Deep Exclusive Reactions

Lecture 1: Nucleon structure studies with the electromagnetic probe

- Elastic scattering: form factors
- DIS: structure function
- Exclusive reactions: Generalized Parton Distributions

Lecture 2: Deeply Virtual Compton Scattering

- GPDs and DVCS
- DVCS on the proton with JLab@6 GeV
- Extraction of GPDs from data
- Proton tomography and forces in the proton

Lecture 3: DVCS and beyond

- DVMP
- New DVCS experiments@12 GeV
- TCS

Lecture 4: Perspectives

- Upgrades at JLab
- GPDs at the EIC

Lecture 5: Tutorial

Data analysis techniques for exclusive reactions





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Data analysis techniques for exclusive reactions



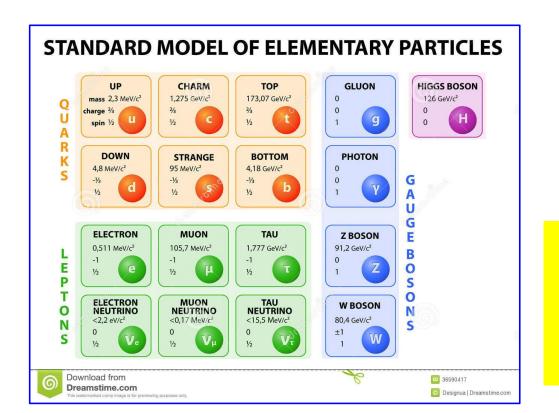


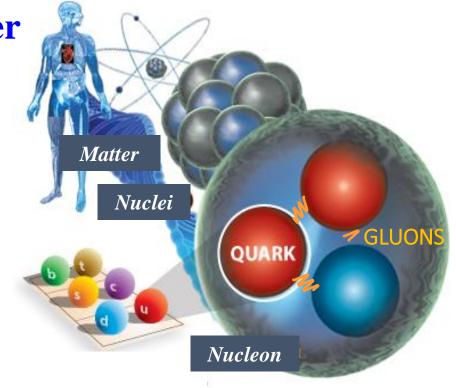
QCD at the heart of matter

- Protons and neutrons are the building blocks of atomic nuclei
- Nucleons provide ~99% of the mass of the visible universe
- ~99% of nucleon mass arises from the dynamics and interactions between its constituents (quarks and gluons)



Quantum Chromodynamics (QCD)





$$L = -\frac{1}{4} \; G^a_{\mu\nu} G^{a\mu\nu} + \sum_f \bar{\psi}_f \bigg(i \not\!\! \partial + g \not\!\! A - m_f \bigg) \psi_f \label{eq:L}$$

QCD contains in principle all the answers BUT...

QCD is still not unsolvable in non-perturbative regions

Insights into "soft" phenomena, such as the structure of hadrons, exist through qualitative **models** and quantitative **numerical calculations** (lattice)

The proton: QCD at work!

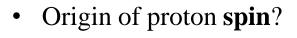
What we know about the content of the proton:

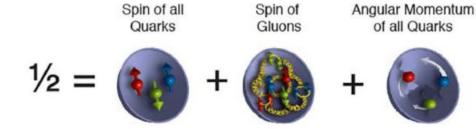
- 2 up quarks $(q_u = 2/3 e) + 1 down quark (q_d = -1/3 e)$
- Any number of quark-antiquark pairs (sea)
- Any number of gluons

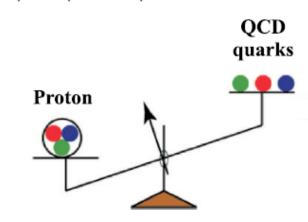
$$|p\rangle = |uud\rangle + |uudq\bar{q}\rangle + |uudg\rangle + \dots$$

Fundamental questions:

- Origin of proton mass?
- → Only a small fraction comes from the actual quark masses
- → Most of it comes from the *motion of quarks and gluons*







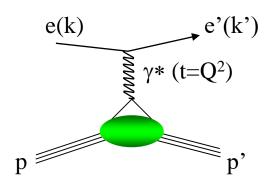
Angular Momentum of Gluons

Electron scattering: the ideal tool to study nucleon structure

Electrons are **structureless** and interact only **electromagnetically** via the exchange of **virtual photons**

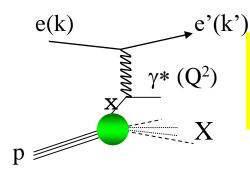
Resolution of the probe (in the one-photon exchange approximation): $Q^2 = -(k-k')^2$

➤ 1956: Elastic scattering ep→e'p' (Hofstadter, Nobel Prize 1961)

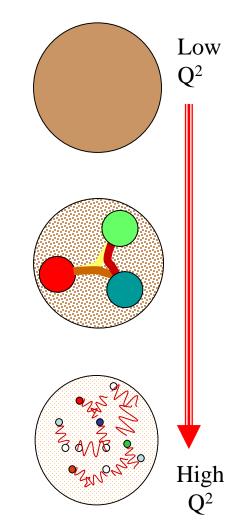


- The proton is **not a point-like object**
- Measurement of charge distributions of the proton: **Electromagnetic form factors** $F_1(t)$, $F_2(t)$

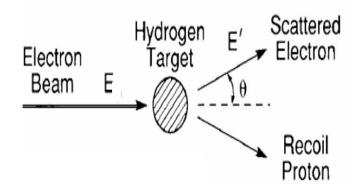
➤ 1967: Deep inelastic scattering (DIS) ep→e'X (Friedman, Kendall, Taylor, Nobel Prize 1990)



- Discovery of the quarks (or "partons")
- Measurement of the momentum and spin distributions of the partons: Parton Distribution Functions (PDF) q(x), $\Delta q(x)$



Elastic scattering: the origins



$$\sigma_M = \frac{\alpha_{QED}^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}$$

Mott cross section: scattering off a point-like object

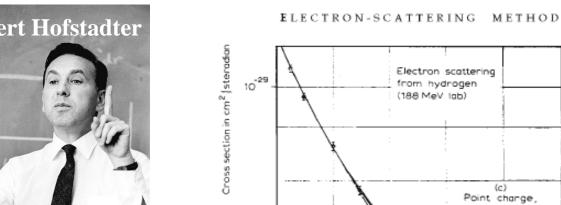
$$\frac{d\sigma}{d\Omega} = \sigma_M [(F_1^2 + \kappa^2 \tau F_2^2) + 2\tau (F_1 + \kappa F_2)^2 \tan^2(\frac{\theta_e}{2})]$$

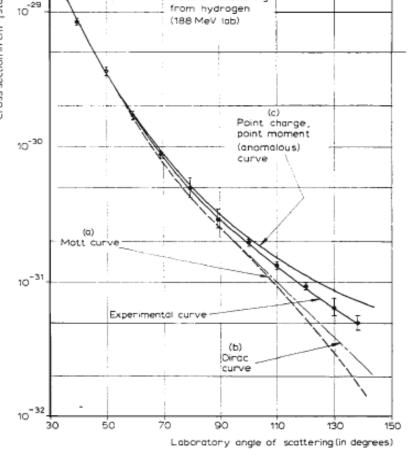
$$\frac{d\sigma}{d\Omega} = \sigma_M [\frac{(G_E^p)^2 + \tau (G_M^p)^2}{1 + \tau} + 2\tau (G_M^p)^2 \tan^2(\frac{\theta_e}{2})]$$



Rosenbluth's formulas for elastic cross section:

- F₁, F₂: Dirac and Pauli FFs
- G_E, G_M: electric and magnetic FF (Sachs FFs)



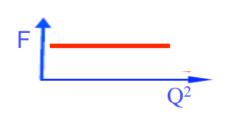


$$\tau = Q^2/(4M_N^2)$$

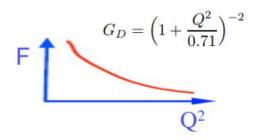
 κ nucleon anomalous magnetic moment

$$\begin{split} G_E(Q^2) &= F_1(Q^2) - \tau \kappa F_2(Q^2); \quad G_E^p(0) = 1; \quad G_E^n(0) = 0; \\ G_M(Q^2) &= F_1(Q^2) + \kappa F_2(Q^2); \quad G_M^{p,n}(0) = \mu_{p,n}, \end{split}$$

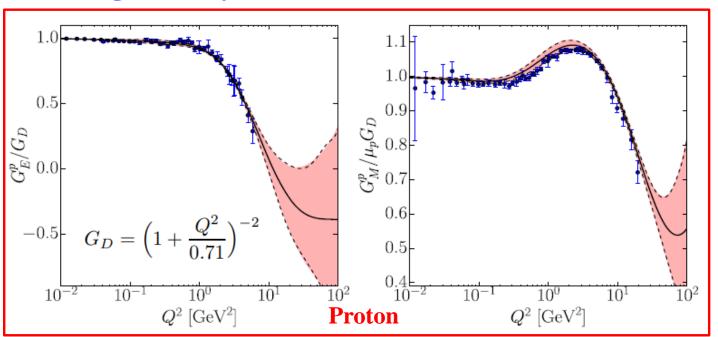
Elastic scattering: today

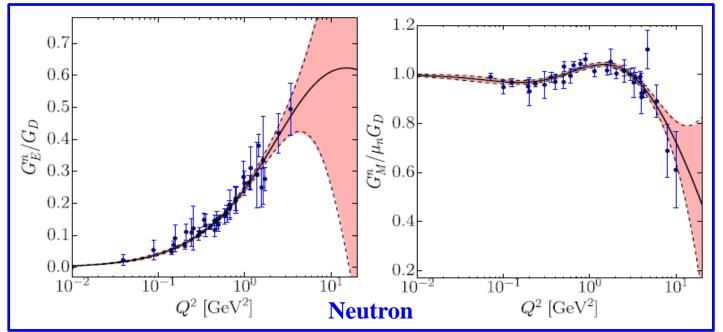


Point charge: $F(Q^2)$ =constant photon always sees all of the proton's charge



Elastic ep scattering:
"dipole" form factor, photon
sees less charge as Q²
increases





Measurements done in: LAL Orsay, Cambridge, Bonn, SLAC, Desy, Mainz, CEA-Saclay, NiKHEF, MIT-Bates, MAMI@Mainz, CEBAF@JLab

Nucleon charge densities from elastic scattering

$$G_E(k^2) = \int_{\mathbb{R}^3} \mathrm{d}^3 r \, e^{-ik \cdot r} \rho_E(r)$$

$$D_y[fm]$$

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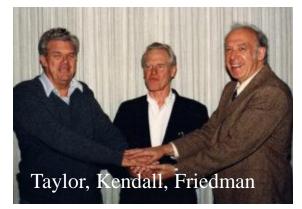
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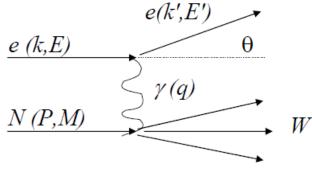
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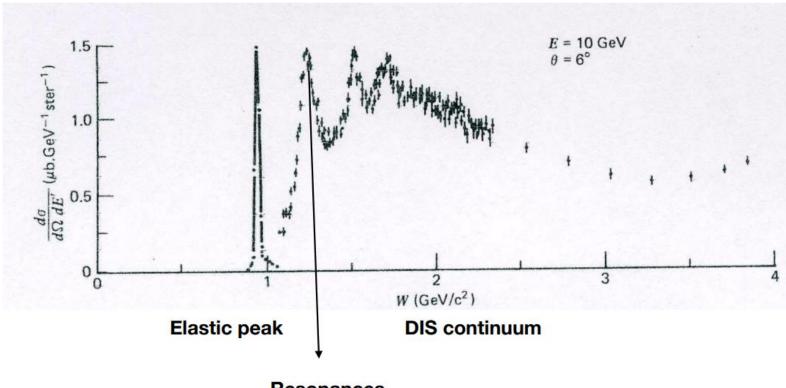


Deep Inelastic Scattering: the origins



$$q^2 = -4EE'\sin^2\frac{\theta}{2} \quad v = \frac{p \cdot q}{M} = E - E'$$

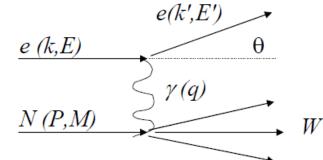
$$W = (P+q)^2 = P^2 + 2P \cdot q + q^2 = M^2 + 2Mv - Q^2$$



Resonances $ep \rightarrow e\Delta$

Taylor, Kendall, Friedman

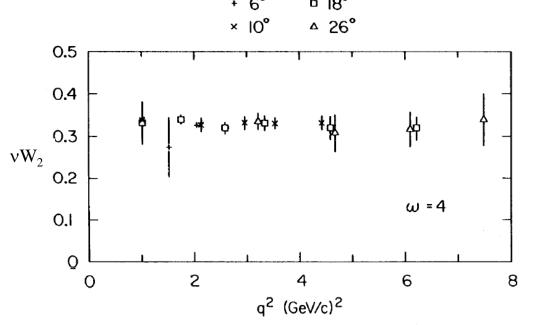
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$$W = (P + q)^{2} = P^{2} + 2P \cdot q + q^{2} = M^{2} + 2Mv - Q^{2}$$

$$\frac{d^2\sigma}{dQdE'}(E,E',\theta) = \sigma_M \left(W_2(v,q^2) + 2W_1(v,q^2) tan^2 \frac{\theta}{2} \right)$$



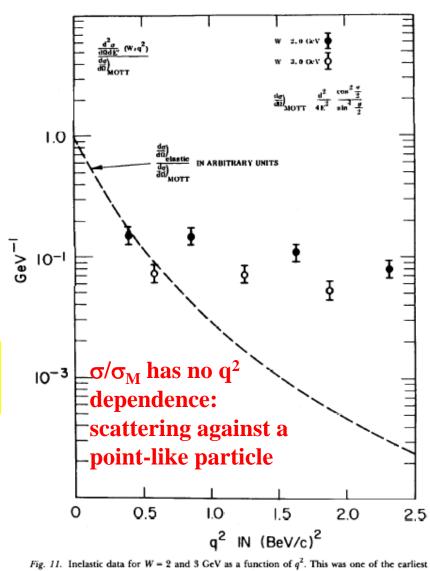
Bjorken scaling

$$2MW_1(v, q^2) = F_1(\omega)$$

$$vW_2(v, q^2) = F_2(\omega)$$

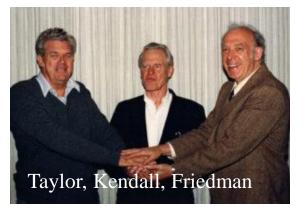
$$\omega = \frac{1}{x} = \frac{2Mv}{O^2}$$

$$x = \frac{Q^2}{2Mv} = \frac{Q^2}{2P \cdot q}$$

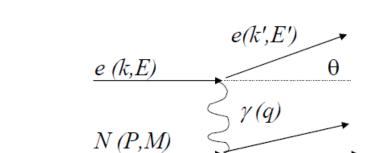


H. W. Kendall

Fig. 11. Inelastic data for W = 2 and 3 GeV as a function of q^2 . This was one of the earliest examples of the relatively large cross sections and weak q^2 dependence that were later found to characterize the deep inelastic scattering and which suggested point-like nucleon constituents. The q^2 dependence of elastic scattering is shown also; these cross sections have been divided by σ_M



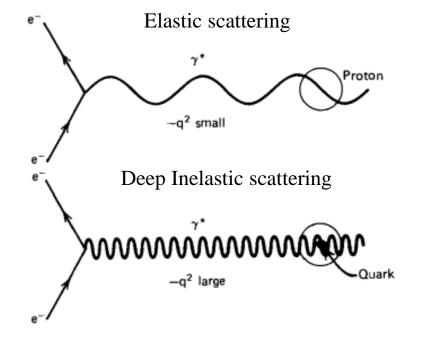
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 $v = \frac{p \cdot q}{M} = E - E'$

$$W = (P+q)^2 = P^2 + 2P \cdot q + q^2 = M^2 + 2Mv - Q^2$$

$$\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta) = \sigma_M \left(W_2(v,q^2) + 2W_1(v,q^2)tan^2\frac{\theta}{2}\right)$$



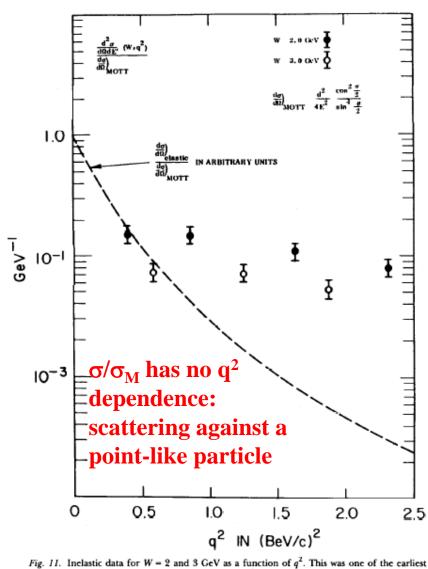
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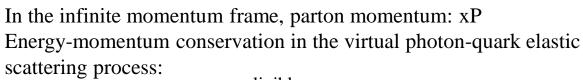
The parton model

Parton model (Feynman, 1969)

- Proton composed of point-like partons, from which the electrons scattered incoherently
- In an infinite momentum frame of reference, time dilation slows down the motion of constituents
- Partons are assumed not to interact with one another while the virtual photon is exchanged (impulse approximation)

In this theory, electrons scatter off "free" constituents and the scattering reflects the properties and motion of the constituents

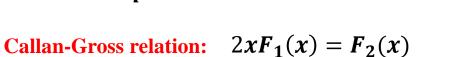
The assumption of near-vanishing parton-parton interaction during electron scattering at high Q² was later shown to be a property of QCD known as asymptotic freedom



$$(x\mathbf{P} + \mathbf{q})^2 = x^2 \mathbf{M}^2 + 2x\mathbf{P} \cdot \mathbf{q} + Q^2$$

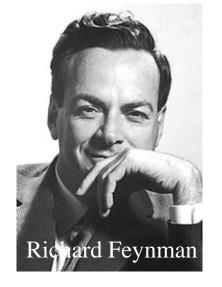
$$\Rightarrow x = \frac{Q^2}{2\mathbf{P} \cdot \mathbf{q}} = x_{Bjorken}$$

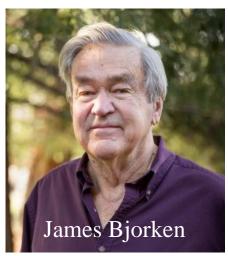
→ In the infinite momentum frame, the fraction $\Rightarrow x = \frac{Q^2}{2P \cdot q} = x_{Bjorken}$ $\Rightarrow x = \frac{Q^2}{2P \cdot q} = x_{Bjorken}$

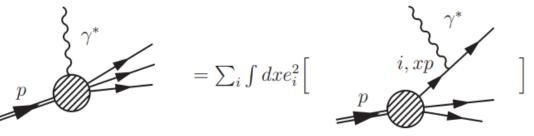


Verified experimentally, it reflects the fact that partons inside the proton are spin-1/2 particles

$$2xF_1(x) = F_2(x) = \sum_{i} e_i^2 x f_i(x)$$





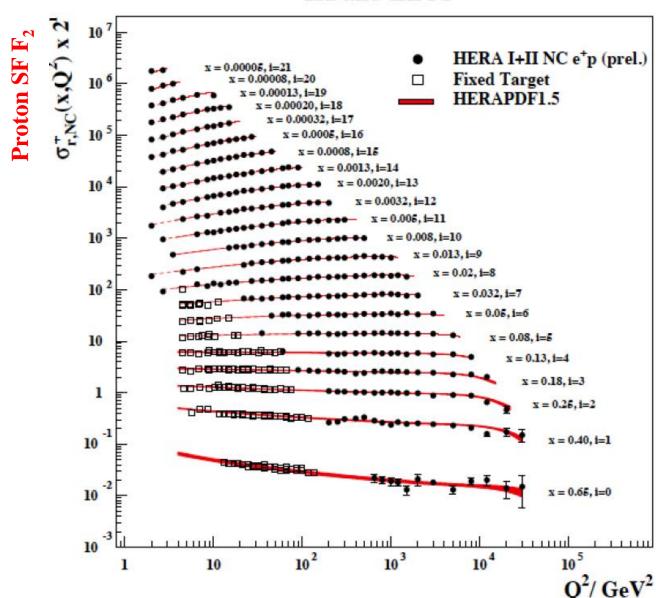


 $f_i(x)$: probability that the struck parton i carries a fraction x of the proton's momentum

$$f_i(x) = \frac{dP_i}{dx} = \frac{1}{p}$$

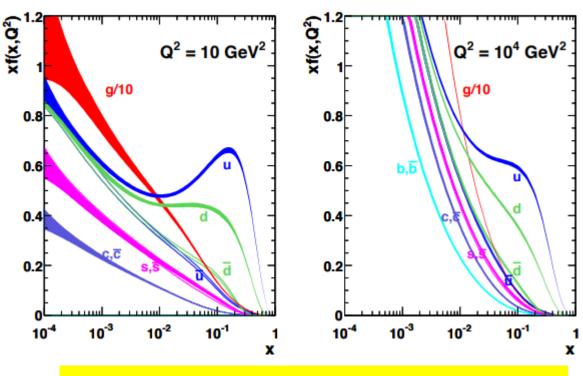
Deep Inelastic Scattering: today

H1 and ZEUS



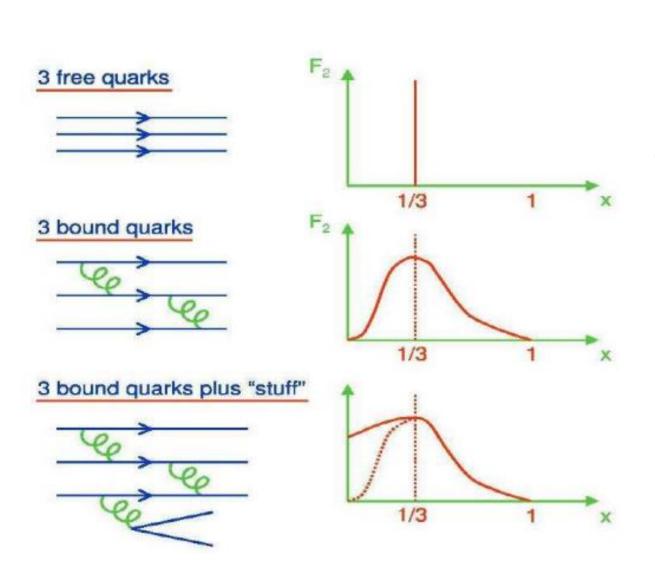
$$F_{2p}(x) = x \left[\frac{4}{9} (u_p(x) + \overline{u}_p(x)) + \frac{1}{9} (d_p(x) + \overline{d}_p(x)) + \frac{1}{9} (s_p(x) + \overline{s}_p(x)) \right]$$

MSTW 2008 NLO PDFs (68% C.L.)



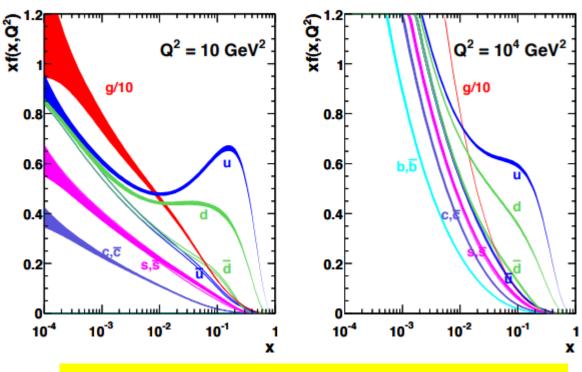
Momentum distributions of the valence *u* and *d* quarks, quark-antiquark sea and gluons

Deep Inelastic Scattering: today



$$F_{2p}(x) = x \left[\frac{4}{9} (u_p(x) + \overline{u}_p(x)) + \frac{1}{9} (d_p(x) + \overline{d}_p(x)) + \frac{1}{9} (s_p(x) + \overline{s}_p(x)) \right]$$

MSTW 2008 NLO PDFs (68% C.L.)



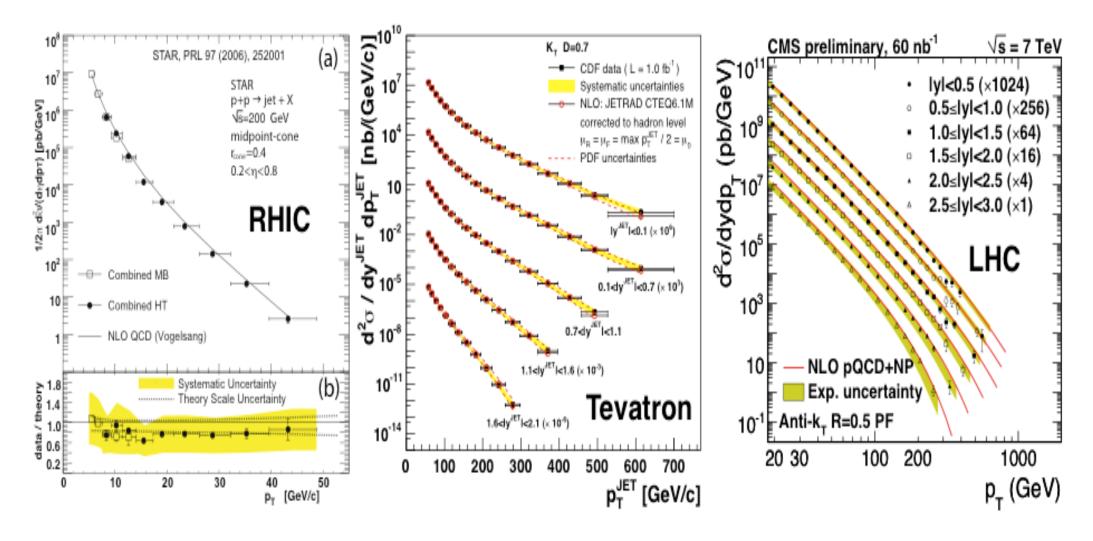
Momentum distributions of the valence *u* and *d* quarks, quark-antiquark sea and gluons

Valence quarks: x>0.1; sea quarks and gluons: x<0.1

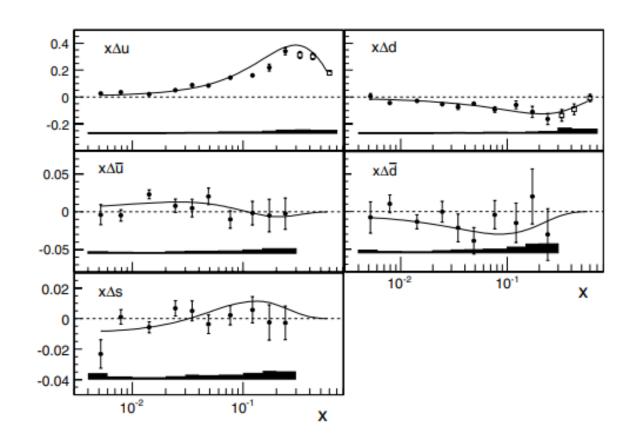
DIS: Successes...

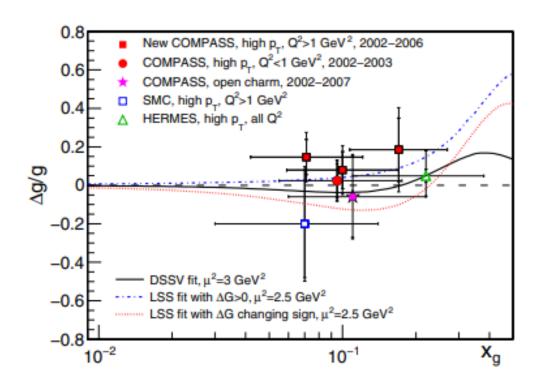
Measurements of F_2 in e-p at 0.3 TeV (HERA)

- → extraction quark and gluon PDFs
- \rightarrow pQCD fits for p-p and p-p at 0.2, 1.96, and 7 TeV



...and open questions





Nucleon spin: $\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L + \Delta G + L_g$

Intrinsic spin of the quarks $\Delta\Sigma \approx 30\%$

Intrinsic spin on the gluons $\Delta G \approx 20\%$

Orbital angular momentum of the quarks L?

Deep Inelastic Scattering: the QCD perspective

QCD factorization: at high virtuality Q² of the exchanged photon, the photon interacts with a single quark of the nucleon.

The complex, non-perturbative quarkgluon structure of the nucleon is encoded in the **structure functions** (**PDFs**).

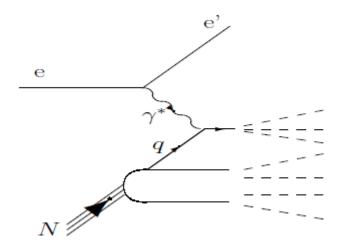
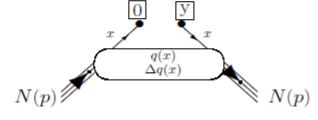


Illustration of the associated **non-local diagonal** matrix element accessed in DIS



The PDFs are **QCD operators** depending on spacetime coordinates. They are obtained as **one-dimensional Fourier transforms in the lightlike coordinate y^-** (at zero values of the other coordinates) as:

$$q(x) = \frac{p^{+}}{4\pi} \int dy^{-} e^{ixp^{+}y^{-}} \langle p|\bar{\psi}_{q}(0)\gamma^{+}\psi_{q}(y)|p\rangle \Big|_{y^{+}=\vec{y}_{\perp}=0},$$

$$\Delta q(x) = \frac{p^{+}}{4\pi} \int dy^{-} e^{ixp^{+}y^{-}} \langle pS_{\parallel}|\bar{\psi}_{q}(0)\gamma^{+}\gamma_{5}\psi_{q}(y)|pS_{\parallel}\rangle \Big|_{y^{+}=\vec{y}_{\perp}=0},$$

 Ψ_q : quark field of flavor q; p: initial and final nucleon momentum (it is the same for DIS by virtue of the optical theorem) x: momentum fraction of the struck quark; S_{\parallel} is the longitudinal nucleon spin projection

Light-front frame: the initial and final nucleons are collinear along the z-axis and the light-front components are:

$$a^{\pm} = (a^0 \pm a^3)/\sqrt{2}$$

 $a_{\perp} = (a^1, a^2) = a^i$

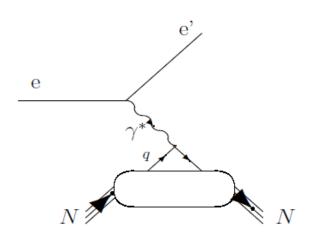
Non-local: the space-time coordinates of the initial and final quark are different

Diagonal: the momenta of the initial and final nucleon are the same

Elastic scattering: the QCD perspective

Elastic scattering: the struck quark remains in the nucleon.

The nucleon has changed its momentum in the process but it remains a nucleon



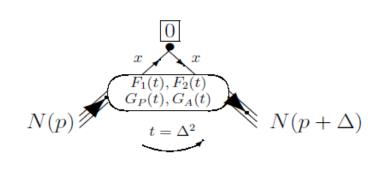


Illustration of the associated local non-diagonal matrix element accessed in elastic lepton-nucleon scattering.

The long-distance "soft" physics is factorized in the Form Factors (FF) $F_1^q(t)$, $F_2^q(t)$, $G_A^q(t)$ and $G_P^q(t)$, where $t = (p_N - p'_N)^2 = -Q^2$. In the **light-front frame**, the squared momentum **transfer** t is the conjugate variable of the impact parameter (= transverse position) \rightarrow the FFs reflect, via a Fourier transform, the spatial distributions of quarks in the plane transverse to the nucleon direction.

The FFs are related to the following **vector** and axial-vector QCD local operators in space-time coordinates:

$$\langle p'|\bar{\psi}_{q}(0)\gamma^{+}\psi_{q}(0)|p\rangle = F_{1}^{q}(t)\bar{N}(p')\gamma^{+}N(p) + F_{2}^{q}(t)\bar{N}(p')i\sigma^{+\nu}\frac{\Delta_{\nu}}{2m_{N}}N(p),$$

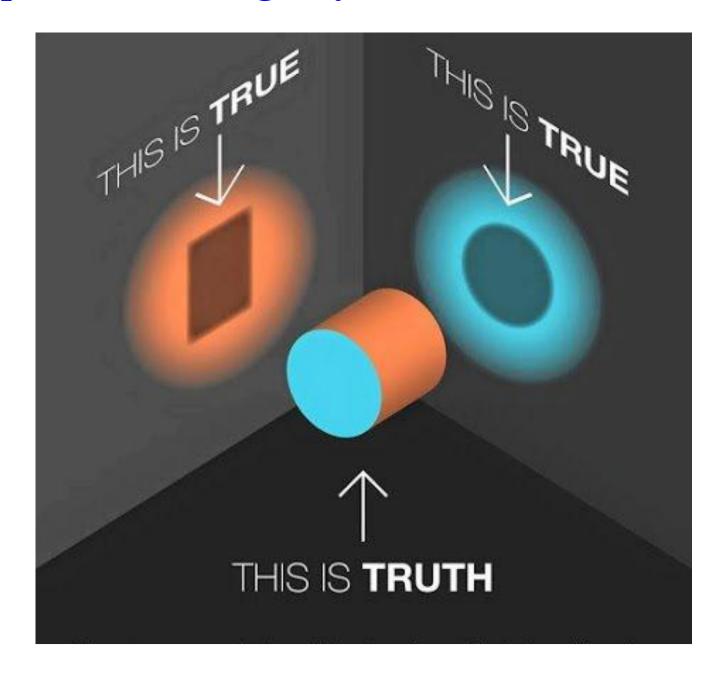
$$\langle p'|\bar{\psi}_{q}(0)\gamma^{+}\gamma_{5}\psi_{q}(0)|p\rangle = G_{A}^{q}(t)\bar{N}(p')\gamma^{+}\gamma_{5}N(p)$$

$$+ G_{P}^{q}(t)\bar{N}(p')\gamma_{5}\frac{\Delta^{+}}{2m_{N}}N(p),$$

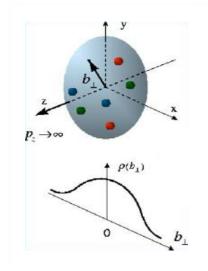
Local: the space-time coordinates of the initial and final quark are the same **Non-diagonal**: the momenta of the initial and final nucleon are different

G_A, G_P: weak interaction; accessed in neutrino scattering

Electron-proton scattering: beyond the one-dimensional picture

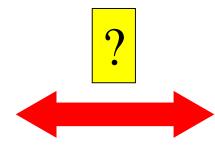


Electron-proton scattering: beyond the one-dimensional picture



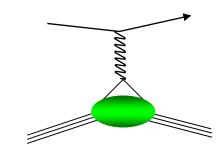
Form factors:

transverse quark distribution in coordinate space

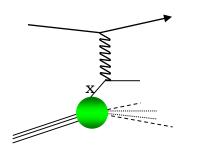




longitudinal quark distribution in momentum space

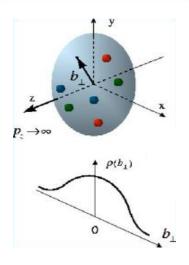


Elastic scattering



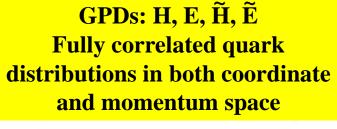
DIS

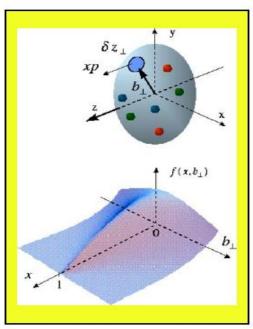
Electron-proton scattering: beyond the one-dimensional picture

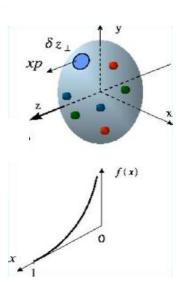


Form factors:

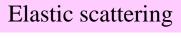
transverse quark distribution in coordinate space

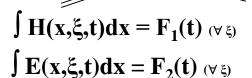






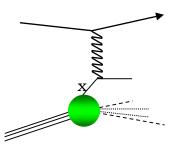
Parton distributions:
longitudinal
quark distribution
in momentum space





Accessible in hard exclusive processes

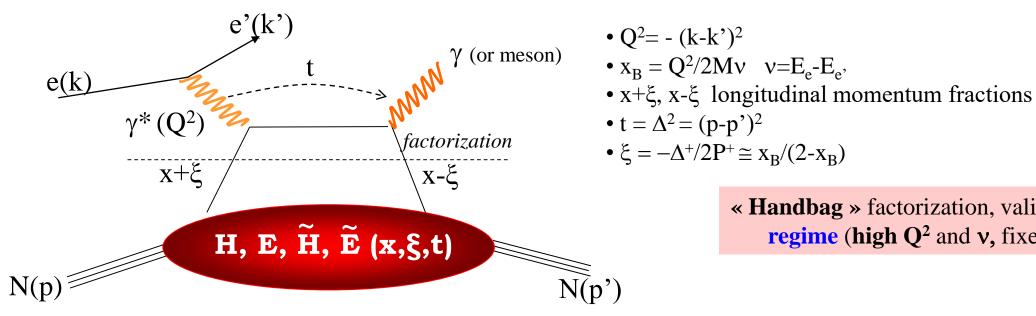
At high Q²
The final state is fully known



$$\mathbf{H}(\mathbf{x},\mathbf{0},\mathbf{0}) = \mathbf{q}(\mathbf{x}), \\ \mathbf{H}(\mathbf{x},\mathbf{0},\mathbf{0}) = \Delta \mathbf{q}(\mathbf{x})$$

DIS

Generalized Parton Distributions (GPDs)



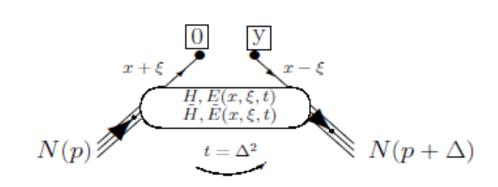
- $Q^2 = -(k-k')^2$

- $t = \Delta^2 = (p-p')^2$
- $\xi = -\Delta^+/2P^+ \cong x_B/(2-x_B)$

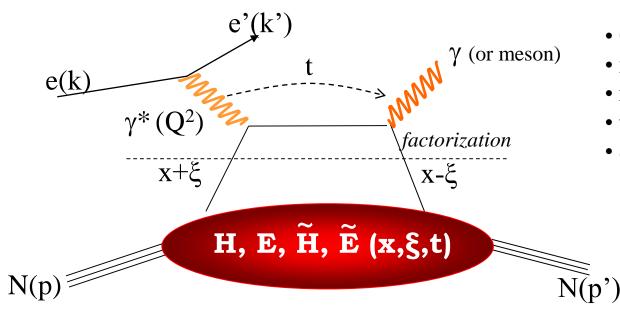
« Handbag » factorization, valid in the Bjorken regime (high Q^2 and ν , fixed x_B), t < Q^2

GPDs: Fourier transforms of non-local, non-diagonal QCD operators

$$\begin{split} &\frac{P^{+}}{2\pi} \int dy^{-}e^{ixP^{+}y^{-}} \langle p^{'} | \bar{\psi}_{q}(0) \gamma^{+} \psi_{q}(y) | p \rangle \bigg|_{y^{+}=\vec{y}_{\perp}=0} \\ &= H^{q}(x,\xi,t) \; \bar{N}(p^{'}) \gamma^{+} N(p) \; + \; E^{q}(x,\xi,t) \; \bar{N}(p^{'}) i \sigma^{+\nu} \frac{\Delta_{\nu}}{2m_{N}} N(p) \; , \\ &\frac{P^{+}}{2\pi} \int dy^{-}e^{ixP^{+}y^{-}} \langle p^{'} | \bar{\psi}_{q}(0) \gamma^{+} \gamma^{5} \psi_{q}(y) | p \rangle \bigg|_{y^{+}=\vec{y}_{\perp}=0} \\ &= \tilde{H}^{q}(x,\xi,t) \; \bar{N}(p^{'}) \gamma^{+} \gamma_{5} N(p) \; + \; \tilde{E}^{q}(x,\xi,t) \; \bar{N}(p^{'}) \gamma_{5} \frac{\Delta^{+}}{2m_{N}} N(p) \; , \end{split}$$



Generalized Parton Distributions (GPDs)



- $Q^2 = -(k-k')^2$
- $x_B = Q^2/2M\nu \quad \nu = E_e E_e$
- $x+\xi$, $x-\xi$ longitudinal momentum fractions
- $t = \Delta^2 = (p-p')^2$

Vector

• $\xi = -\Delta^+/2P^+ \cong x_B/(2-x_B)$

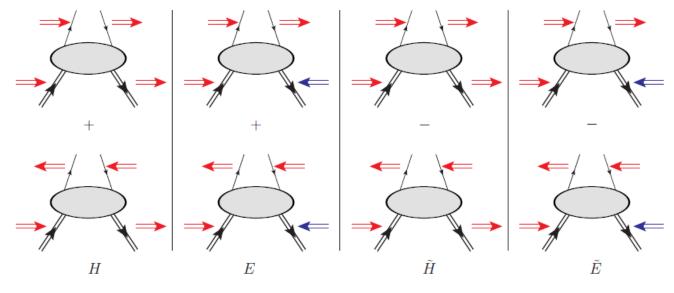
4 GPDs for each quark flavor

(leading order, leading twist, quark-helicity conservation)

conserve nucleon spin

flip nucleon spin

$$\begin{split} &\frac{P^{+}}{2\pi} \int dy^{-} e^{ixP^{+}y^{-}} \langle p^{'} | \bar{\psi}_{q}(0) \gamma^{+} \psi_{q}(y) | p \rangle \bigg|_{y^{+} = \vec{y}_{\perp} = 0} \\ &= H^{q}(x, \xi, t) \; \bar{N}(p^{'}) \gamma^{+} N(p) \; + \; E^{q}(x, \xi, t) \; \bar{N}(p^{'}) i \sigma^{+\nu} \frac{\Delta_{\nu}}{2m_{N}} N(p) \; , \\ &\frac{P^{+}}{2\pi} \int dy^{-} e^{ixP^{+}y^{-}} \langle p^{'} | \bar{\psi}_{q}(0) \gamma^{+} \gamma^{5} \psi_{q}(y) | p \rangle \bigg|_{y^{+} = \vec{y}_{\perp} = 0} \\ &= \tilde{H}^{q}(x, \xi, t) \; \bar{N}(p^{'}) \gamma^{+} \gamma_{5} N(p) \; + \; \tilde{E}^{q}(x, \xi, t) \; \bar{N}(p^{'}) \gamma_{5} \frac{\Delta^{+}}{2m_{N}} N(p) \; , \end{split}$$



Axial-vector

Ps.scalar

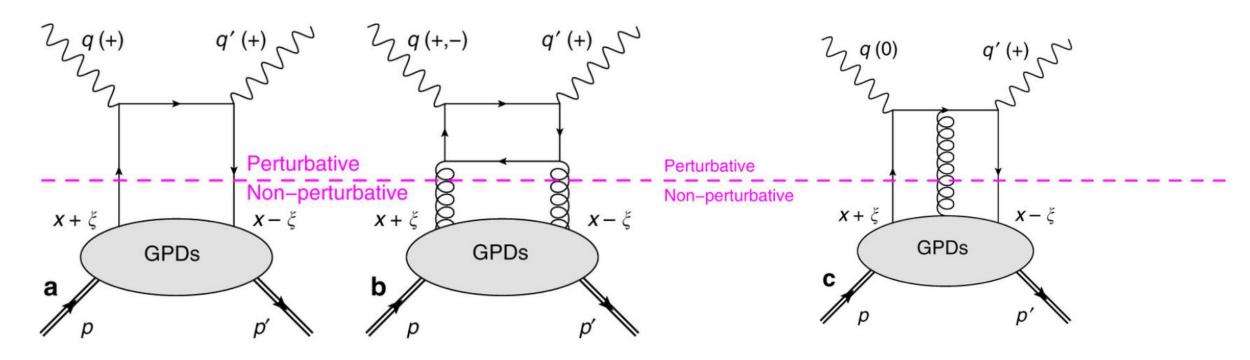
Tensor

Order, twist: examples for DVCS

Leading order, leading twist

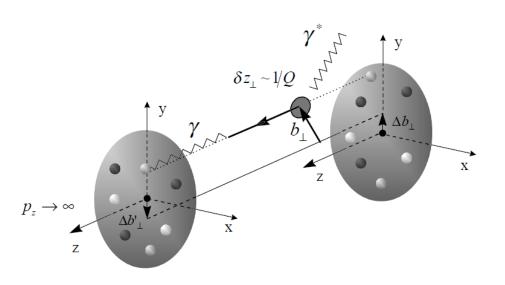
Next-to-leading order, leading twist

Leading order, twist 3

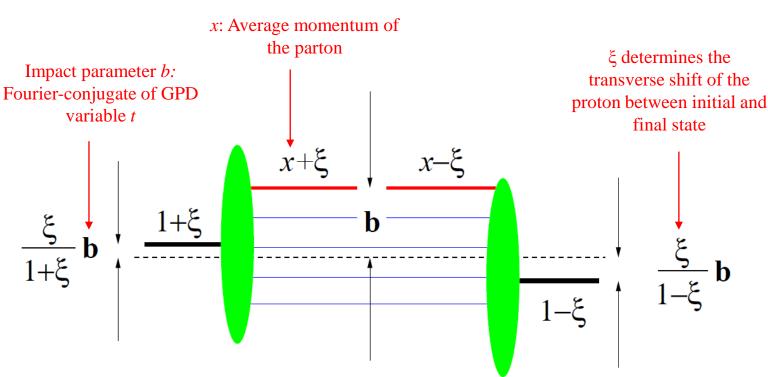


- Twist appears as powers of $\frac{1}{\sqrt{Q^2}}$ in the DVCS amplitude
- Order appears as powers of α_s
- General definition of **twist** of an operator: difference between the operator's dimension d and its spin s: $\tau = d$ -s
- Leading twist: twist = 2

Physical interpretation of GPDs



The 'blobs' represent the light-cone wave functions of the incoming or the outgoing proton



→ GPDs correlate wave functions for different parton configurations and thus are quantum-mechanical interference terms

Properties of GPDs

$$H(x,0,0) = q(x)$$
 Forward limit: **PDFs**
 $\tilde{H}(x,0,0) = \Delta q(x)$ (not for E, \tilde{E})

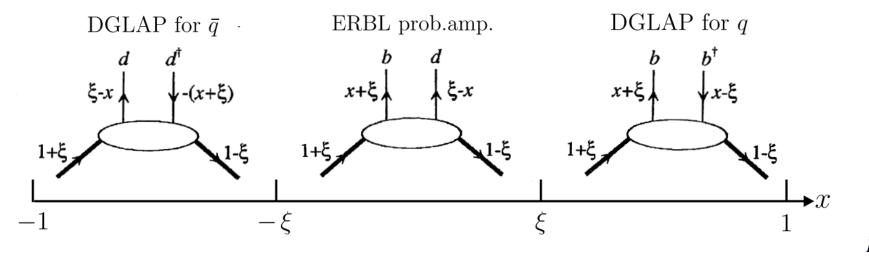
$$\int H(x,\xi,t)dx = F_1(t) \quad \forall \xi$$

$$\int E(x,\xi,t)dx = F_2(t) \quad \forall \xi$$

$$\int \tilde{H}(x,\xi,t)dx = G_A(t) \quad \forall \xi$$

$$\int \tilde{E}(x,\xi,t)dx = G_P(t) \quad \forall \xi$$
Link with **FFs**

$$\int_{-1}^{1} dx \, x^{n} H(x, \xi, t) = a_{0} + a_{2} \xi^{2} + a_{4} \xi^{4} + \dots + a_{n} \xi^{n} \quad \text{Polinomiality}$$



- DGLAP region: scattering from quarks $(x>\xi>0)$ or antiquarks $(x<\xi<0)$
- ERBL region ($-\xi < x < \xi$): scattering results in a $q\overline{q}$ pair

DGLAP: Dokshitzer, Gribov, Lipatov, Altarelli, Parisi ERBL: Efremov, Radyushkin, Brodsky, Lepage

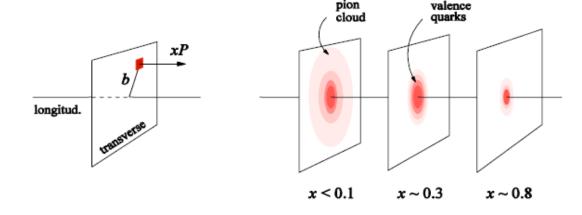
What we can learn from GPDs

Nucleon tomography

$$q(x, \mathbf{b}_{\perp}) = \int_{0}^{\infty} \frac{d^{2} \Delta_{\perp}}{(2\pi)^{2}} e^{i\Delta_{\perp} \mathbf{b}_{\perp}} H(x, 0, -\Delta_{\perp}^{2})$$

$$\Delta q(x, \mathbf{b}_{\perp}) = \int_{0}^{\infty} \frac{d^{2} \Delta_{\perp}}{(2\pi)^{2}} e^{i\Delta_{\perp} \mathbf{b}_{\perp}} \widetilde{H}(x, 0, -\Delta_{\perp}^{2})$$

M. Burkardt, PRD 62, 71503 (2000)



Probability to find a quark of a given momentum fraction at a given position in the transverse plane

Quark angular momentum (Ji's sum rule)

$$\frac{1}{2} \int_{-1}^{1} x dx (H(x, \xi, t = 0) + E(x, \xi, t = 0)) = J = \frac{1}{2} \Delta \Sigma + \Delta L$$

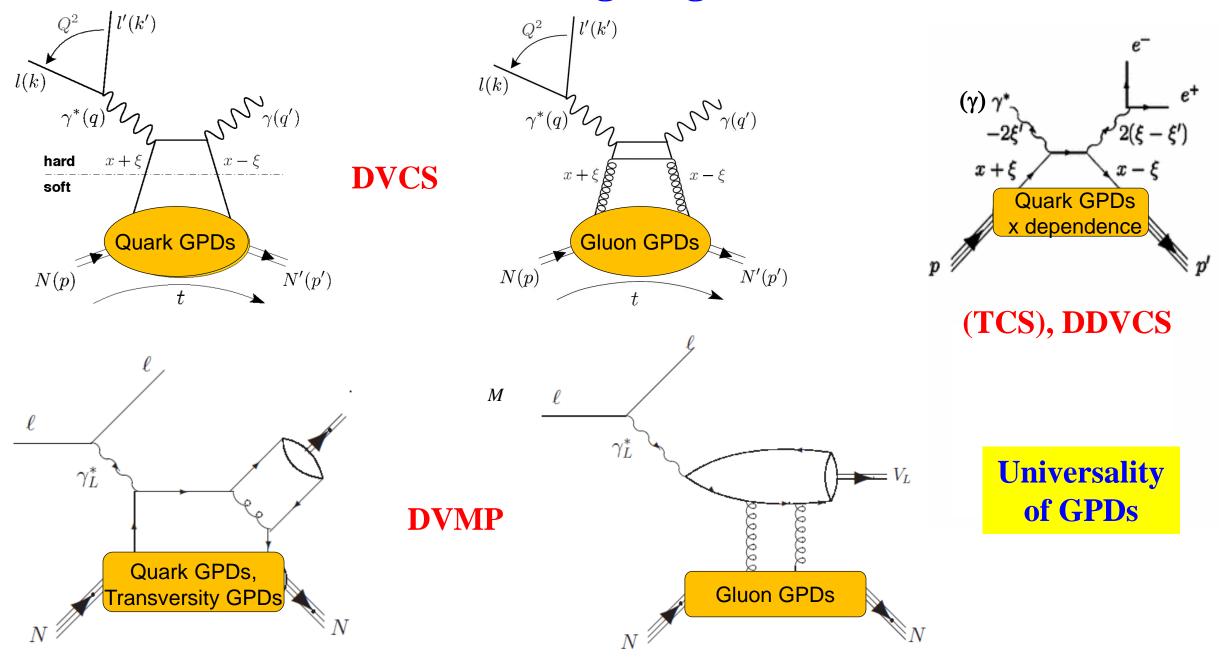
X. Ji, Phy.Rev.Lett.78 (1997)

Nucleon spin:
$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta L + J_G$$

$$\int_{-1}^{1} x H(x, \xi, t) dx = M_2(t) + \frac{4}{5} \xi^2 d_1(t)$$

Gravitational form factor → shear forces and pressure

Exclusive reactions giving access to GPDs



Summary and sources

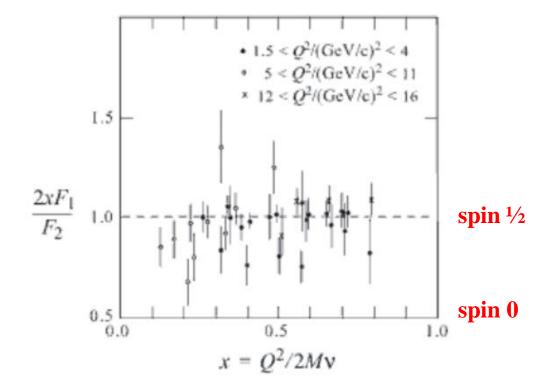
- Electron scattering on nucleons is one of the main tools to study nucleon structure
- Form factors, measured in elastic scattering, give us information on the transverse charge distribution of the partons
- Structure functions, measured in deep inelastic scattering, give us information on the longitudinal momentum distribution of the partons
- Generalized parton distributions correlate information on transverse position and longitudinal momentum of the partons
- The can give access to: OAM of the quarks, tomography, distribution of forces
- They can be measured in exclusive reactions, where the full final state in knows, at high virtuality of the exchanged photon
 - > « Quarks and leptons: an introductory course on modern particle physics », F. Halzen, A. D. Martin
 - ➤ Nobel Lecture, R. Hofstadter
 - Nobel Lecture, H. Kendall
 - Articles by: A. Radyuskin, X.D. Ji, D. Mueller, M. Diehl, M. Burkardt, M. Guidal and M. Vanderhaeghen, J. Arrington, H. Atac, C. Mezrag, C. Lorcé, ...
 - ➤ HUGS lectures of my predecessors and colleagues: C. M. Camacho, D. Sokhan

Back-up slides

Quarks' spin discovery

The Callan-Gross relation

- Bjorken scaling established that DIS must be described in terms of parton-photon processes.
- But what are the properties of these point-like constituents?
- In 1969 Callan and Gross suggested that the two Bjorken's scaling functions are related: $2xF_1(x) = F_2(x)$.
- This reflects the assumption that the partons inside the proton are spin-1/2 particles (spin-0 would lead to $2xF_1(x)/F_2(x) = 0$)
- (It can be derived by comparing the e^-p and $e^-\mu$ differential cross sections)



Charge distribution		Form factor	
point	$f(r) = \delta(r - r_0)$	$F(q^2) = 1$	unity
exponentia1	$f(r) = \frac{a^3}{8\pi} e^{-ar}$	$F(q^{2}) = \left[\frac{1}{1 + q^{2}/a^{2}}\right]^{2}$	dipole
Yukawa	$f(r) = \frac{a^2}{4\pi r} e^{-ar}$	$F(q^2) = \frac{1}{1 + \sigma^2/\sigma^2}$	pole
Gaussian	$f(r) = \left(\frac{a^2}{2\pi}\right)^{\frac{3}{2}} e^{-(a^2r^2/2)}$	$F(q^2) = e^{-(q^2/2a^2)}$	Gaussian