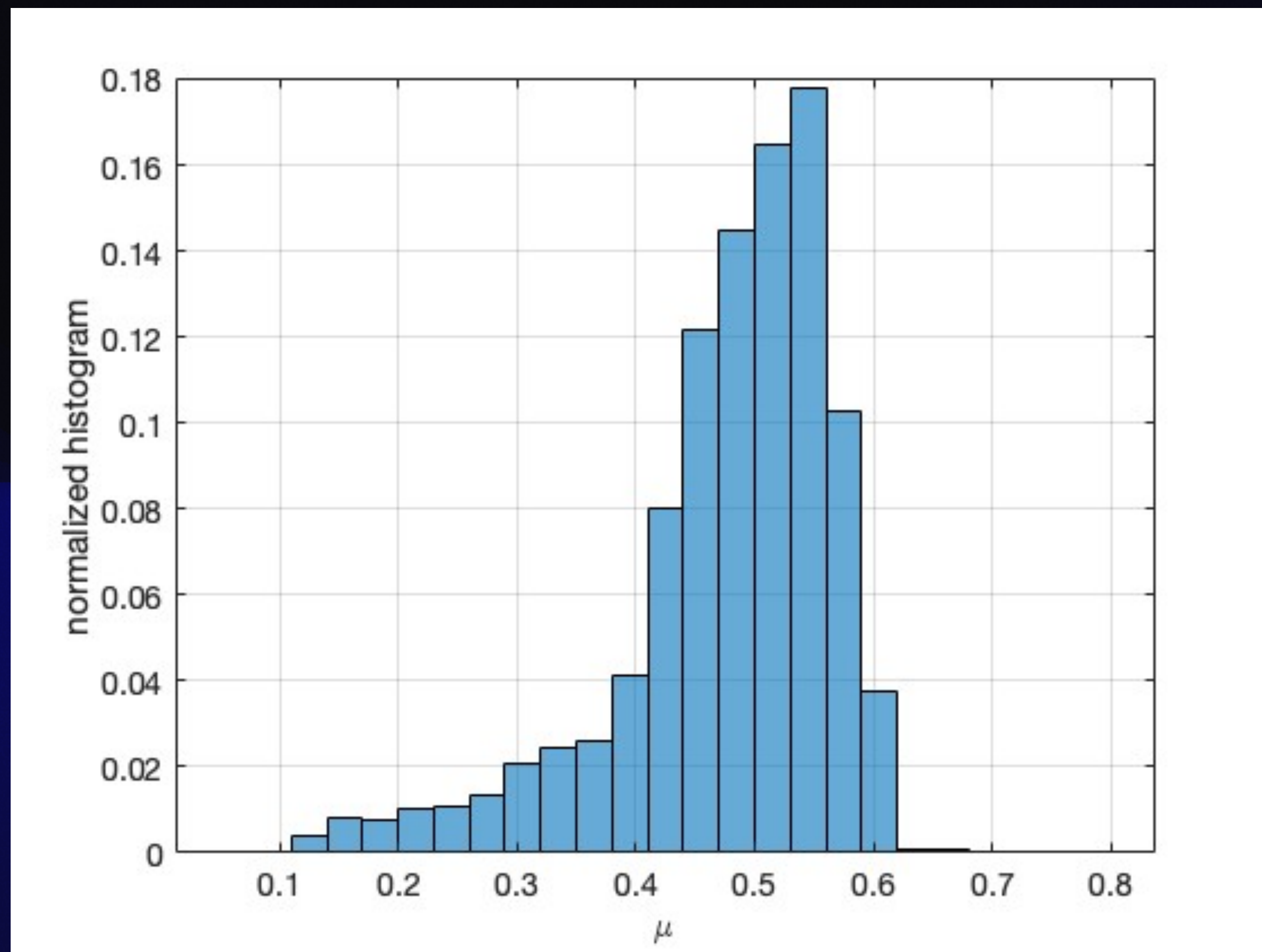
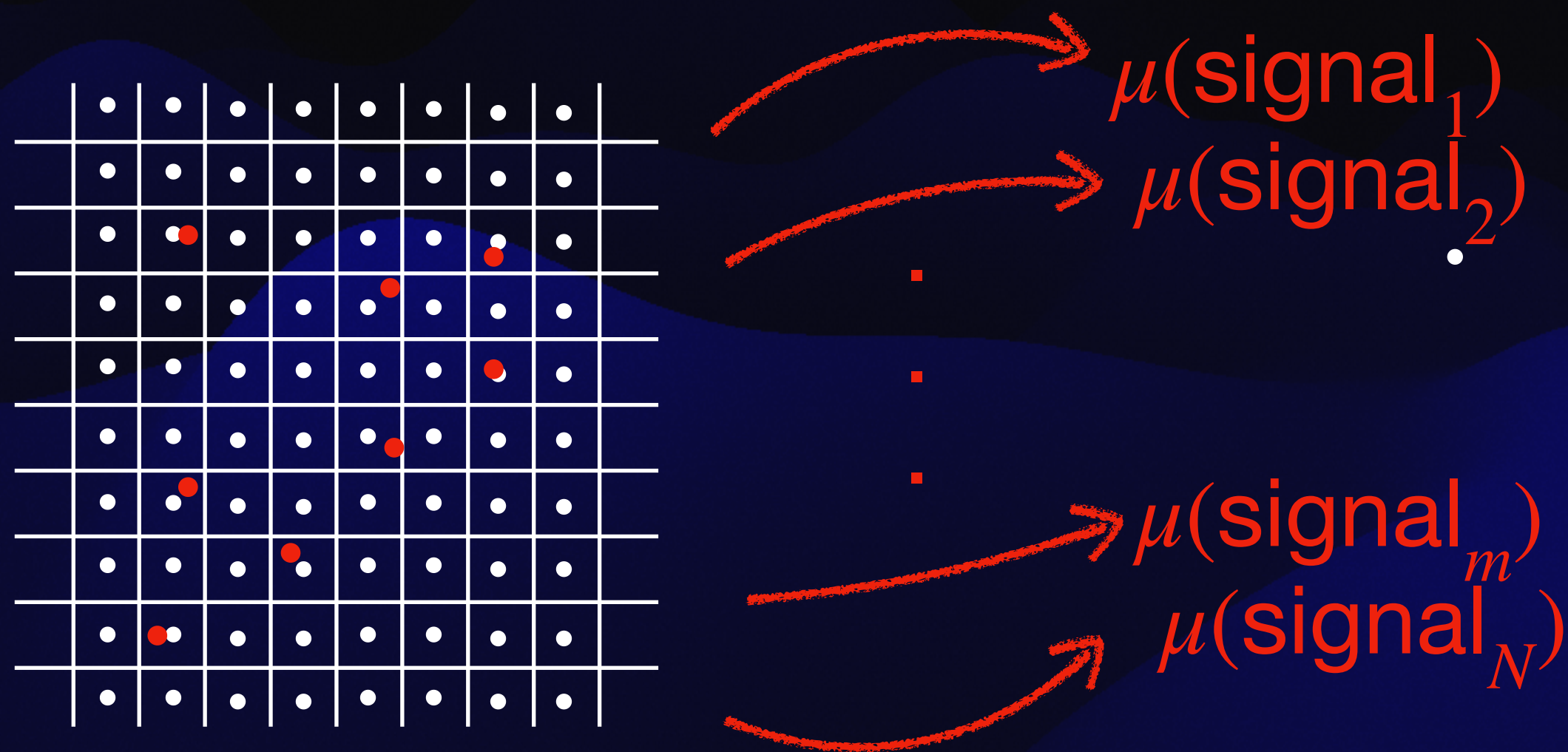
Mismatch distribution

signal



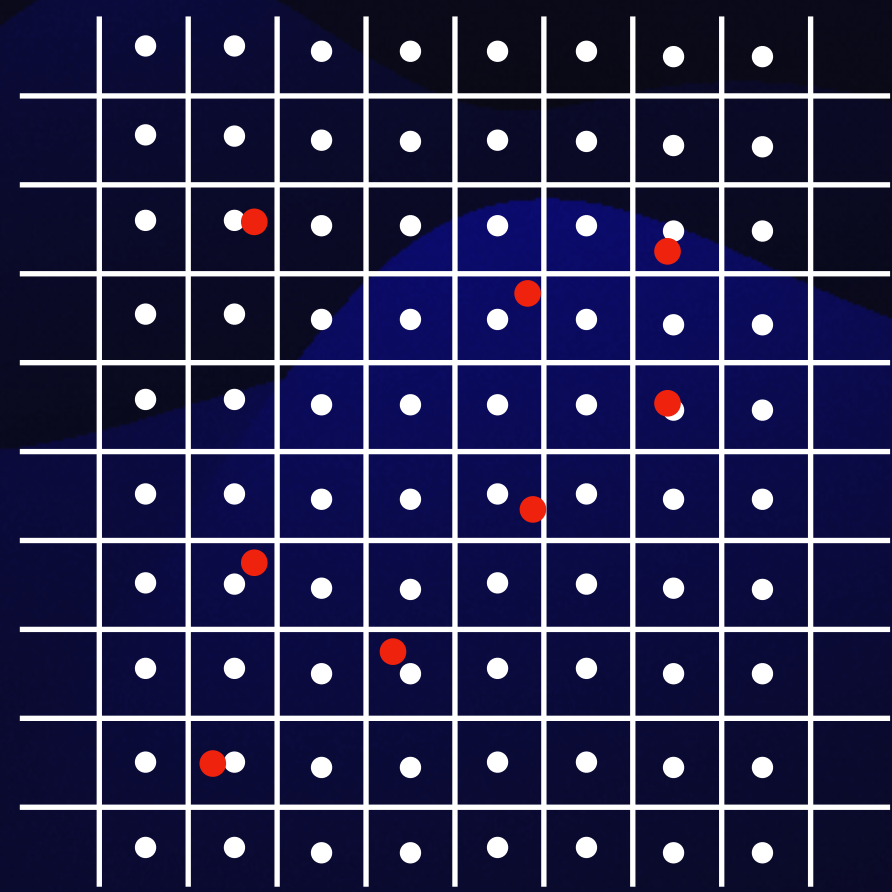
Mismatch distribution all-sky search

signal



Mismatch distribution targeted search

signal

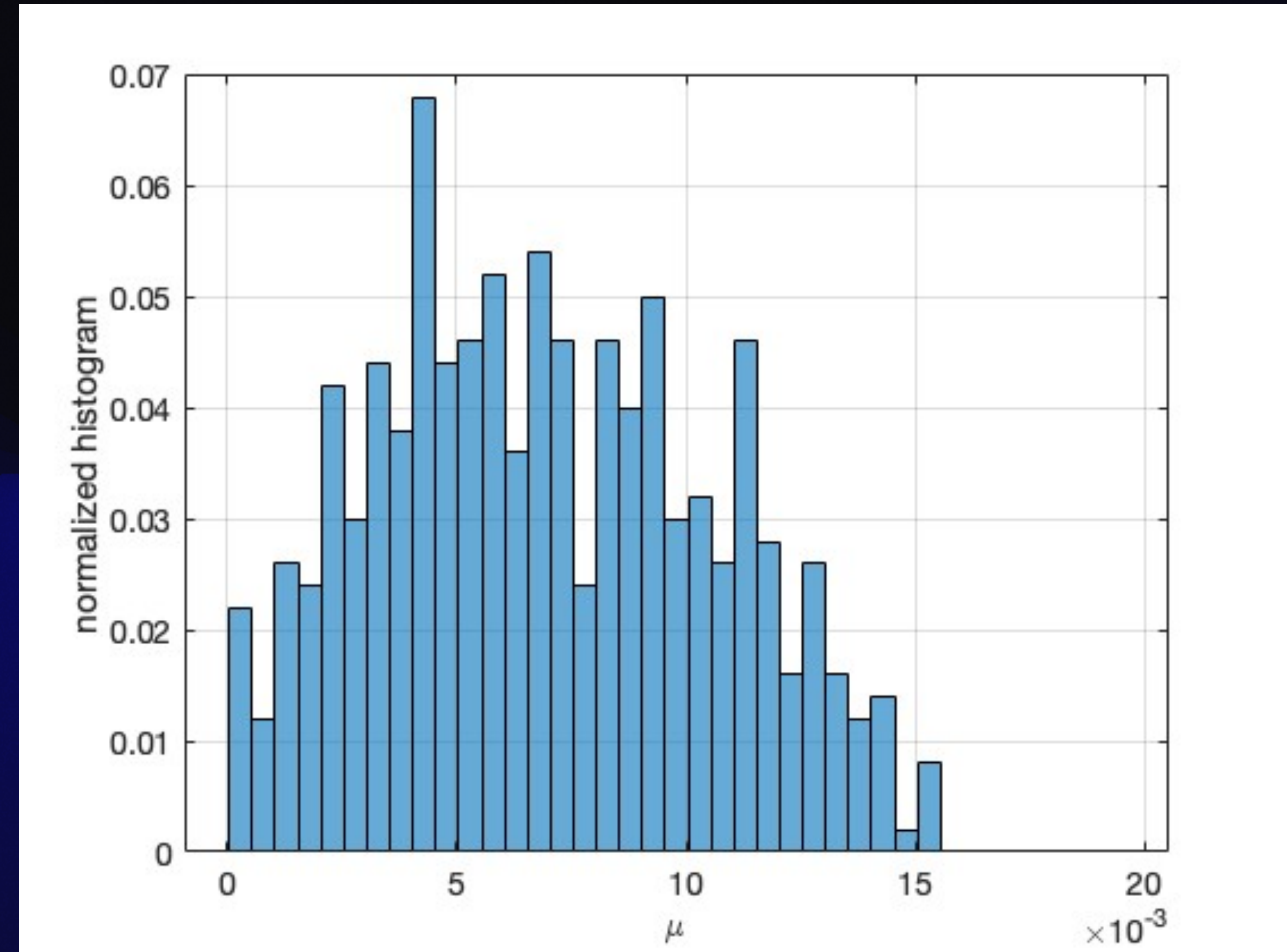


$\mu(\text{signal}_1)$

$\mu(\text{signal}_2)$

$\mu(\text{signal}_m)$

$\mu(\text{signal}_N)$




The grid is representative of the uncertainties in template parameters from EM observations.

**The detector data is not always
Gaussian noise**

The data contains loud signals

... just not GW ones

- $2\mathcal{F} = \max_{\{\mathcal{A}_i\}} \frac{\text{Prob}(x; H_S)}{\text{Prob}(x; H_G)} \longrightarrow \frac{\text{Prob}(H_S; x)}{\text{Prob}(H_G; x)}$
- When combining data from more than one detector, a disturbance in one detector can generate high values of the overall $2\mathcal{F}$
 - \mathcal{F} -stat consistency veto [1]: if $\mathcal{F}^{\text{single}} \geq \mathcal{F}^{\text{comb.ed}}$ \longrightarrow discard result.
 - Quite successful veto.
 -  Gave idea on how to construct a more robust statistic

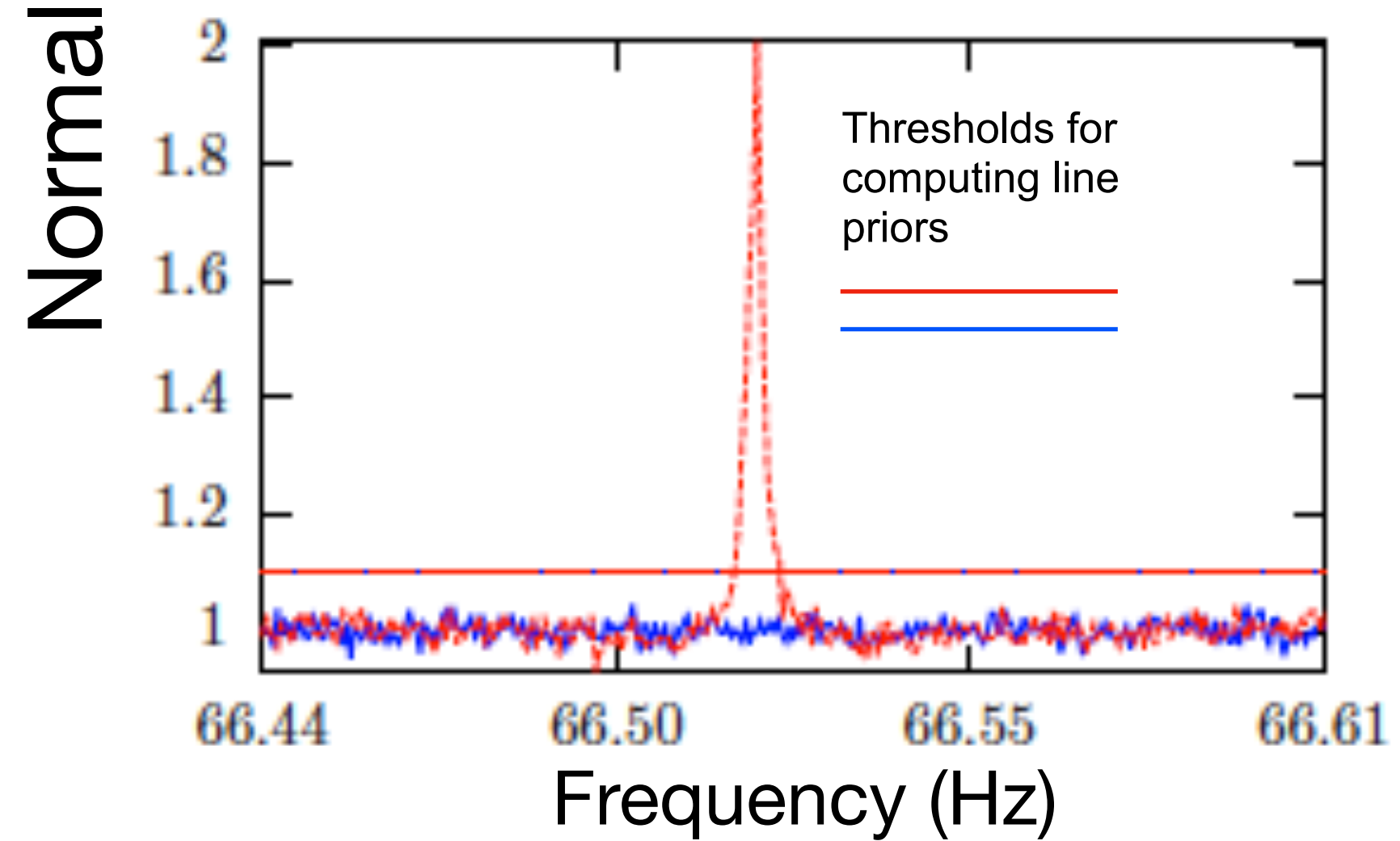
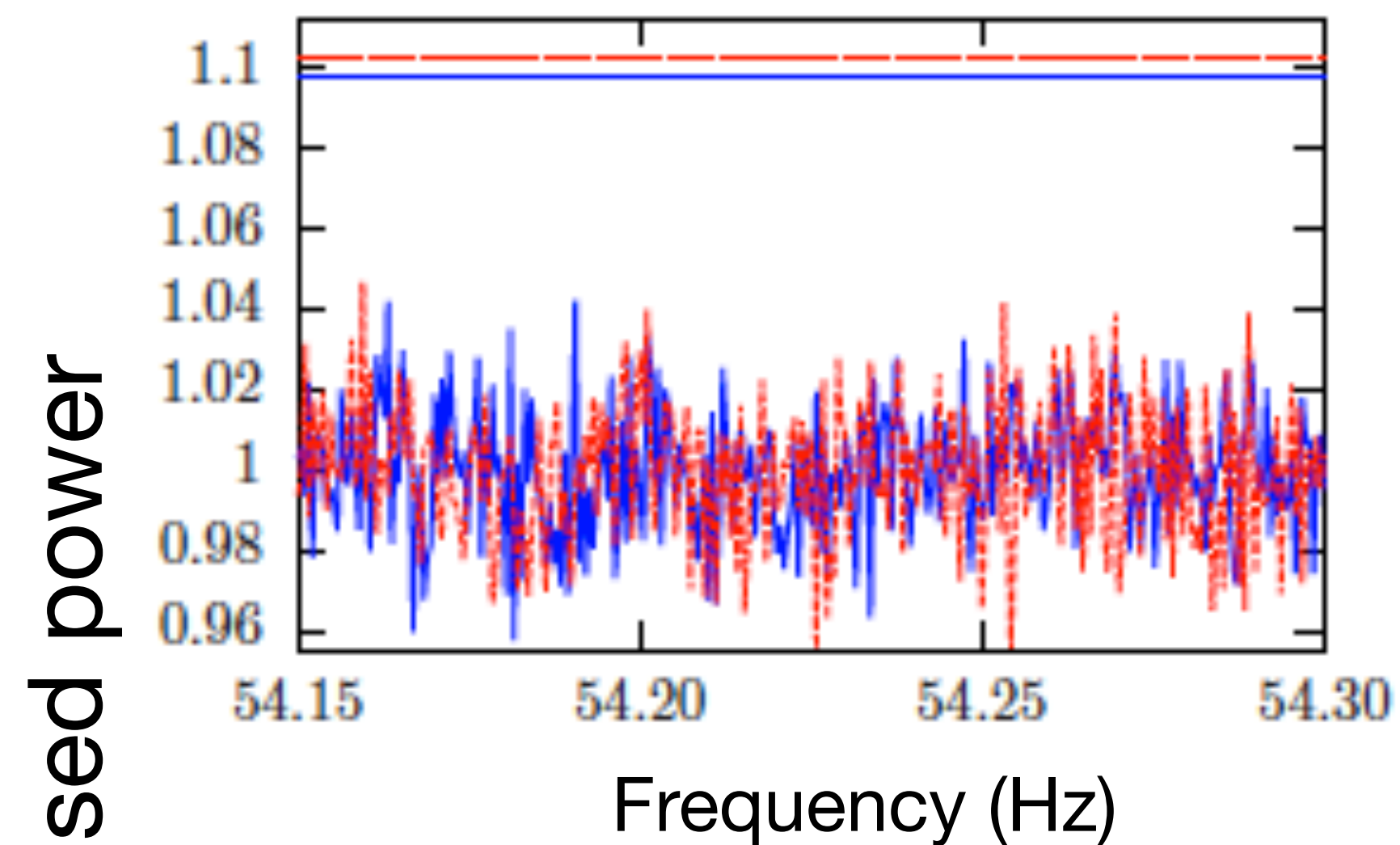
Noise contains loud signals

... just not GW ones

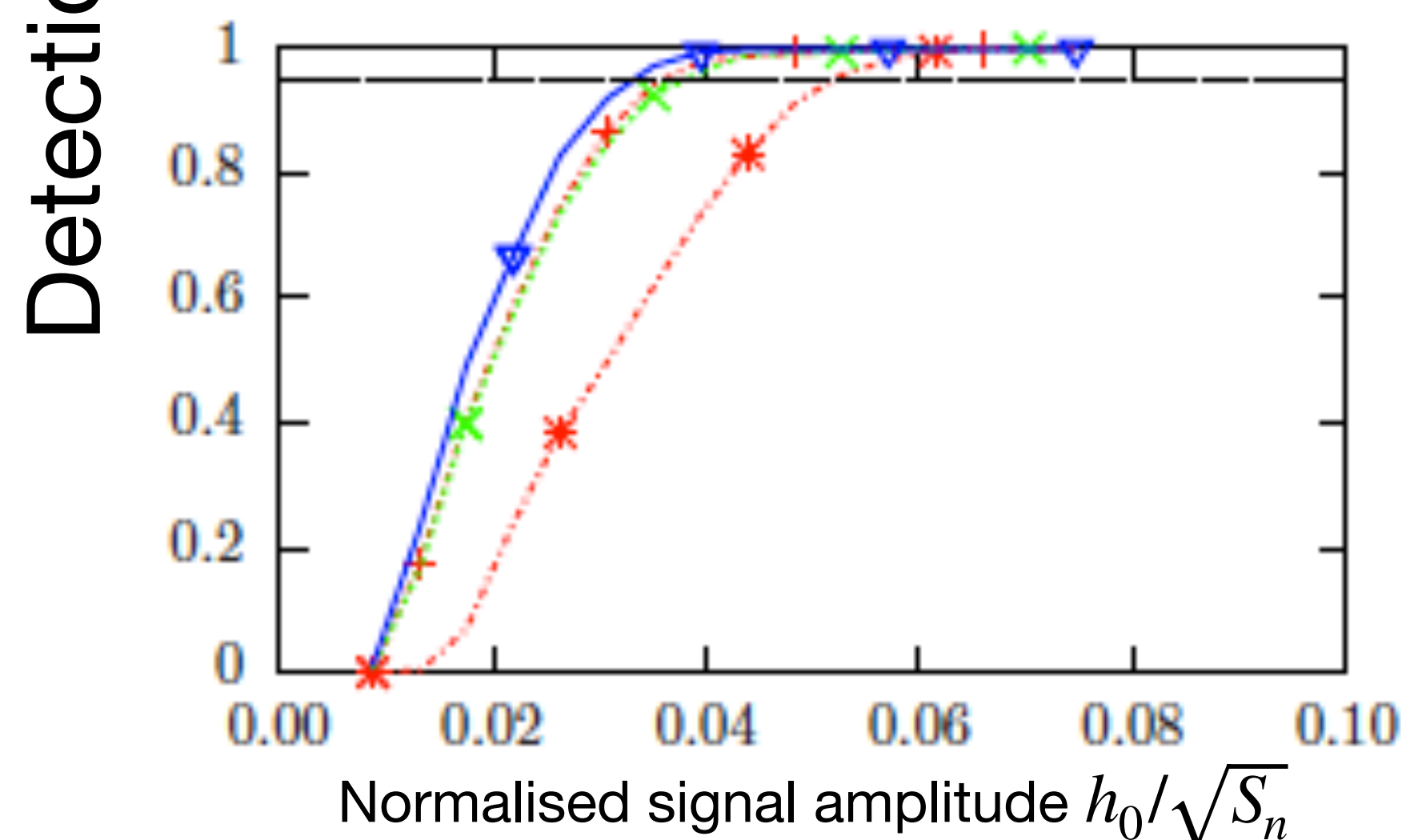
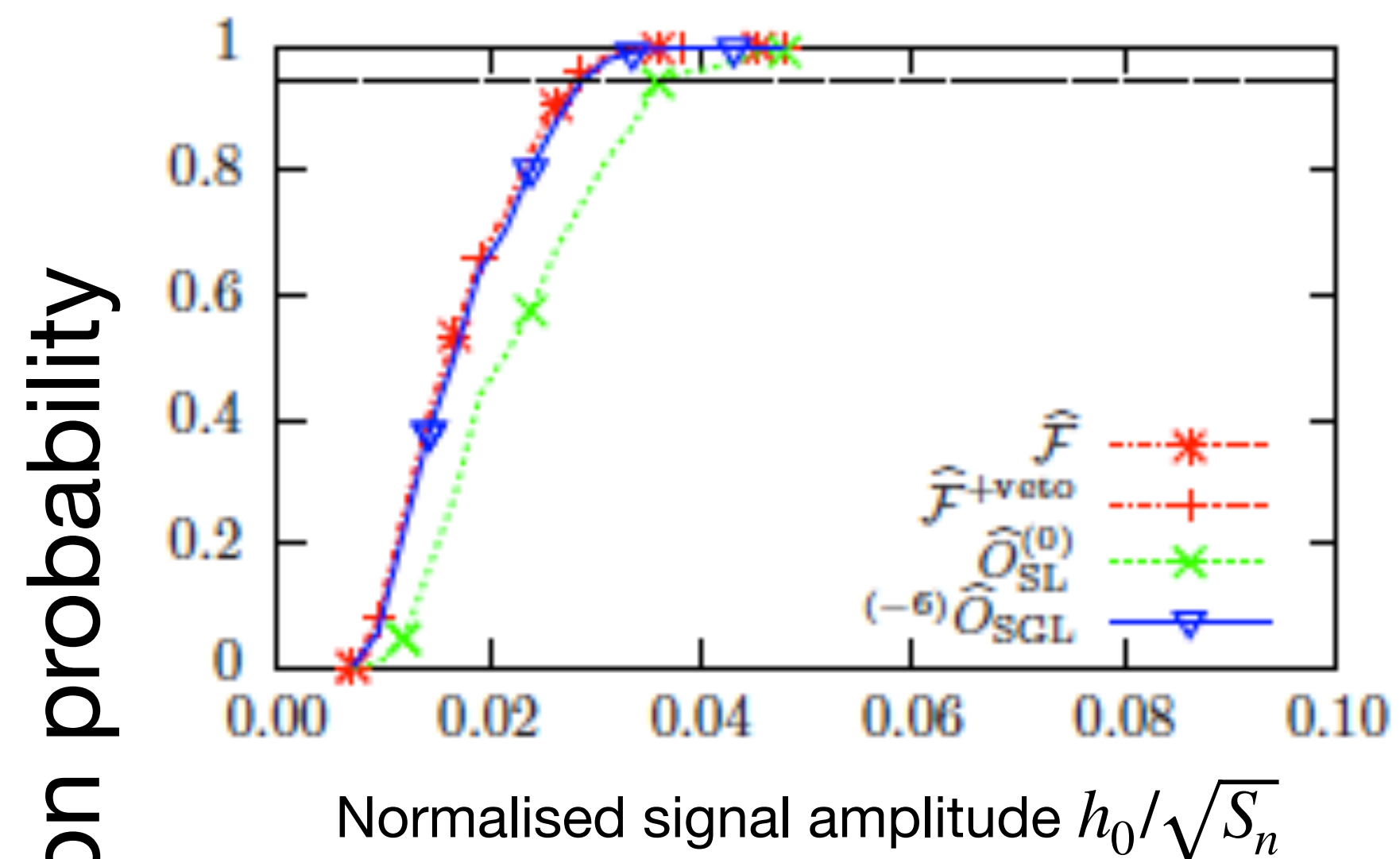
- $2\mathcal{F} \longrightarrow \frac{\text{Prob}(H_s; x)}{\text{Prob}(H_G; x)}$

- Robust statistic: $\frac{\text{Prob}(H_s; x)}{\text{Prob}(H_{GL}; x)}$

Extended noise hypothesis: Gaussian noise or noise-line, with line defined as a high-value of the single-detector \mathcal{F} , only in one of the detectors.



Noise (real detector data): L1 in red, H1 in blue



Detection probability for injected signals of different amplitudes in that noise.

The future king's sibling



The royal family decides to have two kids, and gets twins.

We know for sure that one of them is a boy, and will be future king.

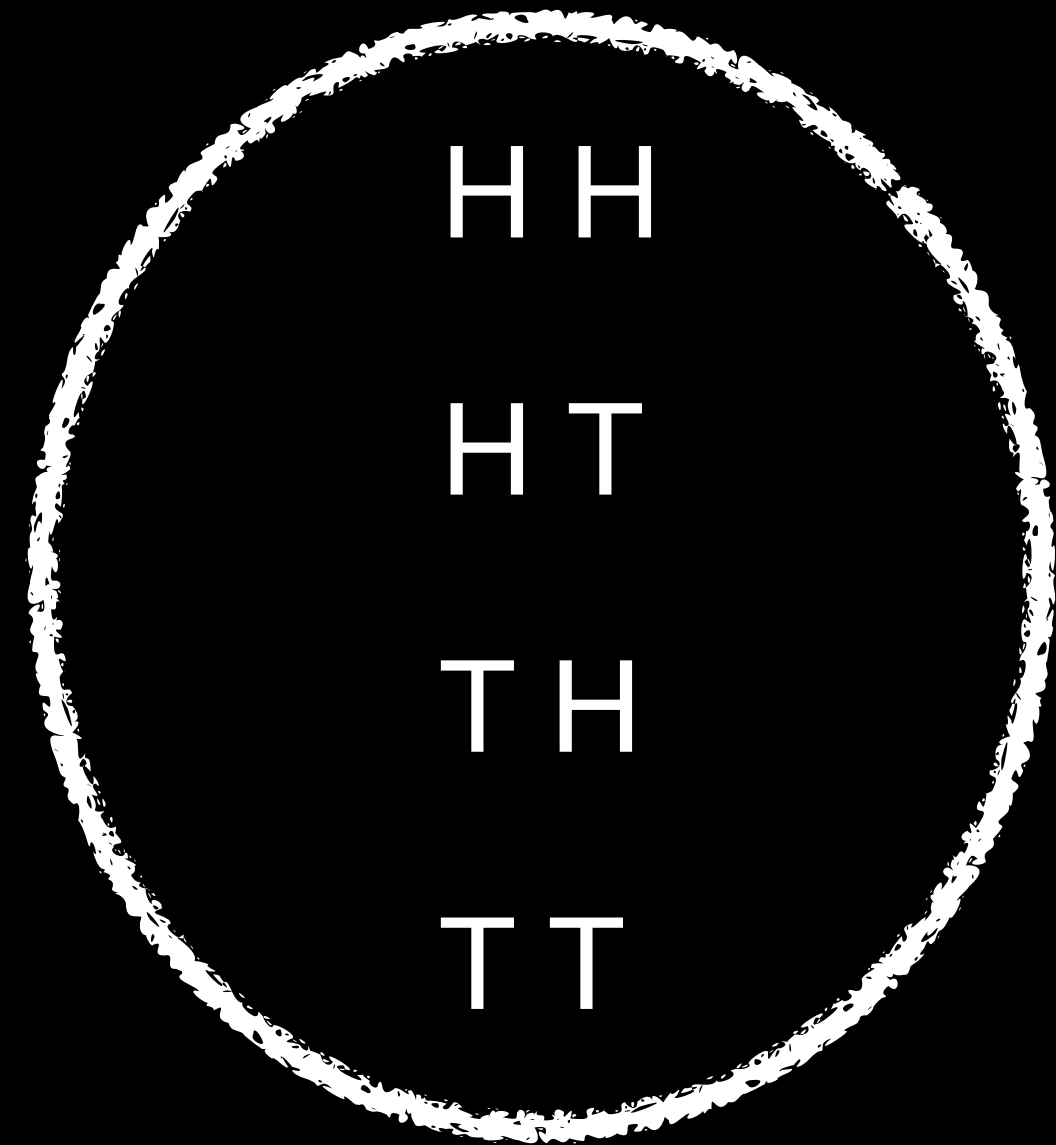
What are the chances that the future king has a sister ?

(Let's assume that baby-boys and baby-girls are equally likely.)

The Bayesian approach

SOMEONE TOSSES A COIN TWICE

Sample space has 4 events:



coins are fair

Probability

$1/4$

$1/4$

$1/4$

$1/4$



NEW INFORMATION: 1ST TOSS WAS TAILS

NEW UNIVERSE:
CONDITIONED BY THE
INFORMATION THAT WE
HAVE GOTTEN



CONDITIONED UNIVERSE:

probability

1/4

HH

1/4

HT

1/4

TH

1/4

TT



TH

1/2

TT

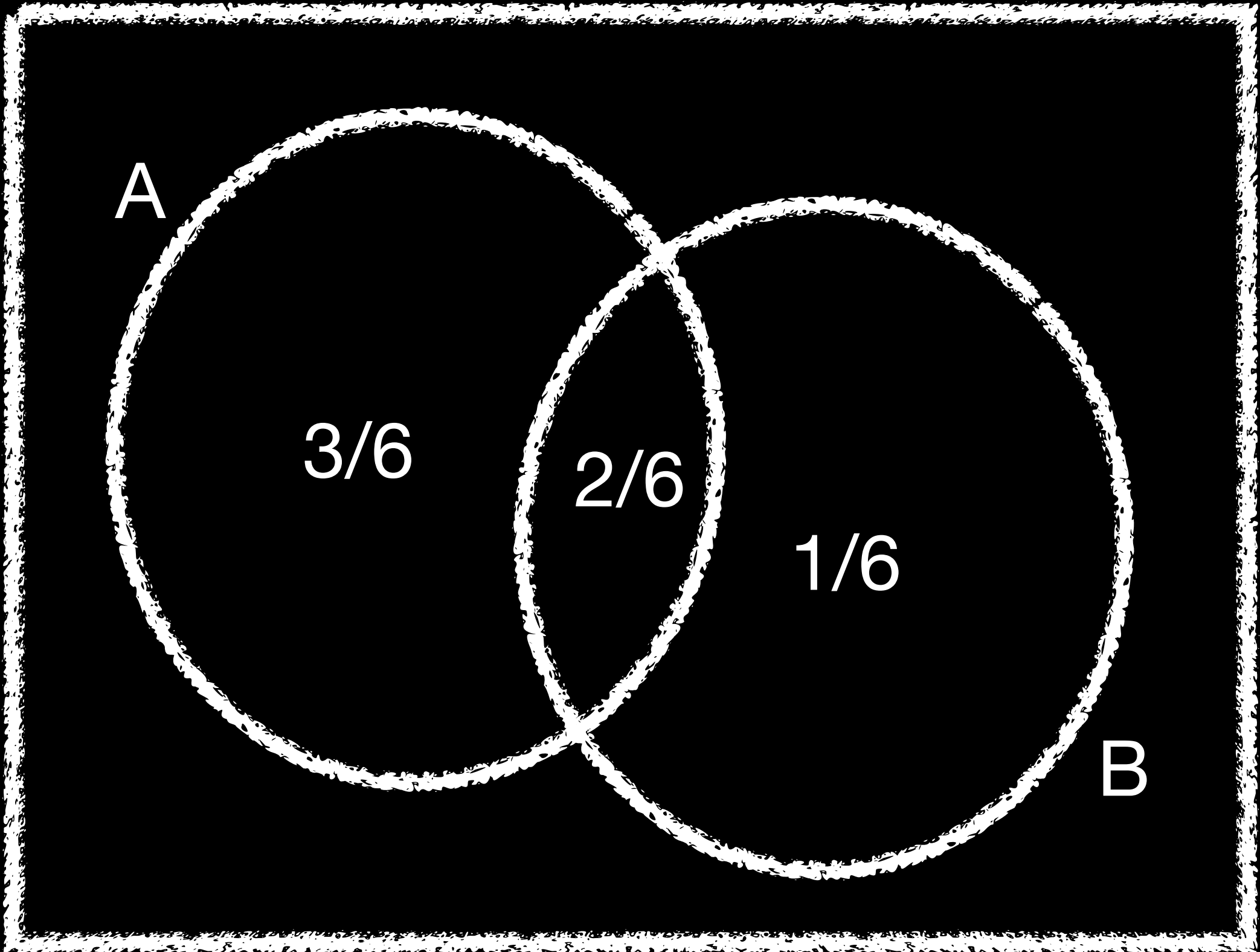
1/2

conditional
probabilities

$P(\text{TH} \mid \text{first toss}=\text{T})$

$P(\text{TT} \mid \text{first toss}=\text{T})$

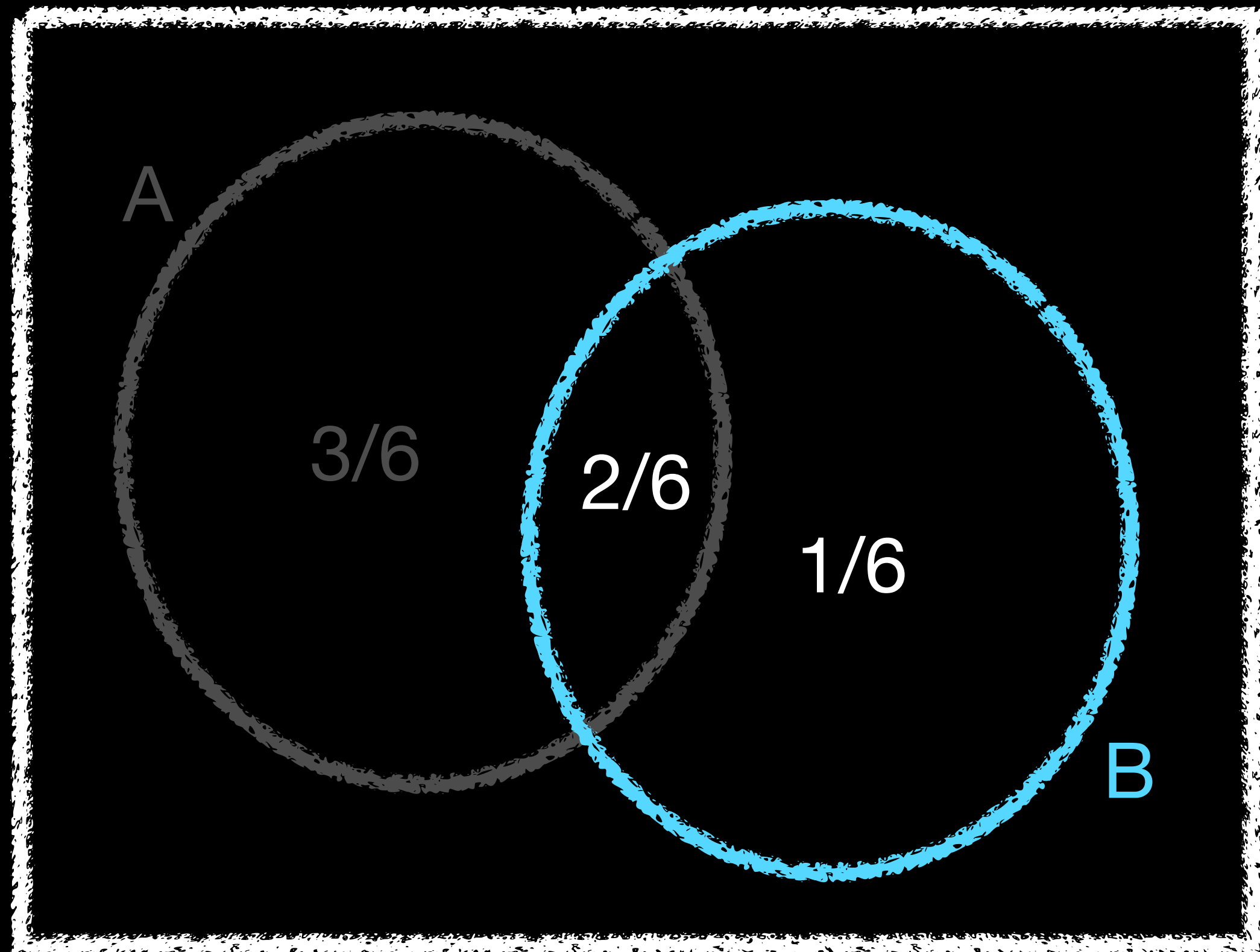
ANOTHER EXAMPLE



$$\begin{cases} P(A) & = 5/6 \\ P(B) & = 3/6 \\ P(A \cap B) & = 2/6 \end{cases}$$

NEW INFORMATION: B HAS HAPPENED

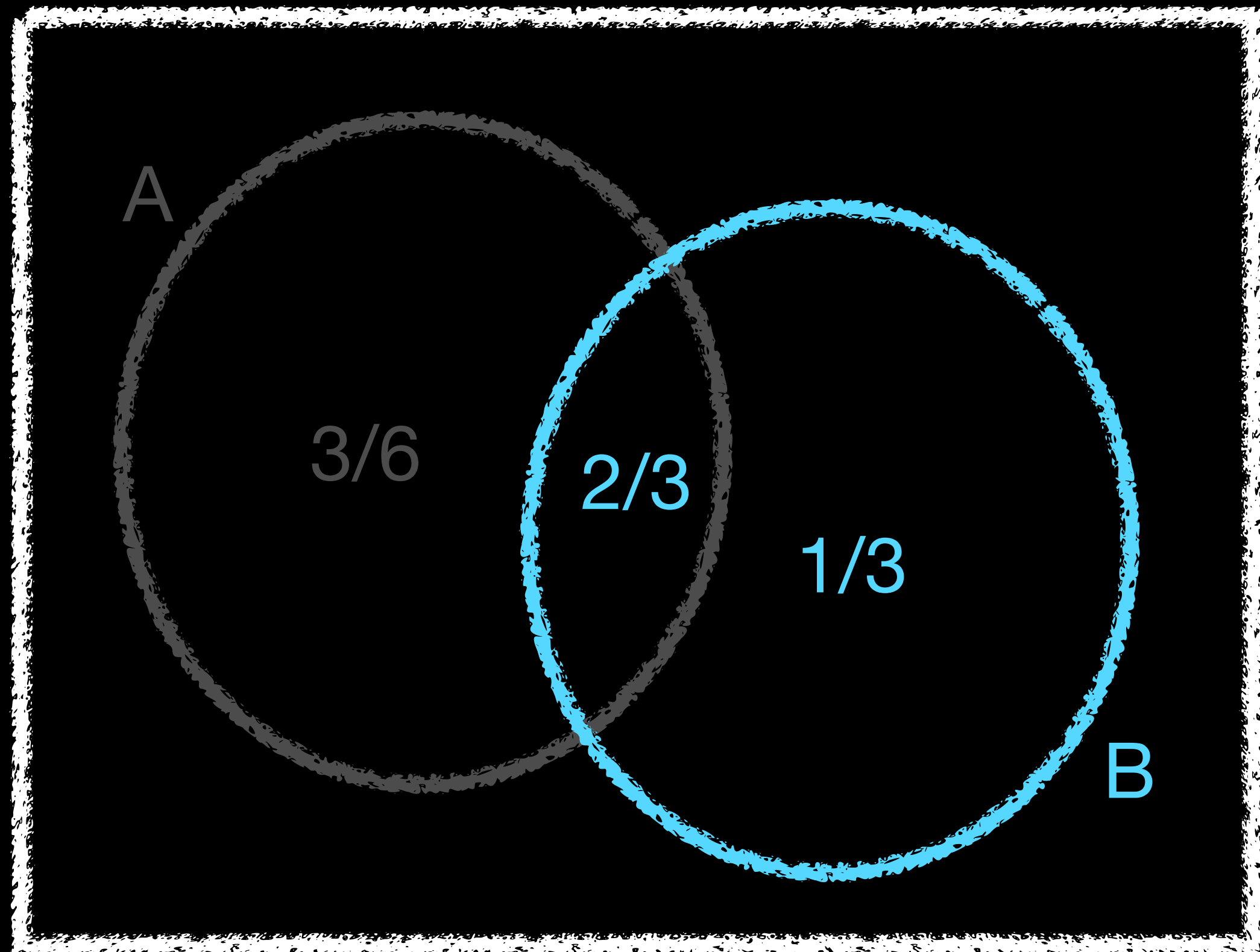
probabilities need to be revised -> conditional probabilities



- maintain same proportions
- has to add up to 1

NEW INFORMATION: B HAS HAPPENED

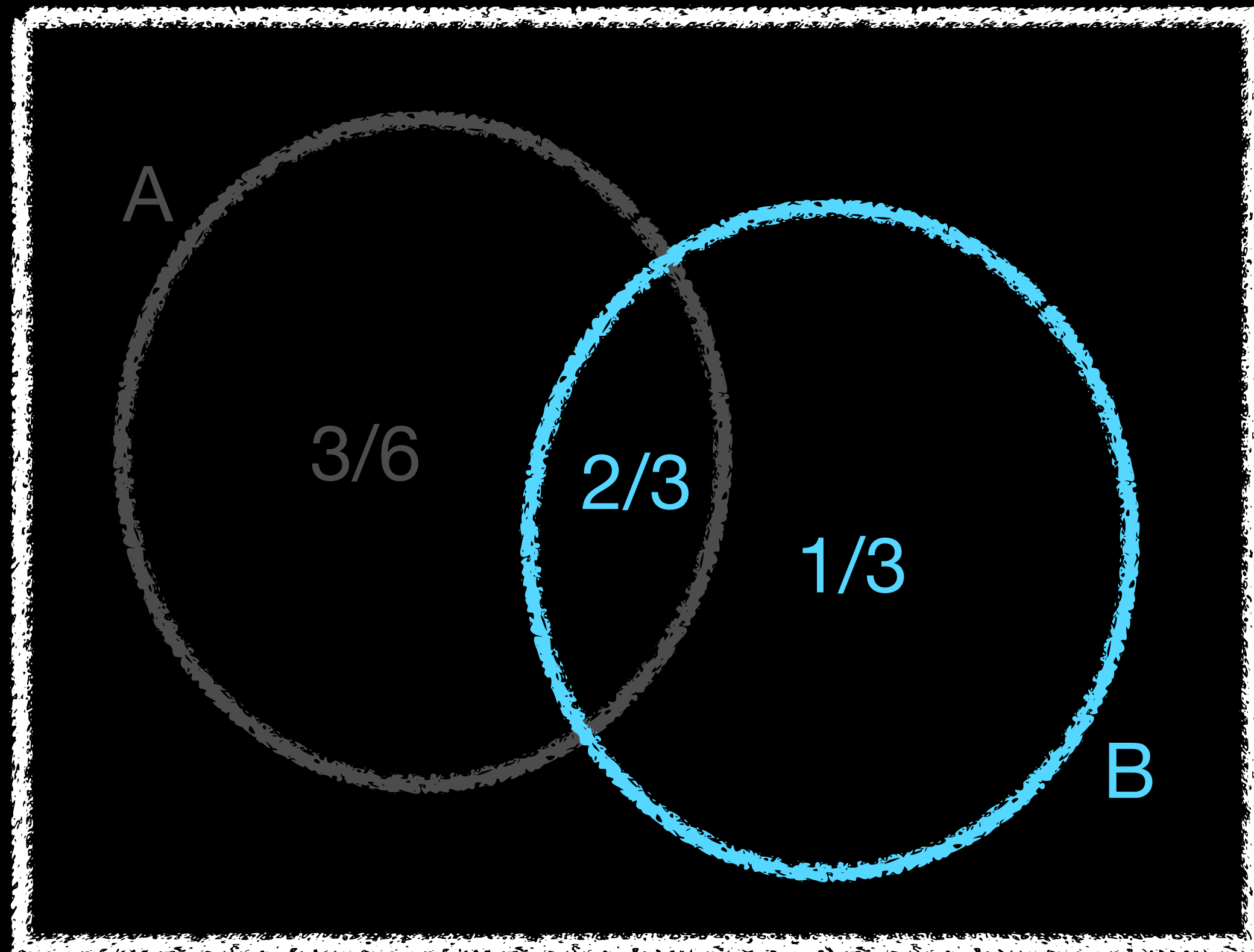
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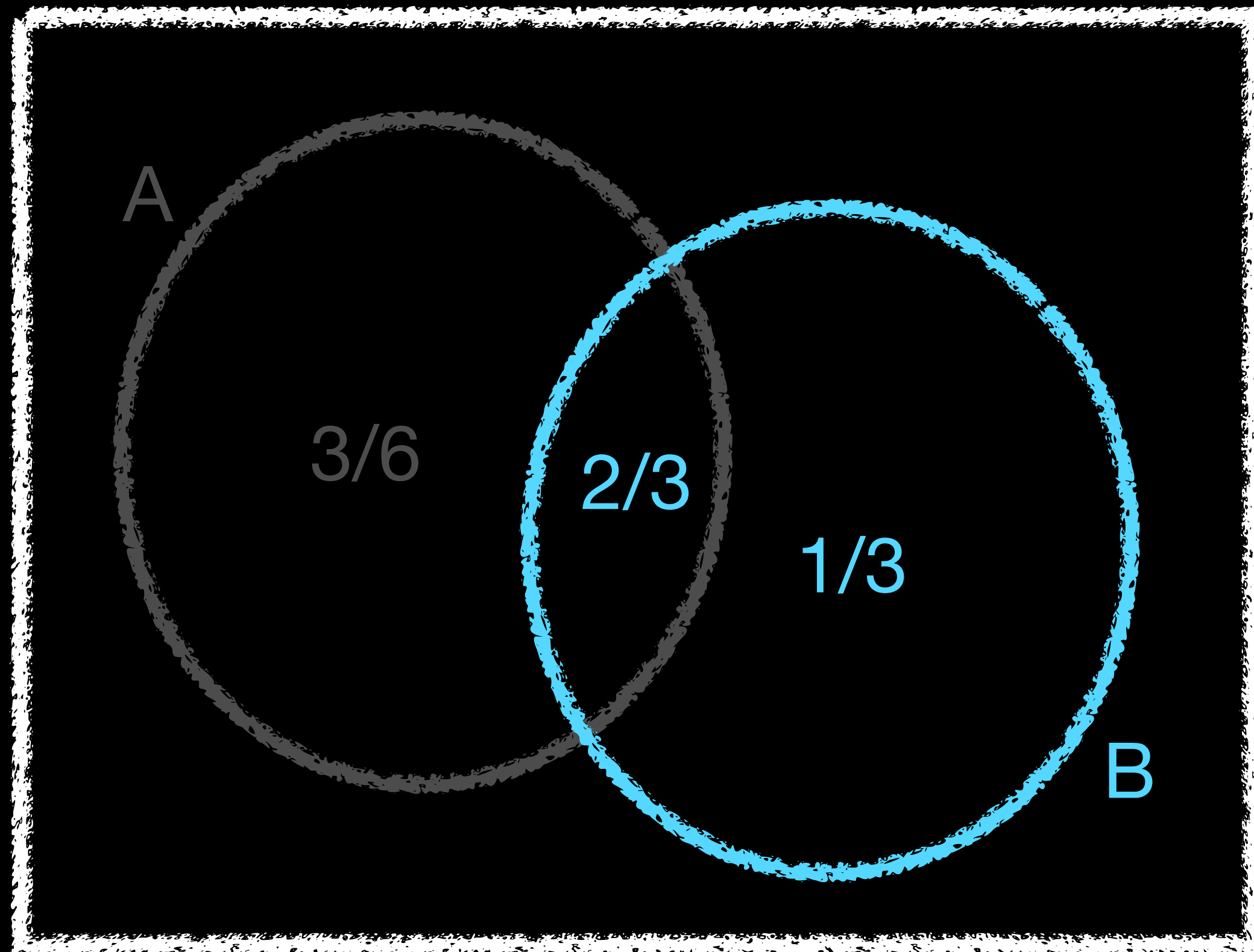


- maintain same proportions
- has to add up to 1

$$\frac{P^{new}(A)}{P^{new}(B)} = \frac{P(A \cap B)}{P(B)}$$

NEW INFORMATION: B HAS HAPPENED

probabilities need to be revised -> conditional probabilities

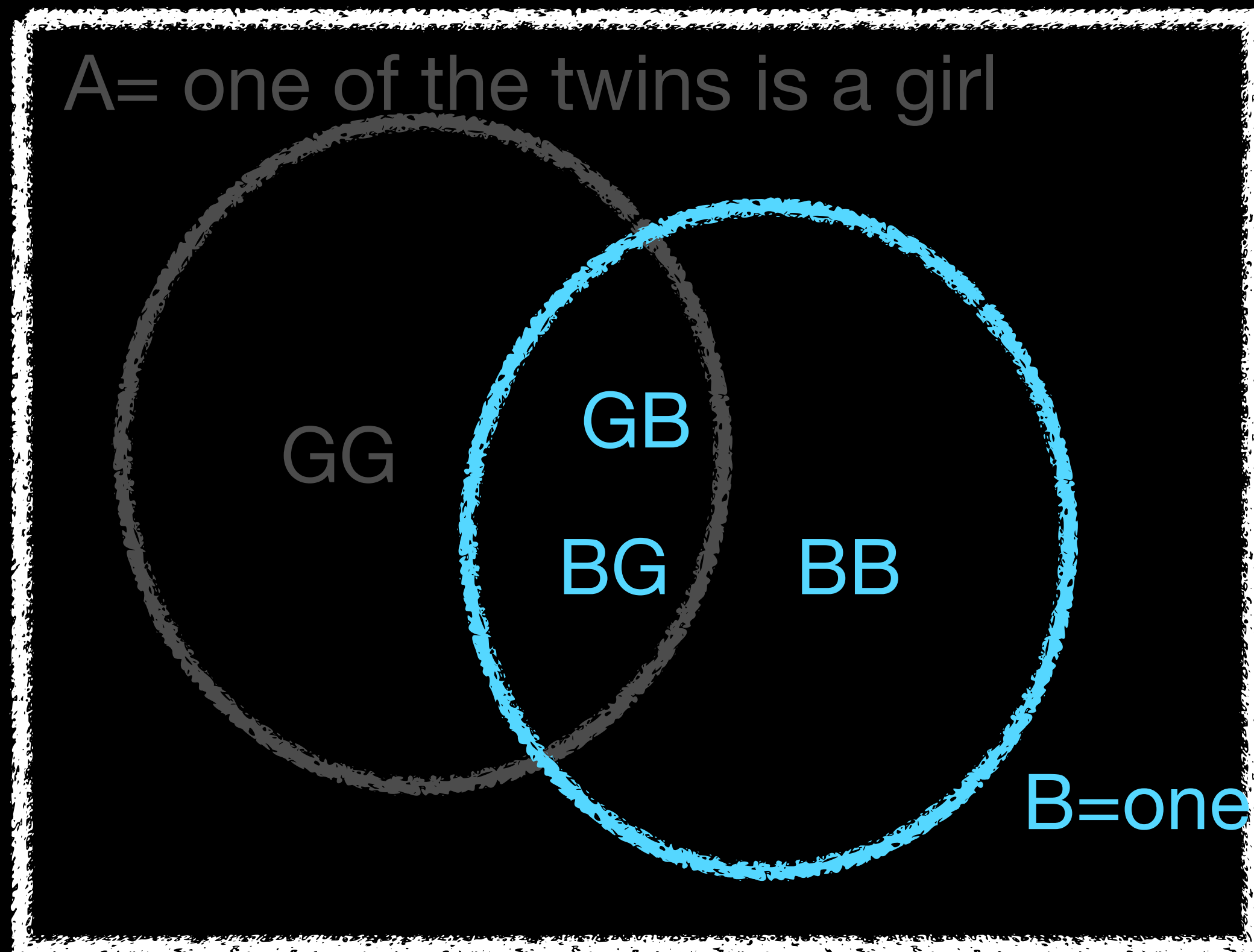


- maintain same proportions
- has to add up to 1

$$\frac{P(A|B)}{\underbrace{P(B|B)}_{\Rightarrow}} = \frac{P(A \cap B)}{P(B)}$$

NEW INFORMATION: B HAS HAPPENED

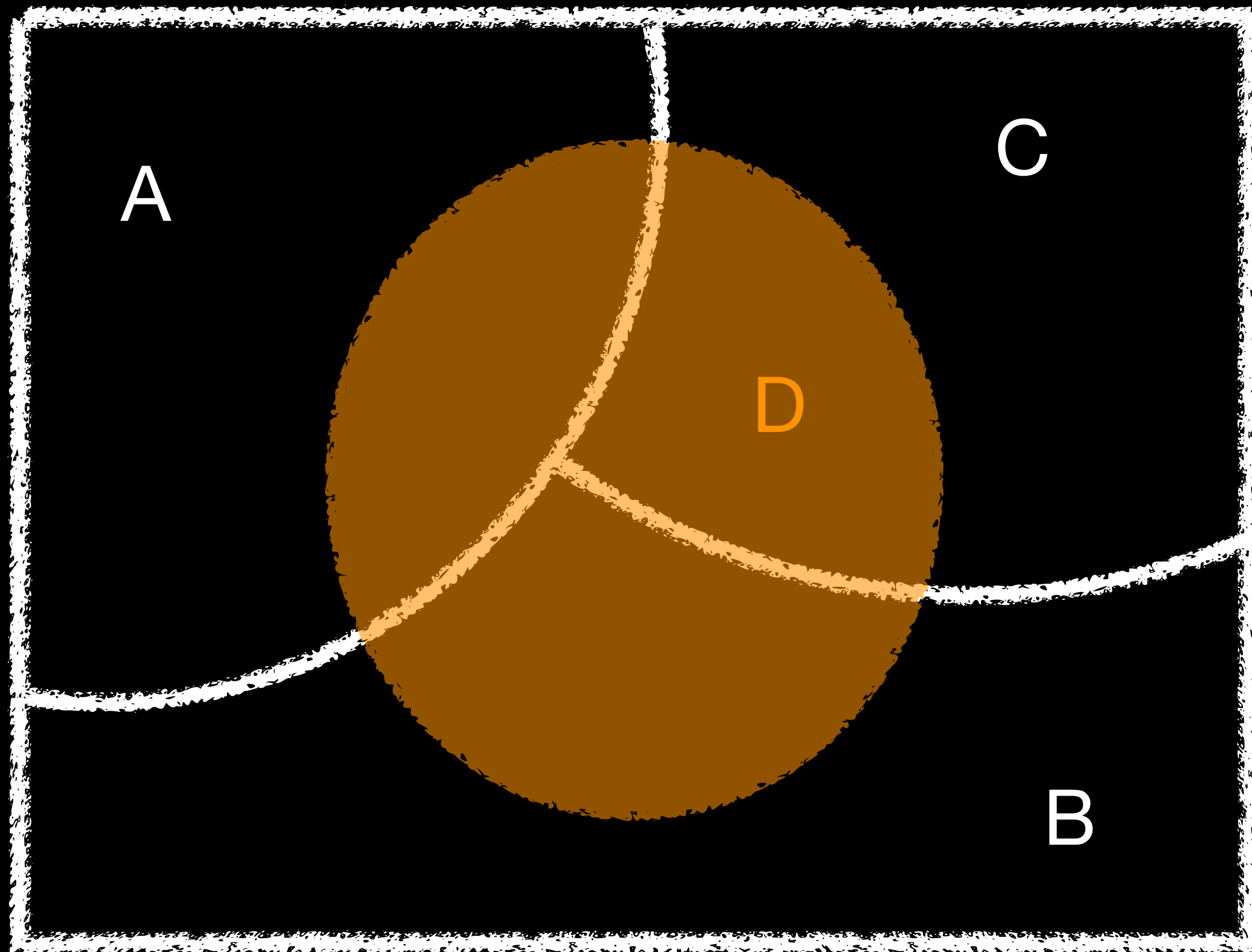
probabilities need to be revised -> conditional probabilities



- maintain same proportions
- has to add up to 1

$$\frac{P(A|B)}{P(B|B)} = \frac{P(A \cap B)}{P(B)} = \frac{1/4 + 1/4}{3/4} = 2/3$$

TOTAL PROBABILITY THEOREM



$$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$$

INFERENCE WITH BAYES' THEOREM

$$P(A_i|D) = \frac{P(A_i \cap D)}{P(D)} \quad \text{and} \quad P(D|A_i) = \frac{P(A_i \cap D)}{P(A)} \quad \longrightarrow \quad P(A_i \cap D) = P(A)P(D|A_i)$$

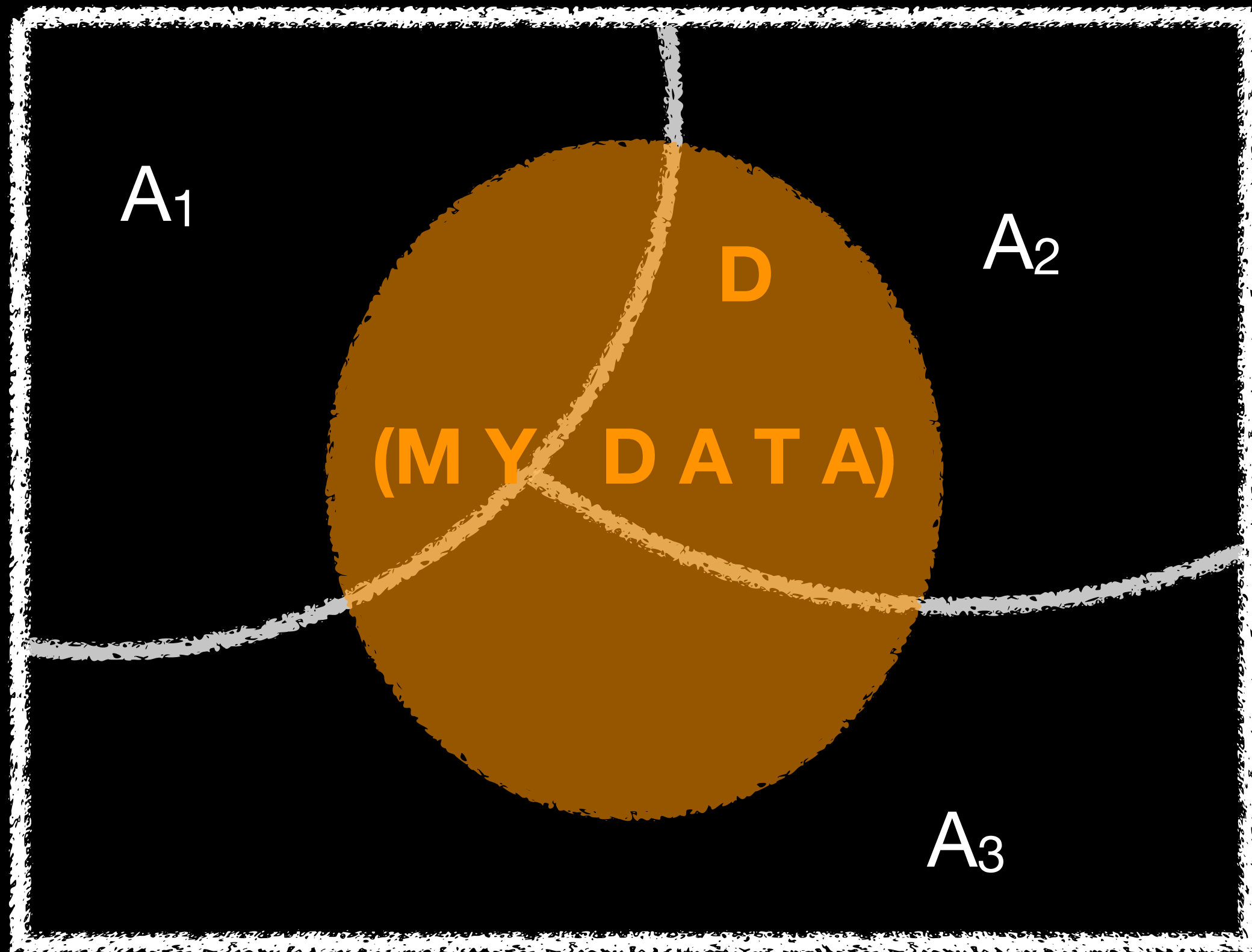
$$P(A_i|D) = \frac{P(D|A_i) P(A_i)}{P(D)}$$

$$P(A_i|D) = \frac{P(D|A_i) P(A_i)}{\sum_j P(A_j) P(D|A_j)}$$

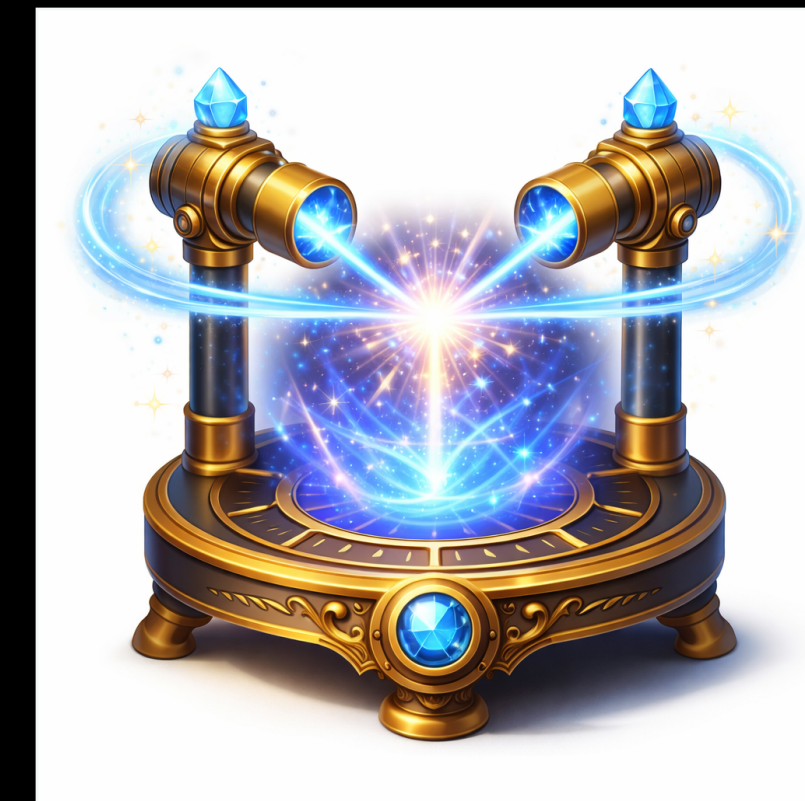
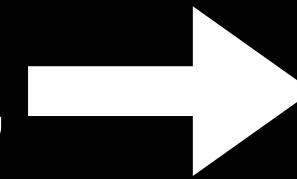
INFERENCE WITH BAYES THEOREM

A_i : different scenarios

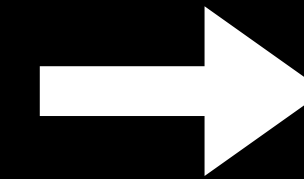
D: the event that has happened, my data



some physical scenario, A_1 or A_2 or A_3



measuring device,

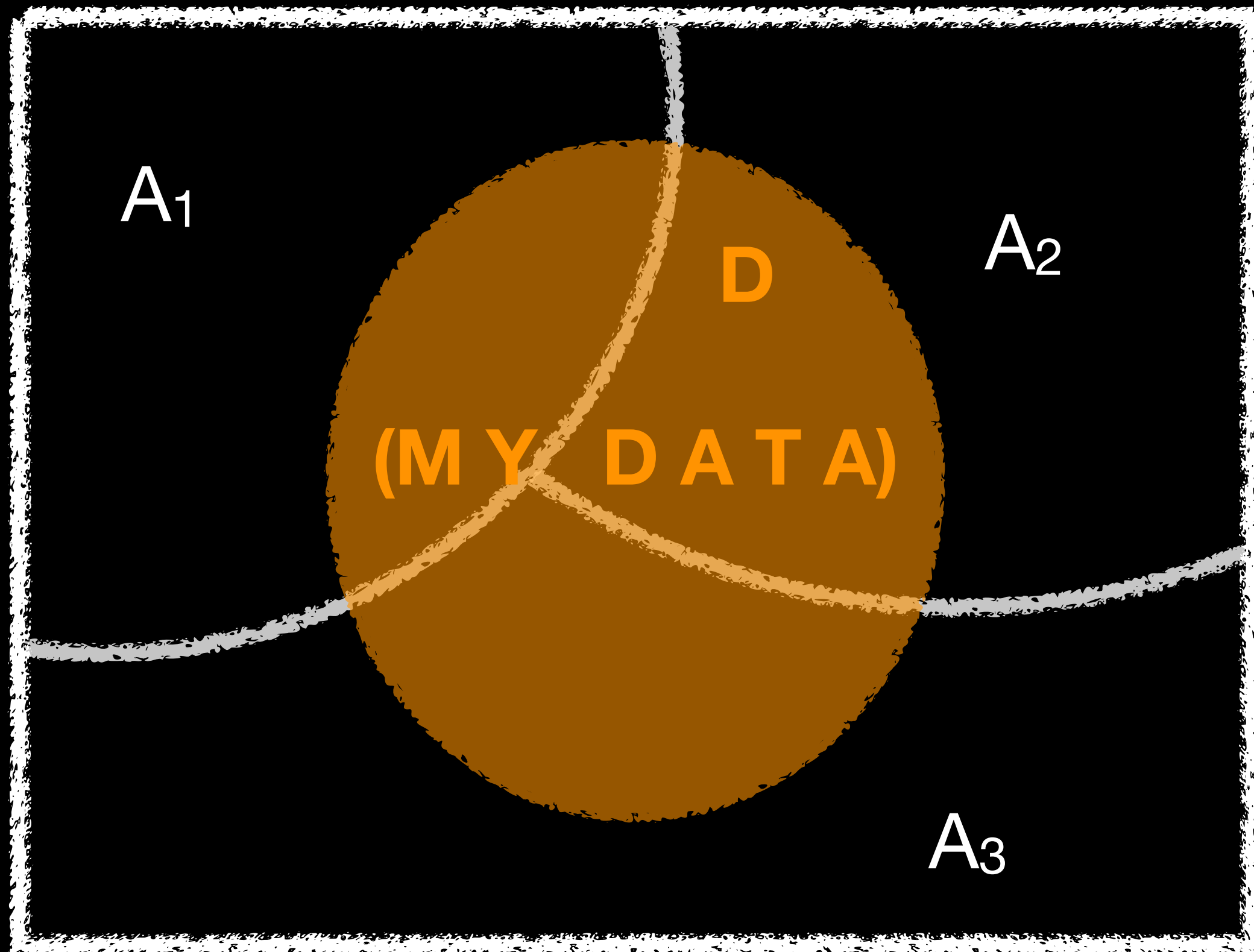


A measurement, an observation, my data D

INFERENCE WITH BAYES THEOREM

A_i : different scenarios

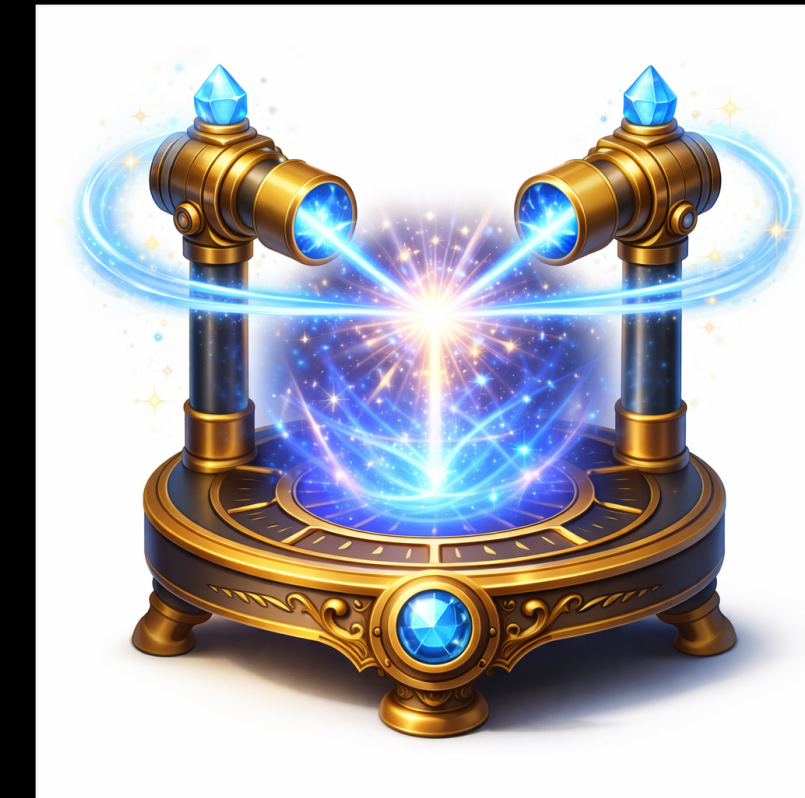
D: the event that has happened, my data



some physical scenario, A_1 or A_2 or A_3

I have prior beliefs on the likelihood of these scenarios: $P(A_i)$

known



measuring device, defined by $P(D|A_i)$

known

A measurement, an observation, my data D

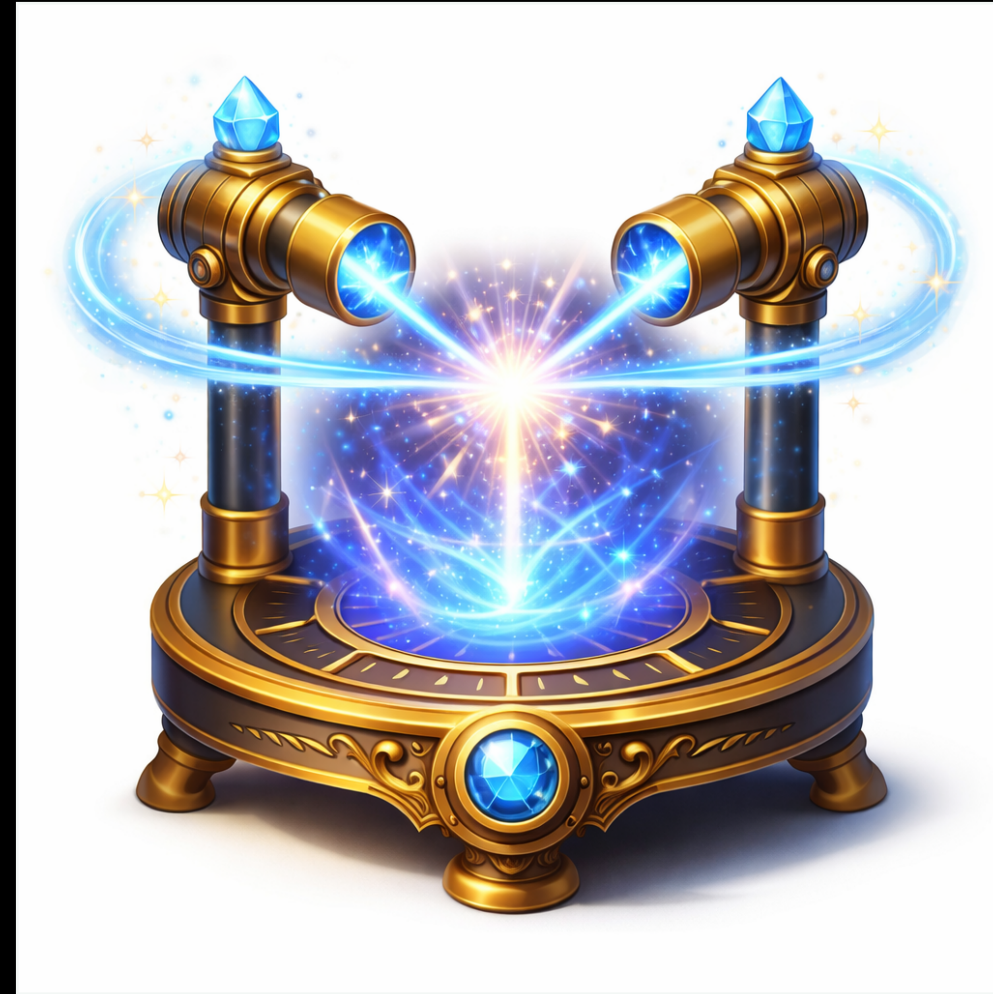
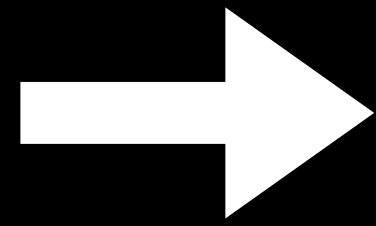
I want to infer something on the likelihood of the scenarios, given my data: $p(A_i|D)$

I want

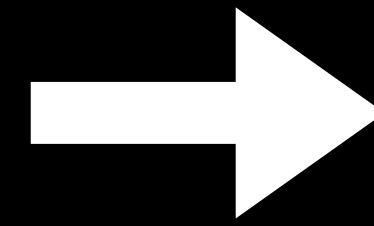
INFERENCE WITH BAYES THEOREM

some physical scenario, A_1 or A_2 or A_3

I have prior beliefs on the likelihood of these scenarios: $P(A_i)$



measuring device, defined by $P(D|A_i)$



A measurement, an observation, my data D

I want to infer something on the likelihood of the scenarios, given my data: $p(A_i|D)$

$$P(A_i | D) = \frac{P(D | A_i) P(A_i)}{\sum_j P(A_j) P(D | A_j)}$$

LET'S USE THIS IN THE CW DETECTION CONTEXT
to estimate amplitude parameters

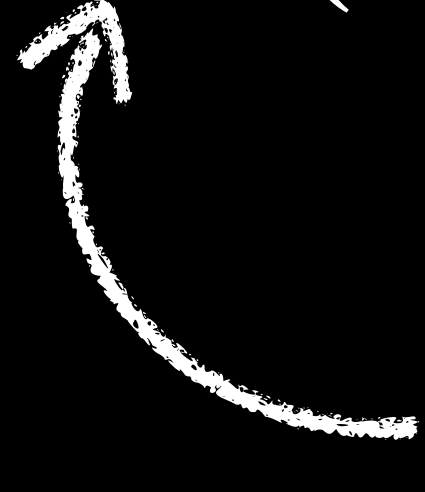
we want the posteriors on the amplitude parameters:

$$P(A | x, \mathcal{H}_s, \lambda)$$

LET'S USE THIS IN THE CW DETECTION CONTEXT
to estimate amplitude parameters

posteriors on the amplitude parameters:

$$P(A | x, \mathcal{H}_s, \lambda) = P(A | \mathcal{H}_s, \lambda) \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_s, \lambda)}$$

 Bayes' theorem

LET'S USE THIS IN THE CW DETECTION CONTEXT
to estimate amplitude parameters

$$P(A | x, \mathcal{H}_s, \lambda) = P(A | \mathcal{H}_s, \lambda) \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_s, \lambda)}$$

we use the
likelihood to get
this

likelihood:

$$\Lambda(x; \mathcal{H}_s, A, \lambda) = \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_N)}$$

LET'S USE THIS IN THE CW DETECTION CONTEXT
to estimate amplitude parameters

$$P(A | x, \mathcal{H}_s, \lambda) = P(A | \mathcal{H}_s, \lambda) \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_s, \lambda)}$$

we use the likelihood to get this

likelihood:

$$\Lambda(x; \mathcal{H}_s, A, \lambda) = \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_N)}$$

$$\rightarrow P(A | x, \mathcal{H}_s, \lambda) = P(A | \mathcal{H}_s, \lambda) \frac{\Lambda(x; \mathcal{H}_s, A, \lambda) P(x | \mathcal{H}_N)}{P(x | \mathcal{H}_s, \lambda)}$$

LET'S USE THIS IN THE CW DETECTION CONTEXT
to estimate amplitude parameters

$$P(A | x, \mathcal{H}_s, \lambda) = P(A | \mathcal{H}_s, \lambda) \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_s, \lambda)}$$

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this

likelihood:

$$\Lambda(x; \mathcal{H}_s, A, \lambda) \equiv \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_N)}$$

$$\rightarrow P(A | x, \mathcal{H}_s, \lambda) = P(A | \mathcal{H}_s, \lambda) \frac{\Lambda(x; \mathcal{H}_s, A, \lambda) P(x | \mathcal{H}_N)}{P(x | \mathcal{H}_s, \lambda)}$$

$$\equiv k P(A | \mathcal{H}_s, \lambda) \Lambda(x; \mathcal{H}_s, A, \lambda)$$

LET'S USE THIS IN THE CW DETECTION CONTEXT
to estimate amplitude parameters

$$P(A | x, \mathcal{H}_s, \lambda) = P(A | \mathcal{H}_s, \lambda) \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_s, \lambda)}$$

we use the likelihood to get this

likelihood:

$$\Lambda(x; \mathcal{H}_s, A, \lambda) = \frac{P(x | \mathcal{H}_s, A, \lambda)}{P(x | \mathcal{H}_N)}$$

$$\rightarrow P(A | x, \mathcal{H}_s, \lambda) = P(A | \mathcal{H}_s, \lambda) \frac{\Lambda(x; \mathcal{H}_s, A, \lambda) P(x | \mathcal{H}_N)}{P(x | \mathcal{H}_s, \lambda)}$$

$$\equiv k P(A | \mathcal{H}_s, \lambda) \Lambda(x; \mathcal{H}_s, A, \lambda) \text{ with } \int dA P(A | \dots) d^4A = 1 \rightarrow k$$

LET'S USE THIS IN THE CW DETECTION CONTEXT
to estimate amplitude parameters

$$P(A | x, \mathcal{H}_s, \lambda) = k P(A | \mathcal{H}_s, \lambda) \underbrace{\Lambda(x; \mathcal{H}_s, A, \lambda)}_{\text{with } \int dA P(A | \dots) d^4A = 1 \rightarrow k}$$

remember:

$$\begin{aligned} \log \Lambda(x; A_i, \lambda) &= A^i x_i(\lambda) - \frac{1}{2} A^i M_{ij}(\vec{n}_{sky}) A^j = \\ &= \sqrt{2\gamma} (A^1 F_A^{\mathcal{R}} + A^2 F_B^{\mathcal{R}} - A^3 F_A^{\mathcal{J}} - A^4 F_B^{\mathcal{J}}) - \frac{1}{3} \rho_{opt}^2(A, \lambda) \end{aligned}$$

LET'S USE THIS IN THE CW DETECTION CONTEXT

to estimate amplitude parameters

$$P(A | x, \mathcal{H}_s, \lambda) = k \underbrace{P(A | \mathcal{H}_s, \lambda)}_{\text{priors on}} \Lambda(x; \mathcal{H}_s, A, \lambda) \quad \text{with } \int dA P(A | \dots) d^4A = 1 \rightarrow k$$

$$A = (h_0, \iota, \psi, \Phi_0)$$

lacking prior
information:

$$\begin{cases} P(\cos \iota) & \sim \mathcal{U}[-1, 1] \\ P(\psi) & \sim \mathcal{U}[-\pi/4, \pi/4] \\ P(\Phi_0) & \sim \mathcal{U}[0, 2\pi] \\ P(h_0) & ??? \end{cases}$$

LET'S USE THIS IN THE CW DETECTION CONTEXT to estimate amplitude parameters

$$P(A | x, \mathcal{H}_s, \lambda) = k \underbrace{P(A | \mathcal{H}_s, \lambda)}_{\text{priors on}} \Lambda(x; \mathcal{H}_s, A, \lambda) \quad \text{with } \int dA P(A | \dots) d^4A = 1 \rightarrow k$$

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not so interesting
physically, can marginalise
it out by

$$\int_0^{2\pi} d\Phi_0 P(\Phi_0) \Lambda(x; \mathcal{H}_s, A, \lambda)$$

PRIOR ON GW INTRINSIC AMPLITUDE

depends on specifics of the search

$$h_0 \in [h_0^{min}, h_0^{max}]$$

if from known pulsar

$$\begin{cases} h_0^{max} = \min[h_0^{spdwn}, h_0^{UL}] \\ h_0^{min} = h_0(B) \end{cases}$$

if from unknown neutron star

$$\begin{cases} h_0^{max} = \min[h_0^{UL}, h_0(\epsilon^{max})] \\ h_0^{min} = h_0(B) \end{cases}$$

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What about $P(h_0)$?

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$$\begin{cases} P(\log(h_0)) & \sim \mathcal{U}[\log(h_0^{\min}), \log(h_0^{\max})] \\ P(h_0) & \sim \mathcal{U}[h_0^{\min}, h_0^{\max}] \end{cases}$$

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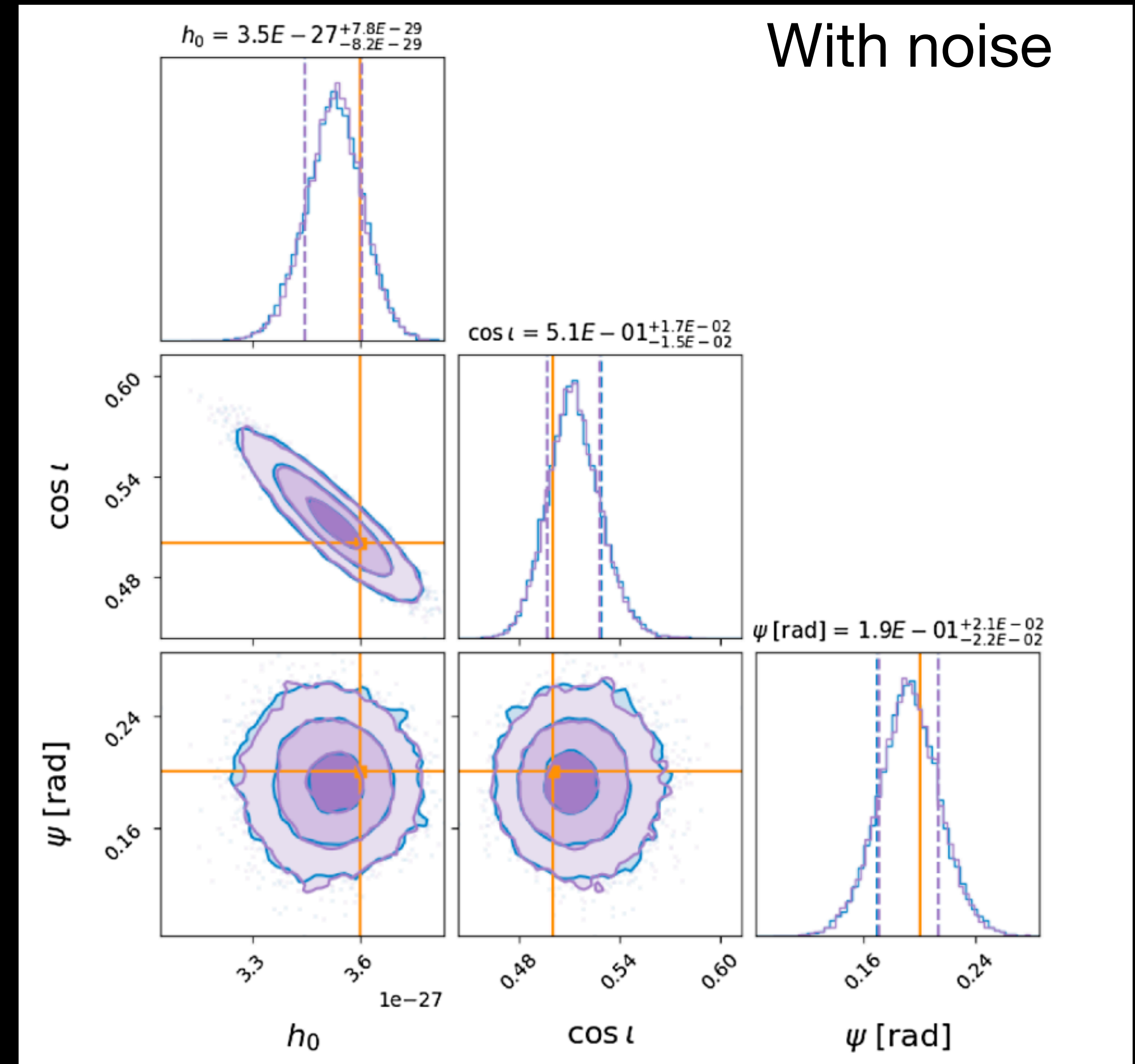
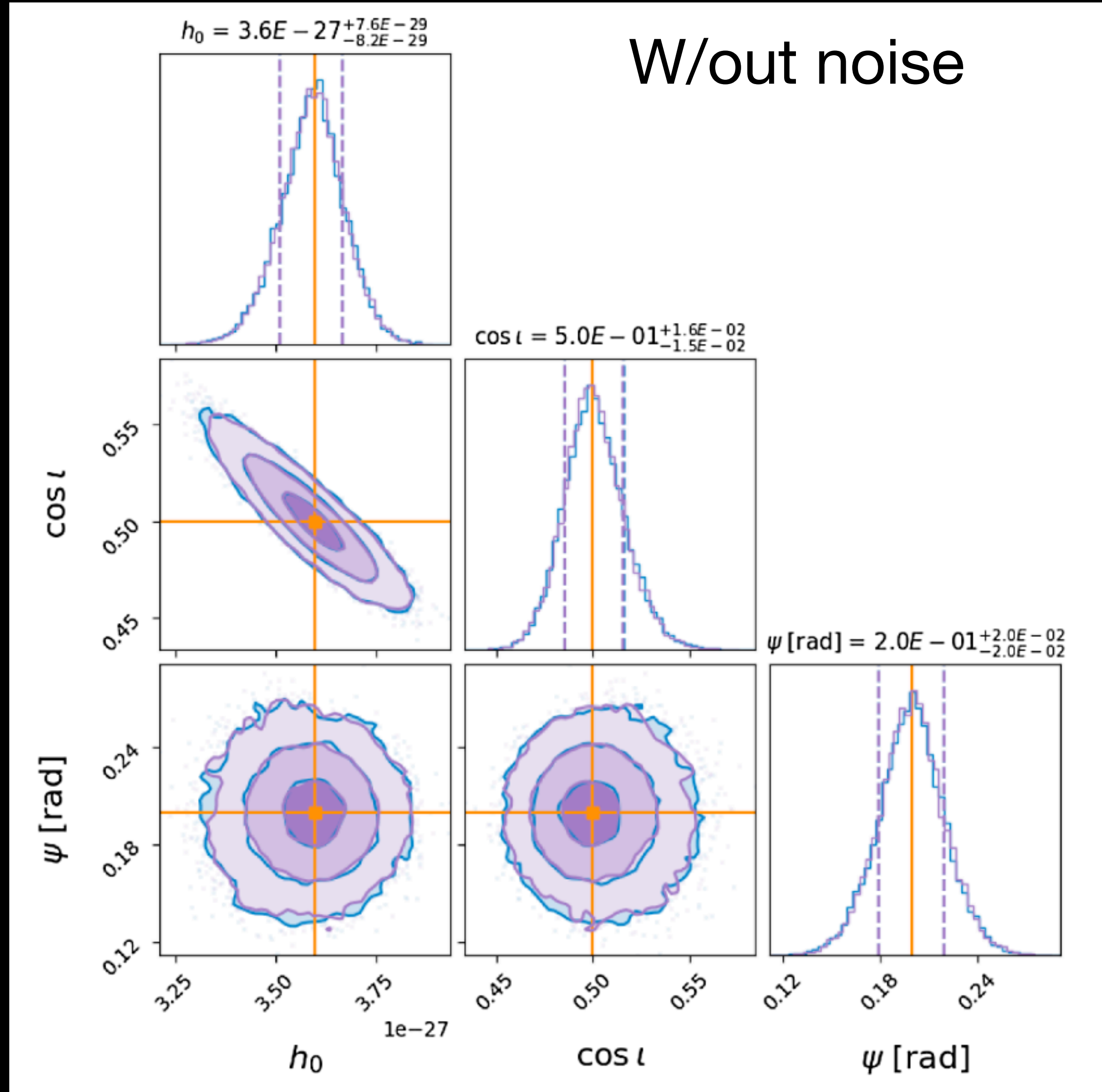
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$$\begin{cases} P(\log(h_0)) \sim \mathcal{U}[\log(h_0^{min}), \log(h_0^{max})] \\ P(h_0) \sim \mathcal{U}[h_0^{min}, h_0^{max}] \end{cases}$$

more representative of our ignorance, even if yields slightly less conservative upper limits.

AMPLITUDE POSTERIORIORS

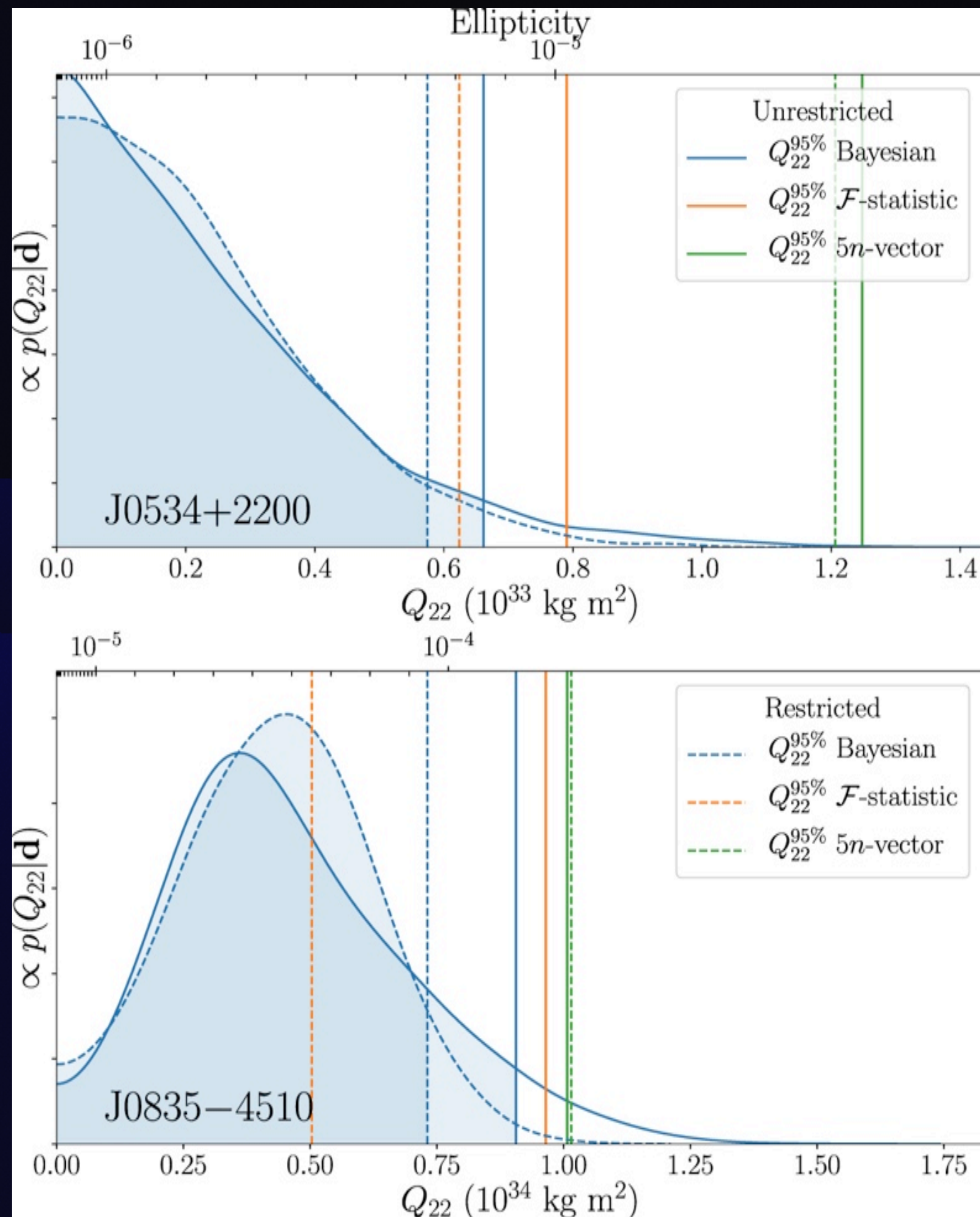
Example



Using Bayesian h_0 posteriors

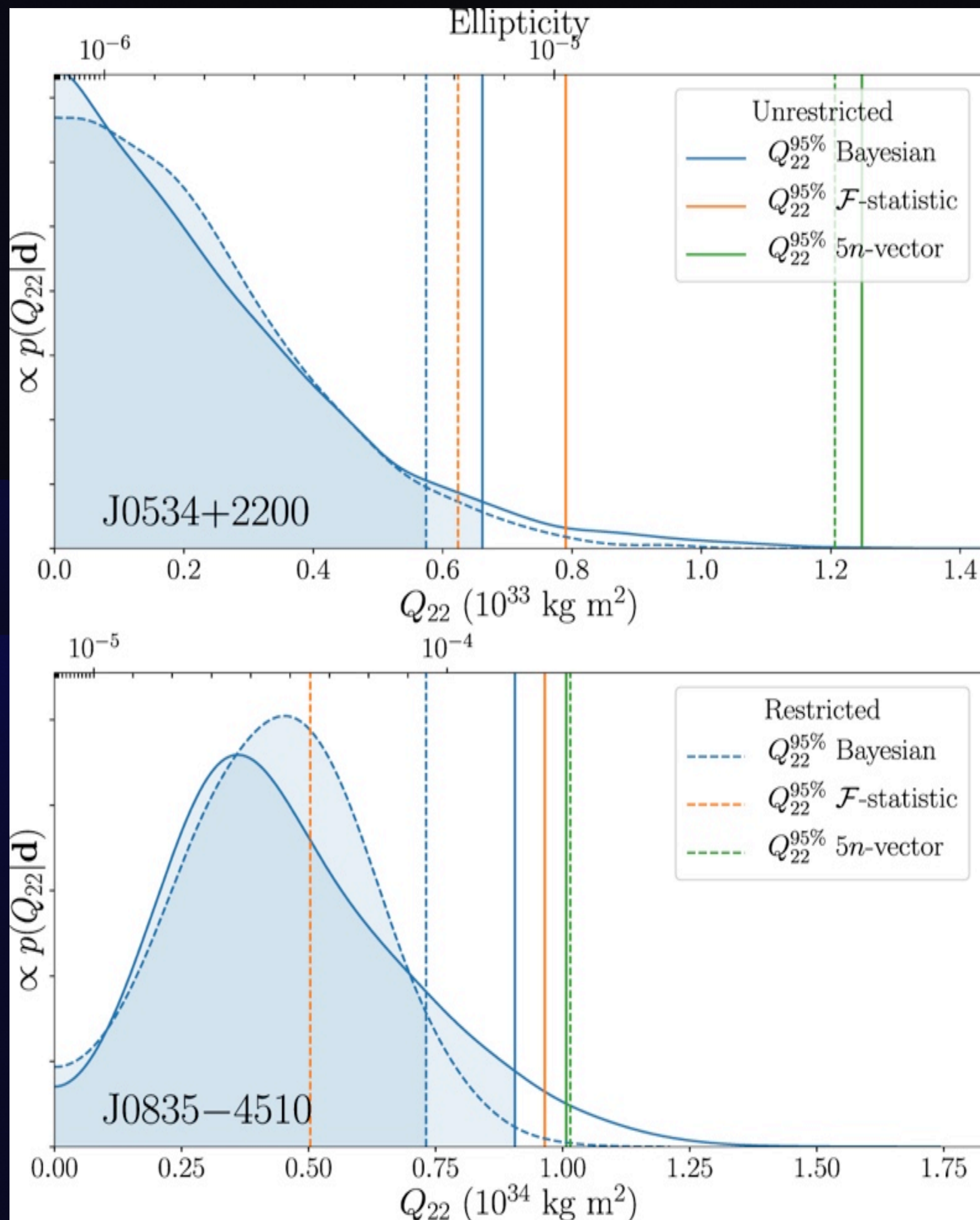


Example posteriors for the Crab and Vela pulsars

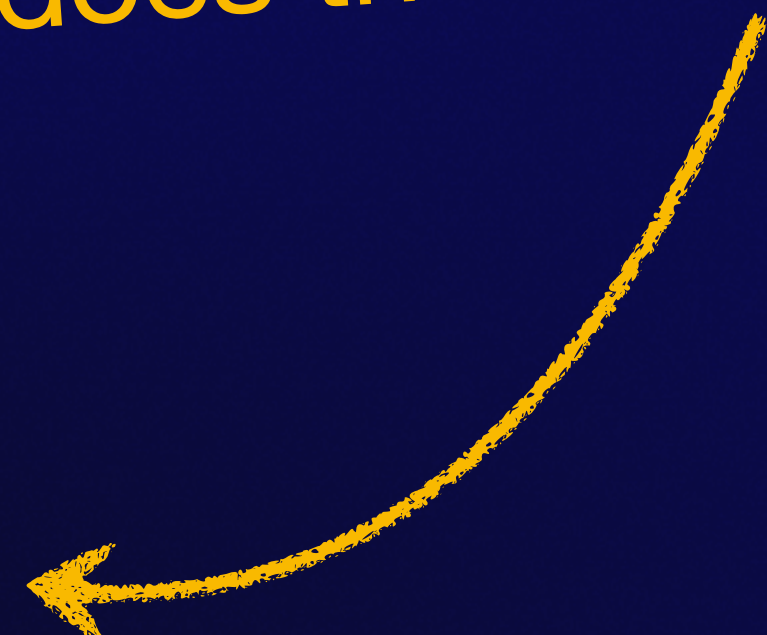


and actually $h_0 \longrightarrow Q_{22}$

Example posteriors for the Crab and Vela pulsars

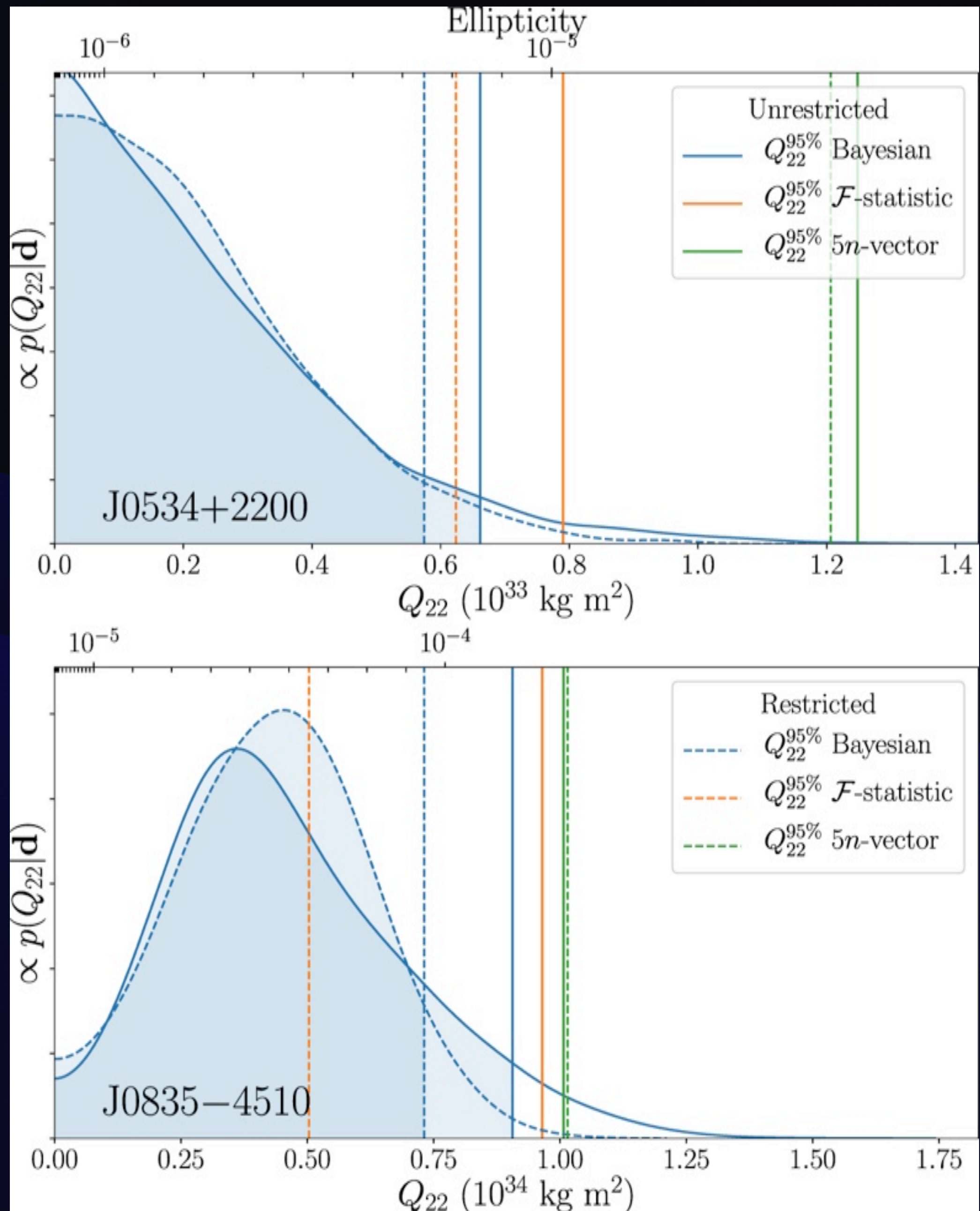


does this look like a signal ?



Establishing detection confidence

- Would it be significant in Gaussian noise ?
- can we exclude a noise disturbance (instrumental/environmental) in the data causing such result ?
- Does the result stay significant if we evaluate it against search results from real detector noise ?
 - Estimating the background

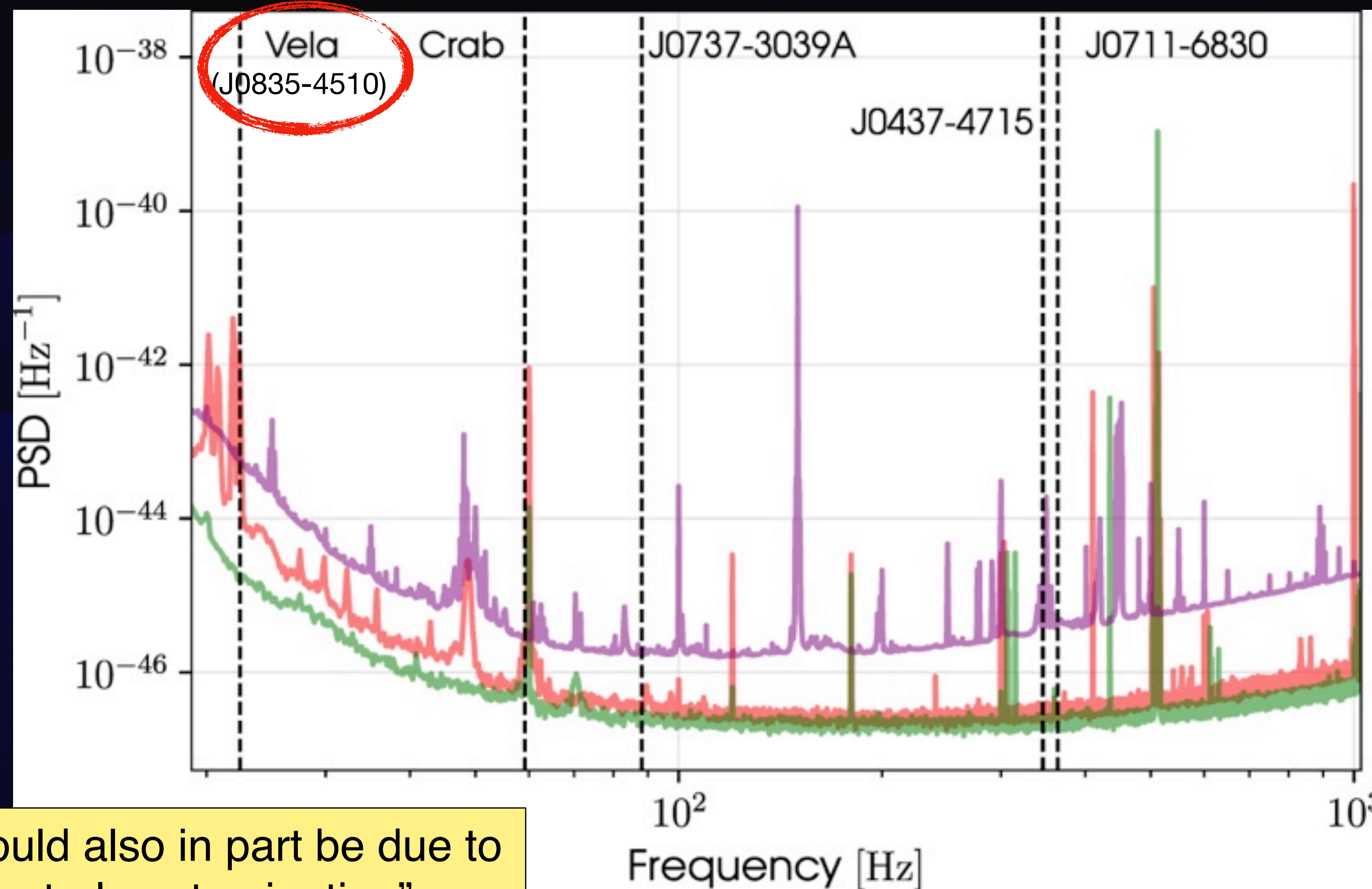


“not disjoint from zero”
 “not uncommon for pure Gaussian noise”

Establishing detection confidence

- Would it be significant in Gaussian noise ?
- can we exclude a noise disturbance (instrumental/environmental) in the data causing such result ?
- Does the result stay significant if we evaluate it against search results from real detector noise ?
 - Estimating the background

Establishing detection confidence



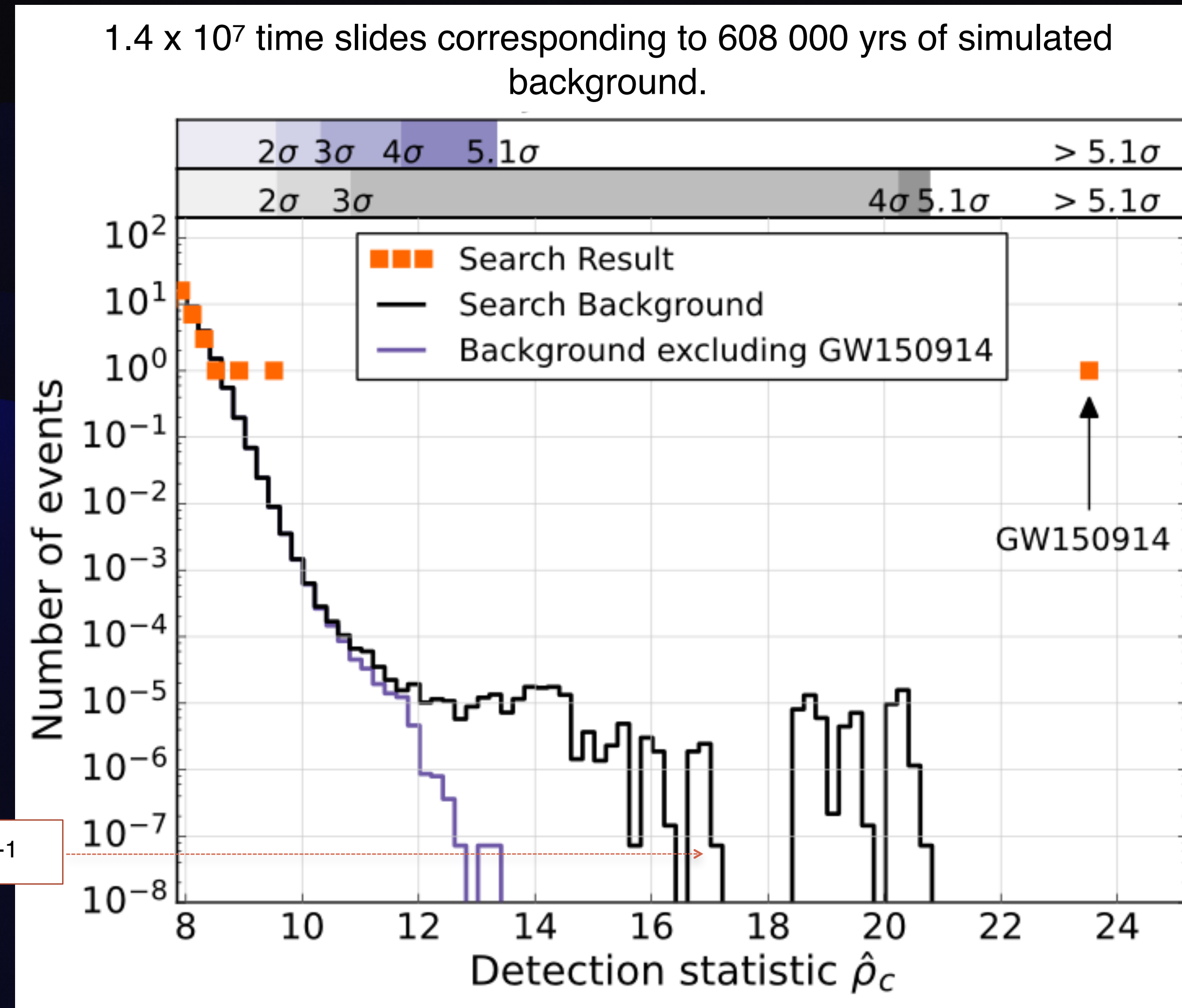
“could also in part be due to spectral contamination”

Establishing detection confidence

- Would it be significant in Gaussian noise ?
- can we exclude a noise disturbance (instrumental/environmental) in the data causing such result ?
- Does the result stay significant if we evaluate it against search results from real detector noise ?
 - Estimating the background

The first GW detection

[Observation of Gravitational Waves from a Binary Black Hole Merger](#)
Phys.Rev.Lett. 116 (2016)



$$7 \times 10^{-8} \approx (1.4 \times 10^7)^{-1}$$

Establishing detection confidence

Background estimate

- For a search for emission from a known pulsar it should be possible to estimate the background:
 - Repeating the same search many times “off-source”
 - [near-by frequencies \(extensive literature\)](#)
 - [different sky positions, Isi et al, arXiv:2010.12612 \(2020\)](#)
- Not so simple for other types of continuous wave searches

End lecture 2