# Theory predictions for $\bar{B} \to D^{(*)}\ell\bar{\nu}$

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Based on arXiv: 2412.04388, 2507.03569
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Emmy Noether-Programm

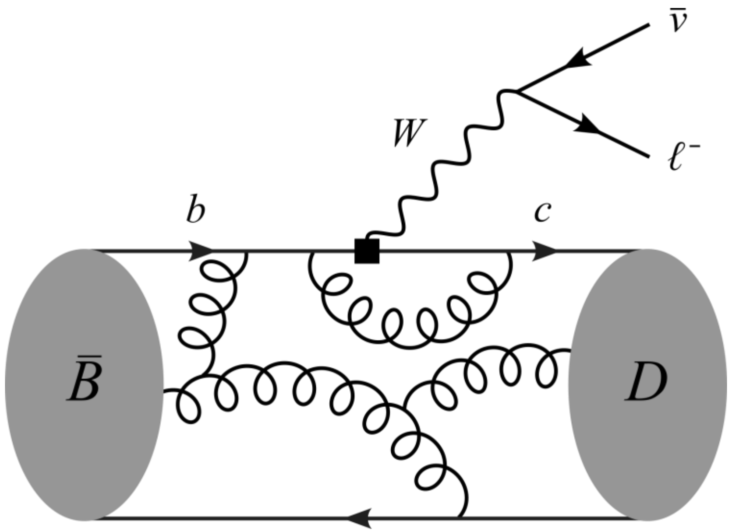
DFG Deutsche Forschungsgemeinschaft

# In this talk



Study transitions $b \to c\ell\nu$

- look for New Physics (NP) (i.e. new particles and forces)
- extract CKM matrix element $V_{cb}$




No free quarks in Nature $\Longrightarrow$ study $B \to D\ell\nu$ decays

States considered in this talk:

- $B$ mesons $(B, B_s, B^*, B_s^*)$ and $D$ mesons $(D, D_s, D^*, D_s^*)$

Problem: non-perturbative QCD limits precision in $B \to D\ell\nu$ predictions

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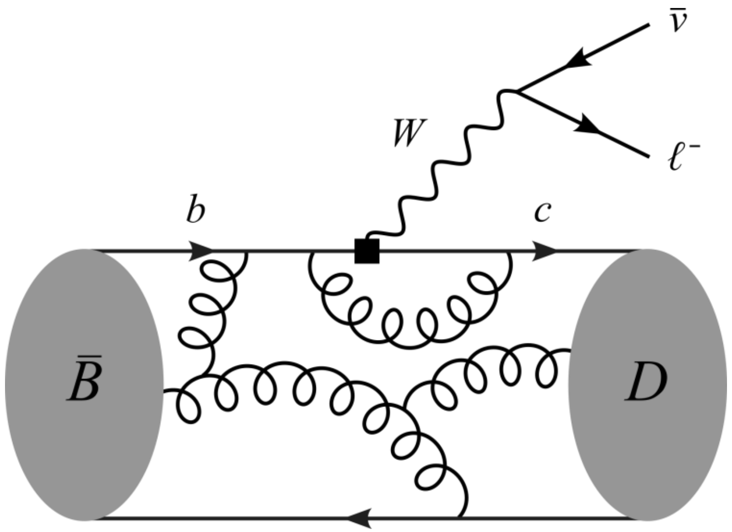


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Problem: non-perturbative QCD limits precision in $B \to D\ell\nu$ predictions

Focus of this talk: reduce theory uncertainties to make progress

# Introduction

# Precision flavour tests of the SM



No NP evidence from direct searches so far (too heavy?)

$\Longrightarrow$ LHC has reached its maximum energy

$\Longrightarrow$ Direct NP discovery difficult in coming decades

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No NP evidence from direct searches so far (too heavy?)

⟹ LHC has reached its maximum energy

⟹ Direct NP discovery difficult in coming decades

Indirect searches (with flavour)

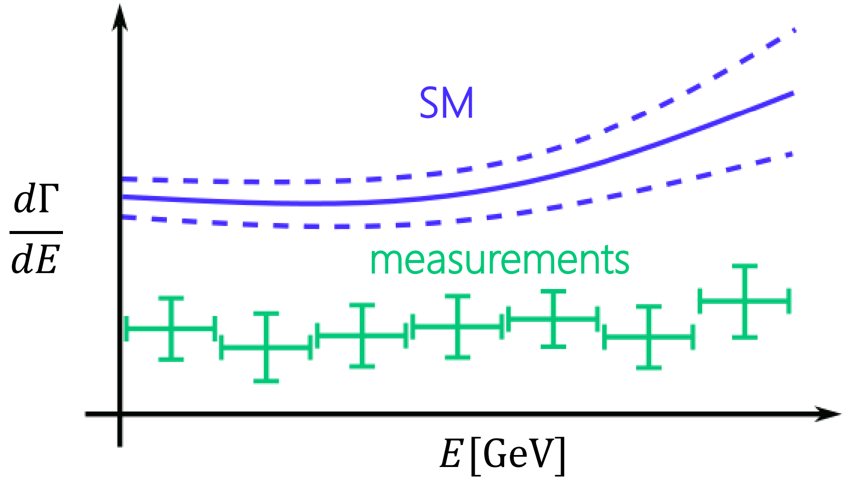
⟹ Probe the SM at **higher energies** than direct searches

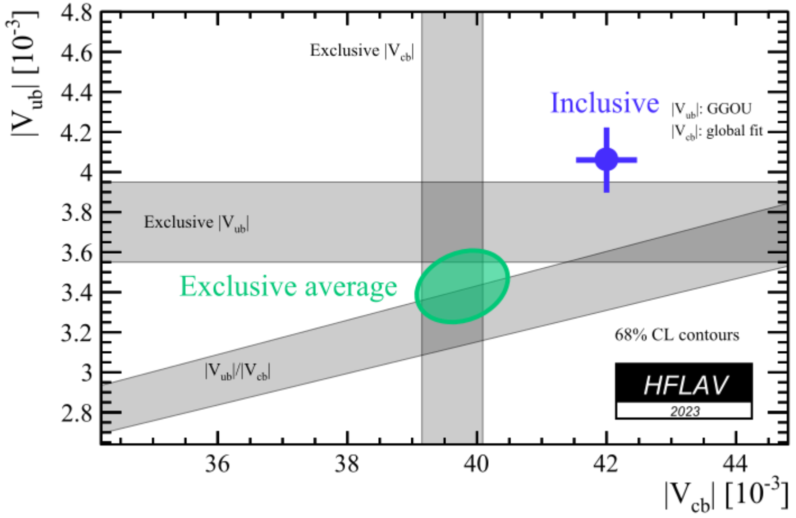
⟹ **Compare** precise measurements and calculations

⟹ Obtain constraints on NP (or new discovery?)

Extraction SM parameters (e.g. $|V_{ub}|$ and $|V_{cb}|$)

⟹ 13 out of 19 are flavour parameters



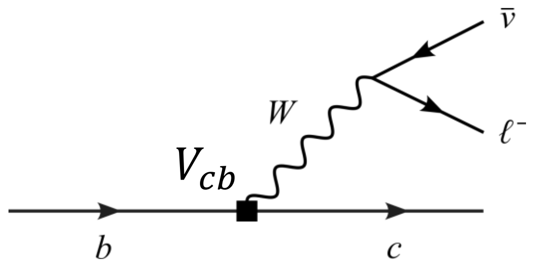

[Adapted from HFLAV]

# The importance of $b \to c\ell\nu$ transitions



$b \to c\ell\nu$ occur at tree level in the SM

$\Longrightarrow$ extract of $|V_{cb}|$



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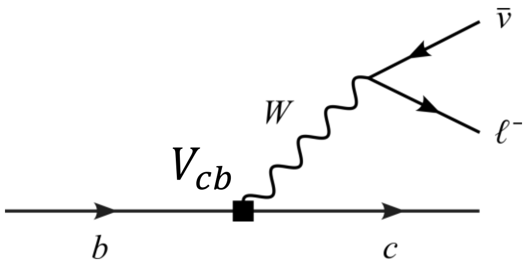
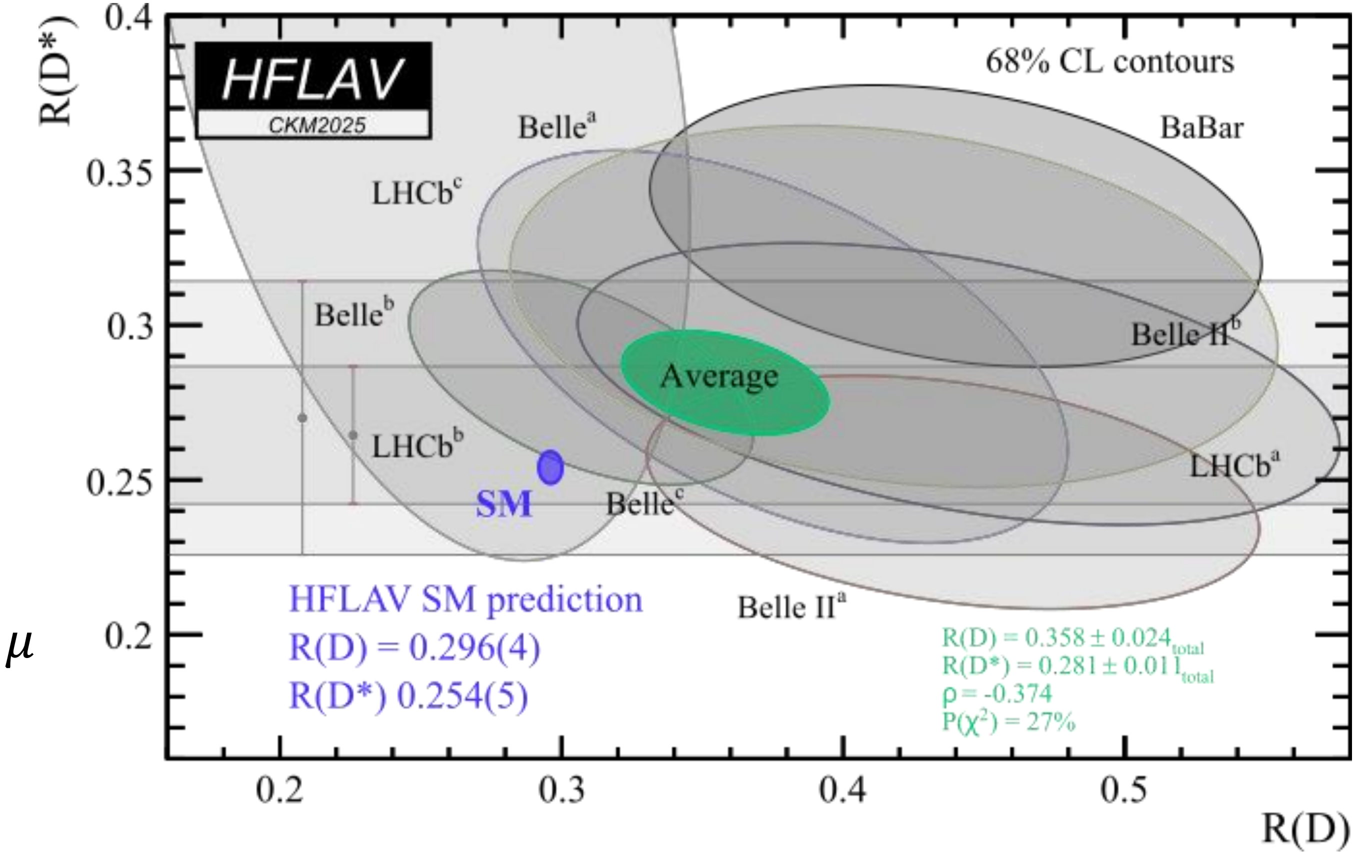



Test lepton flavour universality (LFU)

$$R(D^{(*)}) = \frac{Br(\bar{B} \to D^{(*)}\tau\,\bar{\nu})}{Br(\bar{B} \to D^{(*)}\ell\,\bar{\nu})} \qquad \ell = e, \mu$$

Violations of LFU $\Longrightarrow$ NP

Combined $R(D^{(*)})$ tension $\mathbf{3.8\,\sigma}$

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# Flavour physics at the precision frontier



Precision needed for indirect searches, extraction SM parameters, and clarification of current tensions (or anomalies)

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Precision needed for indirect searches, extraction SM parameters, and clarification of current tensions (or anomalies)

## Experiment status and prospects

- LHC Run 3 is ongoing!
- HiLumi LHC (~ 2030 – 2041)
- Belle II experiment will run until ~2040
- Next big experiment will focus on precision

$\Longrightarrow$ reduction of experimental uncertainties

# Flavour physics at the precision frontier



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## Theory status

Theoretical precision < experimental precision

for many key observables

⟹ reduce theory unc. to make progress

⟹ fully exploit experimental efforts and investments

# Hadronic matrix elements



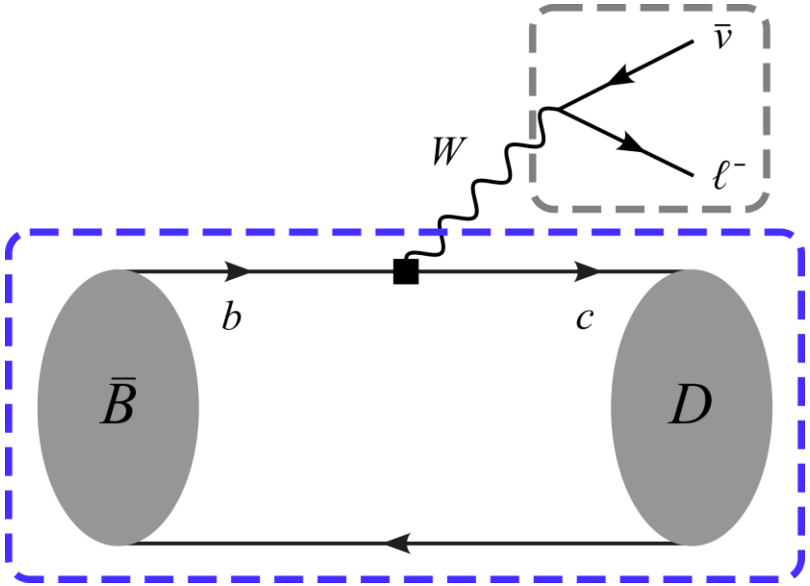
This talk: study $\boldsymbol{B}$-meson decays to test the $b \to c\ell\nu$ transitions

Factorise decay amplitude (neglecting QED corrections)

$$\mathcal{A}^{B\to D\ell\nu} \propto \langle D\ell\nu|\mathcal{O}_{eff}|B\rangle = \langle \ell\nu|\mathcal{O}_{lep}|0\rangle \, \langle D^{(*)}|\mathcal{O}_{had}|B\rangle$$

Leptonic matrix elements: perturbative objects, high accuracy

QED corrections mostly unknown but small (~1%)




Hadronic matrix elements: non-perturbative QCD effects, usually large uncertainties (~10%)

$\Longrightarrow$ biggest challenge for percent precision

Calculate them using lattice QCD or light-cone sum rule

# Form factors definitions



Decompose matrix elements in terms of form factors (FFs) (assume only Lorentz invariance), e.g.

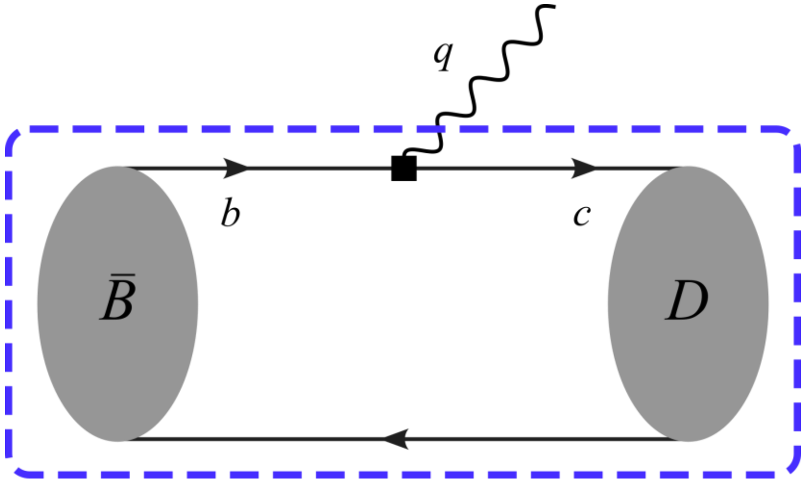
$$\langle D(k) | \bar{c}\, \sigma^{\mu\{q\}} b | B^*(p,\eta) \rangle = 2i\varepsilon^{\mu\{\eta\}\{p\}\{k\}} \bar{T}_1(q^2)$$

FFs are functions of the momentum transfer squared $q^2 = (p-k)^2$

Number of independent vector (tensor) FFs

2(+1) for $B \to D$
4(+3) for $B \to D^*$
4(+3) for $B^* \to D$
10(+7) for $B^* \to D^*$

We define the $B^* \to D$ and $B^* \to D^*$ tensor FFs for the first time



# Theoretical calculations of FFs



Several (new) precise lattice QCD calculations available

- $B \to D$ at high $q^2$
  [FNAL/MILC 2015] [HPQCD 2015]
- $B \to D^*$ at high $q^2$
  [FNAL/MILC 2021] [JLQCD 2023]
  at any $q^2$ (in the physical region)
  [HPQCD 2023]
- $B_s \to D_s$ at any $q^2$
  [HPQCD 2019]
- $B_s \to D_s^*$ at any $q^2$
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Light-cone sum rules (LCSRs) available for the four processes at low $q^2$
[NG/Kokulu/van Dyk 2018] [Bordone/NG/Jung/van Dyk 2019]

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FF calculations from LCSRs and lattice QCD are provided at discrete values of $q^2$ points

To combine theory inputs and obtain FFs for any $q^2$ value, FFs must be parametrized

# Form factors parametrizations



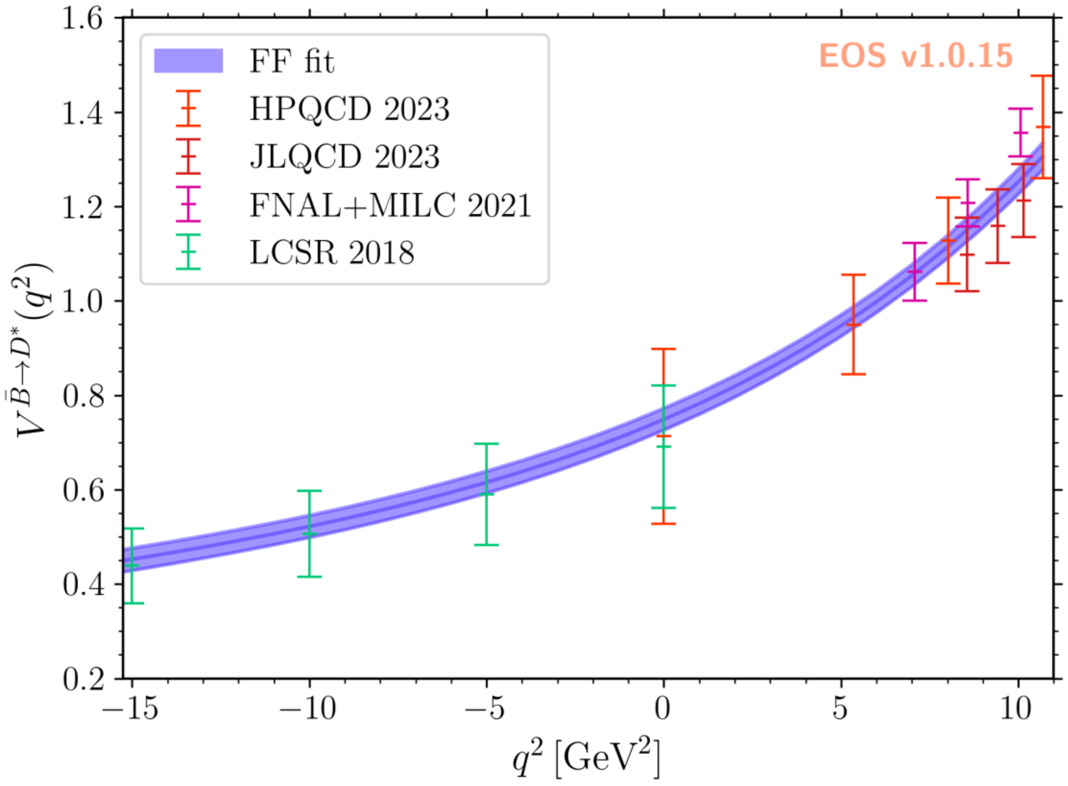
A simple $q^2$ power expansion is perfectly fine

$$\mathcal{F}(q^2) = \sum_{n=0}^{N} a_n (q^2)^n$$

However, with a proper parametrization, additional constraints can be included:

- unitarity bounds
  constraints on the $a_n$ coefficients

- heavy-quark expansion (HQE)
  relations between FFs
  and interplay with unitarity bounds



# Our goal: a comprehensive HQE analysis



Combine all theoretical constraints in a HQE analysis of $B^{(*)} \to D^{(*)}$ and $B_s^{(*)} \to D_s^{(*)}$ FFs
Include tensor FFs and corresponding unitarity bounds for the first time

Steps to perform this analysis:

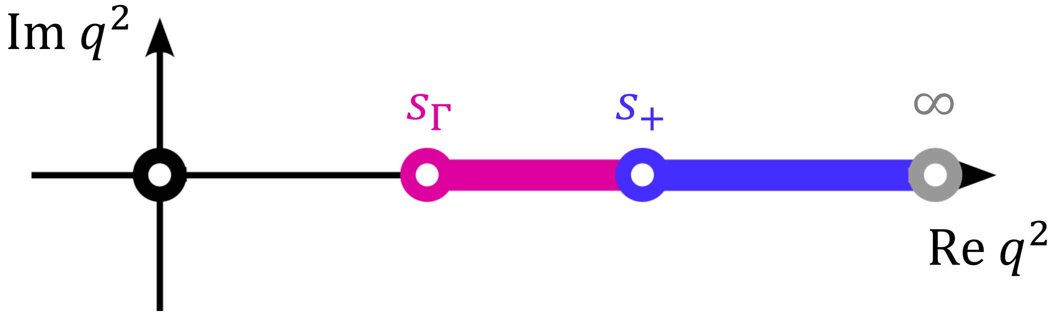
1. find suitable FF definitions
2. expand the $B_{(s)}^{(*)} \to D_{(s)}^{(*)}$ FFs using HQE (i.e. in terms of in terms of Isgur-Wise functions)
3. derive the unitarity bounds for the Isgur-Wise functions
4. fit the Isgur-Wise functions to the theoretical calculations and impose unitarity bounds

# HQE parametrization

# Analytic properties of FFs



Study FF analytic structure to find a suitable parametrization. Example $B \to D$ FFs




**FFs are analytic except for branch cuts** (i.e. lines of discontinuity) starting at

$s_+ = (m_B + m_D)^2$, process threshold

$s_\Gamma = \left(m_{B_c} + m_\pi\right)^2 < s_+$, subthreshold branch cut

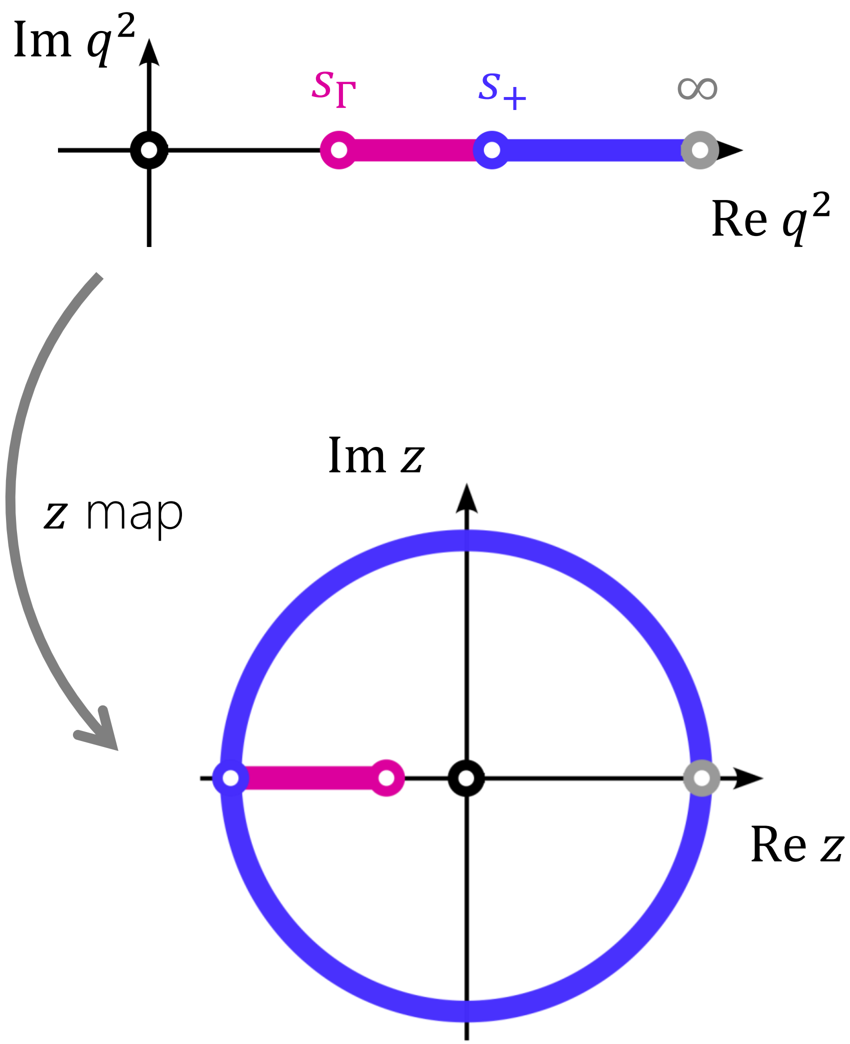
Subthreshold branch cuts are neglected in the common FF parametrizations (BGL, CLN, HQE, …)

See end of the talk [Gopal/NG 2024]

# BGL parametrization

Define the map

$$z(q^2) = \frac{\sqrt{s_+ - q^2} - \sqrt{s_+}}{\sqrt{s_+ - q^2} + \sqrt{s_+}}$$



# BGL parametrization



Define the map

$$z(q^2) = \frac{\sqrt{s_+ - q^2} - \sqrt{s_+}}{\sqrt{s_+ - q^2} + \sqrt{s_+}}$$

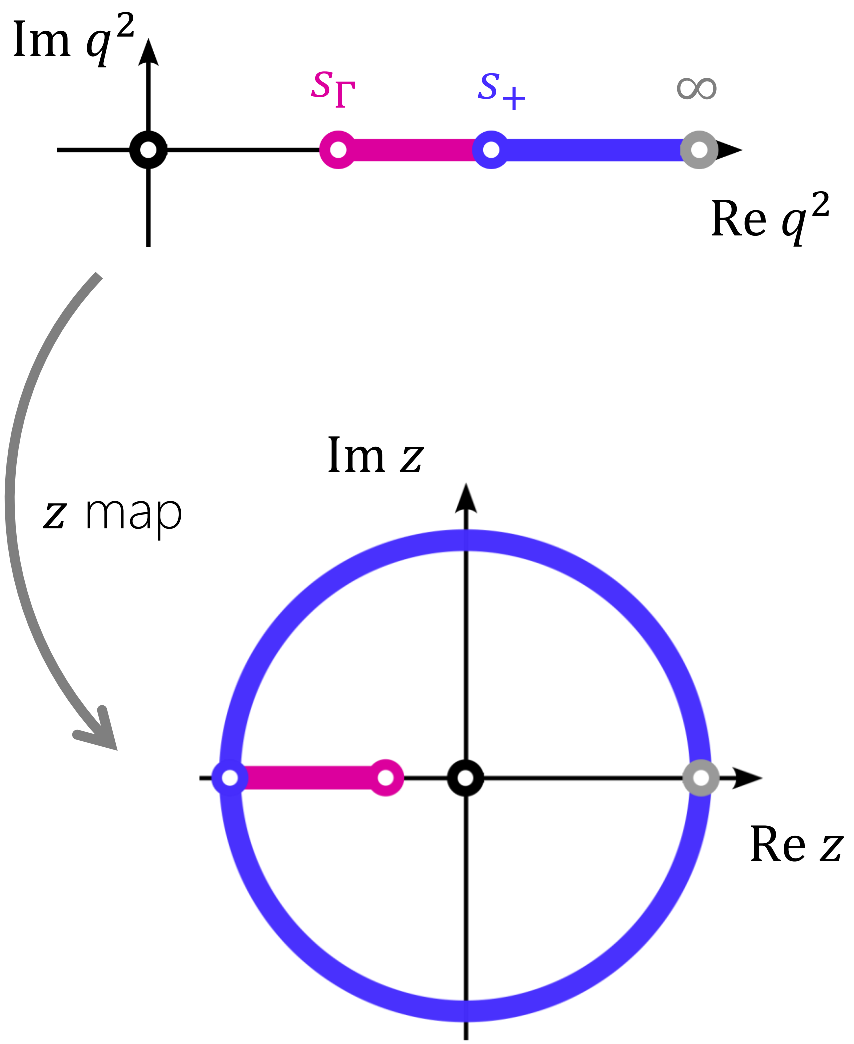
BGL parametrization for $B \to D^{(*)}$ FFs

$$\mathcal{F}(z) = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{n=0}^{\infty} a_n \, z^n$$

with the unitarity bound [Boyd/Grinstein/Lebed 1997]

$$\sum_{n=0}^{\infty} |a_n|^2 < 1$$

Extend approach to tensor $B^* \to D^{(*)}$ FFs



# Heavy Quark Expansion (HQE)



HQE: perform a power series expansion in $\Lambda_{\mathrm{QCD}}/m_c$ and $\Lambda_{\mathrm{QCD}}/m_b$ of FFs ($\Lambda_{\mathrm{QCD}} \sim 300$ MeV)

$$\mathcal{F}^i_{\mathrm{HQE}}(q^2) = \xi(q^2)\left(c_0^i + c_1^i \frac{\alpha_s}{\pi}\right) + c_2^i \frac{1}{m_b} L_k(q^2) + c_3^i \frac{1}{m_c} L_k(q^2) + c_4^i \frac{1}{m_c^2} l_k(q^2)$$

All the $B^{(*)} \to D^{(*)}$ FFs can be expressed in terms of **10 Isgur-Wise functions**
(1 leading, 3 subleading, 6 subsubleading)

Essential to include $\Lambda^2_{\mathrm{QCD}}/m_c^2$ corrections (CLN not sufficient) [Bordone/Jung/van Dyk 2019]

**$\Longrightarrow$ relations between 34 $B^{(*)} \to D^{(*)}$ FFs (34 $B_s^{(*)} \to D_s^{(*)}$ FFs)**

# HQE parametrization



$$\mathcal{F}^i_{\mathrm{HQE}}(q^2) = \xi(q^2)\left(c^i_0 + c^i_1 \frac{\alpha_s}{\pi}\right) + c^i_2 \frac{1}{m_b} L_k(q^2) + c^i_3 \frac{1}{m_c} L_k(q^2) + c^i_4 \frac{1}{m_c^2} l_k(q^2)$$

Expand Isgur-Wise functions around $q^2_{max} = \left(m_{B^{(*)}} - m_{D^{(*)}}\right)^2$ (max recoil)

$$\xi(q^2) \propto \sum_{m=0}^{N} \xi^{(m)} \left(q^2 - q^2_{max}\right)^m$$

$$L_k(q^2) \propto \sum_{m=0}^{M} L_k^{(m)} \left(q^2 - q^2_{max}\right)^m$$

$$l_k(q^2) \propto \sum_{m=0}^{K} l_k^{(m)} \left(q^2 - q^2_{max}\right)^m$$

The derivatives $\xi^{(m)}, L_k^{(m)}, l_k^{(m)}$ are the parameters of our parametrization

$N/M/K$ parametrization $\Longrightarrow$ We consider 3/2/1 and 2/1/0 models

# Weak and strong unitarity bounds



Express the unitarity bound in terms of **Isgur-Wise parameters**

For one FF the bound reads (**weak unitarity bound**)

$$\sum_{n=0}^{\infty} \left| a_n^i \right|^2 \Rightarrow \sum_{n=0}^{\infty} \left| a_n^i(\xi^{(m)}, L_k^{(m)}, l_k^{(m)}) \right|^2 < 1$$

Sum contribution of all channels related by heavy quark symmetry ($B^{(*)} \to D^{(*)}$)

$$\Downarrow$$

strong unitarity bound

$$\sum_{\mathcal{F}^i} \sum_{n=0}^{\infty} \left| a_n^i(\xi^{(m)}, L_k^{(m)}, l_k^{(m)}) \right|^2 < 1$$

# More on unitarity bounds



The HQE parametrization have some crucial advantages

- include the contribution of $B^* \to D^{(*)}$ decays $\Longrightarrow$ increase bound saturation
  not possible at the moment for BGL, as there are no $B^* \to D^{(*)}$ FFs predictions

- relate (axial-)vector and tensor FFs (constrain each other)
  $\Longrightarrow$ theoretical calculations must fulfil HQE relationships

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Other channels not related through HQE symmetry can be added $\Longrightarrow$ *ultra* strong unitarity bound

$$\underbrace{\sum_{\mathcal{F}^i}\sum_{n=0}^{\infty}\left|a_n^i(\xi^{(m)}, L_k^{(m)}, l_k^{(m)})\right|^2}_{B^{(*)}\to D^{(*)}} + \underbrace{\sum_{\mathcal{F}^i}\sum_{n=0}^{\infty}\left|a_n^i(\xi^{s,(m)}, L_k^{s,(m)}, l_k^{s,(m)})\right|^2}_{B_s^{(*)}\to D_s^{(*)}} + \underbrace{\sum_{\mathcal{F}^i}\sum_{n=0}^{\infty}\left|a_n^i(\xi^{\Lambda,(m)}, L_k^{\Lambda,(m)}, l_k^{\Lambda,(m)})\right|^2}_{\Lambda_b\to\Lambda_c} + \cdots < 1$$

# Results

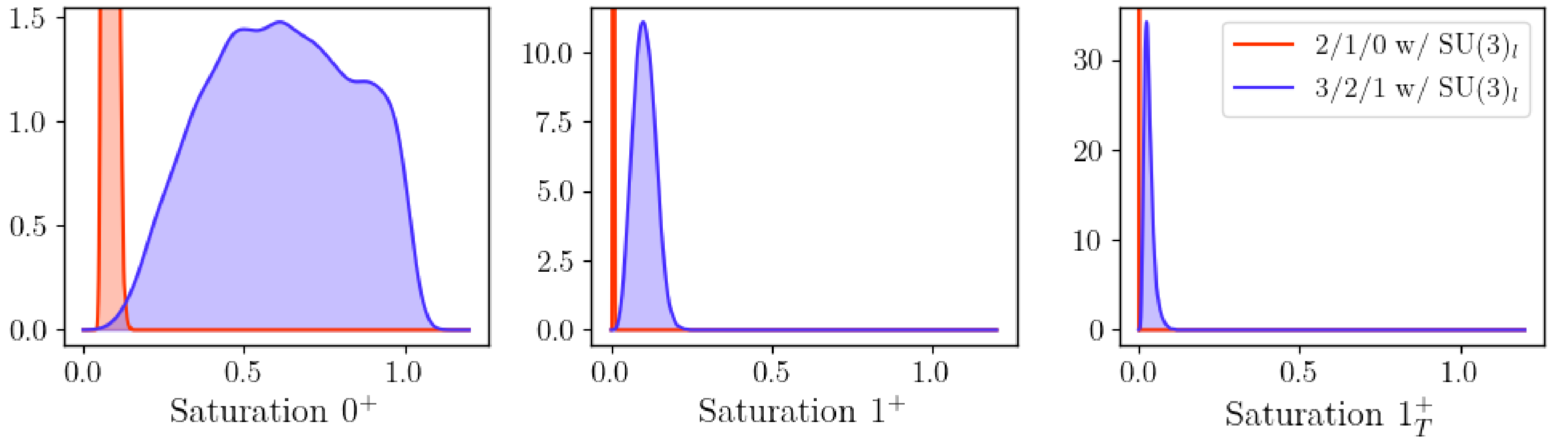
# Nominal fit model



We consider 5 fit models in total (see article)

Nominal fit model: 3/2/1 w/ $\boldsymbol{SU(3)_l}$

- same subsubleading IW functions $l_k$ for $B^{(*)} \to D^{(*)}$ and $B_s^{(*)} \to D_s^{(*)}$ FFs (31 free fit parameters)
- theory inputs used: all lattice QCD and unitarity bounds (no LCSR)
- $p$-value 99.3%
- HQE is free of pathological behavior up to order $\Lambda_{\rm QCD}^2/m_c^2$ (all parameters of $\mathcal{O}(1)$)



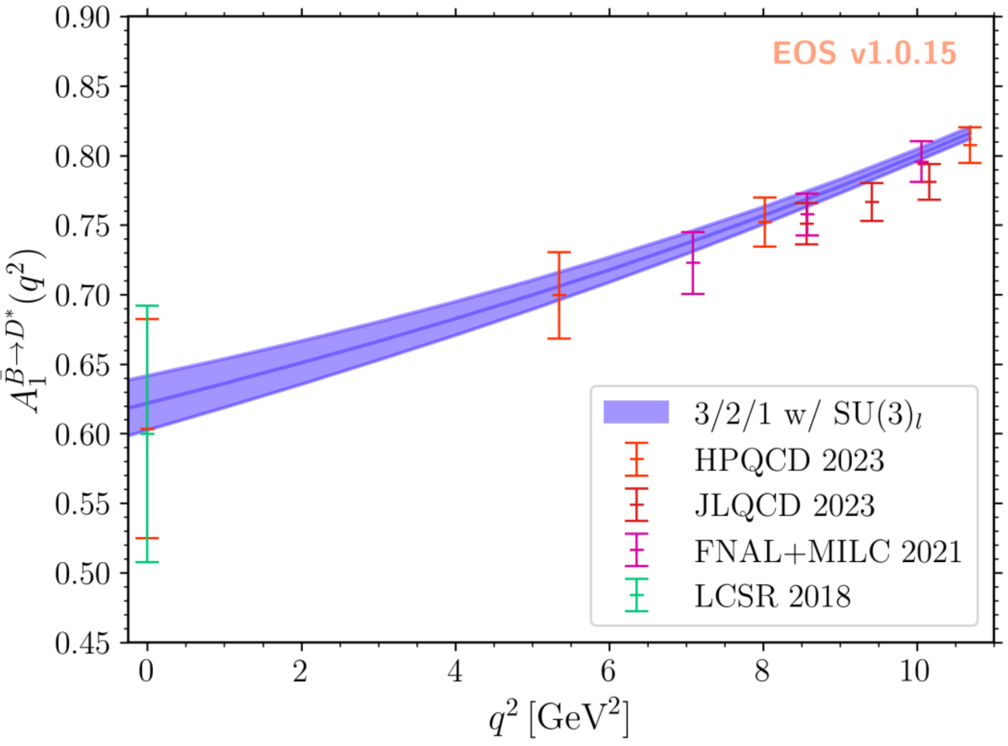
# FFs results

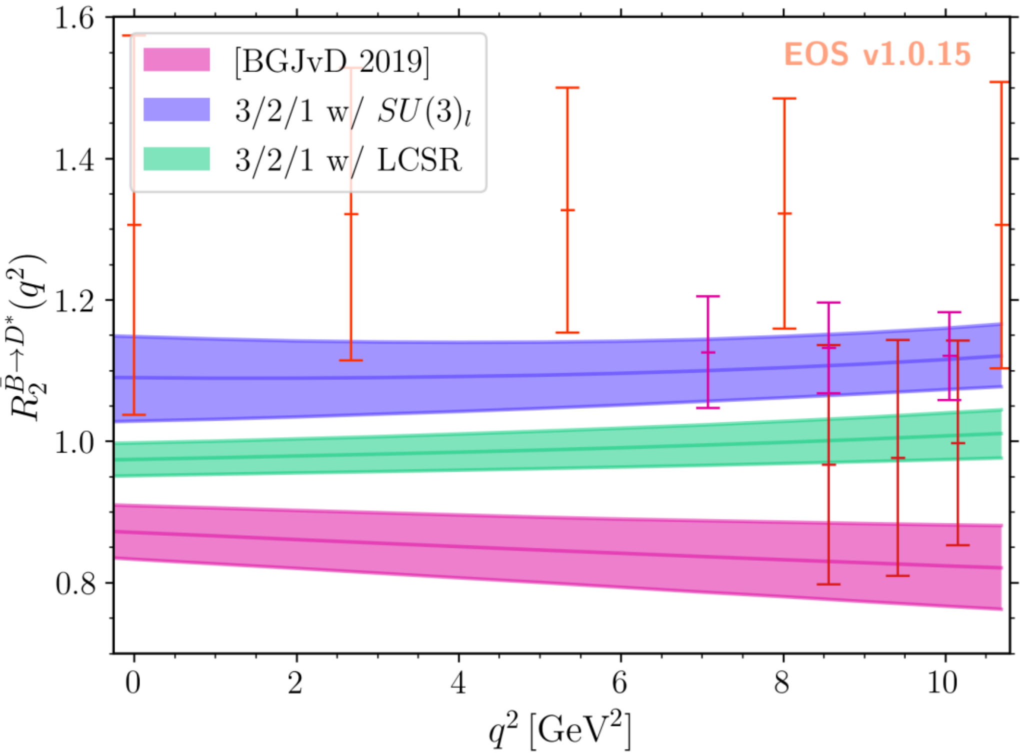


We provide results for all $B^{(*)} \to D^{(*)}$ and $B_s^{(*)} \to D_s^{(*)}$ FFs

Lattice QCD (and LCSR) calculations are mutually compatible

$$R_2(q^2) = \left(1 - \frac{q^2}{s_+}\right)\frac{A_2}{A_1}$$



# Observable predictions



We also predict all branching ratios, angular observables, and LFU ratios

$$R\left(D^{(*)}\right) = \frac{Br(\bar{B} \to D^{(*)}\tau\,\bar{\nu})}{Br(\bar{B} \to D^{(*)}\ell\,\bar{\nu})}$$

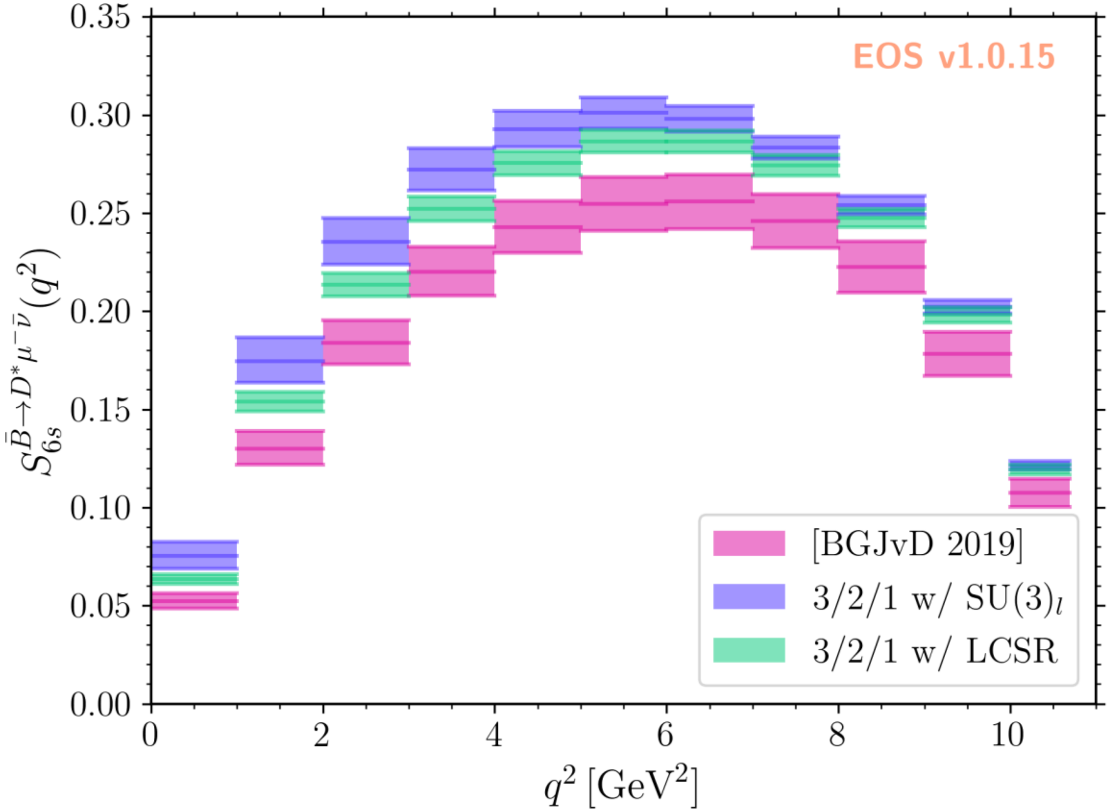
For 3/2/1 w/ $\boldsymbol{SU(3)_l}$

$$R(D) = 0.3022(36) \qquad R(D^*) = 0.2589(42)$$

For 3/2/1 w/ LCSR (most precise predictions)

$$R(D) = 0.2983(31) \qquad R(D^*) = 0.2510(26)$$

Code and results available at zenodo.org/records/17593955



# Subthreshold cuts

# Our approach: GG



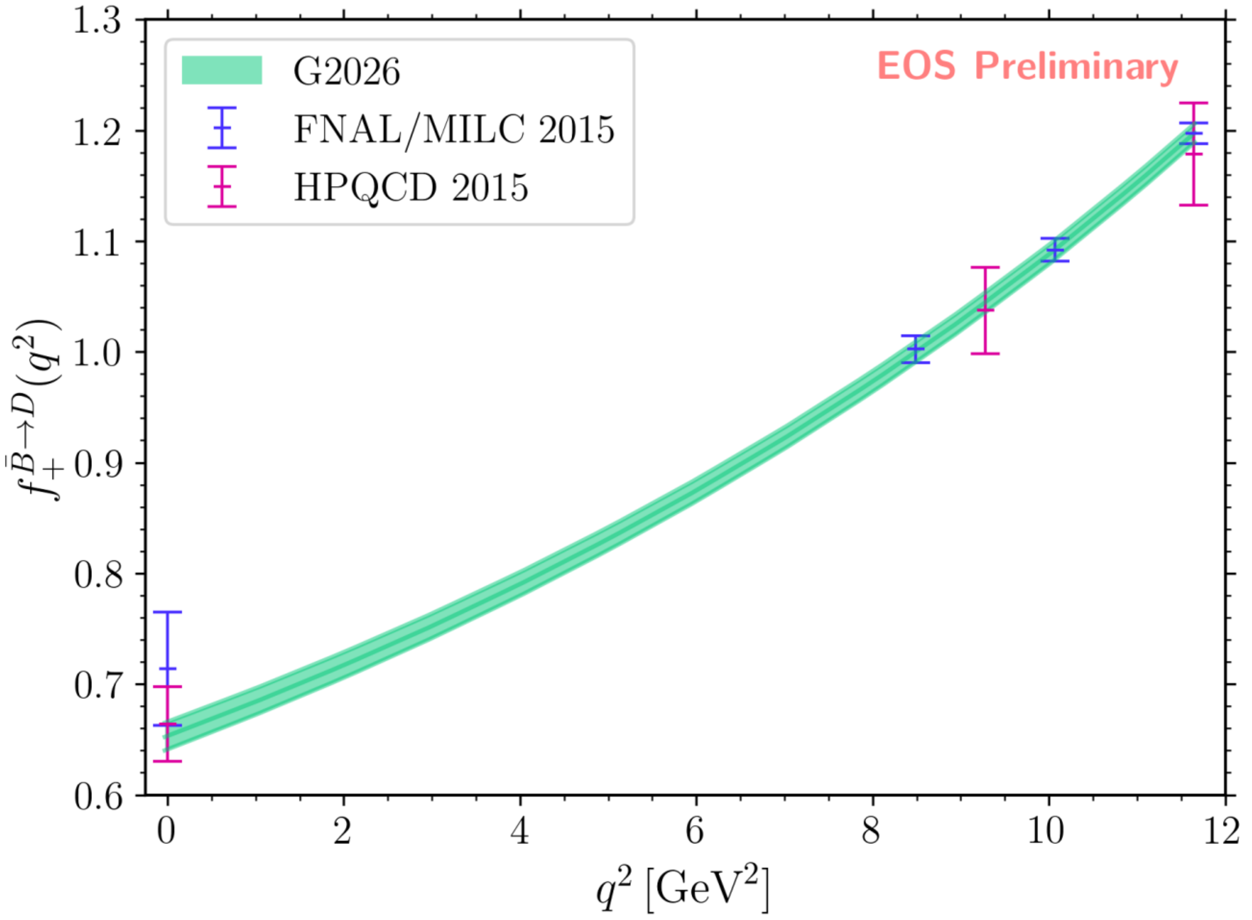
New parametrization that incudes subthreshold cuts [Gopal/NG 2024]

- modified $z$ mapping
- new derivation of the unitarity bound

Preliminary Bayesian fit using our new parametrization

Next steps:

- complete EOS implementation
- impact of subthreshold cuts



# Summary and conclusion

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High theoretical precision needed for indirect searches, extraction SM parameters

HQE analyses: include the contribution of $B^* \to D^{(*)}$ FFs $\Longrightarrow$ increase bound saturation

Relate (axial-)vector and tensor FFs (constrain each other)

Provide precise FF and observable predictions for $B_{(s)} \to D_{(s)}^{(*)}$ processes (code public)

For percent-level accuracy, subthreshold branch cuts cannot be neglected

Systematic approach presented in [Gopal/NG 2024]

Parametrization simple to implement (as simple as BGL)

Thank you!

# Backup slides

# Problems with BGL (and CLN, HQE,...)

Having a branch cut invalidate the expansion for $|z| < 1$

$$\mathcal{F}(z) \neq \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{n=0}^{\infty} a_n z^n \quad \text{for} \quad |z| < 1$$

Issue appears for FFs in $b$-hadron decays except $B \to \pi$

It is crucial to address this issue to accurately estimate uncertainties in $b$-hadron decays

Issue discussed in the literature, but solutions are unsatisfactory:
they do not allow a rigorous estimate of the truncation error (cut modelling, polynomials)

Find a way to recover the unitarity bound:

$$\sum_{n=0}^{\infty} |a_n^2| < 1$$

[Boyd/Grinstein/Lebed 1995]
[Caprini/Neubert 1996]
[NG/van Dyk/Virto 2020]
...

**Essential to estimate truncation error**! (we can only fit a finite number of $a_n$)

# Our approach: GG

Just a reminder: $s_+ = (m_B + m_D)^2$ , $s_\Gamma = (m_{B_c} + m_\pi)^2$

Modify the conformal mapping ($s_+ \mapsto s_\Gamma$)

$$\hat{z}(q^2) = \frac{\sqrt{s_\Gamma - q^2} - \sqrt{s_\Gamma}}{\sqrt{s_\Gamma - q^2} + \sqrt{s_\Gamma}}$$

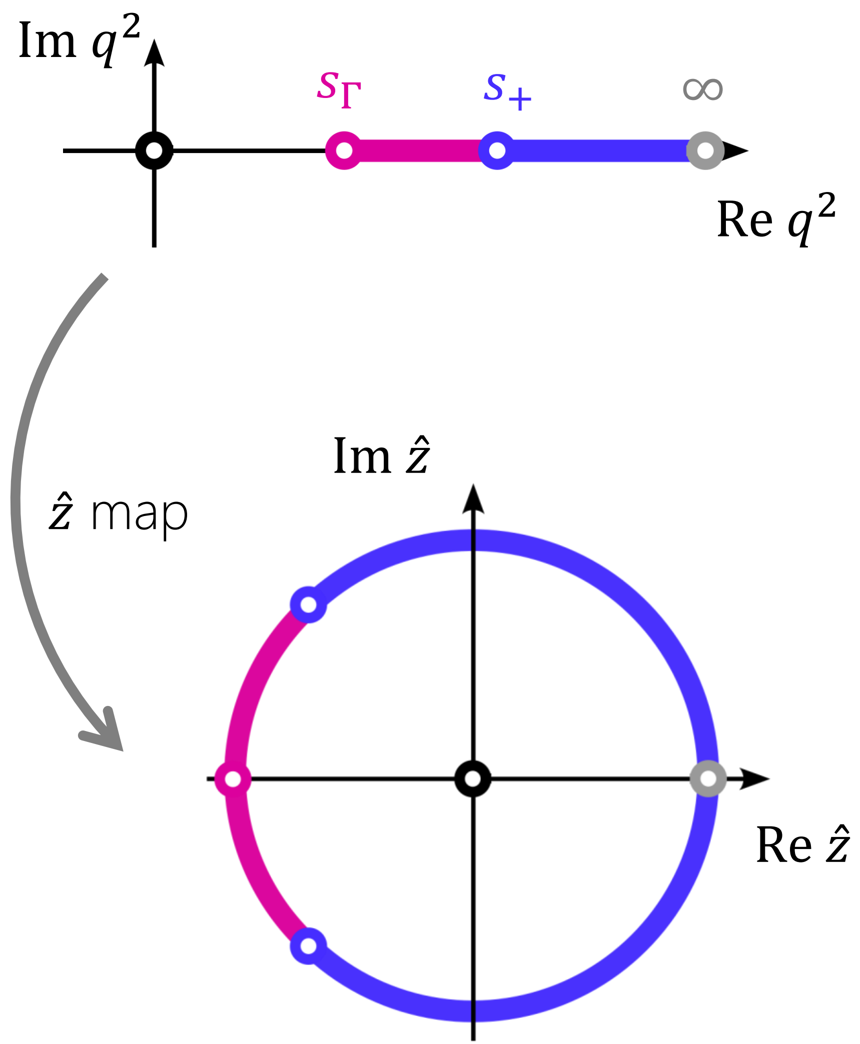
Expand FFs for $|\hat{z}| < 1$ (no singularities now!) as

$$\mathcal{F}(\hat{z}) = \frac{1}{\mathcal{P}(\hat{z})\phi(\hat{z})} \sum_{n=0}^{\infty} b_n \hat{z}^n$$

however

$$\int_{s_+}^{\infty} dq^2 \, |\det J| \, |\phi(q^2)\mathcal{F}(q^2)|^2 < \chi \quad \not\Rightarrow \quad \sum_{n=0}^{\infty} |a_n^2| < 1$$

Integral must over the whole circle!



# Our derivation of the unitarity bound

Start from

$$\int_{s_+}^{\infty} dq^2 \, |\det J| \, |\phi(q^2)\mathcal{F}(q^2)|^2 < \chi$$

add on both sides

$$\Delta\chi \equiv \int_{s_\Gamma}^{s_+} dq^2 \, |\det J| |\phi(q^2)\mathcal{F}(q^2)|^2$$

Estimate $\Delta\chi$ using large $q^2$ scaling behaviour (for $B \to D$ FFs $\frac{\Delta\chi}{\chi} < 0.1\%$)

Obtain the unitarity bound

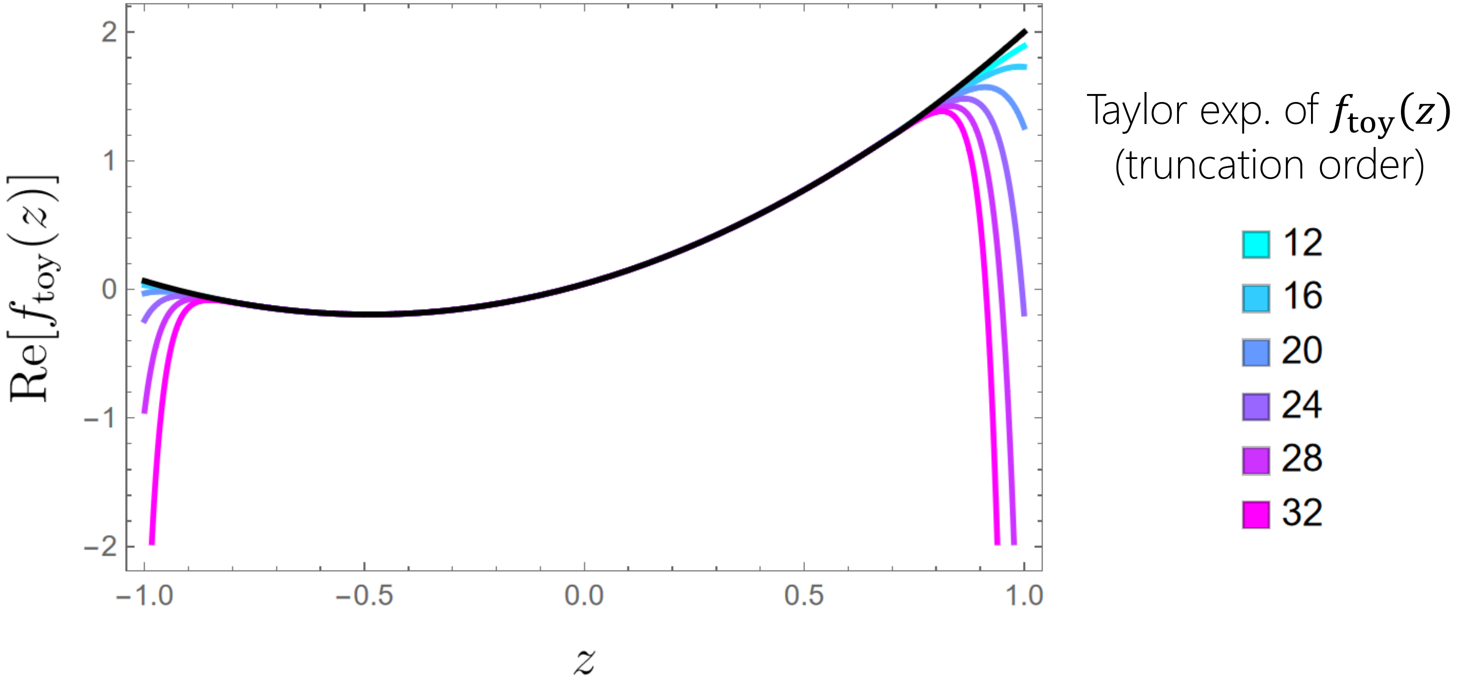
$$\int_{s_\Gamma}^{\infty} dq^2 \, |\det J| |\phi(q^2)\mathcal{F}(q^2)|^2 < \chi + \Delta\chi \quad \Longrightarrow \quad \sum_{n=0}^{\infty} |b_n^2| < 1$$

[Gopal/NG 2024]

New parametrization for FFs that allows to calculate the truncation error!

# Impact of branch cuts in a Taylor expansion

Toy example: $f_{\mathbf{toy}}(z) = z + z^2 + 0.05\sqrt{0.7 - z}$ $\Longrightarrow$ compare with Taylor expansion




Even if the branch cut is suppressed it generates **divergent coefficients**. Hence:

$$\sum_{n=0}^{\infty} b_n^2 \not< \chi$$

# $\Delta\chi$ calculation

Approximate FFs using their large \( $q^2$ \) scaling behaviour calculated in perturbative QCD

E.g. for $B \to K$

[Lepage/Brodsky 1980]
[Akhoury et al. 1994]

$$|\mathcal{F}_+(q^2)|^2 \simeq K\left(\frac{s_\Gamma}{q^2}\right)^2$$

According to [Becher/Hill 2005] $K \sim 1$

Even assuming $K \sim 100$

$$\frac{\Delta\chi}{\chi} \equiv \frac{1}{\chi}\int_{s_\Gamma}^{s_+} dq^2\, |\det J|\, |\phi(q^2)\mathcal{F}(q^2)|^2 \simeq 0.005$$

i.e. smaller than the uncertainty on $\chi$

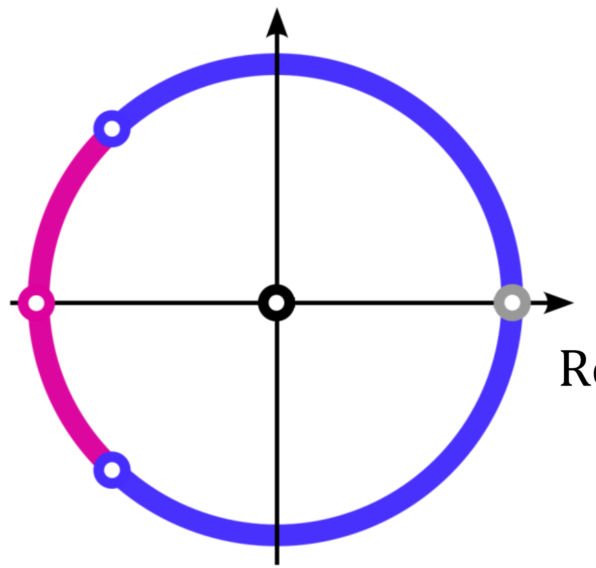
This is due to the fact that $\frac{s_+ - s_\Gamma}{s_\Gamma} \ll 1$ and that $\chi$ is an inclusive quantity while $\Delta\chi$ is exclusive

# Polynomial parametrization

polynomial parametrization ($\hat{z}$ polynomials) [NG/van Dyk/Virto 2020]

$$\mathcal{H}_\lambda(\hat{z}) = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{n=0}^{\infty} \beta_n p_n(\hat{z}) \qquad \sum_{n=0}^{\infty} |\beta_n|^2 < 1$$

$|p_n(\hat{z})| \to \infty$ for $n \to \infty$ some $z$ in the unit disk




$$p_0^{B\to K}(\hat{z}) = \frac{1}{\sqrt{2\alpha_{BK}}}$$

$$p_1^{B\to K}(\hat{z}) = \left(\hat{z} - \frac{\sin(\alpha_{BK})}{\alpha_{BK}}\right)\sqrt{\frac{\alpha_{BK}}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1}}$$

$$p_2^{B\to K}(\hat{z}) = \left(\hat{z}^2 + \frac{\sin(\alpha_{BK})(\sin(2\alpha_{BK})-2\alpha_{BK})}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1}\hat{z} + \frac{2\sin(\alpha_{BK})(\sin(\alpha_{BK})-2\alpha_{BK})}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1}\right)$$

$$p_3^{B\to K}(\hat{z}) = \cdots$$

# Problems with cut modelling and polynomials

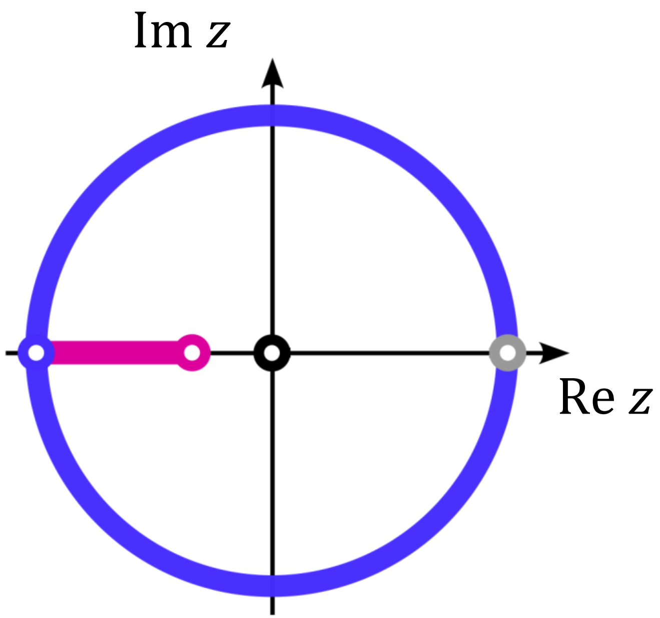
Model the branch cut and subtract it

$$\tilde{\mathcal{F}}(z) \equiv \mathcal{F}(z) - \mathcal{F}_{\text{cut}}(z)$$

expand $\tilde{\mathcal{F}}(z)$ [Boyd/Grinstein/Lebed 1995] [Caprini/Neubert 1996]

Problem: $\mathcal{F}_{\text{cut}}(z)$ is not known

$\Longrightarrow$ cannot rely on exact numerical cancellation of singularities




Expand in polynomials orthogonal on the blue arc [NG/van Dyk/Virto 2020] [Flynn/Jüttner/Tsang 2023]

$$\mathcal{F}(\hat{z}) = \frac{1}{\phi(\hat{z})} \sum_{n=0}^{\infty} b_n p_n(\hat{z})$$

$|\boldsymbol{p_n(\hat{z})}| \to \infty$ for $\boldsymbol{n} \to \infty$ and some $\hat{z}$ in the unit disk