Data features: how to visualize and infer dynamics in a reaction

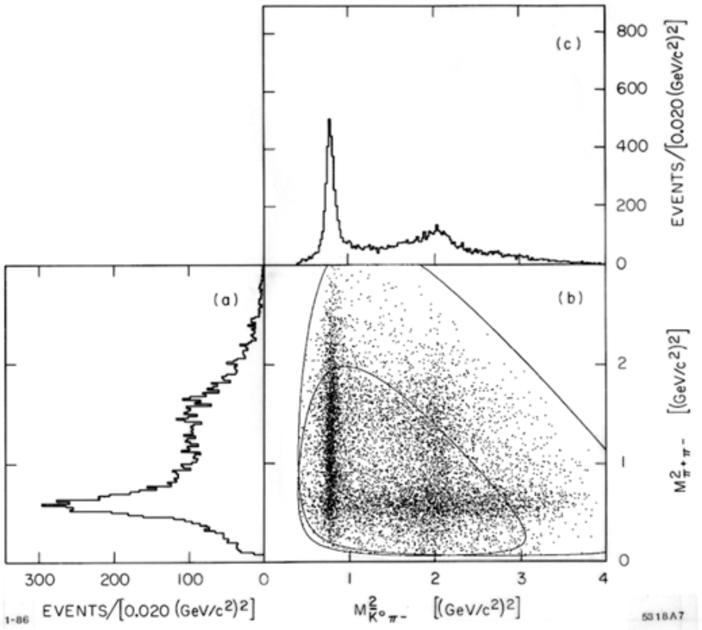
- Dalitz plots: general features and how to read them
- Fake signatures: kinematic reflections vs kinematic peaks
- Combinatorial backgrounds
- Difference spectra
- Acceptance effects
- Many-body approaches
- Hadronic vs electromagnetic processes

Three particle final states

- How many independent observables?
  - Total: 3 particles x4 momentum coordinates
  - Constraints:
    - Conservation laws at reaction vertex: **4**
    - Particle masses: **3**
    - Space isotropy: **3** (2 if reaction in flight)

  ⇒ 12 – 4 – 3 - 3 = 2 independent observables

  ⇒ 2-dim plot: **DALITZ PLOT**



- (most) conventional coordinates: invariant masses of particle pairs, m_{12} and m_{13} (squared)

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M}\left|T\right|^2 dm_{12}^2 dm_{13}^2$$

Mandelstam variables for four points amplitudes

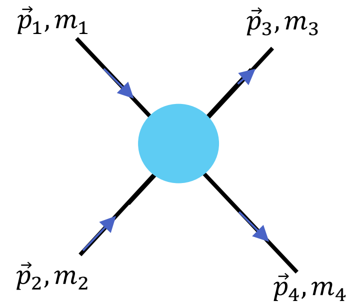
- The scattering amplitude for spinless particles depend on two independent kinematic variables
- One can describe the amplitude through the Mandelstam variables *s*, *t*, *u*:

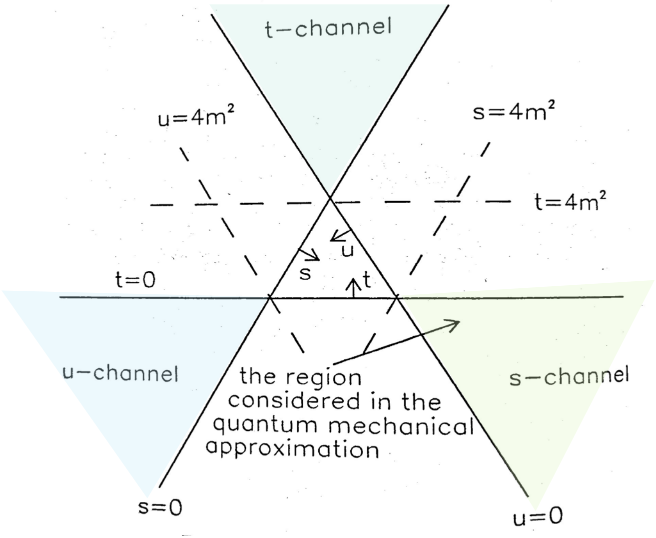
$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$u = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

Which obey the identity

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 = 4m^2$$

(if the mass if the same)



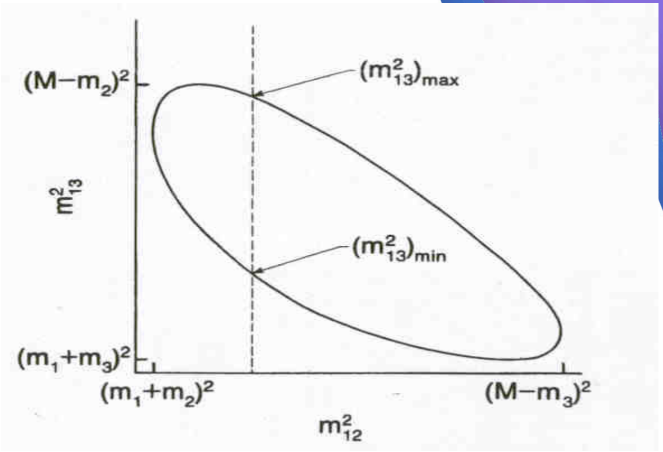


- physical region of *t*-channel: 1 & 3 collide
- physical region of *u*-channel: 1 & 4 collide

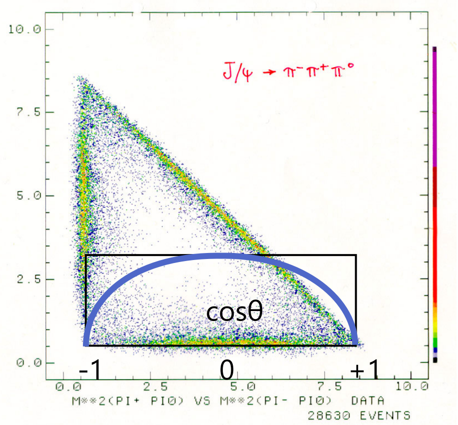
Dalitz plot kinematic features

- When m_{12}^2 is fixed:
  - m_{13}^2 varies in an interval defined by the conditions

$$\vec{p}_1 \uparrow\uparrow \vec{p}_3 \quad \&\& \quad \vec{p}_1 \uparrow\downarrow \vec{p}_3$$



- Final state density constant all over the plot
  - $dN \sim (E_1\, dE_1)(E_2\, dE_2)(E_3\, dE_3)$
  - $E_{tot} = E_1 + E_2 + E_3$
  - $dN/dE_{tot} \sim dE_1\, dE_2\, dE_3$

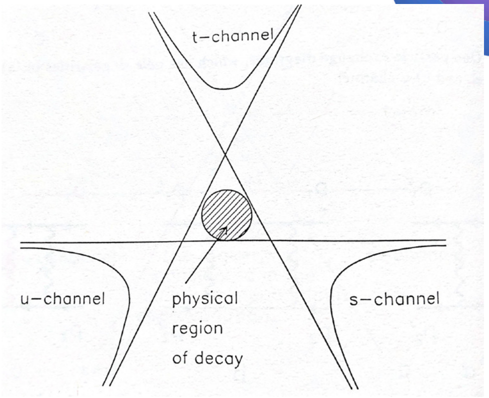


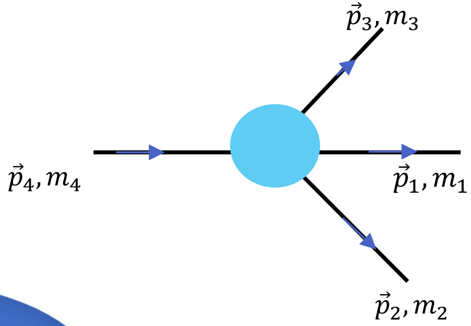
- DP contains all angular information

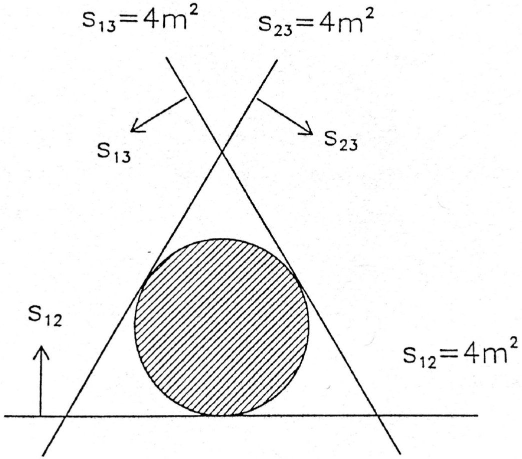
  The bands modulation follows the angular distributions density

Dalitz plot in the Mandelstam plane

- For the decay reaction 4 →1+2+3
- The physical region of the decay process is located at the center of the Mandelstam plane
- The threshold singularities at $s_{ij} = (m_i + m_j)^2$ are tangent to the physical region

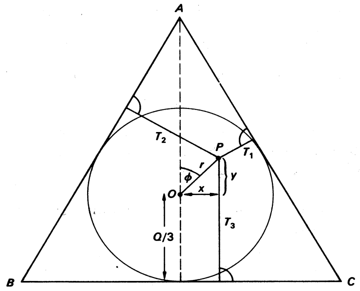


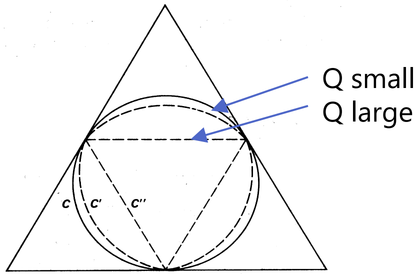




Different Dalitz plot representations

- In a three-particle reaction, a Dalitz plot is defined as the physical region for the decay $P \to p_1\, p_2\, p_3$ described by any variable linked to s_1 and s_2 by means of a linear transformation with constant Jacobian





- Kinetic energies plot:
  - $Q = T_1+T_2+T_3$
  - $x = (T_1-T_2)/\sqrt{3}$
  - $y = T_3-Q/3$

What do you observe in a Dalitz plot?

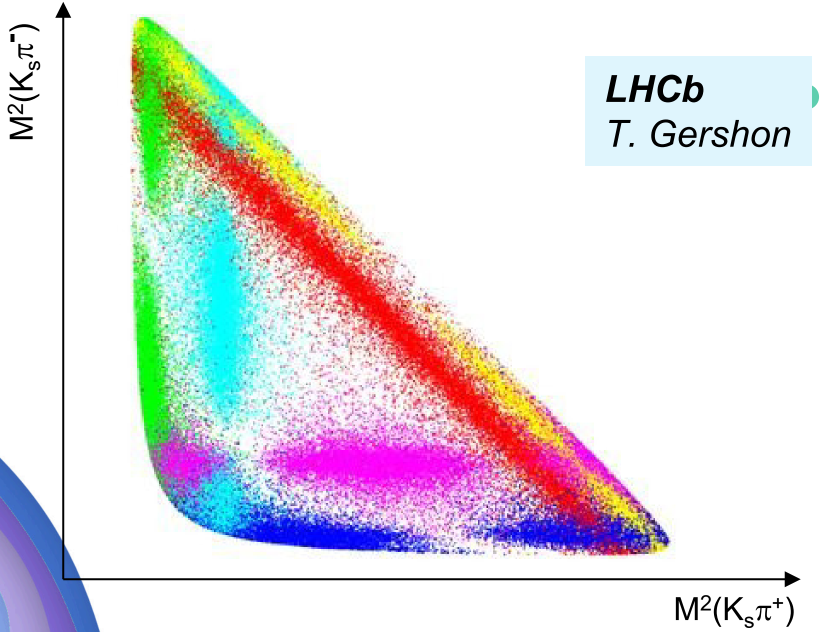
- Bands, bumps $\Rightarrow$ true resonances
- Interference effects
- Angular modulation
- Threshold effects (cusps)

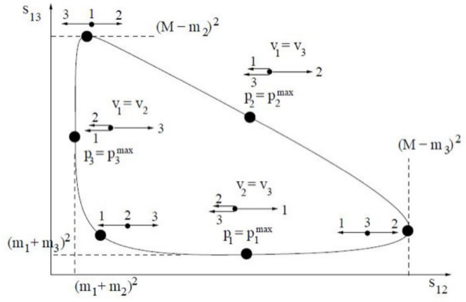
Physics

- Kinematic reflections
- Acceptance peaks
- Kinematic peaks
- Combinatorial background

Spurious (fake) effects

Dalitz plot as kinematics/dynamics visualizer





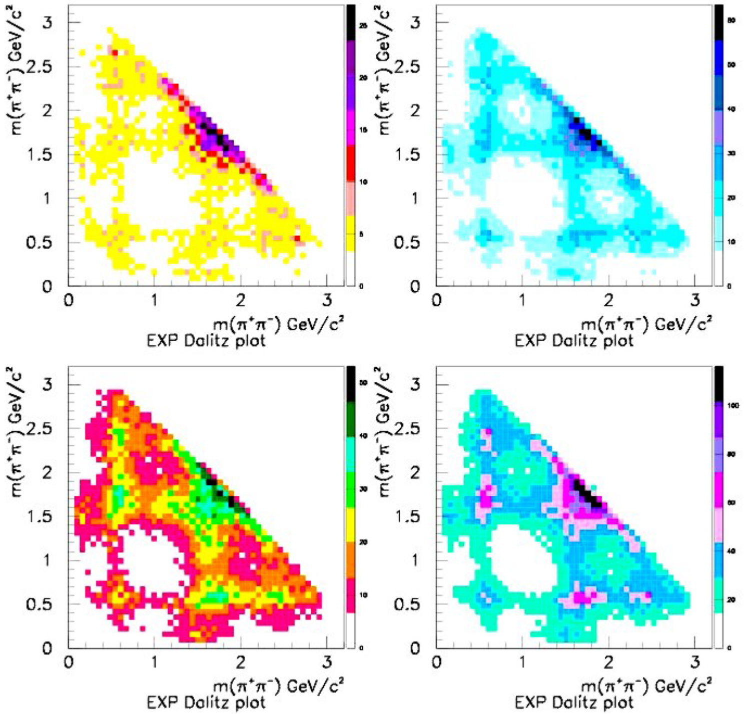
Reaction: $D^0 \rightarrow K_s\pi^+\pi^-$

- $\sqrt{s}$ = 1865 MeV, $I^G(J^P) = ½(0^-)$
- Resonances can be seen as bands and dips
- Holes can be related to kinematics and interference effects
- Which intermetdiate states can be spotted?
  - *Red: f_0(980)* (scalar)
  - *Yellow: ρ^0(770)* (vector)
  - *Green+Blue*: *K*(892)* (vector)
  - *Cyan+Magenta*: *K^*_2(1430)* (tensor)

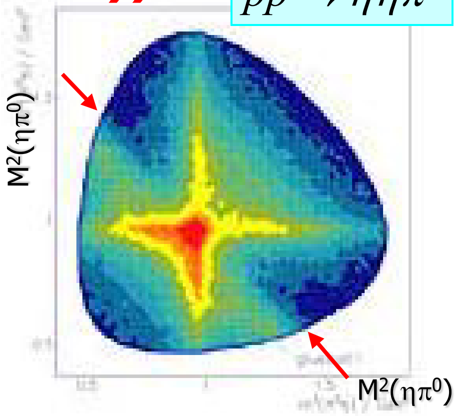
Some examples of Dalitz plots ($f_0(1500)$)

$\bar{n}p \rightarrow \pi^+\pi^+\pi^-$

OBELIX

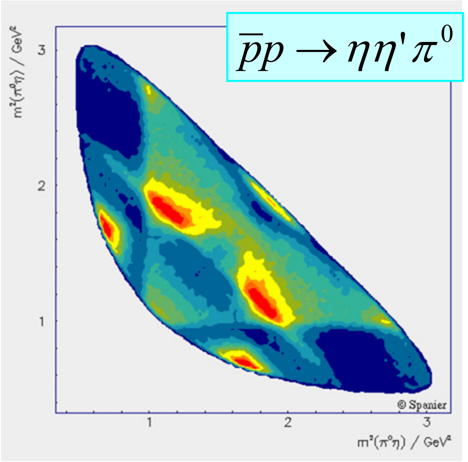


$\bar{p}p \rightarrow \eta\eta\pi^0$



Crystal Barrel

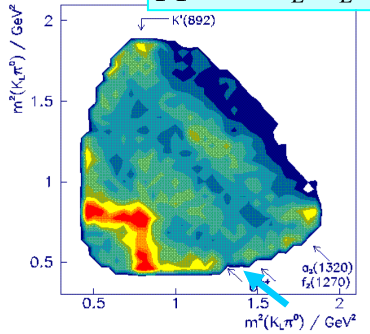
$\bar{p}p \rightarrow \eta\eta'\pi^0$



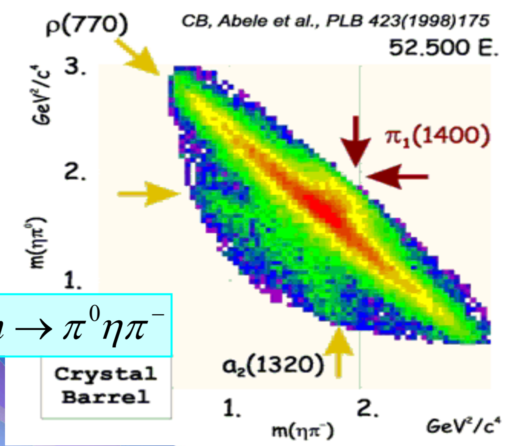
Some other Dalitz plots – $f_0(1500)$ & $\pi_1(1400)$

Crystal Barrel

$\bar{p}p \to K_L^0 K_L^0 \pi^0$



$\bar{p}n \to \pi^0 \eta \pi^-$

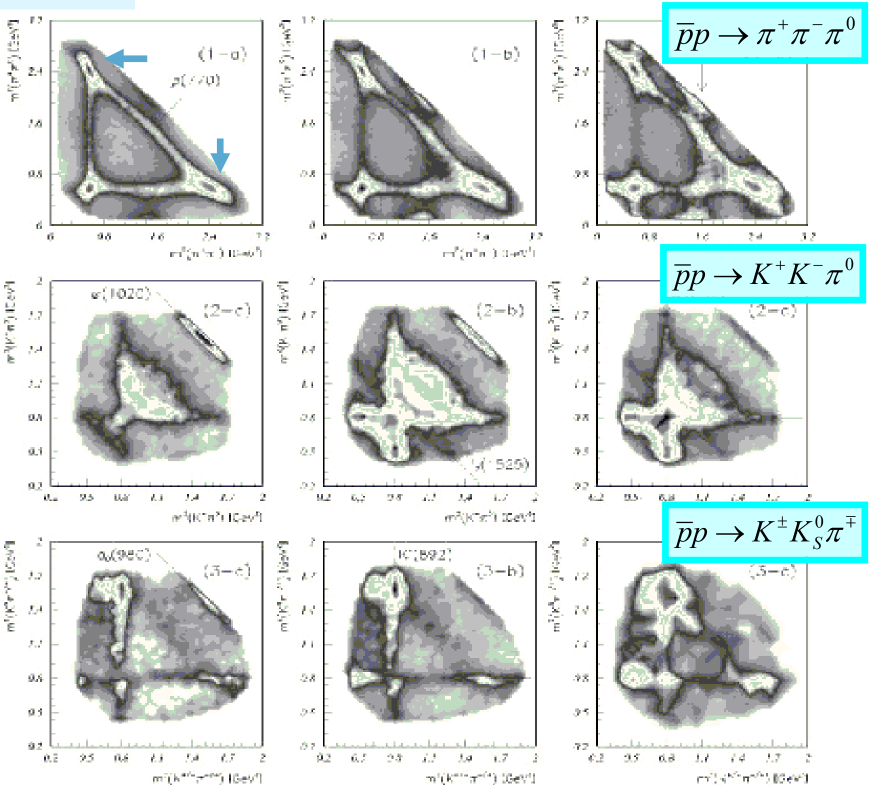


OBELIX

$\bar{p}p \to \pi^+\pi^-\pi^0$

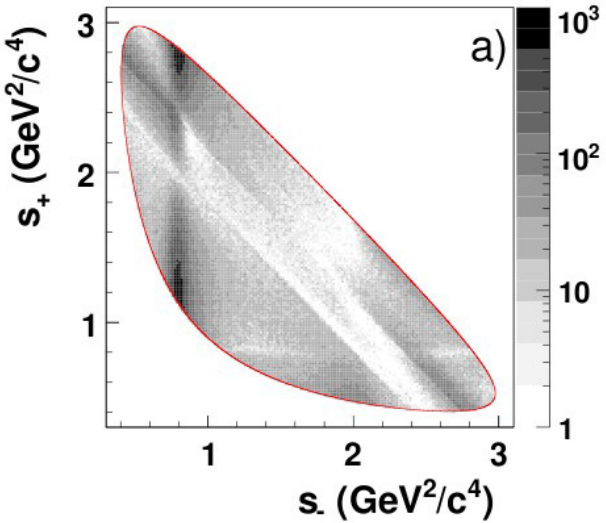
$\bar{p}p \to K^+K^-\pi^0$

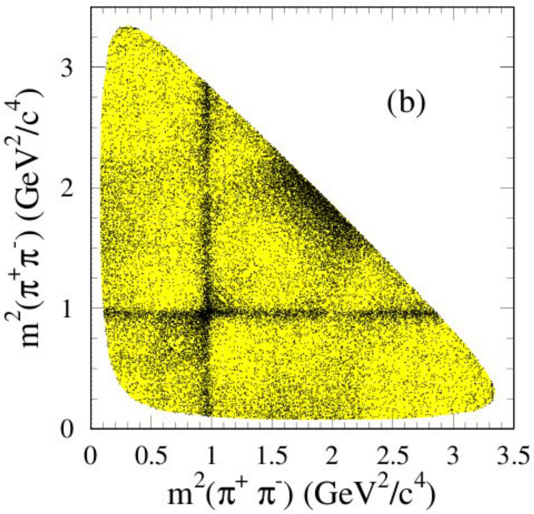
$\bar{p}p \to K^\pm K_S^0 \pi^\mp$

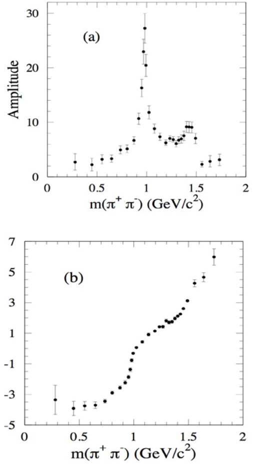


Liquid H_2 | NP gas H_2 | Low pressure H_2

A few more Dalitz plots examples: *BaBar*







- Reaction: $D^0 \to K_s\pi^+\pi^-$
  - *BaBar* PRL**105** (2010), 081803
  - *√s = 1865 MeV, I(J^P) = ½(0⁻)*
  - *K* production and interference patterns*
- Reaction: $D_s^+ \to \pi^+\pi^+\pi^-$
  - *BaBar* PR D**79** (2009), 032003
  - *√s = 1968 MeV, I(J^P) = 0(0⁻)*
  - *f_0 production + ($\pi\pi$) S-wave*

Note: the final states have the same particle content but the shapes are very different!

Let’s work with some data!

- Open and run the following Google Colab notebooks
  - Basics/1_evaluateInvariantMass
  - Basics/2_evaluateInvariantMasses_g14Data
  - Basics/3_relativisticKinematics
  - Basics/4_angles
  - Basics/5_phaseSpaceSimulation

Now you take the steering wheel!

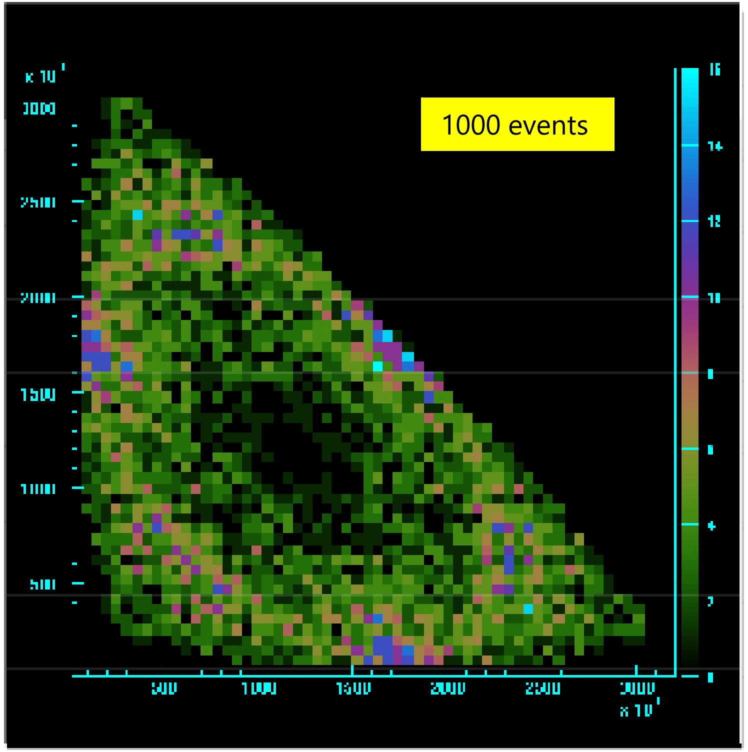
- Make your own notebooks and look at some new data!
  - Get inspiration from exercise 1 and 2
  - Plot the Dalitz Plot distributions and invariant mass projections for the data sets you may find in the folder Data Files/Spectroscopy/FurtherFun
  - You may find data (.csv files format) for the following reactions (already selected, but rather loosely) :
    - CLAS data at 5 GeV
      - exp. data: $\gamma p \rightarrow \pi^+ K^+ K^- n_{miss}$
      - exp. data: $\gamma p \rightarrow p K^+ \gamma\gamma K^-_{miss}$
      - exp. data: $\gamma p \rightarrow p K^+ K^- \eta_{miss}$
      - exp. data: $\gamma p \rightarrow p K^+ K^- \pi^0_{miss}$
      - exp. data: $\gamma p \rightarrow p K^+ \pi^- K_L$
      - exp. data: $\gamma p \rightarrow p \pi^+ \pi^- K^+ \pi^-_{miss}$
    - OBELIX data up to 405 MeV/c: work out a many-body reaction
      - $\bar{n} p \rightarrow \pi^+ \pi^+ \pi^+ \pi^- \pi^-$ exp. data and MC generated data

Statistics is a basic issue...

- Large statistics:
  - Better resolution
  - Cleaner/easier intermediate states assessment

$$\bar{p}p \rightarrow \pi^0\pi^0\pi^0$$

Crystal Barrel @ LEAR
C. Amsler et al., EPJ **C23**(2002), 29



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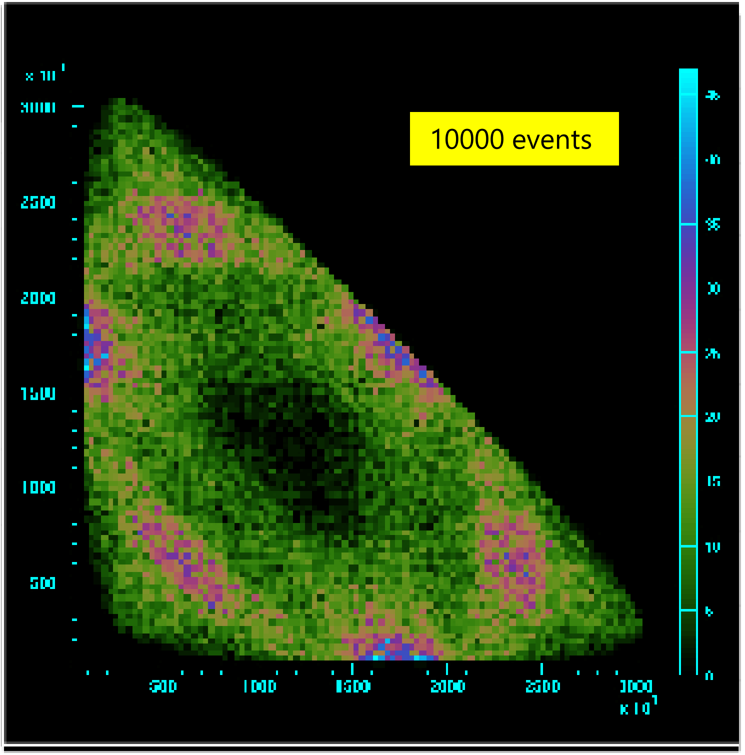

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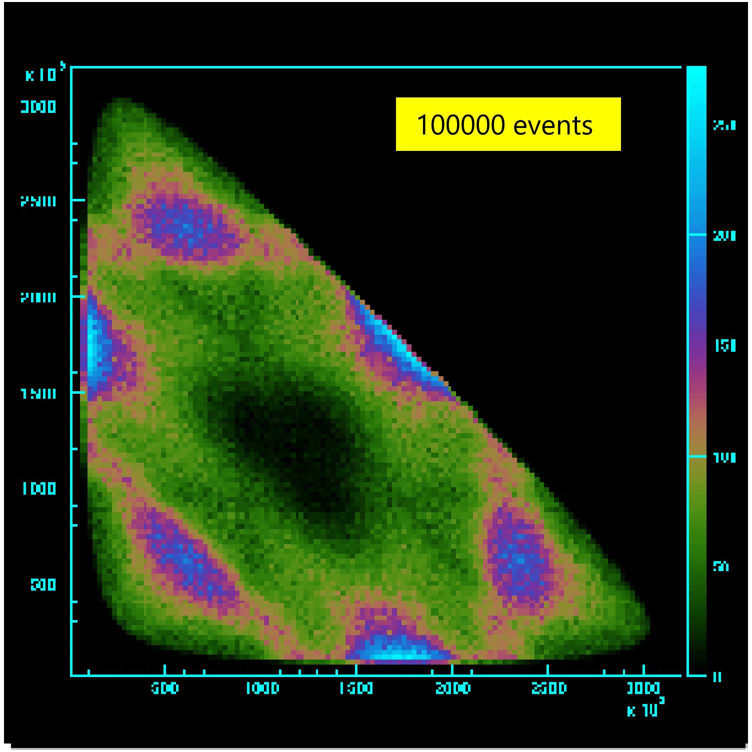
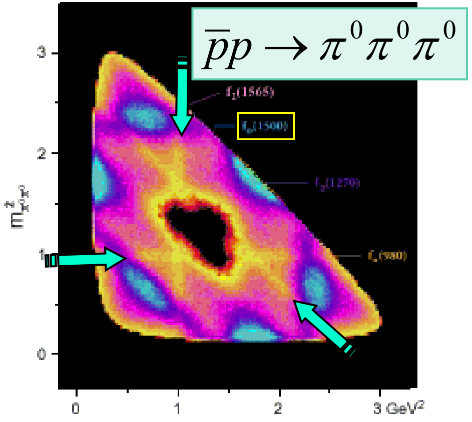
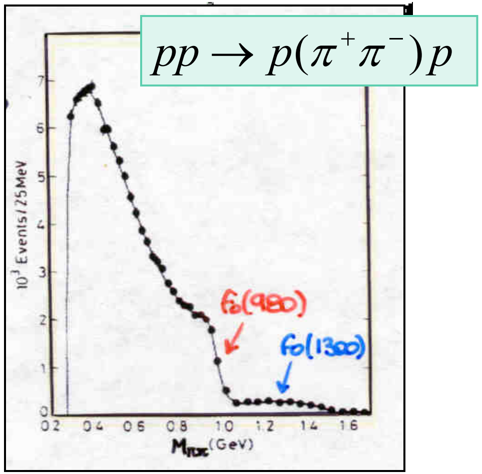
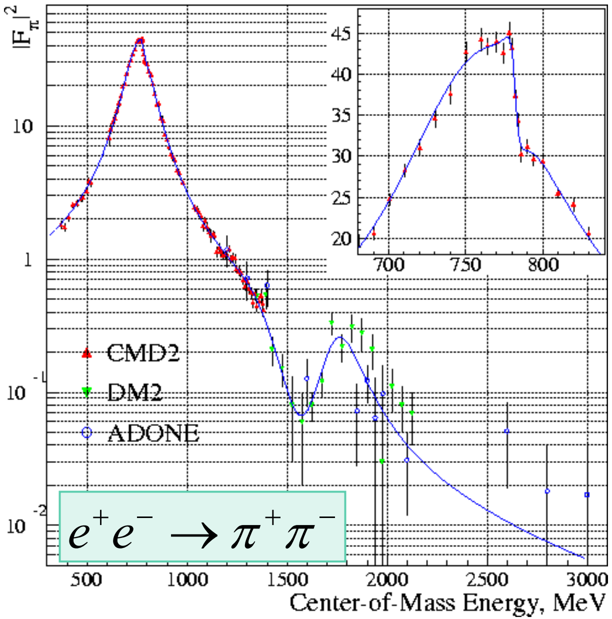


Not only peaks but also holes and dips…





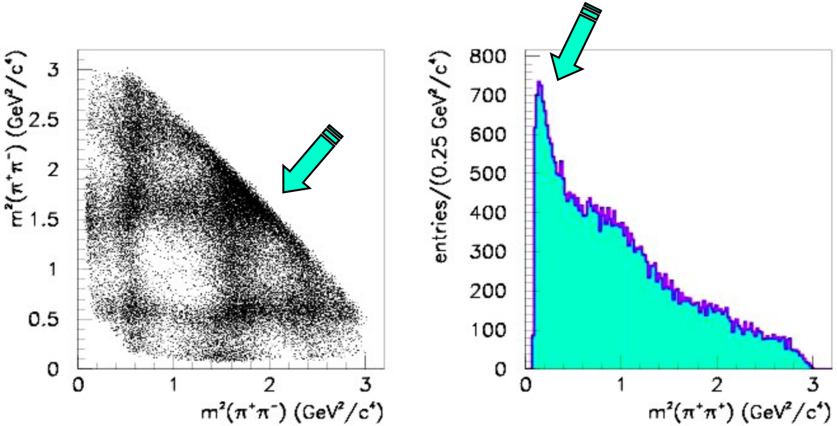


The $f_0(980)$ case: it's not a peak!

Spurious effects:
kinematic reflections & kinematic peaks

- Kinematic reflections
  - Geometrical effects emerging from the projection of a 2D spectrum with non-flat structures along one axis
  - Not linked to any resonance production

- Kinematic peaks
  - If $\beta_{cm} > \beta_{res}$ some momentum accumulations occur due to the relative motion between the system center of mass and the resonance
  - "cusp-effects" at threshold opening



(OBELIX @ LEAR)

$$\overline{n}p \to \pi^+\pi^+\pi^-$$

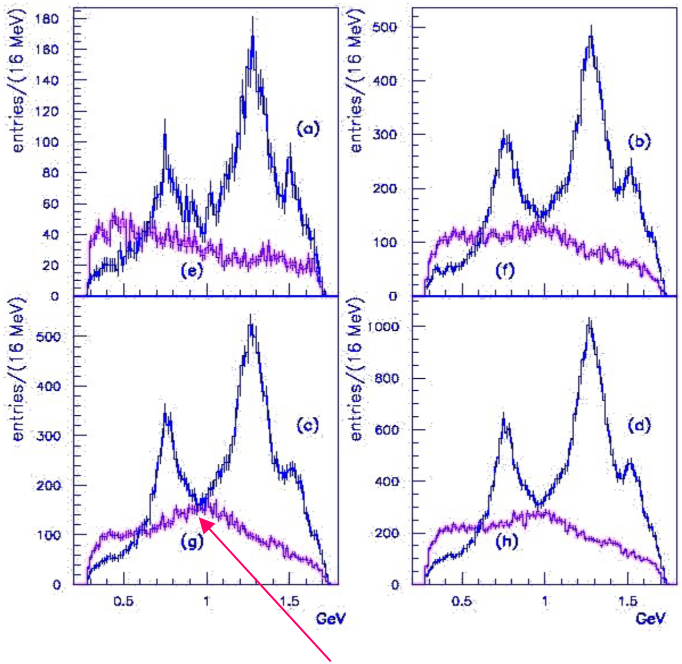
Combinatorial background

- The presence of two (or more) identical particles in the final state gives rise to a large structureless combinatorial background formed by the particles which participate weakly to the resonances

- The combinatorial background is not a dramatic problem since it may be correctly reproduced by the FSI amplitude

- The presence of the combinatorial background can however mask the effective contributions of some resonances

$$\bar{n}p \rightarrow \pi^+\pi^+\pi^-$$



Kinematic reflection!

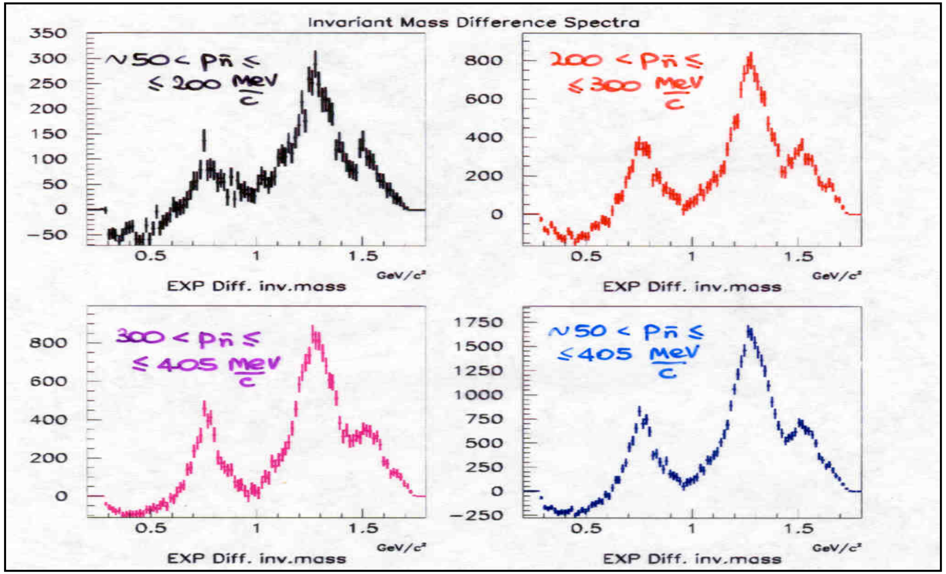
Practical estimation of combinatorial background effects: many-body Final State (3 or more particles of the same kind)

- **Difference spectrum method**
  - Subtraction from the neutral system invariant mass the like-charge combination
- Qualitative method to infer:
  - How many resonant states are present and visible
  - How large is the phase space contribution
  - If a PWA can be useful/useless
- Not suitable to get quantitative evaluations
  - No interference effect taken into account
- Valid under some basic hypotheses
  - Only one particle of the group of like-charged participates to the resonance
  - Change conjugation invariance
- Method reliability improves with:
  - Increase of resonance mass
  - Decrease of its width
  - Increase of final state multiplicity

Example I: difference spectrum in the n̄p→π⁺ π⁺ π⁻ case

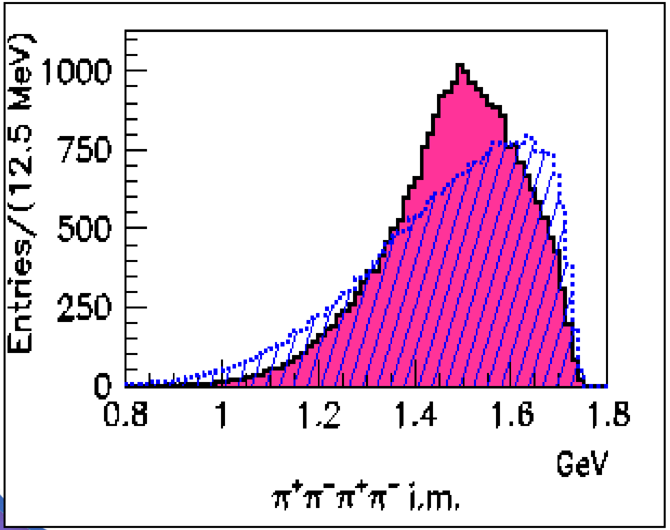
$$
\begin{aligned}
&D(\overline{n}p \to (\pi_1^+\pi_3^-)\,\pi_2^+) \\
&= |BW(\pi_1^+\pi_3^-)|^2 LIPS[(\pi_1^+\pi_3^-)\pi_2^+] + \\
&\quad \cancel{LIPS[(\pi_2^+\pi_3^-)\pi_1^+]} - \cancel{LIPS[(\pi_1^+\pi_2^+)\pi_3^-]} = \\
&\qquad |BW(\pi_1^+\pi_3^-)|^2 LIPS[(\pi_1^+\pi_3^-)\pi_2^+]
\end{aligned}
$$

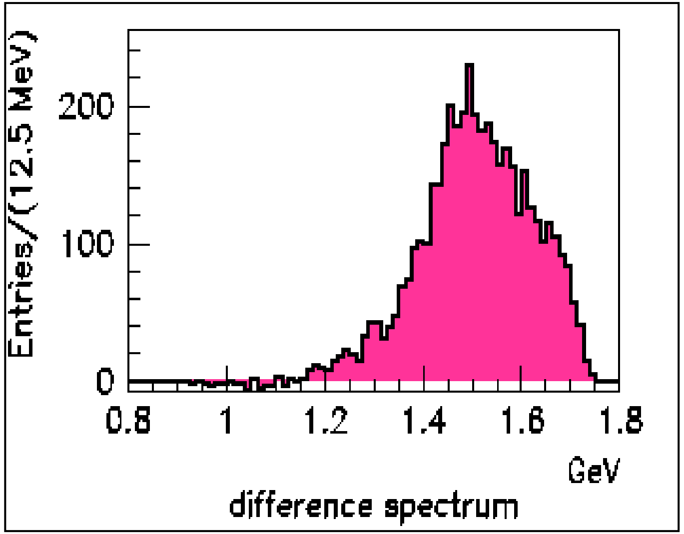
- Peaks appear more definite
- Good qualitative method to judge the presence of resonant structures
- Negative regions: interference contributions



Example II: difference spectrum in the $\bar{n}p \to 3\pi^+ 2\pi^-$ case

- Method to identify the largest contributions to the observed invariant mass distribution





Acceptance effects

- For a perfect apparatus the theoretical Dalitz plot density is given by

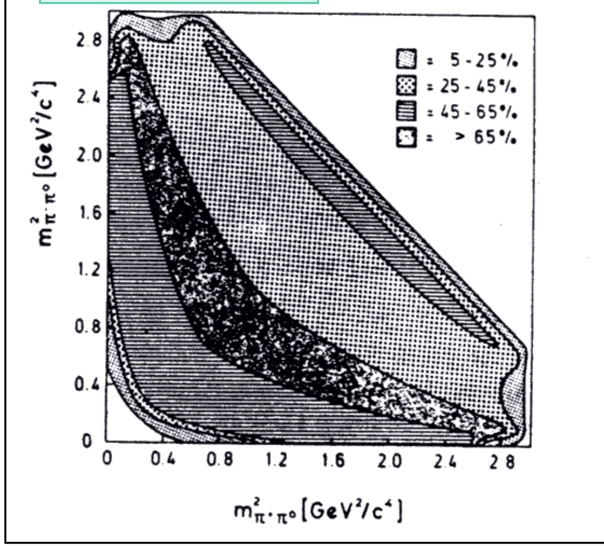
$$D(X,Y) = |T(X,Y)|^2 LIPS(X,Y)$$

  - Case of bubble chamber experiments (4π acceptance)
  - In general: acceptance effects must be properly taken into account in the amplitudes

- The true values X′, Y′ are smeared out by the acceptance function ε to provide the observed values X and Y

$$D(X,Y) = \int_{apparatus} |T(X',Y')|^2 LIPS(X',Y')\varepsilon(X',Y',X,Y)dX'dY'$$

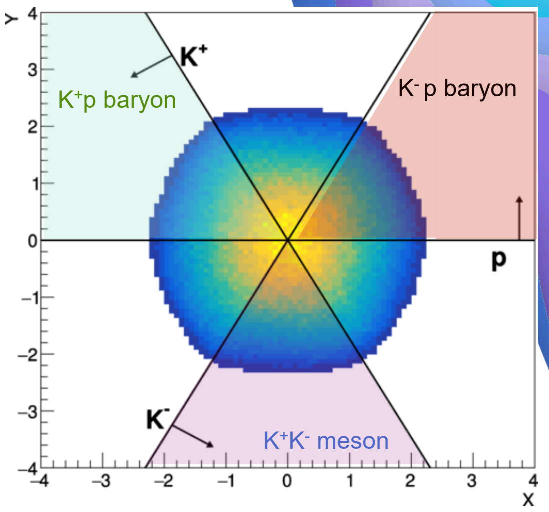
$\bar{p}p \to \pi^+\pi^-\pi^0$



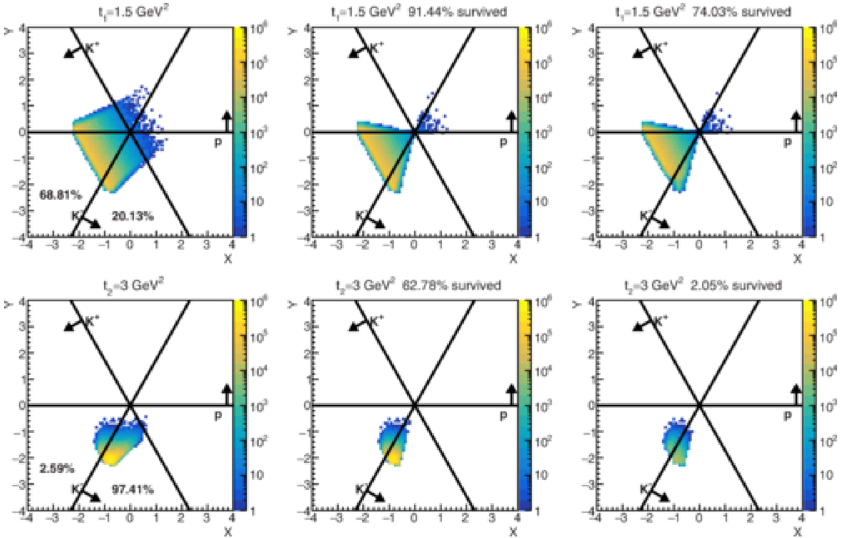
ASTERIX @ LEAR, ZP 46 (1990), 191, 203

Longitudinal plots and Van Hove angles

$\gamma p \rightarrow pK^+K^-$



- Based on the observation that in hadron collisions at high energy (E_{lab} < 8 GeV) the transverse momenta remain small
  - Outgoing particles transverse momenta < 300, 400 MeV/c
- The longitudinal phase space for n particles can be parametrized by n-2 new angular variables



- The scatter plots obtained through these new angular variables enable to observe the signature of meson/baryon bound states, and to apply new phase-space cuts which can enhance their contributions

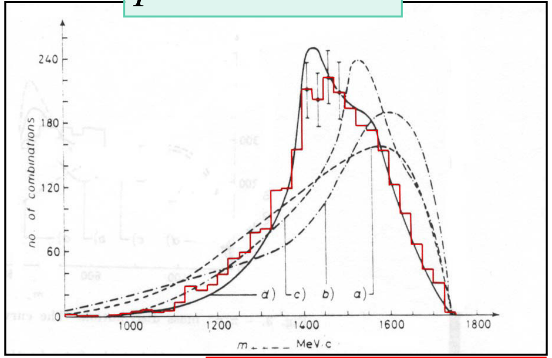
Colab notebook: longitudinal plots

… from the practical point of view …

- Build as many distributions as possible (invariant mass of any particle combination, angular distributions, etc) to infer details for formulating a starting hypothesis

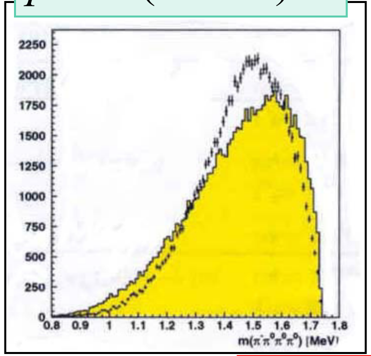
- Example: annihilation in 5 pion final state

$\bar{p}n \to 2\pi^+ 3\pi^-$

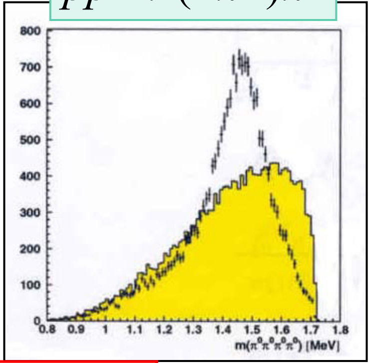


Bettini et al.,1966

$\bar{p}n \to (3\pi^0\pi^-)\pi^0$



$\bar{p}p \to (4\pi^0)\pi^0$



CRYSTAL BARREL

Summary: experimental input features

- With three particles in the final state: build a Dalitz plot
- Correct it by acceptance
- Infer the presence of resonant/bound states from deviations from uniformity
    - Bands (possible modulation according to spin and angular emission)
    - Furrows/dips
    - Holes

- With many particles in the final state: build as many 1-D histogram as possibile, the most uncorrelated as possible
    - Angular distributions
    - Invariant mass/missing mass distributions

- Decide which 2-body intermediate states are needed to describe the amplitude
    - Formulate several hypotheses, check/choose the one which provides the better representation for the data