Multiple axions:

Strong CP, Dark Matter and how to count them

LA THUILE 2026 - Les Rencontres de Physique de la Vallée d'Aoste

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CERN MSCA fellow

Based on **JHEP 04 (2024) 056** 2305.15465 w/ B. Gavela and M. Ramos

JHEP 01 (2026) 077 2507.06287 w/ D. Dunsky, C. Manzari, M. Ramos P. Sørensen

2603.XXXX w/ C. Miró and B. Grinstein

The QCD axion

- ➔ Solves the Strong CP problem
- ➔ Excellent Dark Matter candidate

[Peccei+Quinn 77]
[Weinberg, 78]
[Wilczek, 78]

[Abbot+Sikivie, 83]
[Dine and W. Fischler, 83]
[Preskil et al, 91]

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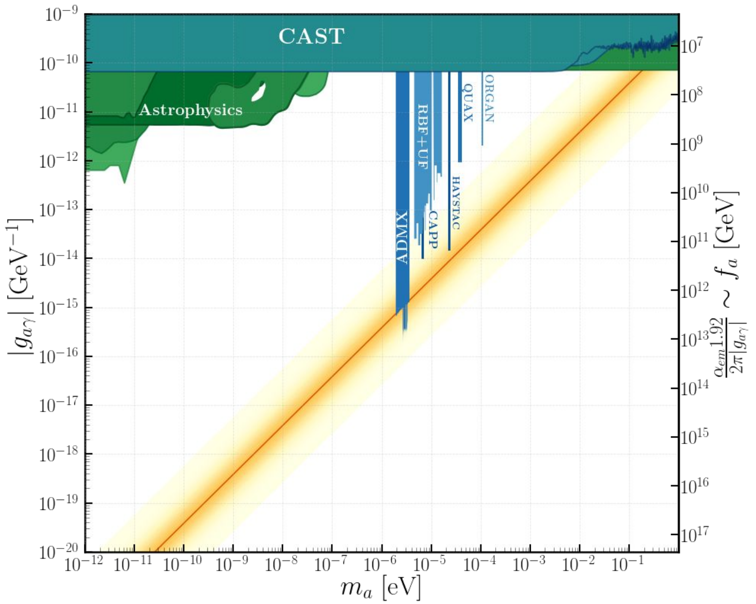

Adapted from AxionLimits
[Ciaran O'hare, 20]

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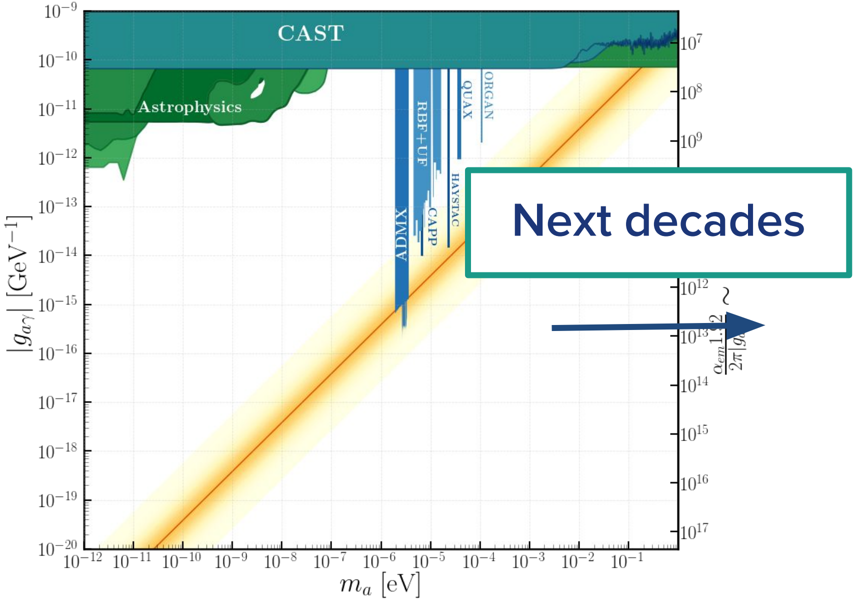

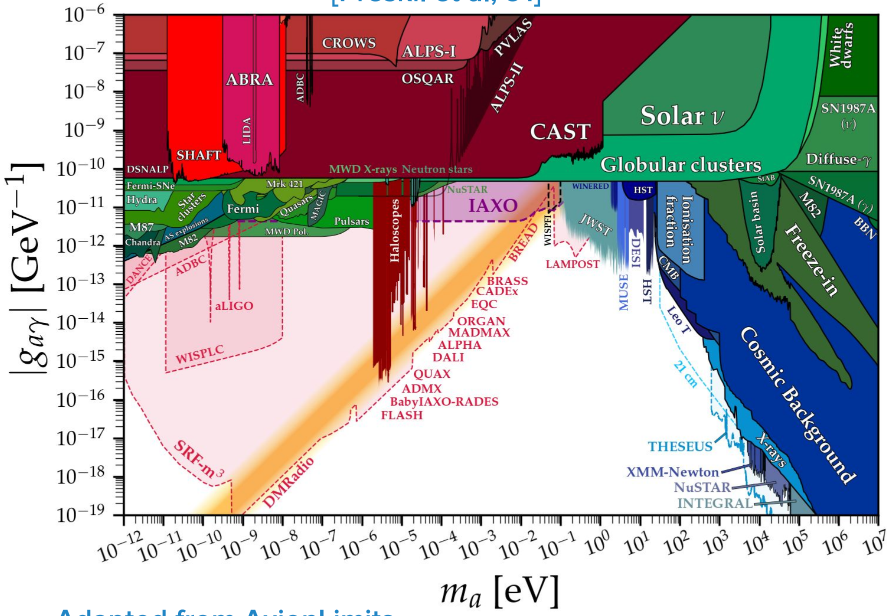

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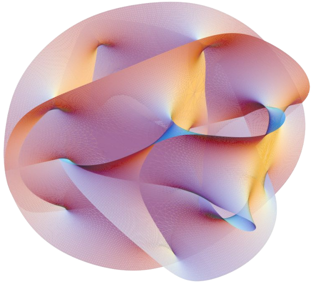
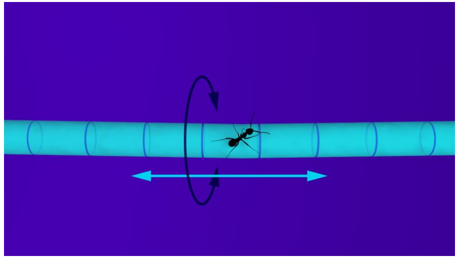
Based on **2305.15465 2507.06287** - Pablo Quílez

Motivation Multiple axions



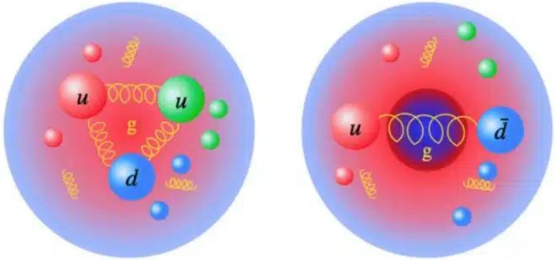
String Theory
axiverse

[Witten 84]
[Arvanitaki+ 10]



Extra dimensions

[Dienes et al, 99]



QCD-like theories
Dark pions

[Cheng et al, 21]

Multiple axions

Strong CP problem

Multiple axions can solve strong CP simultaneously

Dark Matter

Rich cosmological production from multi-axion dynamics

Counting them

How to disentangle multiple axions in helioscopes as IAXO?

[Gavela, Quilez, Ramos JHEP 24] [Dunsky, Manzari, Quilez, Ramos, Sørensen JHEP 26] [Grinstein, Miró, Quilez 26]

The single QCD axion line

$$\mathcal{L} \supset \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

$$\delta\mathcal{L} \equiv -\frac{i}{2}\frac{0.011\,e}{m_n}\frac{a}{f_a}\bar{n}\sigma_{\mu\nu}\gamma_5 n F^{\mu\nu}$$

$$\equiv g_{a\gamma n}$$

$$m_a^2 f_a^2 \simeq m_\pi^2 f_\pi^2 \frac{m_u\, m_d}{(m_u + m_d)^2}$$

Coupling to the nEDM

Axion mass

The single QCD axion line

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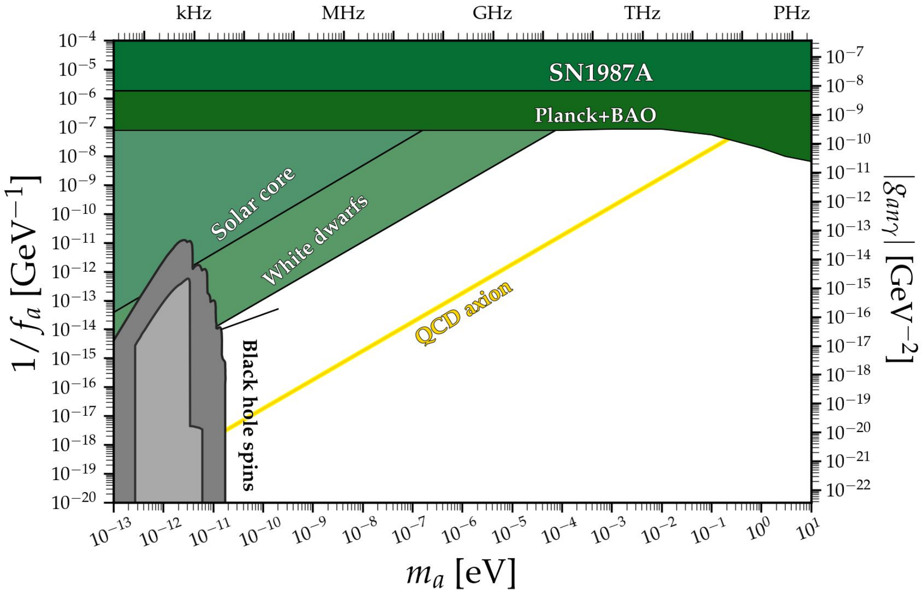


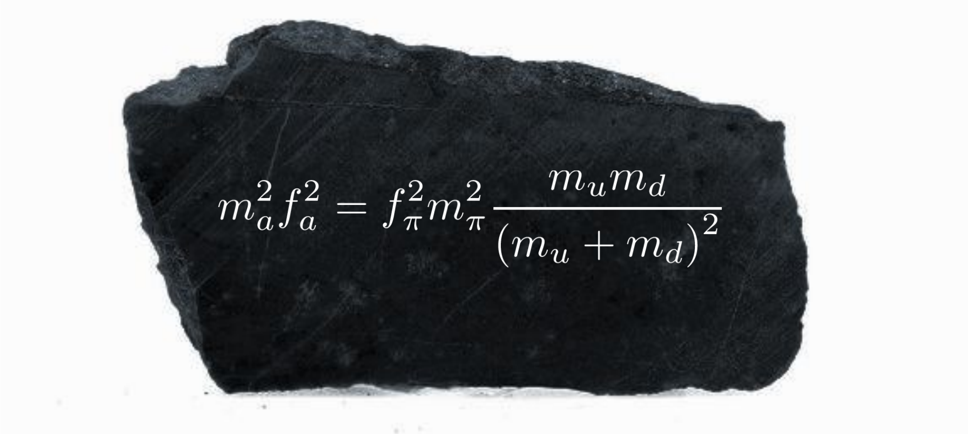
Adapted from AxionLimits [Ciaran O'hare, 20]

“

Can the QCD axion deviate from the standard m_a-f_a relation being QCD the only source of PQ breaking?

“

Or is it written in stone?



Multiple **QCD** axions: Strong CP problem

Multiple QCD axions

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta}\right) G\widetilde{G} - V_B(\hat{a}_1,\, \hat{a}_2, \ldots, \hat{a}_N)\,,$$

$$\exists\ \mathit{true}\ U(1)_{\rm PQ} \Longleftrightarrow \text{classically exact and only broken by QCD}$$

$a_{\rm PQ}$ $\qquad$ $a_{G\tilde{G}}$

Multiple QCD axions

$$\mathcal{L} = -\frac{1}{2}\chi_{\rm QCD}\Bigg(\underbrace{\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k}}_{\hat{a}_{G\widetilde{G}}/F}\Bigg)^2 - V_B(\hat{a}_1,\, \hat{a}_2, \ldots, \hat{a}_N)\,,$$

Diagonalize mass matrix

$$\mathcal{L} \supset -\frac{1}{2}m_i^2 a_i^2$$

Compute coupling to gluons of the N mass eigenstates

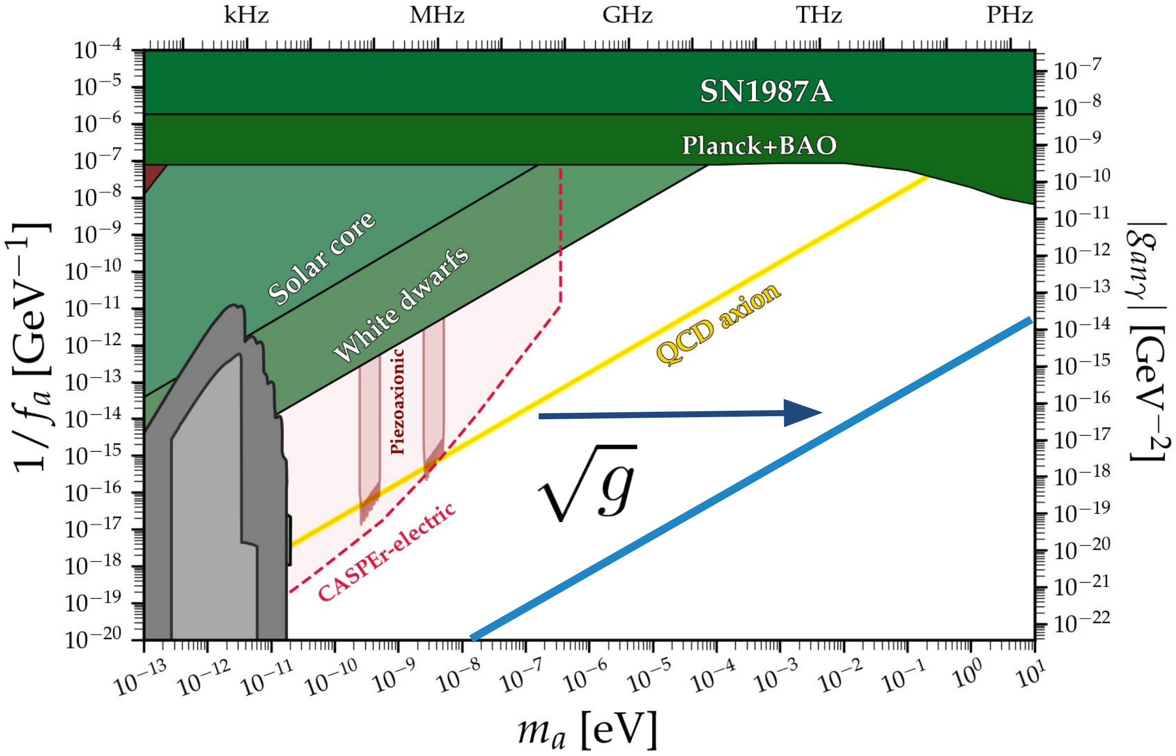
$$\frac{1}{f_i} \equiv \frac{\langle a_i \mid \hat{a}_{G\widetilde{G}}\rangle}{F}$$

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\frac{a_i}{f_i}G\widetilde{G}$$

Based on **2305.15465 2507.06287** - Pablo Quílez

Deviation from the QCD line: g_i-factor

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \times g_i$$



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Toy example: N=2

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \times g_i$$

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}} + \bar{\theta}\right) G\widetilde{G} - \frac{1}{2}\hat{m}_2^2\, \hat{a}_2^2\,. \qquad \longrightarrow V_{N=2} = \frac{1}{2}\chi_{\rm QCD}\left[\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}}\right)^2 + r\,\hat{a}_2^2\right]$$

$$a_{\rm PQ} = \hat{a}_1\,, \qquad \hat{a}_{G\widetilde{G}} = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2)$$

Toy example: N=2

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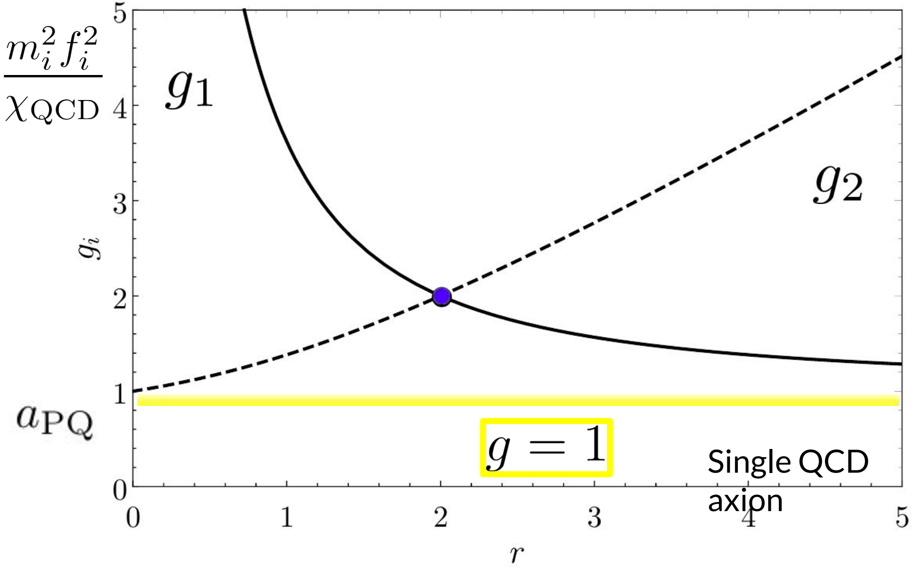
$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$



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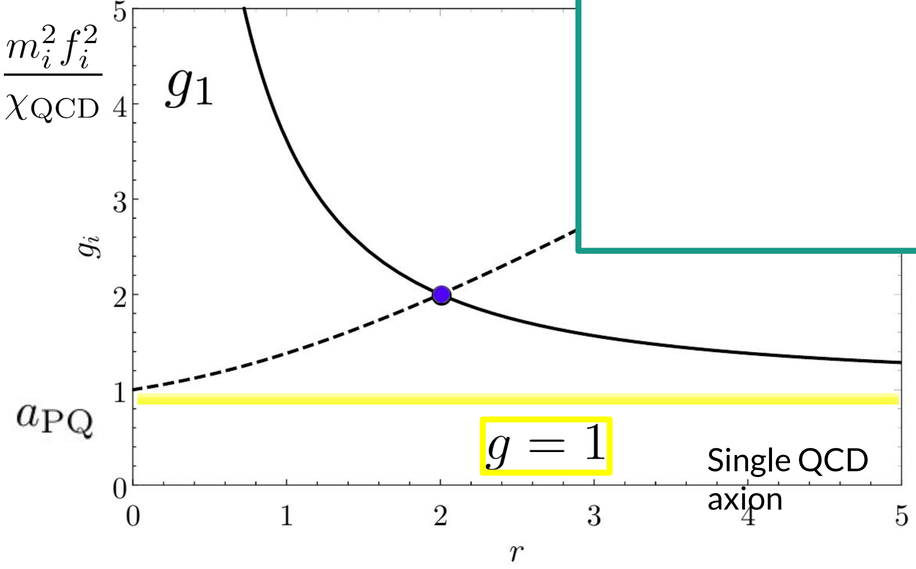
- ➔ **There seems to be a see-saw like pattern, is there any conserved quantity or sum rule?**

$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$



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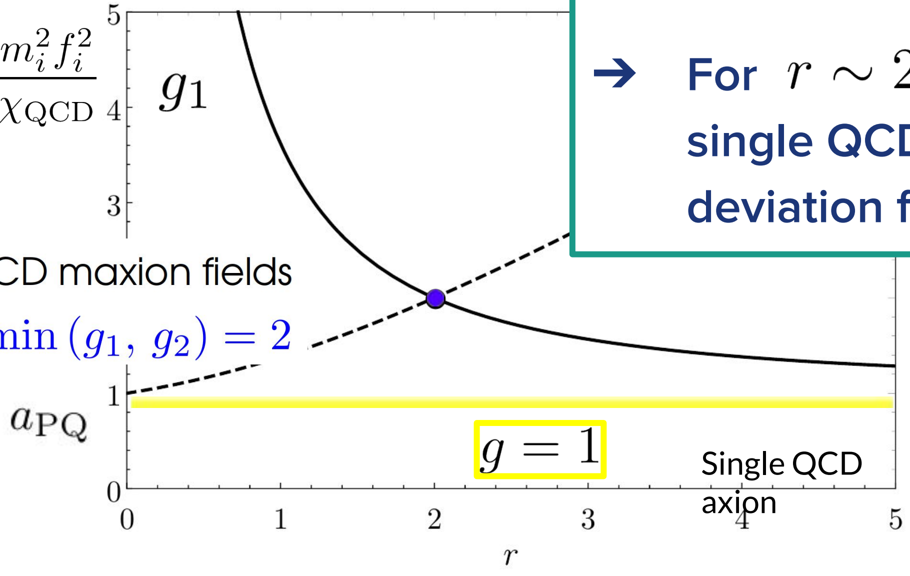
- ➔ There seems to be a see-saw like pattern, is there any conserved quantity or sum rule?
- ➔ For $r \sim 2$ the axions deviate from the single QCD line, what is the maximum deviation for N axions?

$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$



- 2 QCD maxion fields

$$\max\min(g_1, g_2) = 2$$

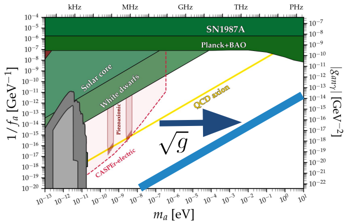
$$\hat{a}_{G\tilde{G}} = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2)$$

QCD-axionness

$$\frac{1}{g_i} = f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \frac{1}{m_i^2 f_i^2} = \frac{m_a^2 f_a^2}{m_i^2 f_i^2} \Big|_{\text{single QCD axion}}$$

$$\frac{1}{g_i} = \frac{\chi_{\text{QCD}}}{m_i^2 f_i^2}$$

- Inverse of the distance to QCD line
- Fraction of its mass stemming from QCD



QCD-axionness: a sum rule from true PQ

$$\frac{1}{g_i} = \frac{\chi_{\rm QCD}}{m_i^2 f_i^2}$$

- Inverse of the distance to QCD line
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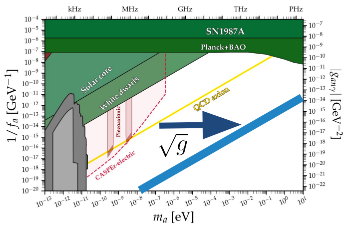


$$\exists\, U(1)_{PQ} \implies \sum_{i=1}^{N} \frac{1}{g_i} = 1\,,$$

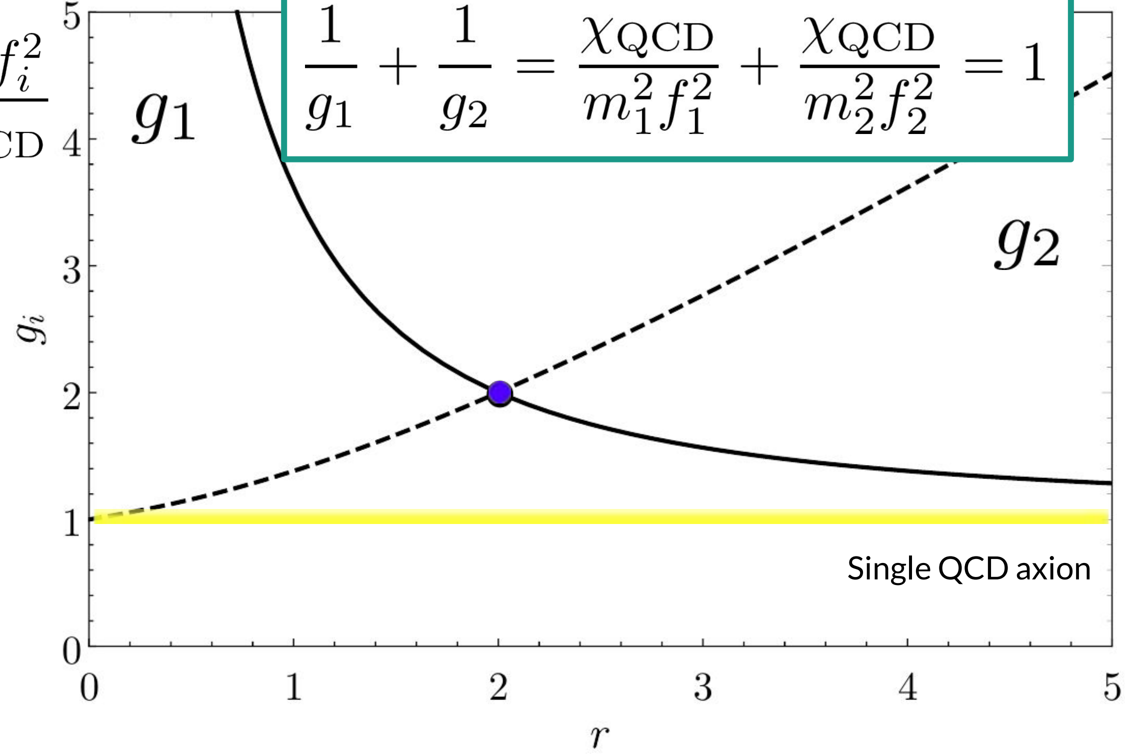
Toy example: N=2

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2} \times g_i$$

$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$

$$\frac{1}{g_1}+\frac{1}{g_2} = \frac{\chi_{\rm QCD}}{m_1^2 f_1^2}+\frac{\chi_{\rm QCD}}{m_2^2 f_2^2} = 1$$

$$g_{1(2)} = \frac{2\sqrt{4+r^2}}{\sqrt{4+r^2}\pm(r-2)},$$



$$g = 1$$

Toy example: N=2

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$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$

$$\frac{1}{g_1} + \frac{1}{g_2} = \frac{\chi_{\rm QCD}}{m_1^2 f_1^2} + \frac{\chi_{\rm QCD}}{m_2^2 f_2^2} = 1$$

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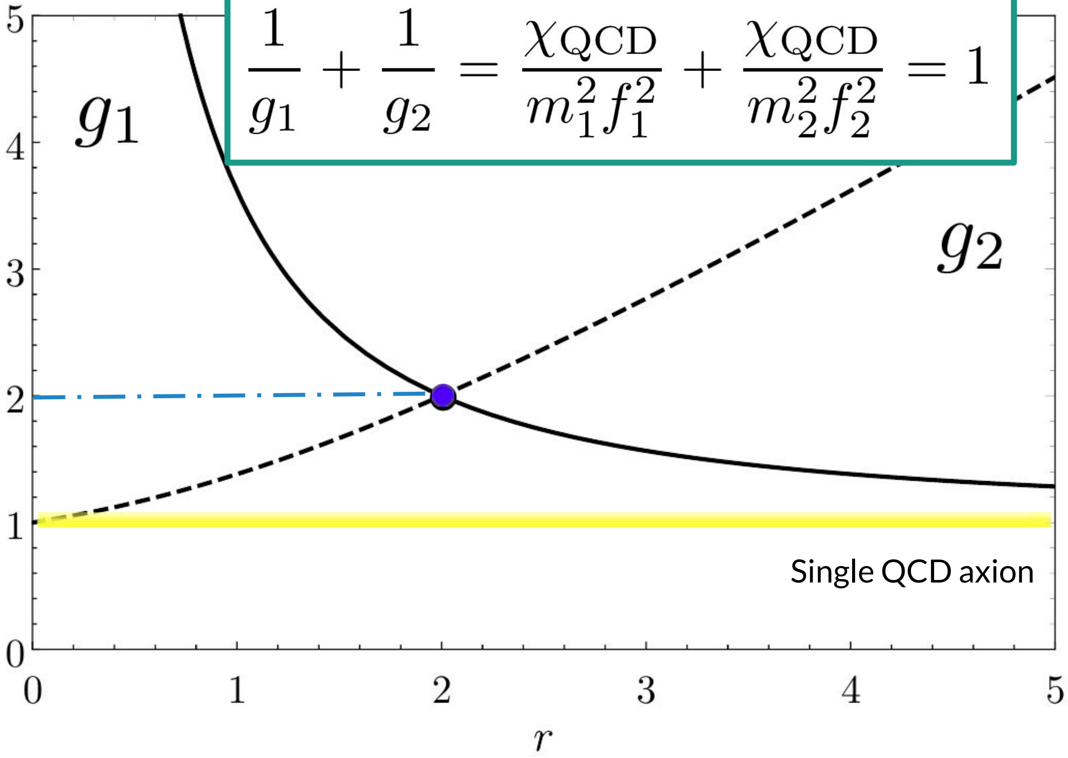


$g = 1$

Based on **2305.15465 2507.06287** - Pablo Quílez

Gluon interaction basis $\neq$ Mass basis $\neq$ PQ basis

Multiple QCD axions will deviate from the standard relation

Combination whose shift sym. is only broken by QCD

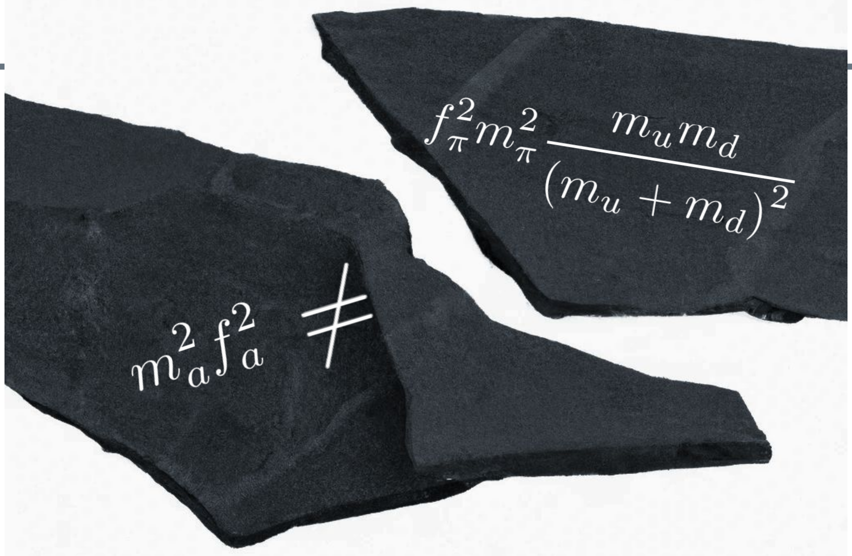
Mass Eigenstates

Combination that couples to gluons

$$\frac{1}{g_i} = \frac{\langle a_{\mathrm{PQ}} \mid a_i \rangle \langle a_i \mid a_{G\tilde{G}} \rangle}{\langle a_{\mathrm{PQ}} \mid a_{G\tilde{G}} \rangle}$$

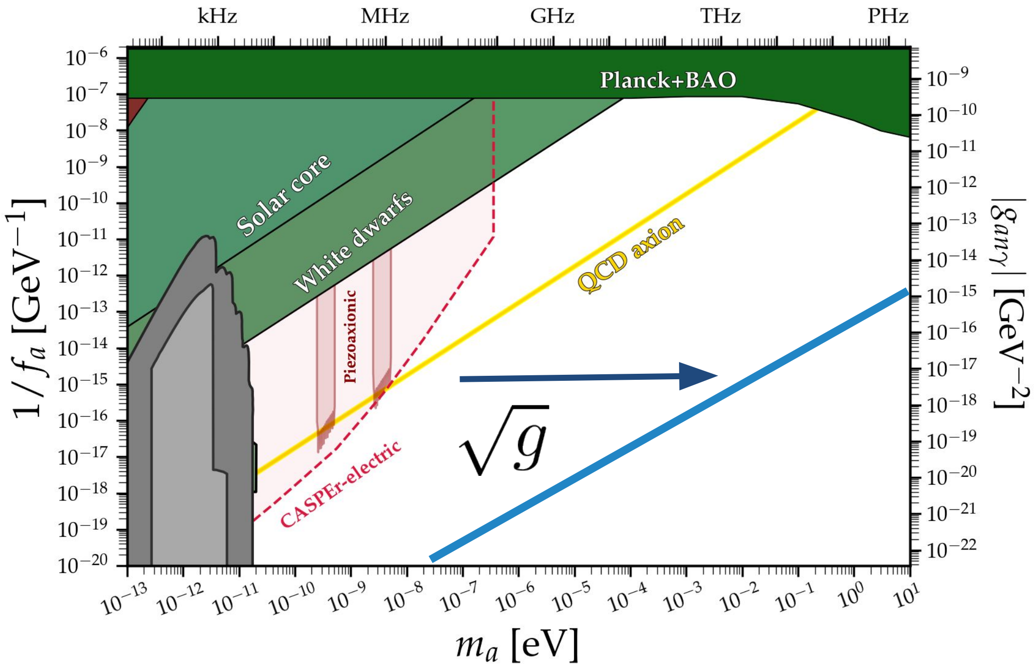
Gluon interaction basis $\neq$ Mass basis $\neq$ PQ basis

Multiple QCD axions will deviate from the standard relation



Experimental consequences $\frac{1}{g_i} = \frac{\chi_{QCD}}{m_i^2 f_i^2}$ $\sum_{i=1}^{N} \frac{1}{g_i} = 1$

1) $g_i \geq 1 = \left(1 + \frac{F^2}{\chi_{QCD}} \frac{\langle a_i|\mathbf{M}_B^2|a_i\rangle}{|\langle a_i|\hat{a}_{G\tilde{G}}\rangle|^2}\right)$



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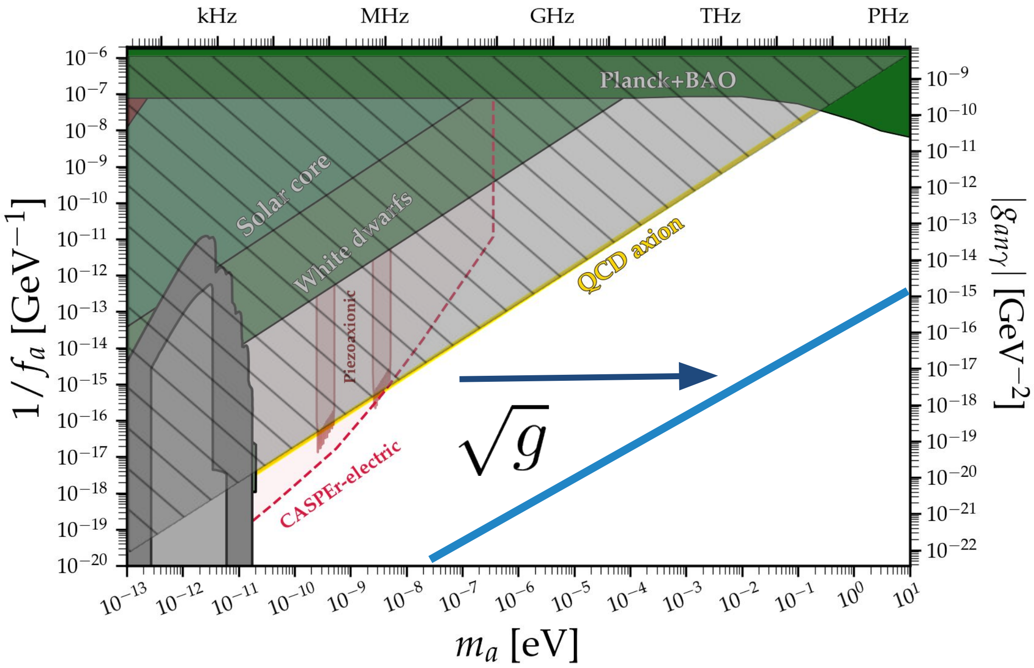

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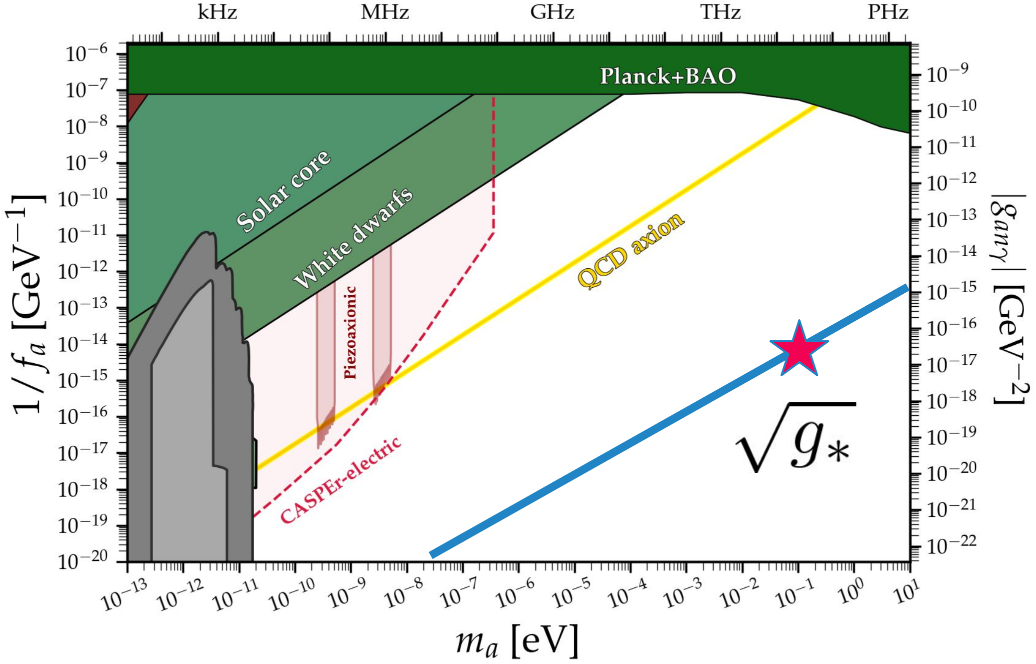

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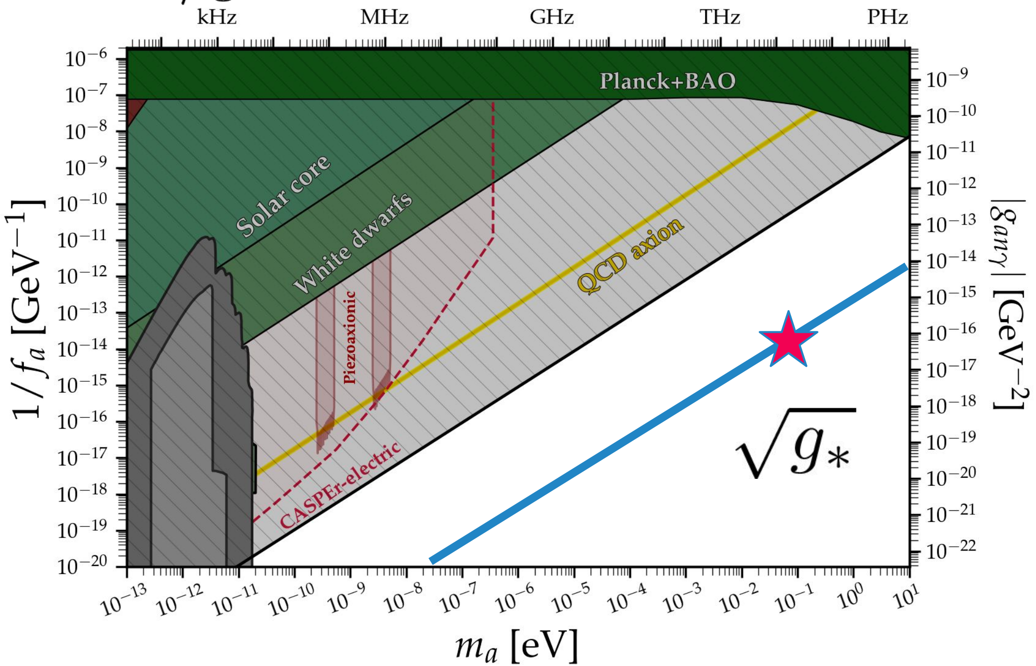

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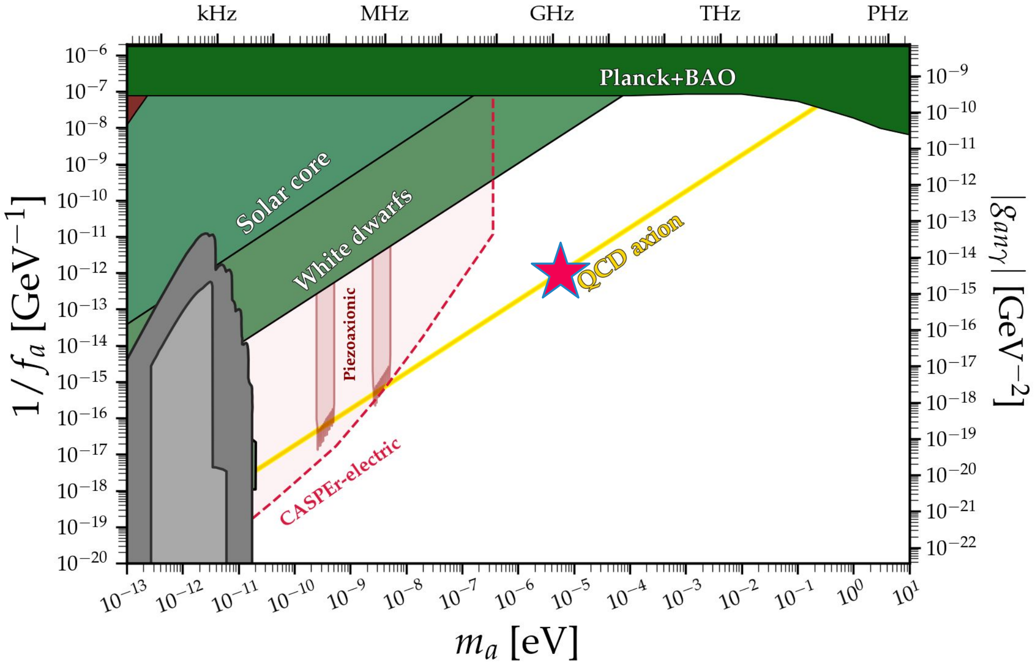

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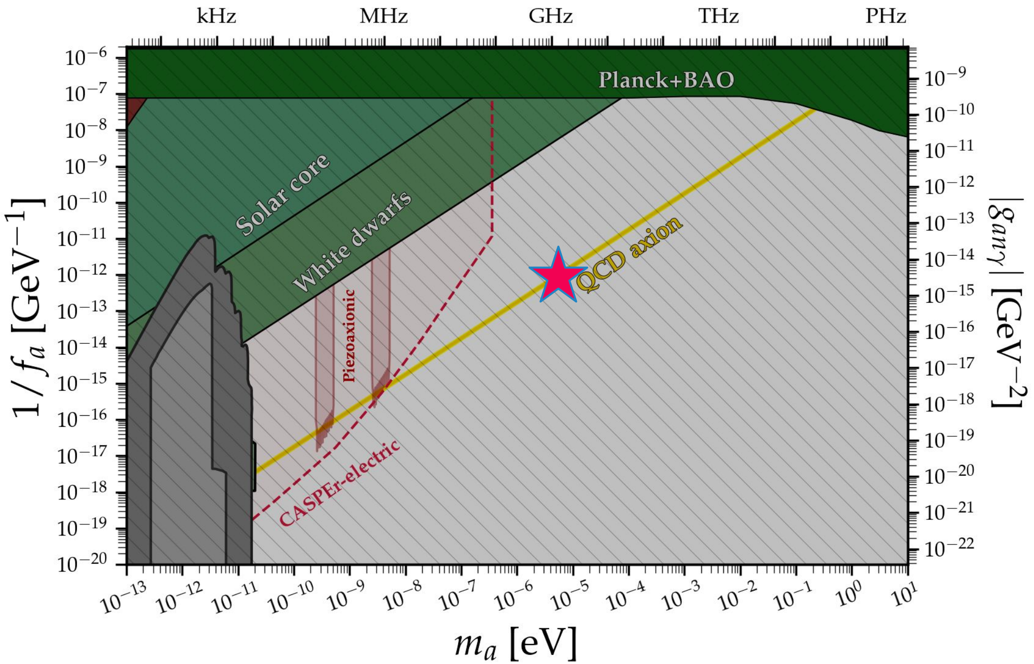

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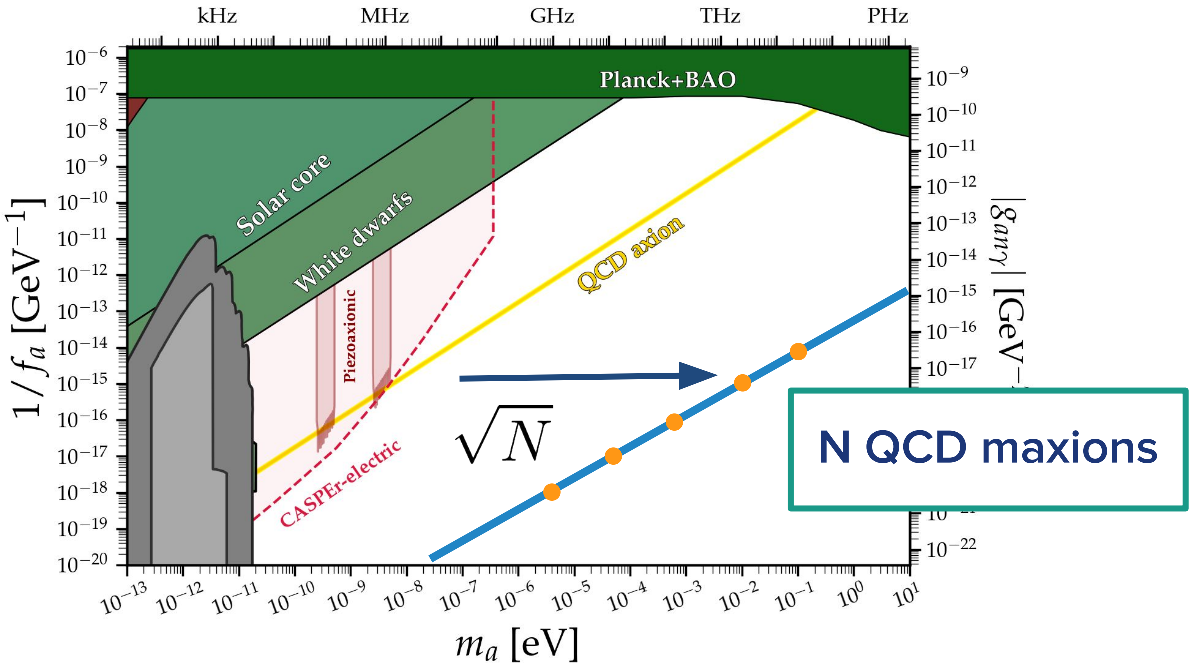
4) $$\max_{\mathbf{M}^2}\left\{\min_i\{g_i\}\right\} = N \quad \Longrightarrow \quad g_i = N,\ \forall\, i\,.$$

QCD Maxions

- Maximally deviated QCD axions
- Multiple QCD axions

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

4) $$\max_{\mathbf{M}^2}\left\{\min_i\{g_i\}\right\} = N \quad \Longrightarrow \quad g_i = N,\ \forall i\,.$$



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MATHEMATICAL PHYSICS

Neutrinos Lead to Unexpected Discovery in Basic Math

65

Three physicists wanted to calculate how neutrinos change. They ended up discovering an unexpected relationship between some of the most ubiquitous objects in math.

PETER B. DENTON, STEPHEN J. PARKE, TERENCE TAO, AND XINING ZHANG

ABSTRACT. If A is an $n \times n$ Hermitian matrix with eigenvalues $\lambda_1(A), \ldots, \lambda_n(A)$ and $i, j = 1, \ldots, n$, then the j^{th} component $v_{i,j}$ of a unit eigenvector v_i associated to the eigenvalue $\lambda_i(A)$ is related to the eigenvalues $\lambda_1(M_j), \ldots, \lambda_{n-1}(M_j)$ of the minor M_j of A formed by removing the j^{th} row and column by the formula

$$|v_{i,j}|^2 \prod_{k=1;k\neq i}^{n} (\lambda_i(A) - \lambda_k(A)) = \prod_{k=1}^{n-1} (\lambda_i(A) - \lambda_k(M_j)) .$$

We refer to this identity as the *eigenvector-eigenvalue identity* and show how

Coupling to photons

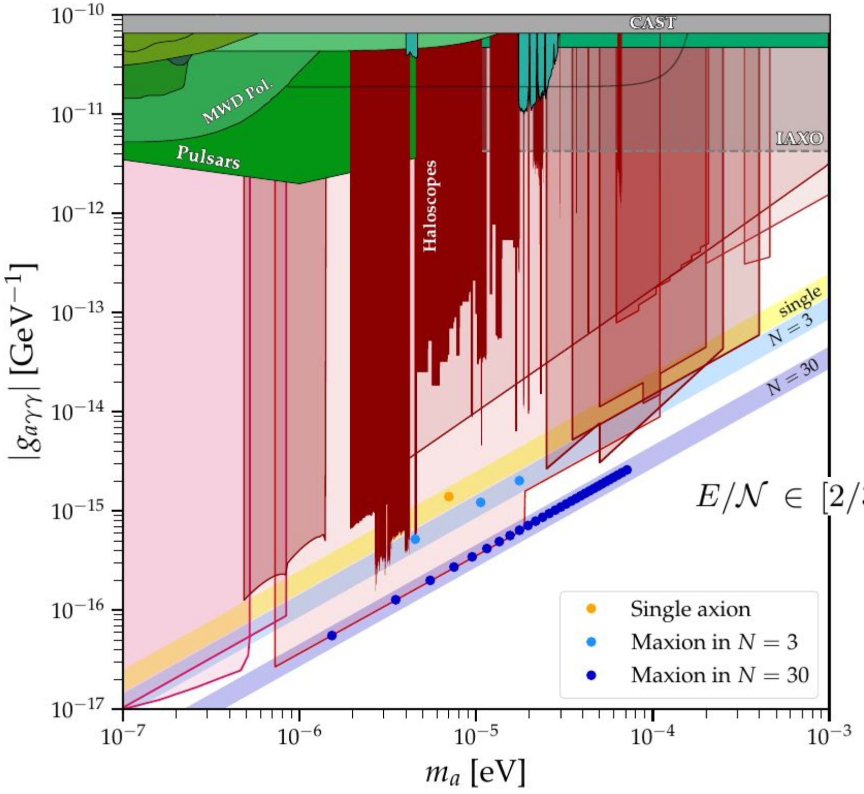
Same E/N

$$\delta\mathcal{L} = \frac{1}{4}\sum_{k=1}^{N} g^0_{\hat{a}_k\gamma\gamma}\,\hat{a}_k F\widetilde{F} \equiv \frac{\alpha_{em}}{8\pi}\sum_{k=1}^{N}\frac{E}{\mathcal{N}}\frac{\hat{a}_k}{\hat{f}_k}F\widetilde{F}\,,$$

All the results apply to photons if all a_k have the same E/N

$$\frac{m_i^2}{g^2_{a_i\gamma\gamma}} = \left.\frac{m_a^2}{g^2_{a\gamma\gamma}}\right|_{\text{single QCD axion}} \times g_i\,.$$

$$\frac{(2\pi)^2\chi_{\rm QCD}}{\alpha_{em}^2}\left[\frac{E}{\mathcal{N}} - 1.92\right]^{-2}\sum_{i=1}^{N}\frac{g_{a_i\gamma\gamma}}{m_i^2} = 1\,.$$



$E/\mathcal{N} \in [2/3, 8/3]$.

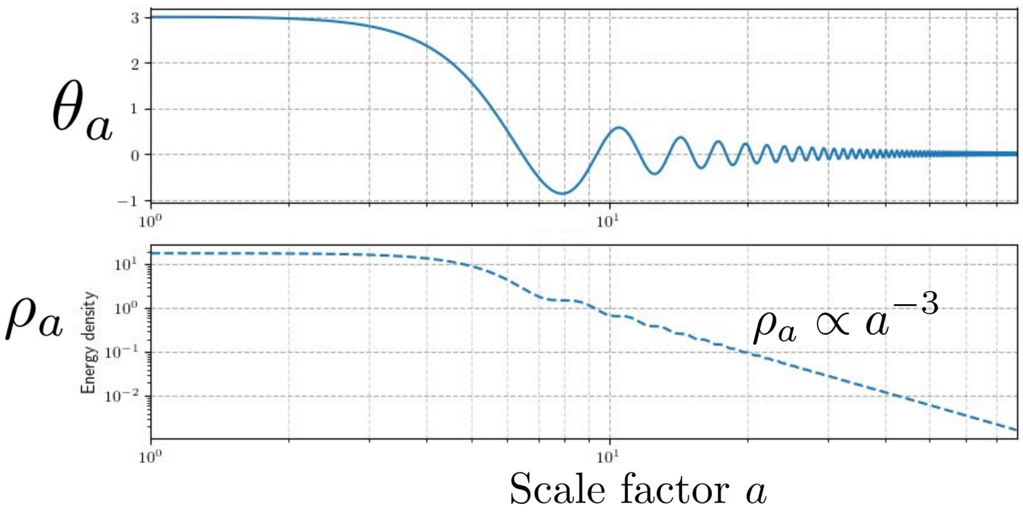
Multiple axions: **Dark Matter**

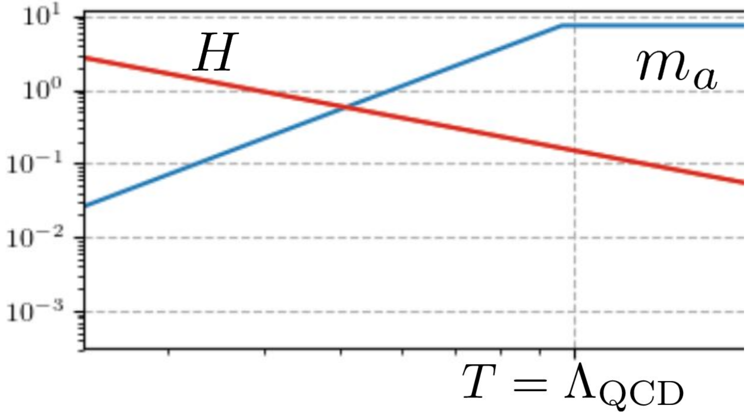
Axion DM: Misalignment mech.

[Abbot+Sikivie, 83]
[Dine and W. Fischler, 83]
[Preskil et al, 91]

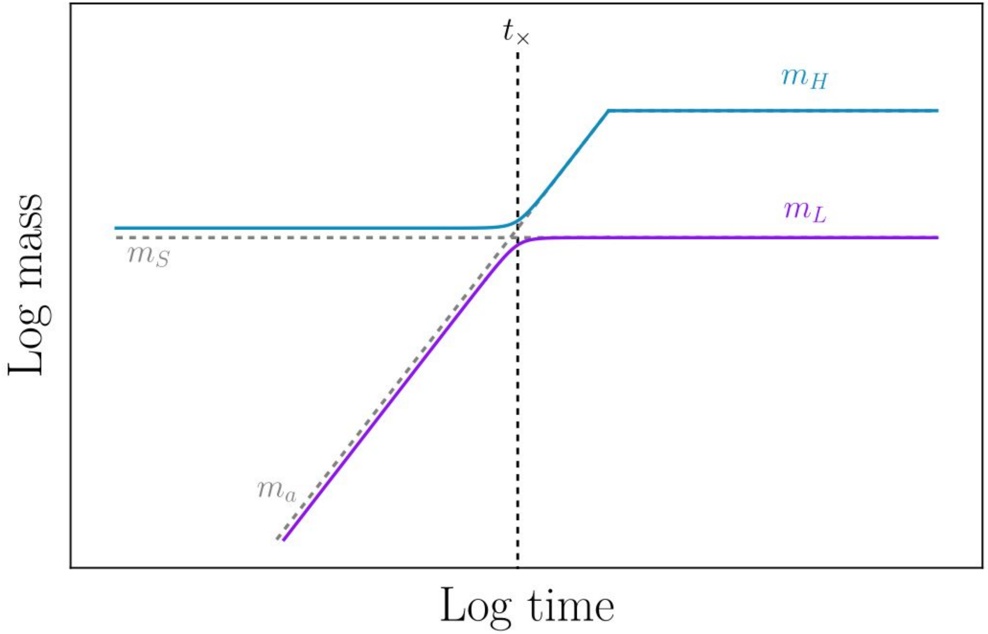
$$\ddot{\theta}_a + 3H\dot{\theta}_a + m_a^2 \sin(\theta_a) = 0$$

~Damped harmonic oscillator: $\left(\ddot{x} + \gamma\dot{x} + \omega^2 x = 0\right)$





Multiple Axion DM: Avoided crossing



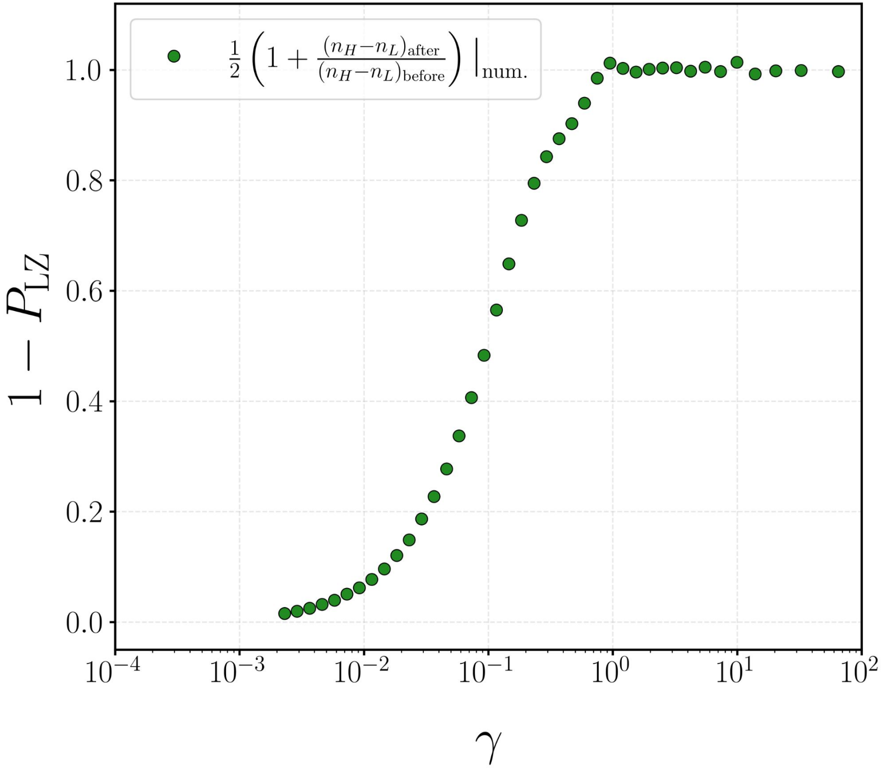
Adiabatic theorem: a slow-acting perturbation in the Hamiltonian does not affect the energy level populations

[Kitajima+Takahashi, 14]

Non-adiabatic: Landau-Zener conversion

Author	Date	System
Majorana	02-1932	Spin 1/2 in a magnetic field
Landau	06-1932	Inelastic, adiabatic atomic collisions
Zener	06-1932	Crossing of polar/homopolar states
Stückelberg	11-1932	Inelastic, adiabatic atomic collision

Multiple Axion DM: Landau-Zener



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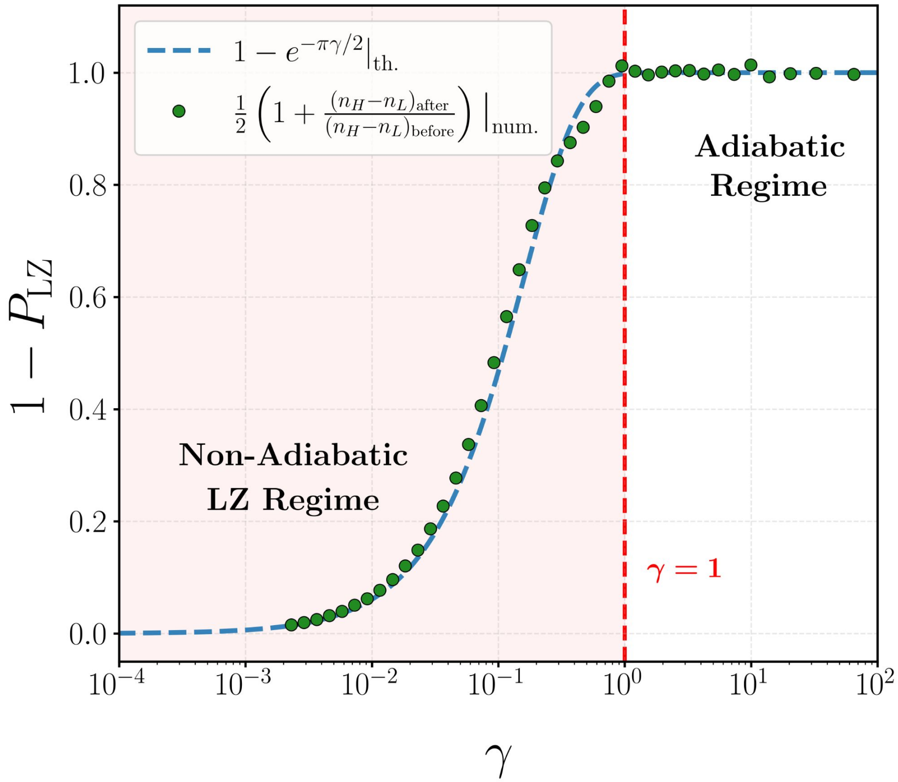
Multiple Axion DM: Landau-Zener



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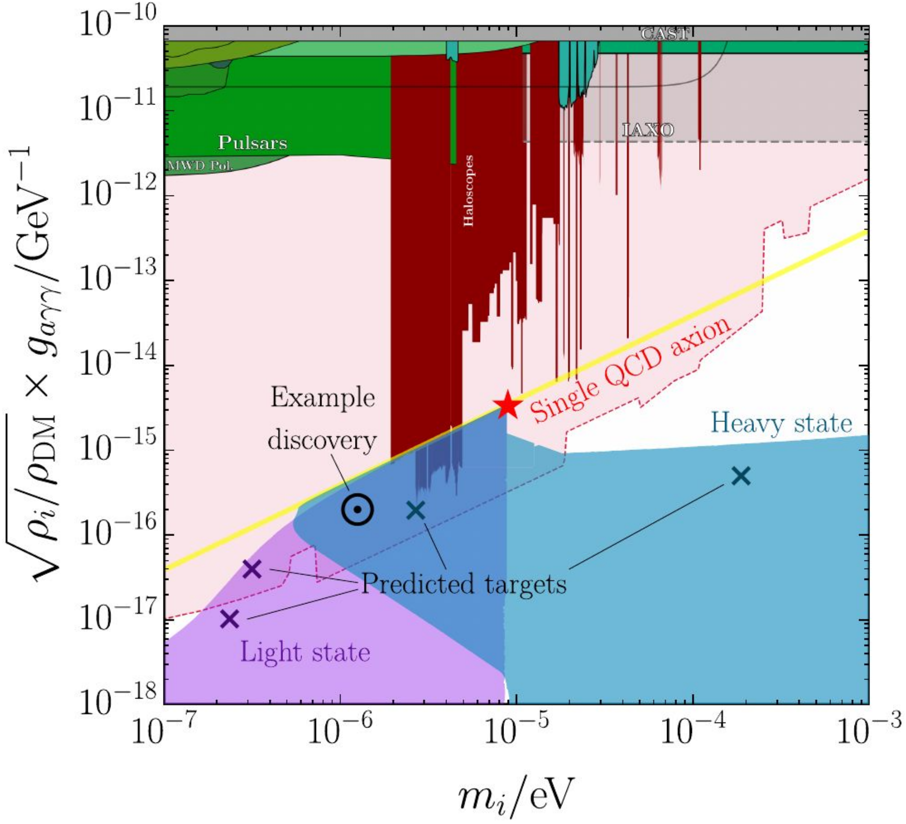
Multiple Axion DM: Landau-Zener



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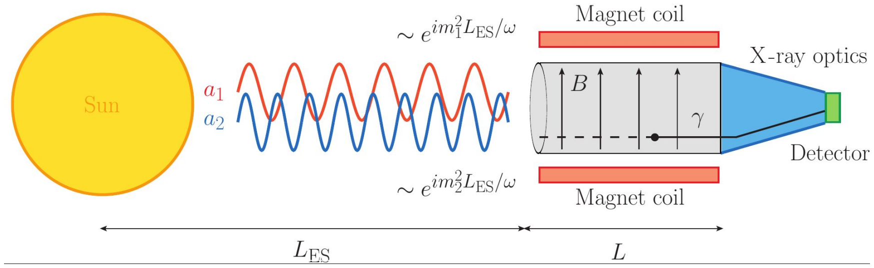
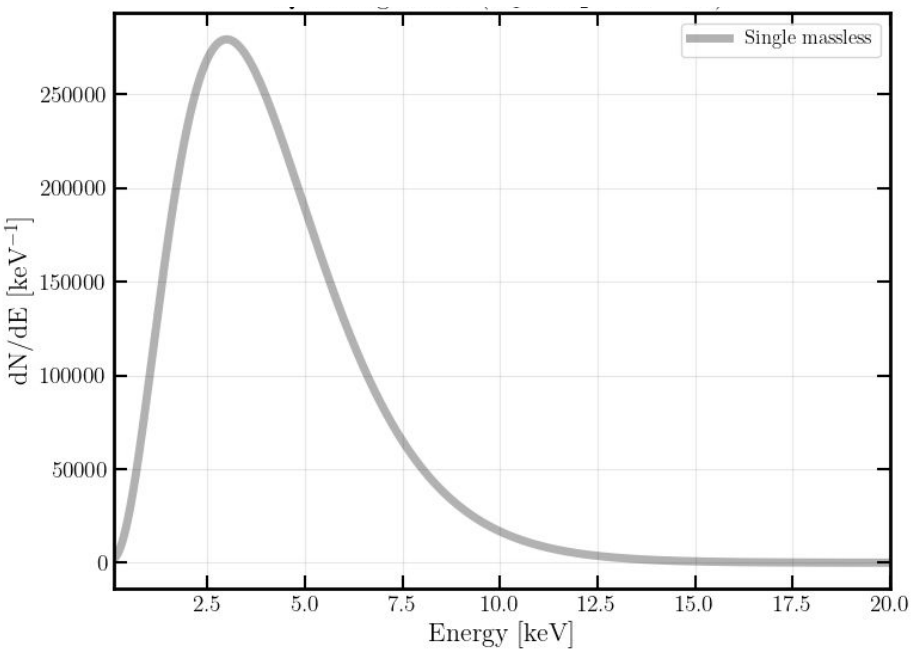
Multiple axions:
How to count them

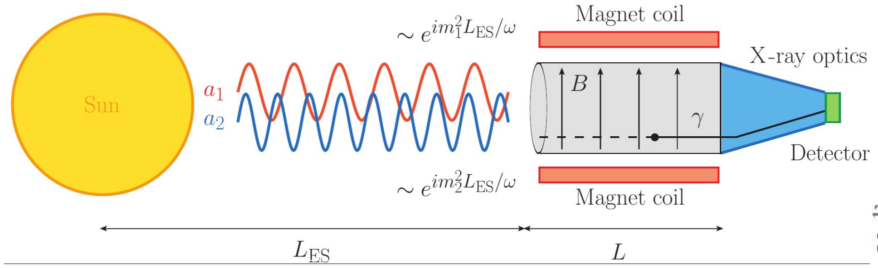
Counting axions with IAXO



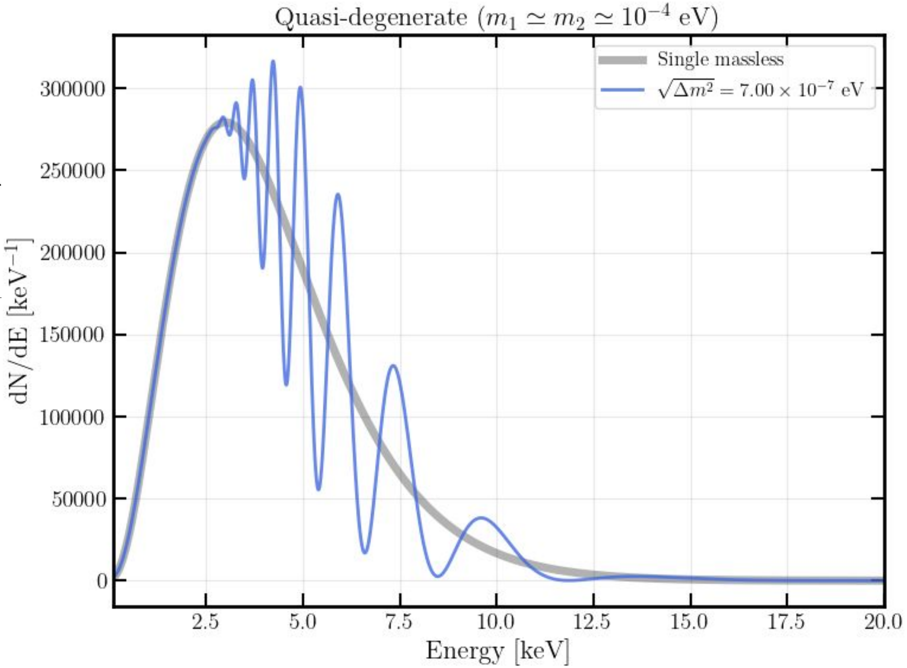
$$P_{\{a_1, a_2\}\to\gamma} = \left(\frac{gBL}{4}\right)^2 \left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{\Delta m_{21}^2 L_{ES}}{2\omega}\right)\right]$$



Counting axions with IAXO



$$P_{\{a_1,a_2\}\to\gamma} = \left(\frac{gBL}{4}\right)^2 \left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{\Delta m_{21}^2 L_{ES}}{2\omega}\right)\right]$$



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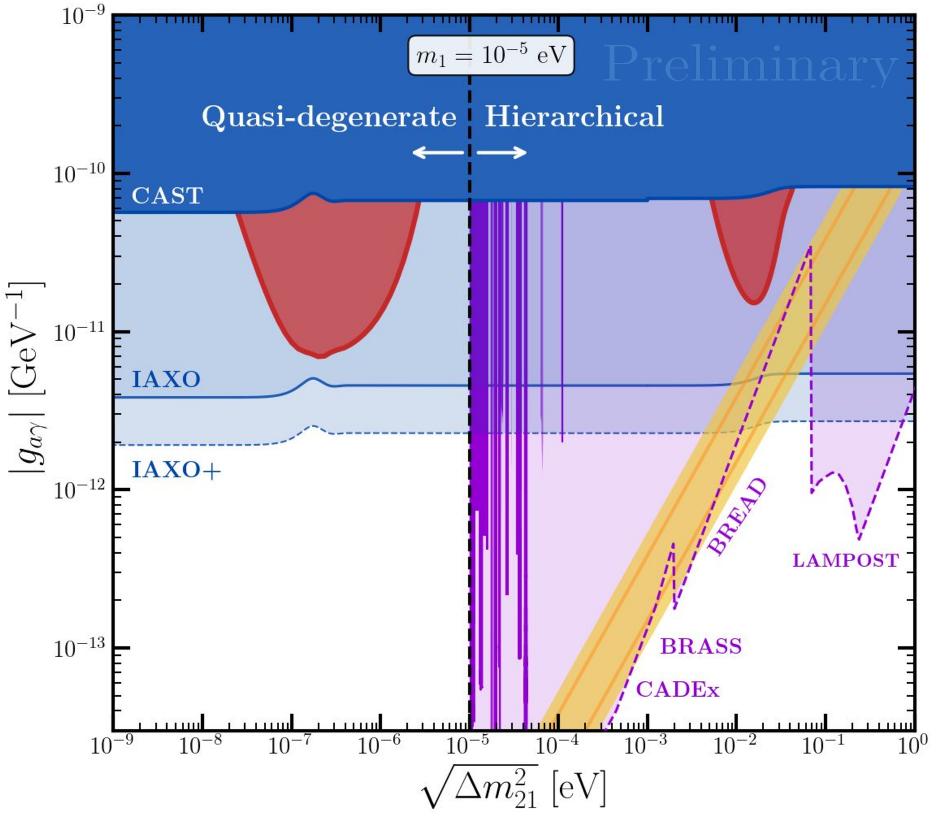
Counting axions with IAXO



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Conclusions

- ➔ A plethora of BSM scenarios predict <u>multiple axions</u>
- ➔ **Strong CP**: A sum rule from $U(1)_{PQ}$ links the possible values $\{m_i, f_i\}$.
- ➔ **Dark Matter**: Avoided crossing + LZ modifies Mis. mech. prediction
- ➔ **Pheno**: Spectral information @IAXO can distinguish them (axion osc.)

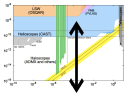
Thank you

Back up slides

Beyond the canonical band

Summary slide from Patras 2021 Review talk: "True axions beyond the canonical band" P. Quilez

$g_{a\gamma}$



A) Photophilic/photophobic axions

1. Single scalar: Playing with fermionic representations

"Preferred axion window"

[Di Luzio, Mescia, Nardi, 16]
[Di Luzio, Mescia, Nardi, 18]

"Axion from monopoles"

[Sokolov, Ringwald, 21]

2. Multiple scalars: Alignment in field space

"Clockwork axion"

[Farina et al, 17]
[Coy, Frigerio, 17]
[Kim et al, 04]
[Choi et al, 14 and 16]
[Kaplan et al 16]
[Giudice et al 16]

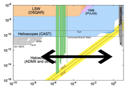
"KNP alignment"

[Agrawal et al 17]
[Kim et al, 04]
+ Refs in FIPs report [2102.12143]

"Multi-higgs models"

[Di Luzio, Mescia, Nardi, 17]
[Di Luzio, Giannotti, Nardi, Visinelli, 16]
[Darmé, Di Luzio, Giannotti, Nardi, 20]

m_a



B) Heavy/even lighter axions

1. Heavy axions: extra instantons

[Rubakov, 97]
[Berezhiani et al ,01]
[Fukuda et al, 01]
[Hsu et al, 04]
[Gianotti, 05]
[Hook et al, 14]
[Chiang et al, 16]
[Khobadize et al,]
[Dimopoulos et al, 16]
[Gherghetta et al, 16]
[Agrawal et al, 17]
[Gaillard, Gavela, Houtz, Rey PQ, 18]
[Fuentes-Martin et al, 19]
[Csaki et al, 19]
[Gherghetta et al, 20]

2. Even lighter QCD axion

[Hook, 18]
[Luzio, Gavela, PQ, Ringwald, 21]
[Luzio, Gavela, PQ, Ringwald, 21]

More PQ breaking

$$\partial_\mu j^\mu_{\rm PQ} = G\tilde{G} + \ldots$$

Exact diagonalization: N=2

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2} \times g_i$$

$\equiv \chi_{\rm QCD}$

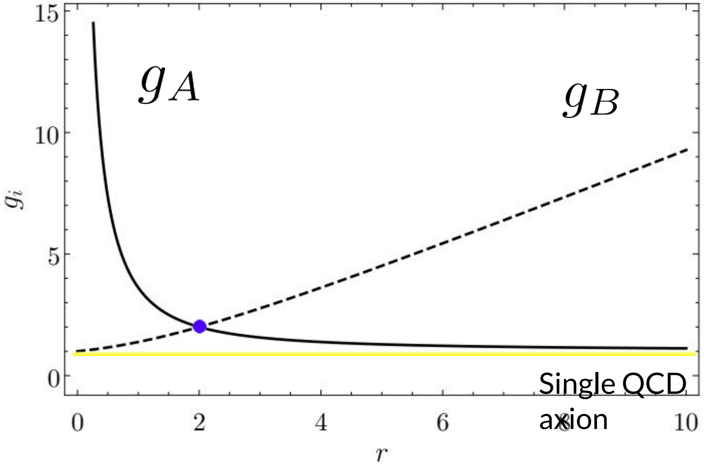
$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}} + \bar{\theta}\right) G\widetilde{G} - \frac{1}{2}\hat{m}_2^2\,\hat{a}_2^2\,. \quad \longrightarrow V_{N=2} = \frac{1}{2}\chi_{\rm QCD}\left[\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}}\right)^2 + r\,\hat{a}_2^2\right]$$

$$m_{1,2}^2 = \frac{\chi_{\rm QCD}}{2\hat{f}^2}\left(2 + r \mp \sqrt{4+r^2}\right)$$

$$g_{1(2)} = \frac{2\sqrt{4+r^2}}{\sqrt{4+r^2} \pm (r-2)}\,,$$

$$a_1 = \frac{2\hat{a}_1 + \hat{a}_2\left(r - \sqrt{4+r^2}\right)}{\sqrt{2}\sqrt{4+r^2 - r\sqrt{4+r^2}}}\,,$$

$$a_2 = \frac{2\hat{a}_2 + \hat{a}_1\left(-r + \sqrt{4+r^2}\right)}{\sqrt{2}\sqrt{4+r^2 - r\sqrt{4+r^2}}}\,.$$



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QCD Maxion conditions

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

4) $$\max_{\mathbf{M}^2} \left\{ \min_i \{g_i\} \right\} = N \qquad \Longrightarrow \quad g_i = N, \ \forall\, i\,.$$

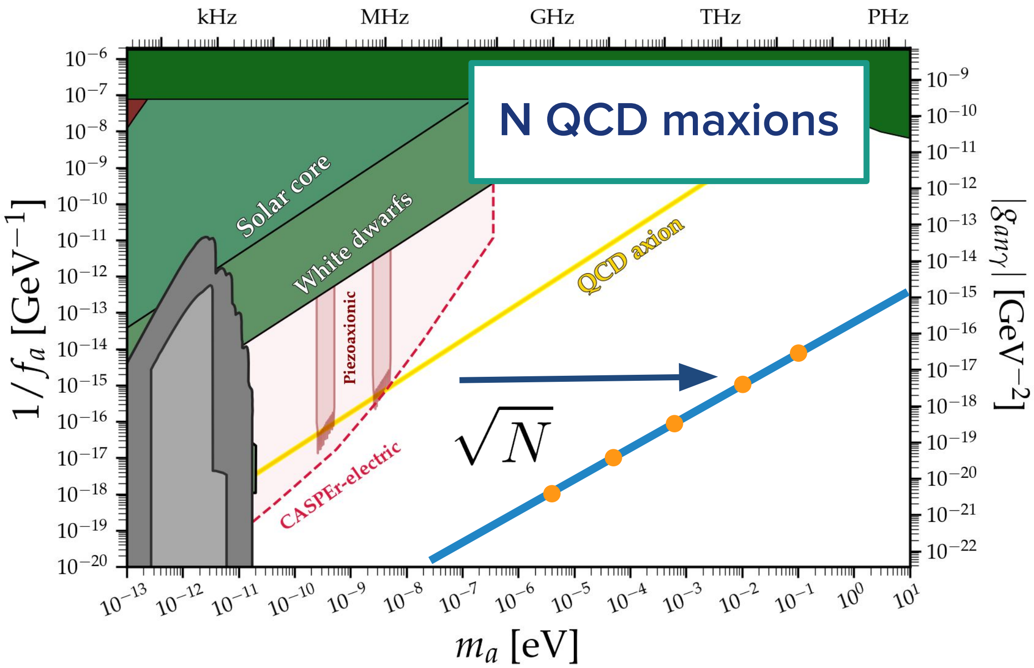
$$\mathcal{B}^{\mathbf{M}^2}_{N-k} = N\,\frac{\chi_{\rm QCD}}{F^2}\,\mathcal{B}^{\mathbf{M}^2_1}_{N-k-1}\,.$$

$$p_{\mathbf{M}^2}(\lambda) = \sum_{k=0}^{N} \frac{(-1)^{N-k}}{(n-k)!}\,\mathcal{B}^{\mathbf{M}^2}_{n-k}\,\lambda^k$$

$$\operatorname{tr}\mathbf{M}^2 = \sum_{i=1}^{N} m_i^2 = N\,\frac{\chi_{\rm QCD}}{F^2}\,.$$

$$m = N(N+1)/2.$$

m-parameter family of Maxion matrices,



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As a particular case of the QCD Maxion

If the starting extra potential is diagonal

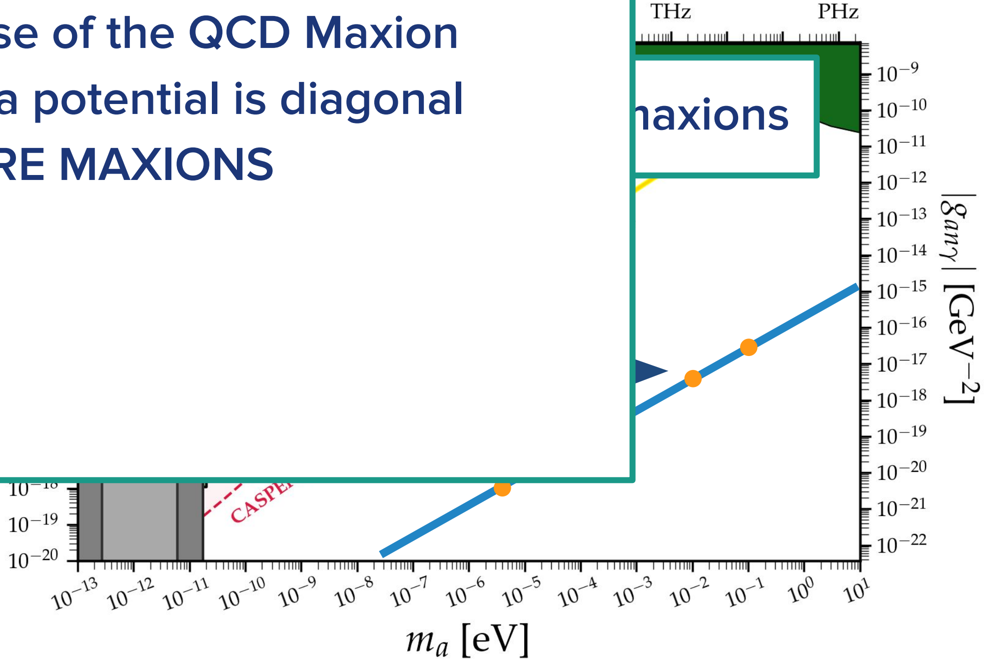
LAGUERRE MAXIONS

$\mathcal{B}^{\mathbf{M}^2}_{N-k} =$

$p_{\mathbf{M}^2}(\lambda)$

$\mathrm{tr}\, \mathbf{M}^2$

$$m = N(N+1)/2.$$



m-parameter family of Maxion matrices,

QCD Maxion conditions

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

4) $\max_{\mathbf{M}} \left\{ \min\{g_i\} \right\} = N \implies g_i = N, \ \forall i \, .$

$\mathcal{B}^{\mathbf{M}^2}_{N-k} =$

$p_{\mathbf{M}^2}(\lambda)$

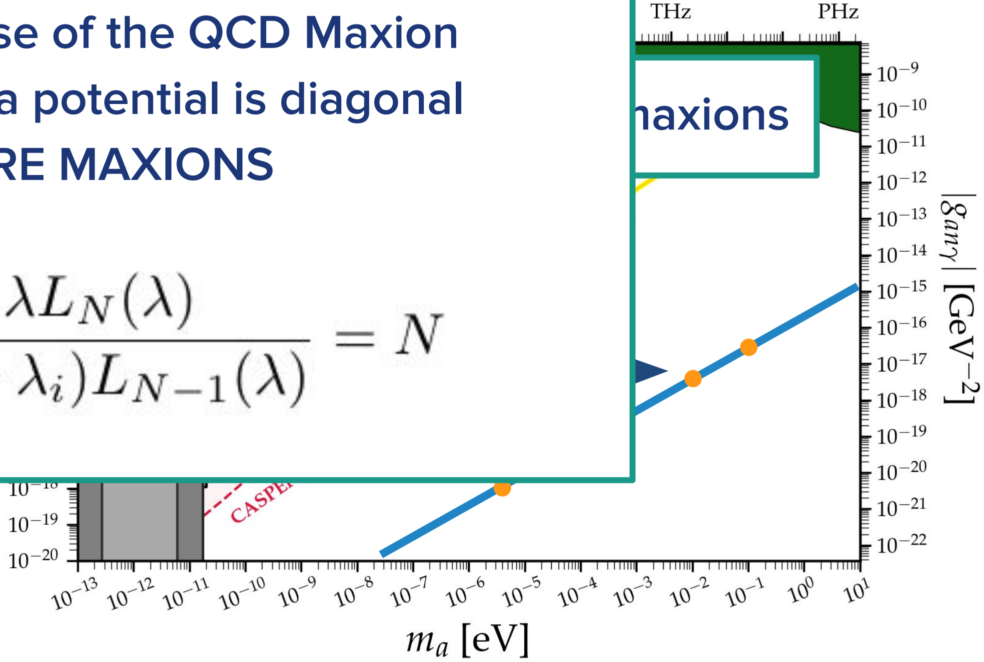
$\mathrm{tr}\, \mathbf{M}^2 =$

As a particular case of the QCD Maxion

If the starting extra potential is diagonal

LAGUERRE MAXIONS

$$g_i \equiv \lim_{\lambda \to \lambda_i} \frac{\lambda L_N(\lambda)}{(\lambda - \lambda_i) L_{N-1}(\lambda)} = N$$



m-parameter family of Maxion matrices,

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