

LA DETERMINAZIONE DI α_s : DISCORSO SUL METODO E I SUOI LIMITI

STEFANO FORTE
UNIVERSITÀ DI MILANO & INFN



UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA



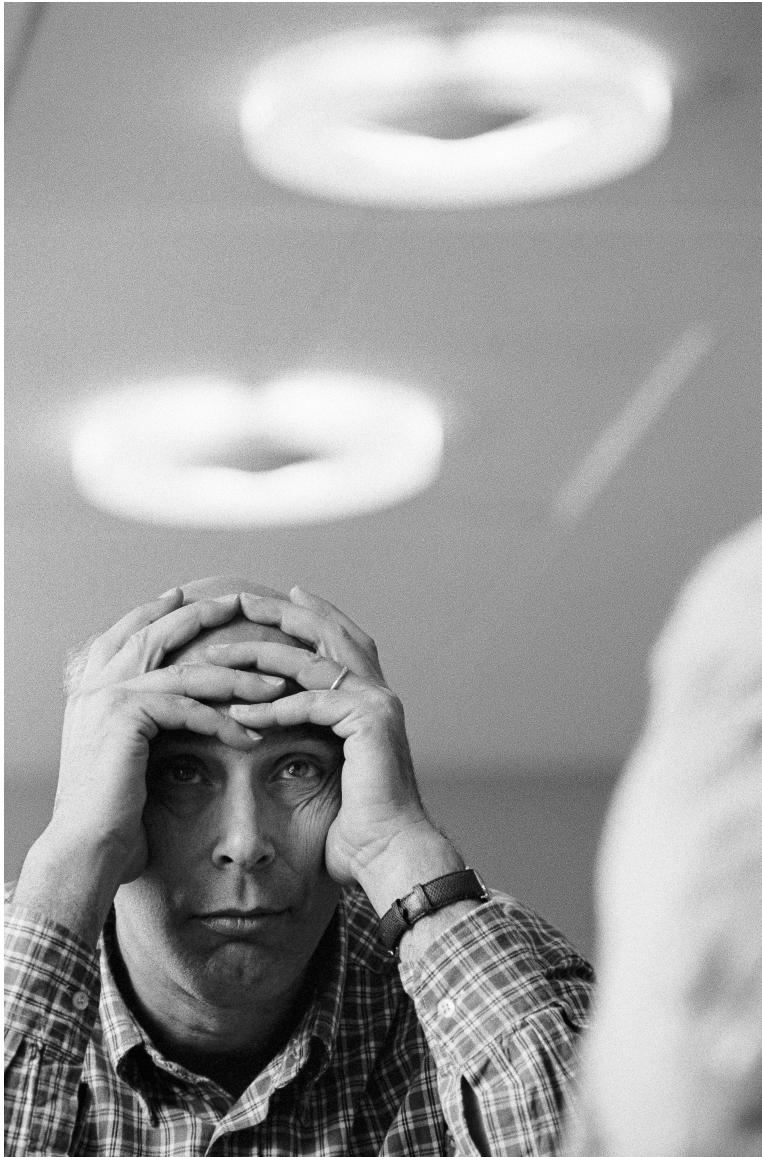
Istituto Nazionale di Fisica Nucleare

Guido Altarelli, 10 anni dopo

Roma, 2-12-2025

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GUIDO AND α_s



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The QCD Running Coupling and its Measurement

Guido Altarelli^{†‡}

*Dipartimento di Fisica 'E. Amaldi', Università di Roma Tre
INFN, Sezione di Roma Tre, I-00146 Rome, Italy*

and

*CERN, Department of Physics, Theory Division
CH-1211 Geneva 23, Switzerland*

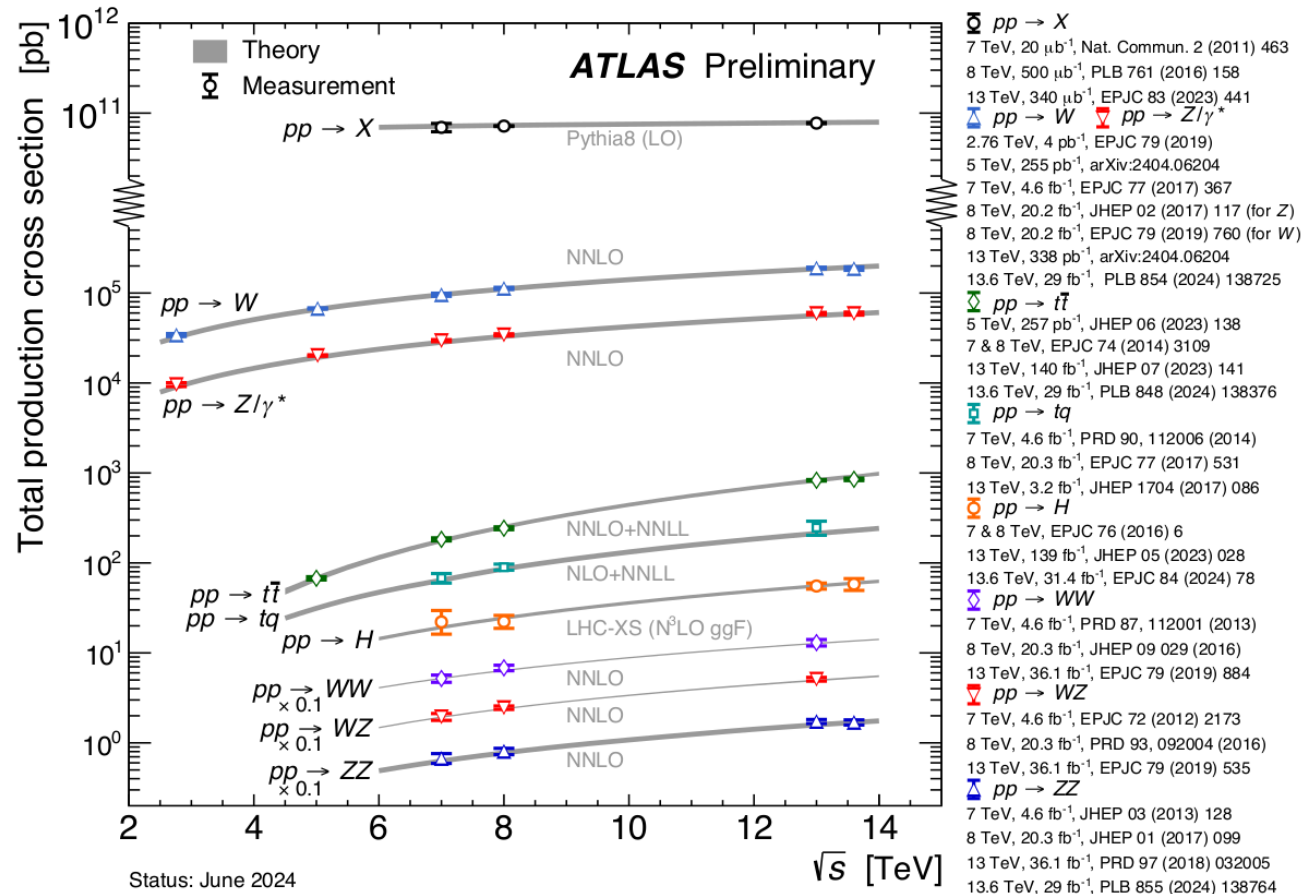
E-mail: guido.altarelli@cern.ch

In this lecture, after recalling the basic definitions and facts about the running coupling in QCD, I present a critical discussion of the methods for measuring α_s and select those that appear to me as the most reliably precise

I think that this is a good example of an underestimated error which is obtained within a given machinery without considering the limits of the method itself.

LHC: THE FIRST PRECISION HADRON COLLIDER!

ATL-PHYS-PUB-2024-011



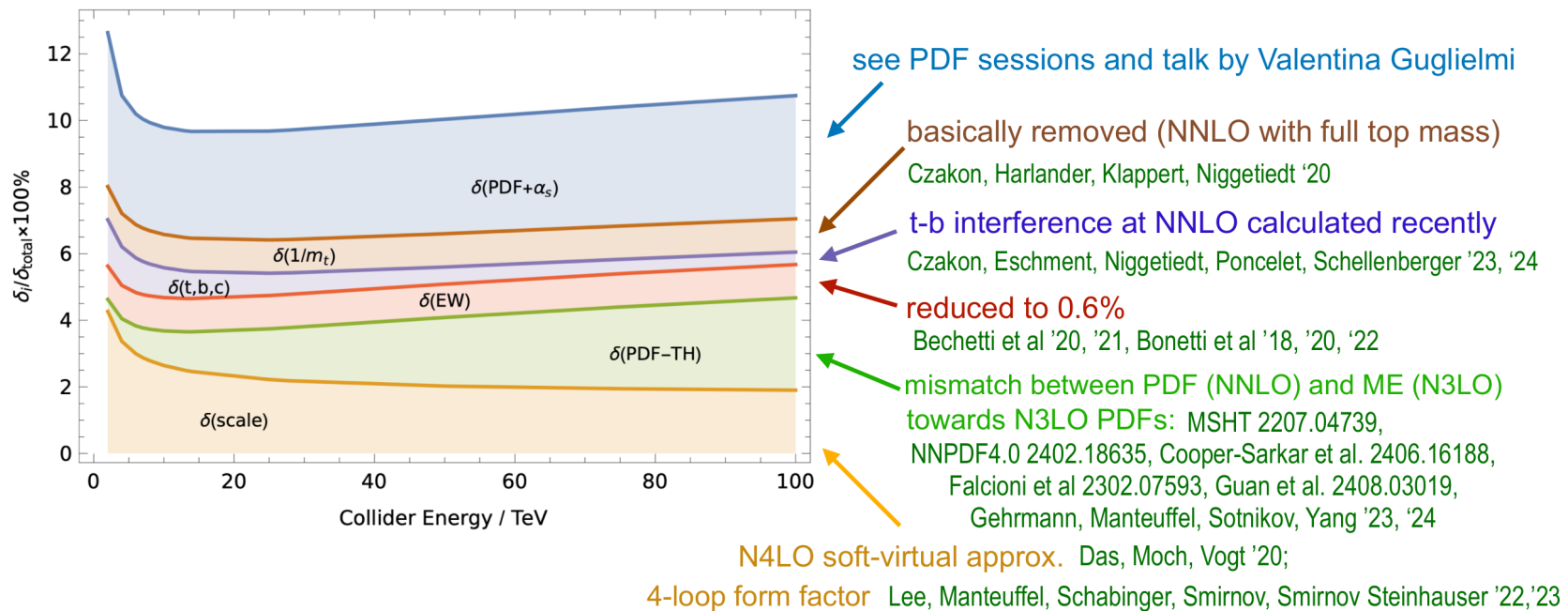
α_s AND THE PRECISION FRONTIER

HIGGS IN GLUON FUSION: UNCERTAINTIES

Higgs production in gluon fusion



uncertainty budget 6 years ago Dulat, Lazopoulos, Mistlberger '18
N3LO in heavy top limit

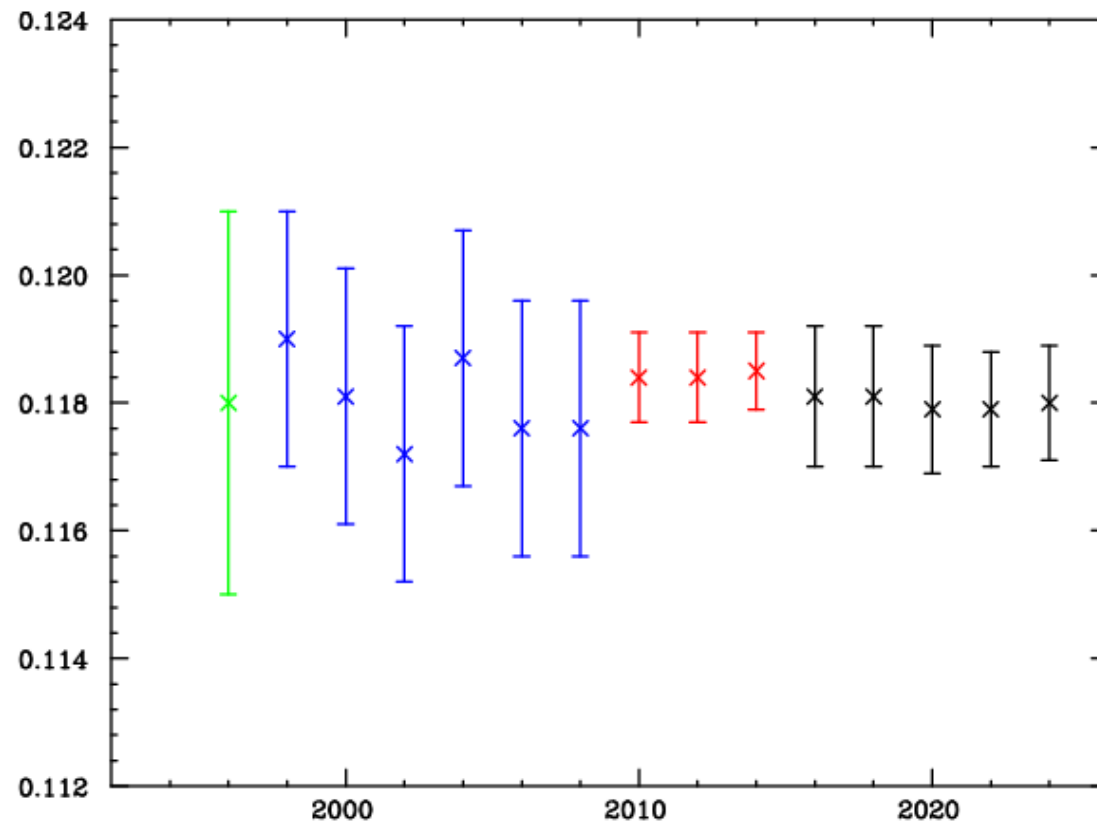


UNCERTAINTY ON $\alpha_s \sim 1\% \Rightarrow \sim 3\%$ ON **CROSS SECTION**

$LO \propto \alpha_s^2$; $NLO \approx LO \approx 2NNLO$

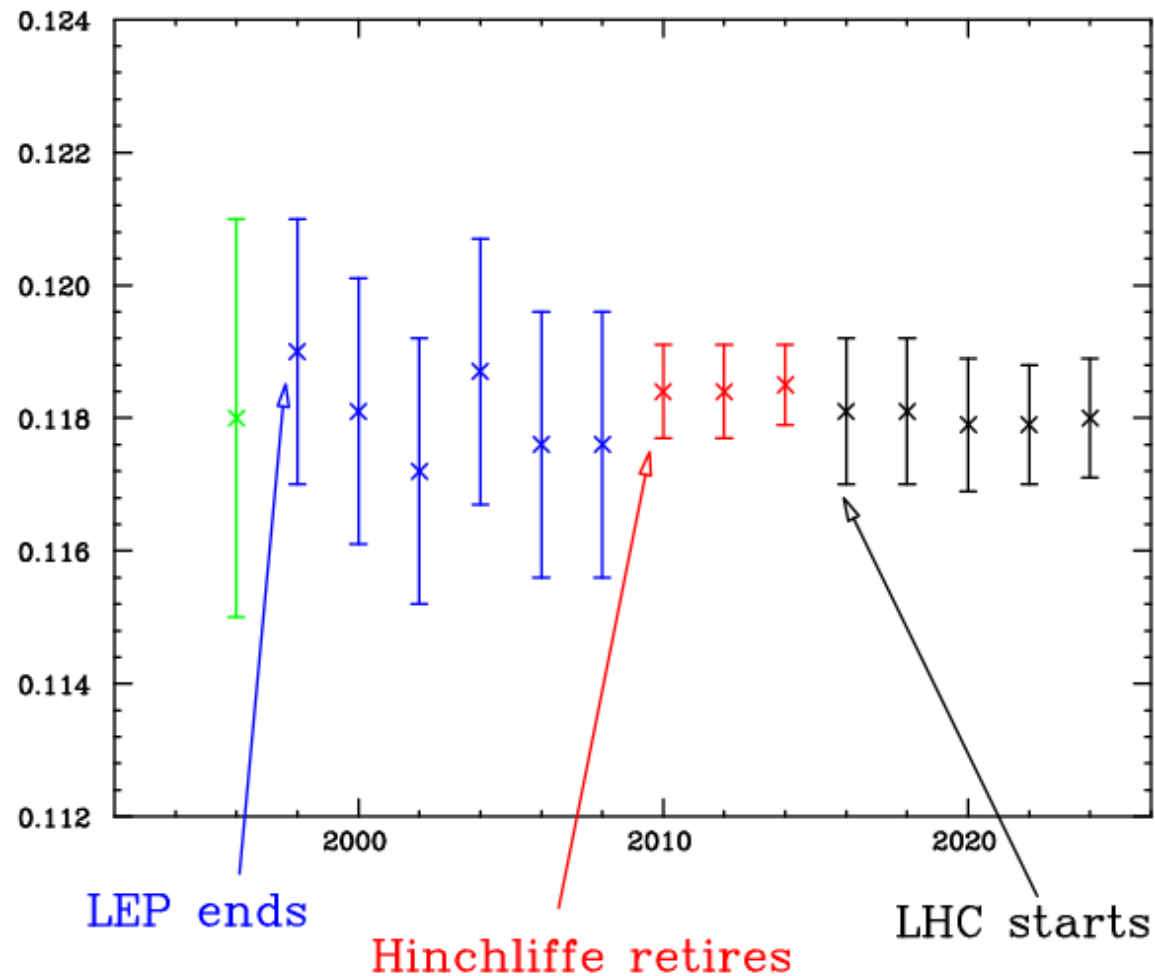
DO WE KNOW THE α_s UNCERTAINTY?

THE PDG VALUE OVER TIME



DO WE KNOW THE α_s UNCERTAINTY?

THE PDG VALUE OVER TIME



- RESULT **DEPENDS ON WHO PERFORMS THE ANALYSIS**
- UNCERTAINTY **INCREASES WITH NEW DATA**

α_s IN THE PDG NOW

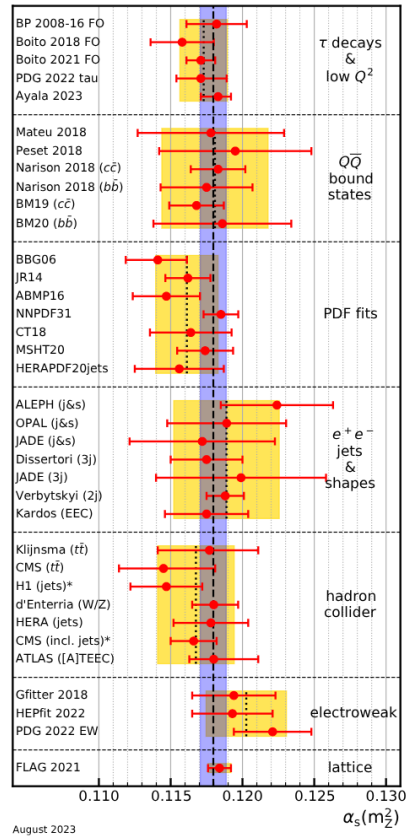
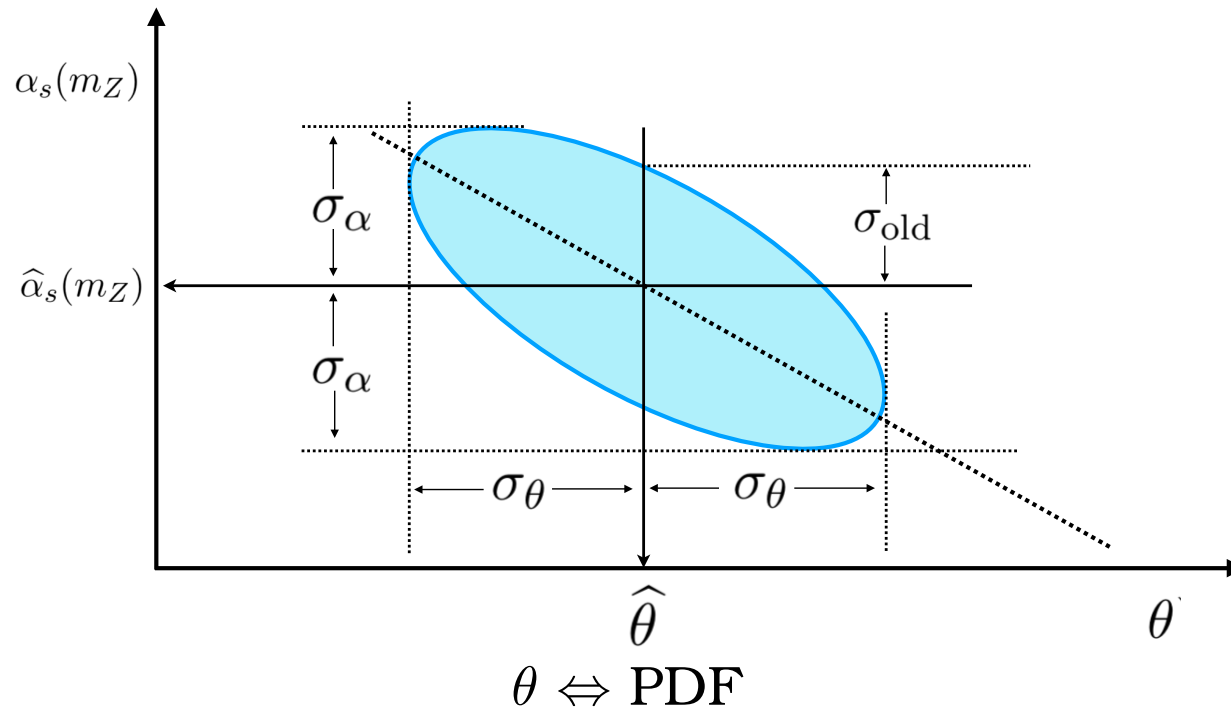


Figure 9.2: Summary of determinations of $\alpha_s(m_Z^2)$ with uncertainty in the seven sub-fields as discussed in the text. The yellow (light shaded) bands and dotted lines indicate the pre-average values of each sub-field. The dashed line and blue (dark shaded) band represent the final world average value of $\alpha_s(m_Z^2)$. The “*” symbol within the “hadron colliders” sub-field indicates a determination including a simultaneous fit of PDFs.

31st May, 2024

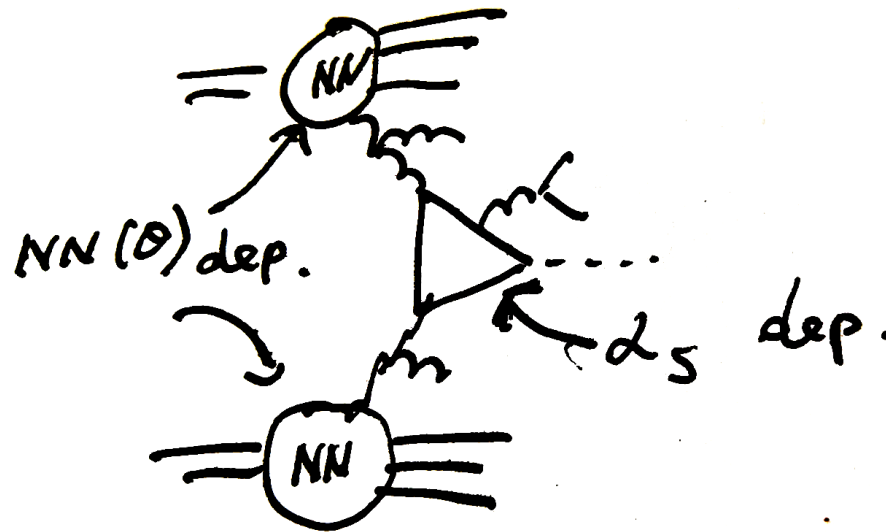
- AVERAGE OF SUB-AVERAGES
- CATEGORIZATION ARBITRARY
- MIXED BAG OF RELIABILITIES

α_s FROM DIS & HADRONIC PROCESSES



- MINIMUM DETERMINED ALONG THE “BEST PDF” LINE $\Rightarrow \sigma_{\text{old}}$
UNDERESTIMATE α_s UNCERTAINTY
- NEED SIMULTANEOUS MINIMIZATION IN (PDF, α_s) SPACE

SIMULTANEOUS PDF- α_s DETERMINATION



$$\sigma = \sum_{ij} \hat{\sigma}_{ij}(\alpha_s) \otimes f_i^{(1)}(\theta) \otimes f_j^{(2)}(\theta)$$

- CANNOT RELY ON SIMPLE PDF MODELS $\Rightarrow$ ML APPROACH
- PDF PARAMETERS θ ENTER ML MODEL OF PDFS $f_i^{(a)}(\theta)$
- α_s ENTERS THEORY PREDICTION $\hat{\sigma}_{ij}(\alpha_s)$

SOLUTIONS

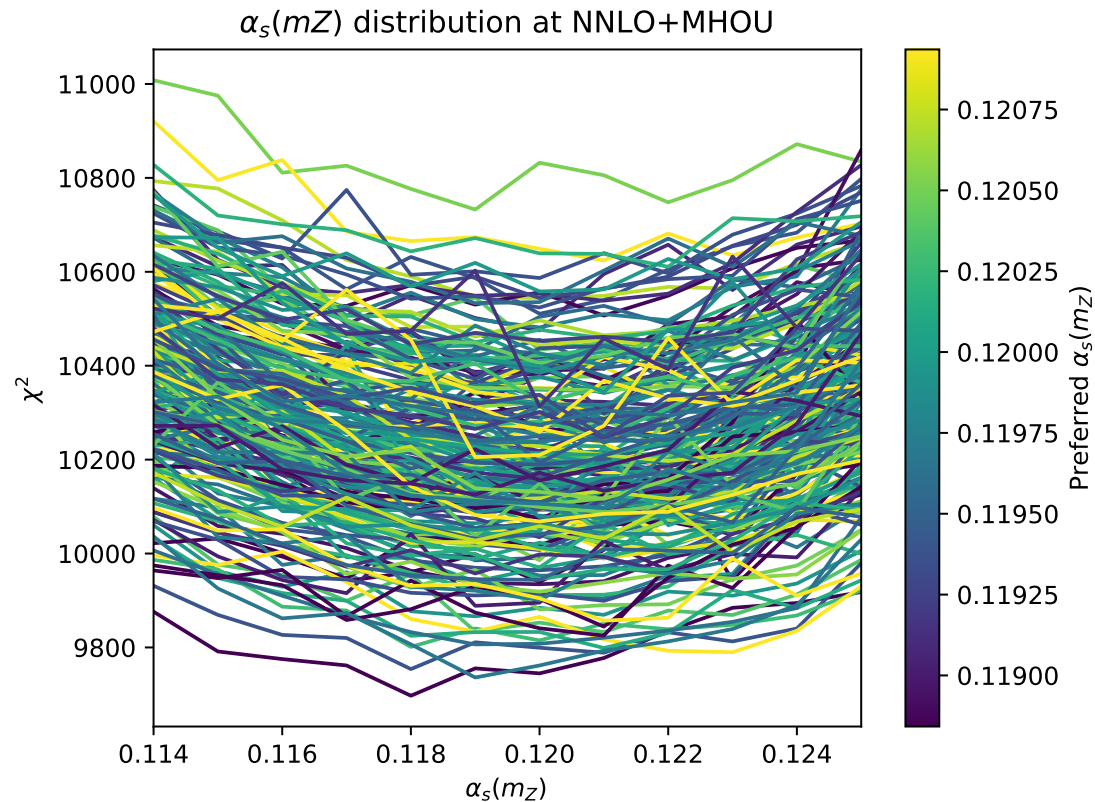
- $\times$ THEORY INTERPOLATION $\Rightarrow$ SIMUNET (S. Iranipour and M. Ubiali, 2022)
- $\checkmark$ FREQUENTIST MC RESAMPLING $\Rightarrow$ CORRELATED REPLICA METHOD (NNPDF, 2018)
- $\checkmark$ BAYESIAN POSTERIOR PARM. ESTIMATION $\Rightarrow$ THEORY COVARIANCE MATRIX METHOD (R. D Ball and R. Pearson, 2021)

THE CORRELATED REPLICA METHOD

NNPDF3.1 (2018)

- DETERMINE BEST-FIT PDF REPLICA TO EACH DATA REPLICA FOR SEVERAL (DISCRETE) α_s VALUES
- EACH DATA-REPLICA $\Rightarrow \chi^2$ PROFILE $\Rightarrow \alpha_s$ VALUE
- MC SAMPLE OF α_s REPLICAS

ENSEMBLE OF α_s VALUES

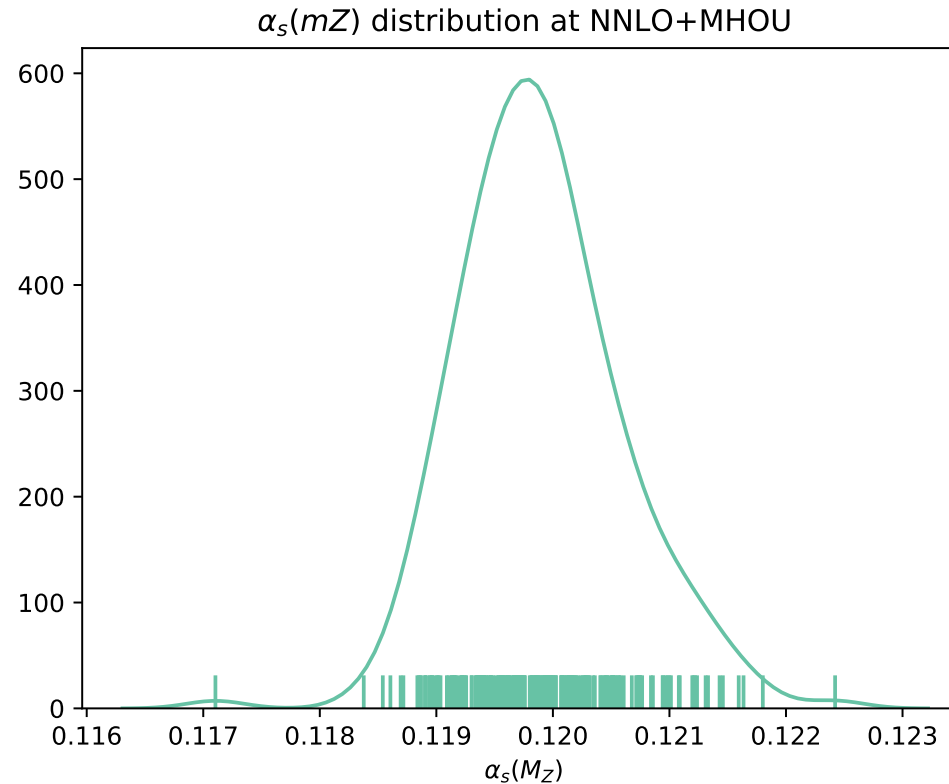


THE CORRELATED REPLICA METHOD

NNPDF3.1 (2018)

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ENSEMBLE OF α_s VALUES



THE THEORY COVARIANCE METHOD

CORRELATED UNCERTAINTY COVMAT $S \Leftrightarrow$ **NUISANCE PARAMETER** λ ON PREDICTION T
 $S_{ij} = \beta_i \beta_j \quad T \rightarrow T + \lambda \beta$

- **PROBABILITY OF PREDICTION** T GIVEN DATA & λ
 $P(T|D, \lambda) \propto \exp \left[-\frac{1}{2} (T + \lambda \beta - D)^T C^{-1} (T + \lambda \beta - D) \right]$
- **UNCERTAINTY ON NUISANCE PARM** $P(\lambda) \propto \exp \left(-\frac{\lambda^2}{2} \right)$
- **PROBABILITY OF PREDICTION** T GIVEN DATA
 $P(T|D) \propto \exp \left[-\frac{1}{2} (T - D)^T (C + S)^{-1} (T - D) \right]$
- **POSTERIOR DISTRIBUTION** OF λ !: $P(\lambda|T, D) \propto \exp \left(-\frac{1}{2} Z^{-1} (\lambda - \bar{\lambda}(T, D))^2 \right)$,
 WITH $\bar{\lambda}(T, D) = \beta^T (C + S)^{-1} (D - T)$, $Z = 1 - \beta^T (C + S)^{-1} \beta$

IDEA

TREAT $\alpha_s - \alpha_s^0$ AS **NUISANCE** PARAMETER WITH **PRIOR** $P(\alpha_s)$
 CENTERED ABOUT **PRIOR** α_s^0

VALIDATION: CLOSURE TESTS

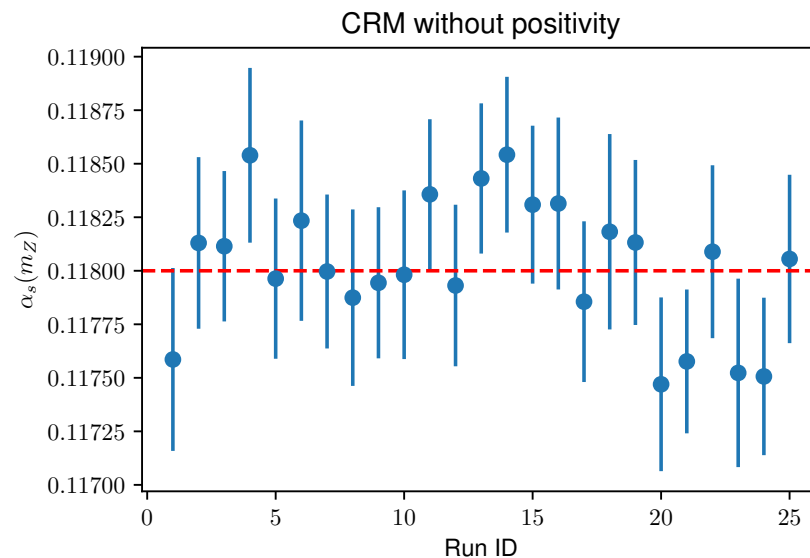
- ASSUME “TRUE” UNDERLYING PDF & α_s VALUE
- GENERATE DATA DISTRIBUTED ACCORDING TO EXPERIMENTAL COVARIANCE MATRIX
- RUN WHOLE METHODOLOGY ON THESE DATA
- DO STATISTICS ON “RUNS OF THE UNIVERSE”:
TURNING BAYESIAN INTO FREQUENTIST
 - BIAS/VARIANCE: MEAN SQUARE DEVIATION WR TO TRUTH VS UNCERTAINTY
$$R_{bv} = \sqrt{\frac{1}{N_r} \sum_{j=1}^{N_r} \left(\frac{\alpha_s^{(j)} - \alpha^{\text{true}}_s}{\sigma_\alpha^{(j)}} \right)^2}$$
 - ONE-SIGMA QUANTILE $\xi_{1\sigma}$: IS TRUTH WITHIN ONE SIGMA 68% OF TIMES?

CLOSURE TEST RUNS

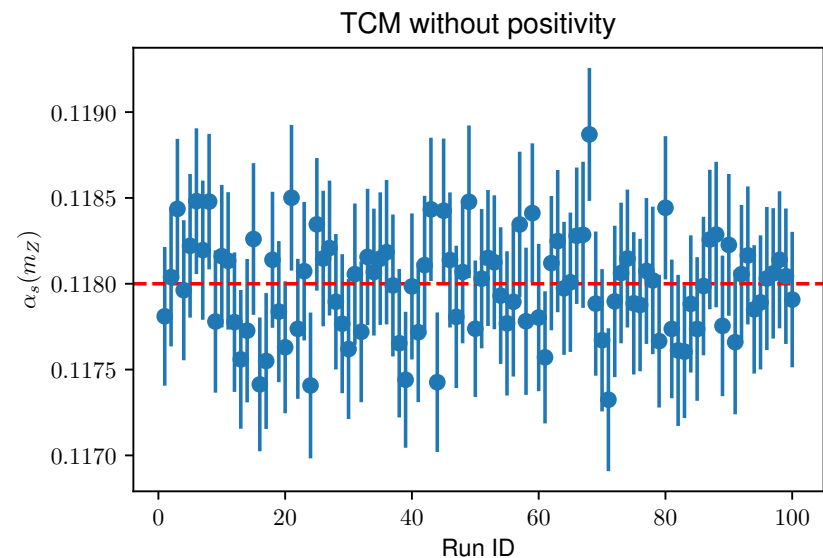
- **CRM** (FREQ.) 250 MC **REPLICAS** $\times$ 12 **VALUES** $0.114 \leq \alpha_s \leq 0.123$;
25 RUNS OF THE UNIVERSE
- **TCM** (BAYS.) 550 MC **REPLICAS**; **100 RUNS** OF THE UNIVERSE

$$N_r \text{ RUNS} \Rightarrow \text{DETERMINE } \langle \alpha_s \rangle, \text{ UNCERTAINTY } \frac{\sigma_\alpha}{\sqrt{N_r}}; \text{ PULL } P = \frac{\langle \alpha_s \rangle - \alpha_s^{\text{true}}}{\sigma_\alpha / \sqrt{N_r}}$$

CRM RUNS



TCM RUNS



CLOSURE TEST RESULTS

INPUT $\alpha_s = 0.118$

TCM 100 RUNS

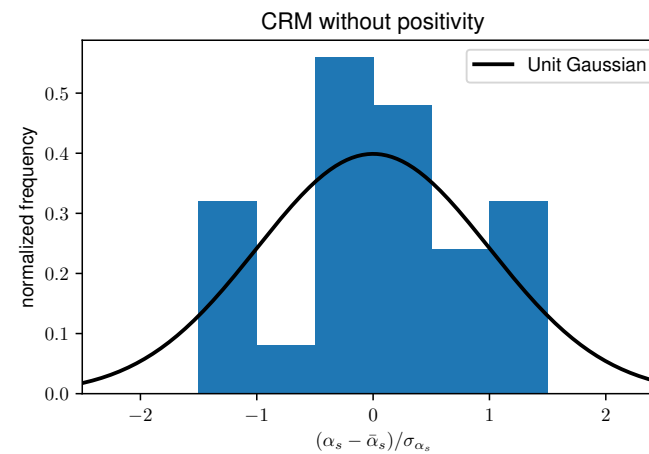
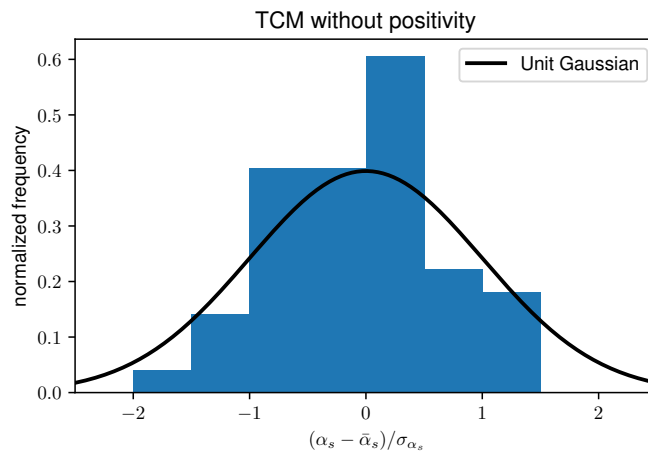
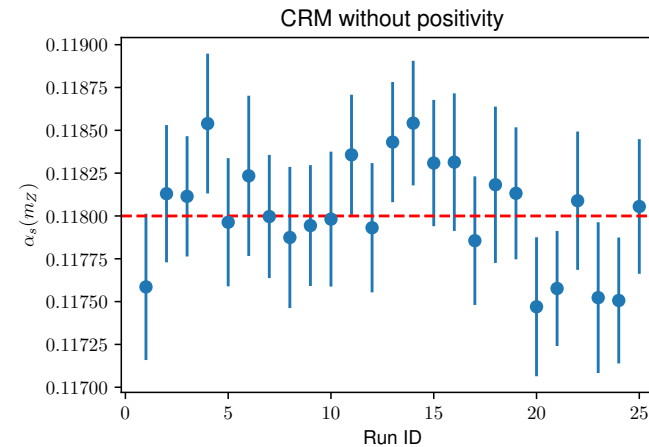
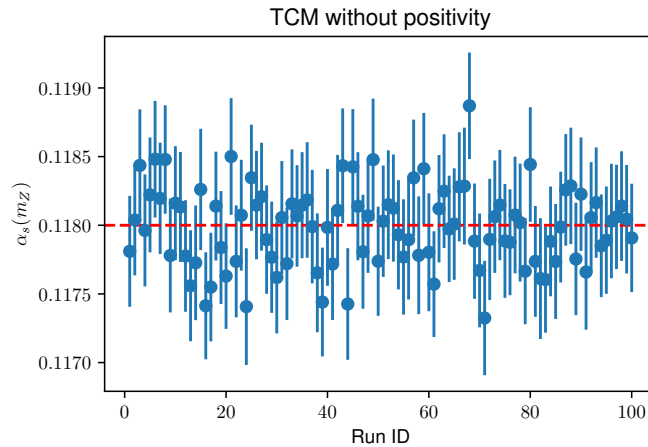
$$\langle \alpha_s \rangle = 0.117984; \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000041$$

$$P = 0.39 \quad R_{\text{bv}} = 0.71 \pm 0.05$$

CRM 25 RUNS

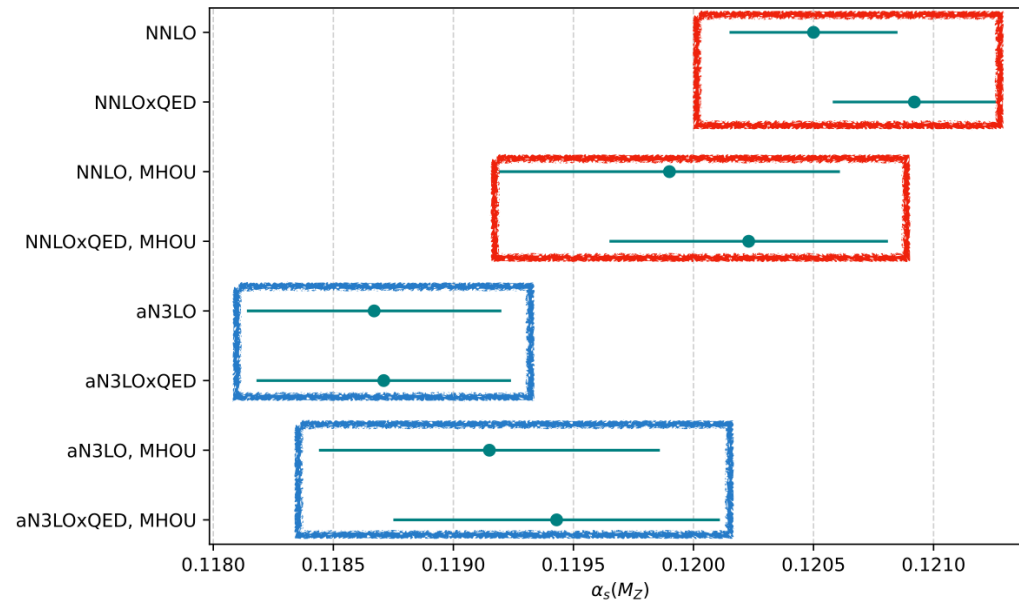
$$\langle \alpha_s \rangle = 0.118029; \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000077$$

$$P = 0.38 \quad R_{\text{bv}} = 0.80 \pm 0.09$$



- BAYESIAN (TCM) AND FREQUENTIST (CRM) IN PERFECT AGREEMENT
- VALIDATION OF THEORY UNCERTAINTIES

FINAL RESULTS REAL WORLD



$$\alpha_s(M_z) = 0.1194 \pm 0.0007$$

(NNPDF, 2025)

- **MHOU** ON PDF EXTRACTION **NEEDED** FOR PERTURBATIVE CONSISTENCY
- **PHOTON** PDF $\Rightarrow$ **IMPACT** AT $\sim 0.5\%$ LEVEL
- **NAIVE** EXTRACTION ALONG BEST-FIT PDF LINE:

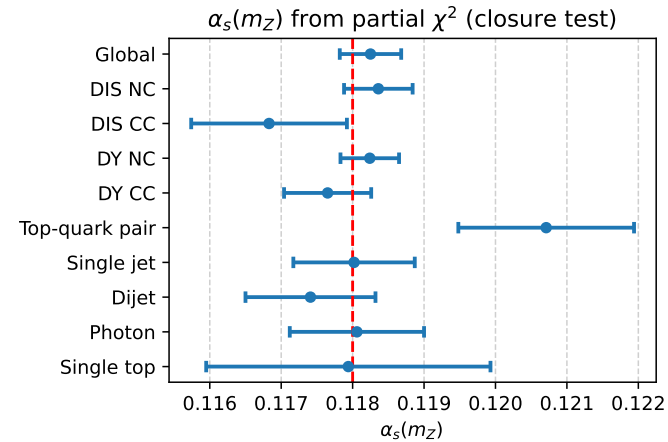
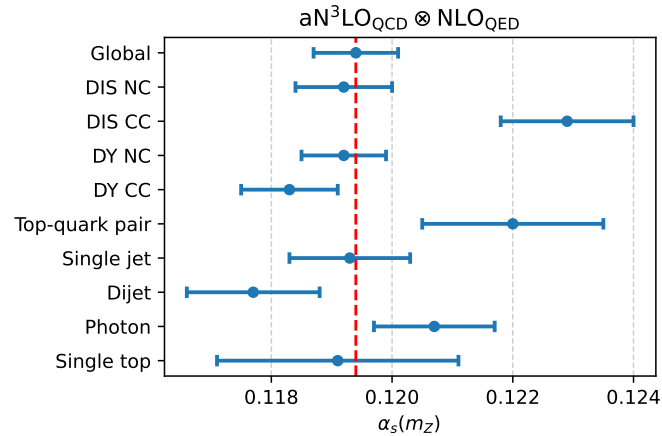
$$\alpha_s(M_z) = 0.1193 \pm 0.0005$$

UNCERTAINTY **UNDERESTIMATED**

THE NEED TO DETERMINE PDFs

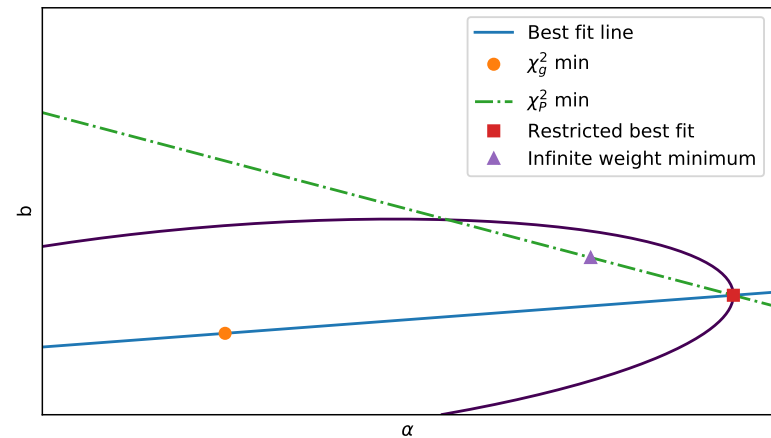
α_s FROM INDIVIDUAL PROCESSES DO NOT AVERAGE TO THE GLOBAL FIT: WHY?

α_s FROM PROFILING OF DATASETS IN GLOBAL FIT
REAL DATA **CLOSURE TEST**



CLOSURE TEST

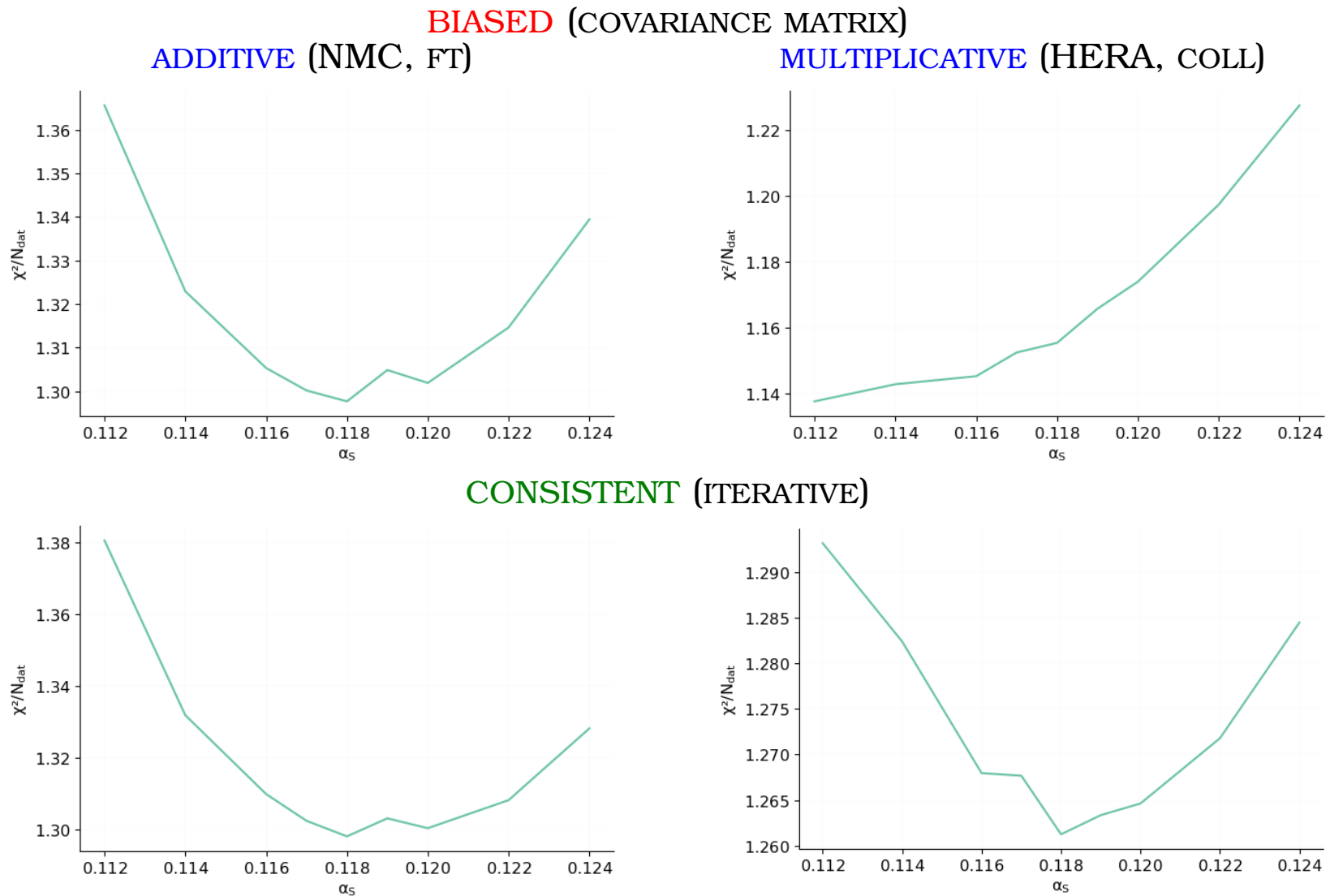
- **TOP DATA PARTIAL:** $\alpha_s^{t\bar{t}}(m_Z) = 0.1207 \pm 0.0012$
- **GLOBAL** $\alpha_s^{\text{global}}(m_Z) = 0.1183 \pm 0.0004$
- **TOP DATA WEIGHTED** $\alpha_s^{t\bar{t}}(m_Z) = 0.1201 \pm 0.0012$



SF, Kassabov, 2020

MULTIPLICATIVE UNCERTAINTIES A KNOWN PROBLEM

- COLLIDER DATA: MULTIPLICATIVE UNCERTAINTIES IN COVARIANCE MATRIX
- EXPERIMENTAL COVARIANCE MATRIX $\Rightarrow$ BIASED RESULT (d'Agostini, 1994)
- ITERATIVE PROCEDURE NEEDED COVARIANCE MATRIX COMPUTED FROM PREVIOUS FIT



THE PROBLEM OF MULTIPLICATIVE UNCERTAINTIES SOLUTION:

- **MULTIPLICATIVE** UNCERTAINTIES (E.G. **LUMI**): $\text{cov}_{ij} = \sigma_i^{\mathcal{L}} \sigma_j^{\mathcal{L}} D_i D_j$; D_i DATA
- CANNOT BE USED IN **LIKELIHOOD** $\Rightarrow$ **BIAS** (d'Agostini, 1994)
- **SOLUTION** (t_0): $\text{cov}_{ij} = \sigma_i^{\mathcal{L}} \sigma_j^{\mathcal{L}} t_i^{(0)} t_j^{(0)}$; $t_i^{(0)}$ THEORY PRED. FROM PREVIOUS

A NEW PROBLEM

- THEORY PREDICTION **DEPENDS** ON α_s
- **VARYING** $t^{(0)}(\alpha_s)$ OR **FIXED** $t^{(0)}$?

INPUT $\alpha_s = 0.118$; CRM 25 RUNS

$\text{VARYING } t^{(0)}(\alpha_s)$ $\langle \alpha_s \rangle = 0.119450; \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000077$ $P = 19 \quad R_{\text{bv}} = 3.8 \pm 0.16$	$\text{FIXED } t^{(0)}$ $\langle \alpha_s \rangle = 0.118152; \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000070$ $P = 2.2 \quad R_{\text{bv}} = 0.97 \pm 0.11$
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PREVIOUSLY UNDETECTED BECAUSE

- HADRON **COLLIDER** DATA **LESS RELEVANT**
- MANY **RUNS** OF UNIVERSE **NOT FEASIBLE**

THE PROBLEM OF PDF POSITIVITY

INPUT $\alpha_s = 0.118$; TCM 100 RUNS

POSITIVITY

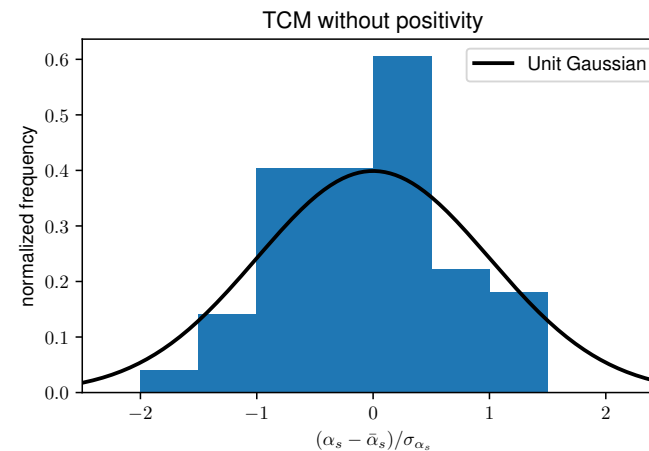
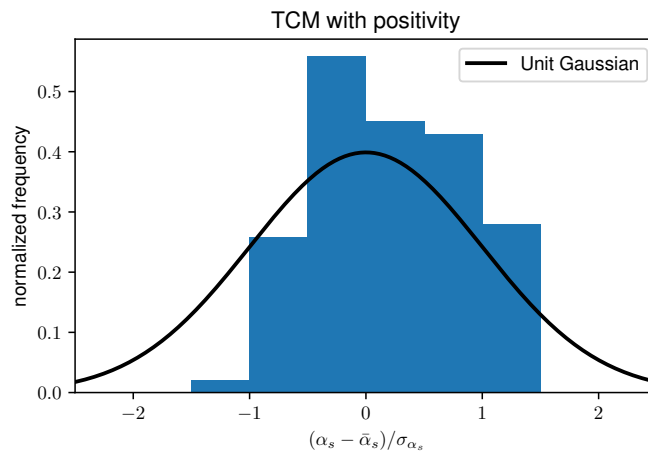
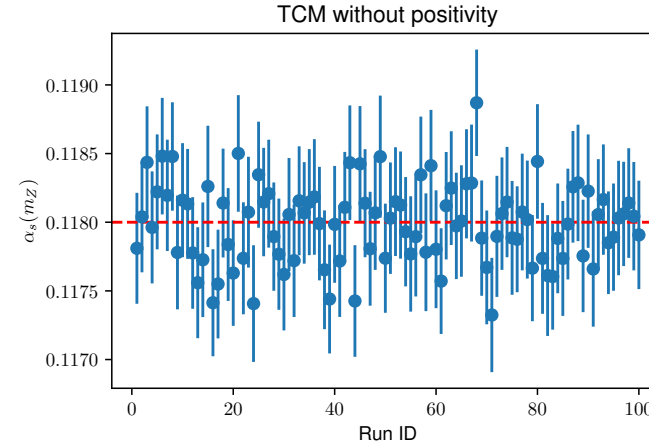
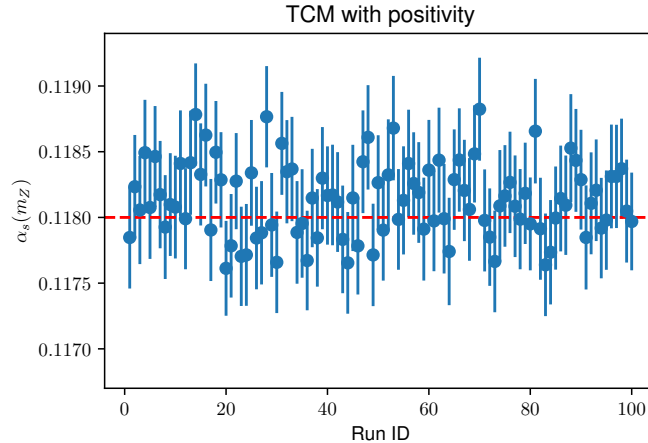
$$\langle \alpha_s \rangle = 0.118132; \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000039$$

$$P = 3.4 \quad R_{bv} = 0.80 \pm 0.06$$

NO POSITIVITY

$$\langle \alpha_s \rangle = 0.117984; \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000041$$

$$P = 0.39 \quad R_{bv} = 0.71 \pm 0.05$$



- POSITIVITY OF OBSERVABLES: PERFECT UNCERTAINTIES; BIASED CENTRAL VALUE
- DETECTED WITH MULTI-RUNS
- POSITIVITY $\Leftrightarrow$ NONGAUSSIAN BEHAVIOR

EXTRA UNCERTAINTY ON REAL-WORLD EXTRACTION

CONCLUSION



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Guido Altarelli[†]

*Dipartimento di Fisica 'E. Amaldi', Università di Roma Tre
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In this lecture, after recalling the basic definitions and facts about the running coupling in QCD, I present a critical discussion of the methods for measuring α_s and select those that appear to me as the most reliably precise

In my opinion one should select few theoretically simplest processes for measuring α_s and consider all other ways as tests of the theory. Note that this is what is usually done in QED where α is measured from one single very precise and theoretically reliable observable (one possible calibration process is at present the electron g-2 [22]). The cleanest processes for measuring α_s are the totally inclusive ones (no hadronic corrections) with light cone dominance, like Z decay, scaling violations in DIS and perhaps τ decay (but, for τ , the energy scale is dangerously low).