QUART&T WP1 Meeting

Simulating correlations and dynamics on a few qubit device: benchmark with some simple theoretical models

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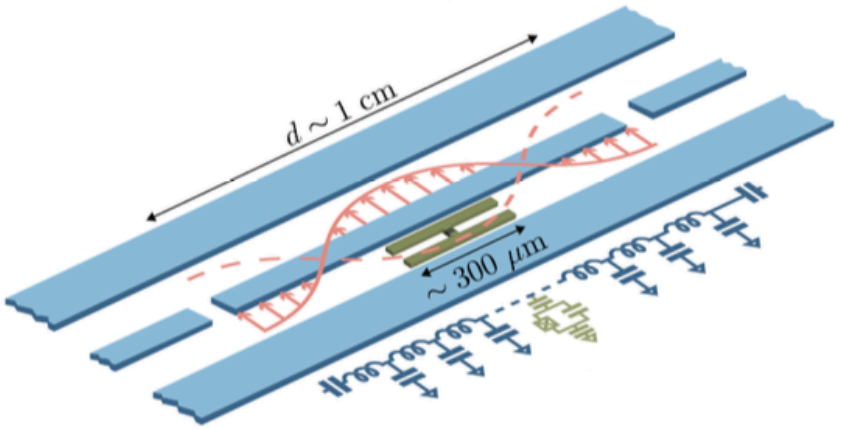
TRASMONS COUPLED TO RESONATORS



N identical transmon qubits (truncated to 2 levels),

coupled to a single mode of a common resonator

$$\hat{H} = \hbar\omega_r \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_q}{2} \sum_{j=1}^{N} \hat{\sigma}_z^{(j)} + \hbar g \sum_{j=1}^{N} \hat{\sigma}_x^{(j)} \left(\hat{a} + \hat{a}^\dagger \right)$$

(Jaynes-Cummings)

RWA approximation (neglecting fast rotating terms) $|g| \ll \omega_r, \omega_q$

dispersive regime (at second order in SW transformation) $|\Delta| = |\omega_q - \omega_r| \gg g$

vacuum field approximation $\langle \hat{a}^\dagger \hat{a} \rangle = 0$

$$\hat{H}_{\text{disp}}^{\text{RWA}} = \sum_{j=1}^{N-1} \frac{J\hbar}{2} \left(\hat{\sigma}_x^{(j)} \hat{\sigma}_x^{(j+1)} + \hat{\sigma}_y^{(j)} \hat{\sigma}_y^{(j+1)} \right) + \sum_{j=1}^{N} \frac{\hbar \omega_q^0}{2} \hat{\sigma}_z^{(j)}$$

XY-model

$$J = g^2/\Delta$$

transverse field

$$H = \frac{\hbar \omega_q^0}{2}$$

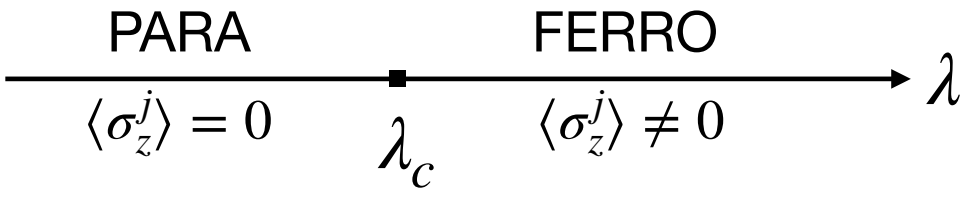
XY model in transverse field

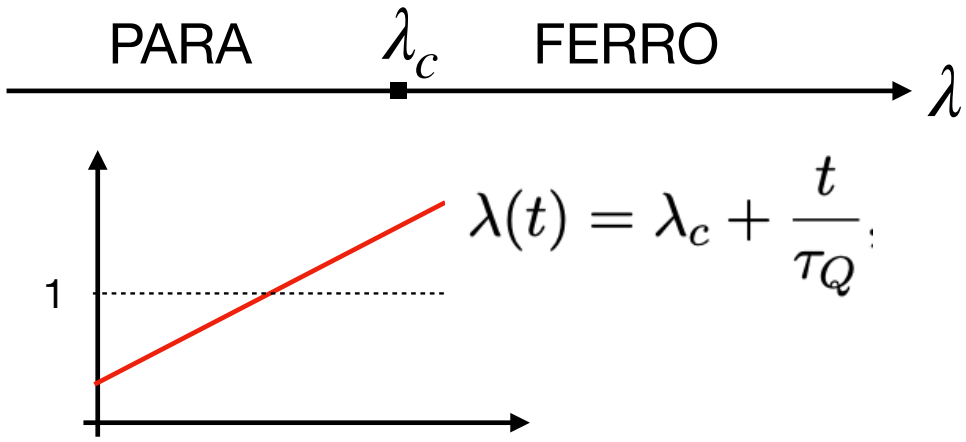
$$H_{XY} = \sum_j (\sigma_x^{(j)}\sigma_x^{(j+1)} + \sigma_y^{(j)}\sigma_y^{(j+1)}) + \lambda \sum_j \sigma_z^{(j)} \qquad \lambda = \frac{\omega_q^0}{2J}$$

- exactly solvable (free fermions with cosine band $\epsilon_k = 2J\hbar(\lambda + \cos k)$

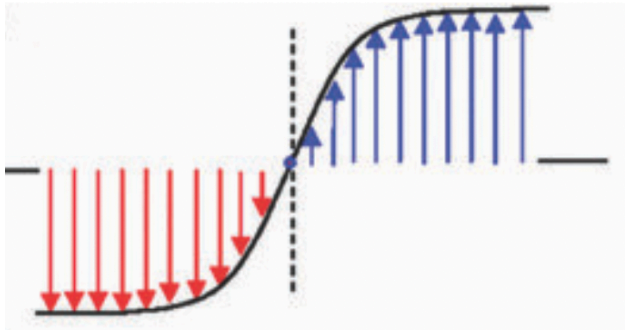
$$H_{\pm} = \sum_k \varepsilon_k c_k^{\dagger} c_k - \frac{\hbar\omega_q^0}{2} N$$

- phase transition (Ising type) at $\lambda_c = 1$, with spontaneous symmetry breaking





KIBBLE-ZUREK MECHANISM



- proliferation of excitations (domain walls; solitons) $n_{\rm exc} \sim \tau_Q^{-1/2}$

ISSUES

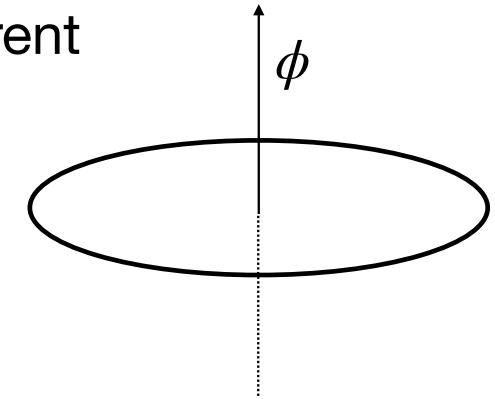
- number of qubits = length of chain (finite size effects for phase transition) + auxiliary qubits
- values of $\lambda \sim 0.2$ -> FLUXONIUM??

VARIATIONS

- Longer range interactions (N.N. + N.N.N.; all to all)
- Observation on the fermionic chain:

on a circle we can but quasi-periodic boundary conditions $c_N = e^{i\phi/N} c_1$

this is equivalent to add a magnetic flux threading the circle and inducing a stationary current



<u>ISSUES</u>

- can make a closed loop?
- can we take $J \to J e^{i\phi/N}$
- can we also modulate it in time?

CORRELATORS

For XY-model in transverse field

$$\langle \sigma^z_\ell \sigma^z_m \rangle = H(0)^2 - H(m-\ell)\, H(\ell - m)$$

$$\langle \sigma^{x/y}_\ell \sigma^{x/y}_m \rangle = \begin{vmatrix} H(-1) & H(-2) & \cdots & H(-r) \\ H(0) & H(-1) & \cdots & H(-r+1) \\ \vdots & \vdots & \ddots & \vdots \\ H(r-2) & H(r-3) & \cdots & H(-1) \end{vmatrix}$$

$$H(r) \equiv \frac{1}{N} \sum_{k \in \mathcal{I}_s} e^{ikr}$$

$$r = m - \ell$$

Correlators (and their time-dependence) are very hard to obtain

Two-Qubit State Tomography using a Joint Dispersive Read-Out

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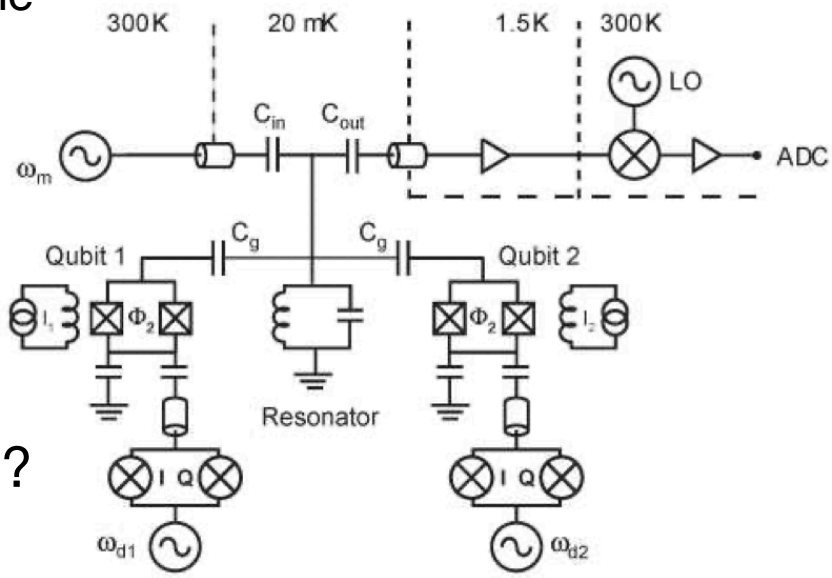
we extract two-qubit correlations from the simultaneous averaged measurement of two qubits dispersively coupled to a common resonator

we employ standard tomographic techniques to reconstruct the density matrix of two-qubit quantum states

$$\hat{M} = \frac{1}{4}\left(\beta_{00}\hat{\mathrm{id}} + \beta_{10}\hat{\sigma}_{z1} + \beta_{01}\hat{\sigma}_{z2} + \beta_{11}\hat{\sigma}_{z1}\hat{\sigma}_{z2}\right)$$



ISSUES

- can we apply it to any couple of/multiple of qubits?
- can we generalize to state tomography of

more than two qubits?