Dynamical systems and Superstring Phases in the Early Universe

2505.14187 (JHEP) and 2510.XXXX

Noelia Sánchez González

Dark Universe Cosmology, Bologna - 22nd October 2025

UNIVERSITY OF OXFORD

Outline

1. Introduction
2. Dynamical system analysis (2505.14187)
3. Gravitational wave signal (2510.XXXX)
4. Conclusions

1. Introduction

1.1 Cosmic (super-)strings

- Cosmological sizes

- Characterised by their **tension** $\mu = E/L$

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- Fundamental objects in String Theory

  e.g. **F-strings**

  Effective strings from wrapped branes

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1.2 Time-dependent tensions

- In the early universe, **moduli could be evolving**

$$\Phi(t) \quad + \; \mu(\Phi) \;\; \Rightarrow \;\; \mu = \mu(t)$$ **time-dependent tensions**

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- Evolution of sub-horizon cosmic string loops with $\mu(t)$

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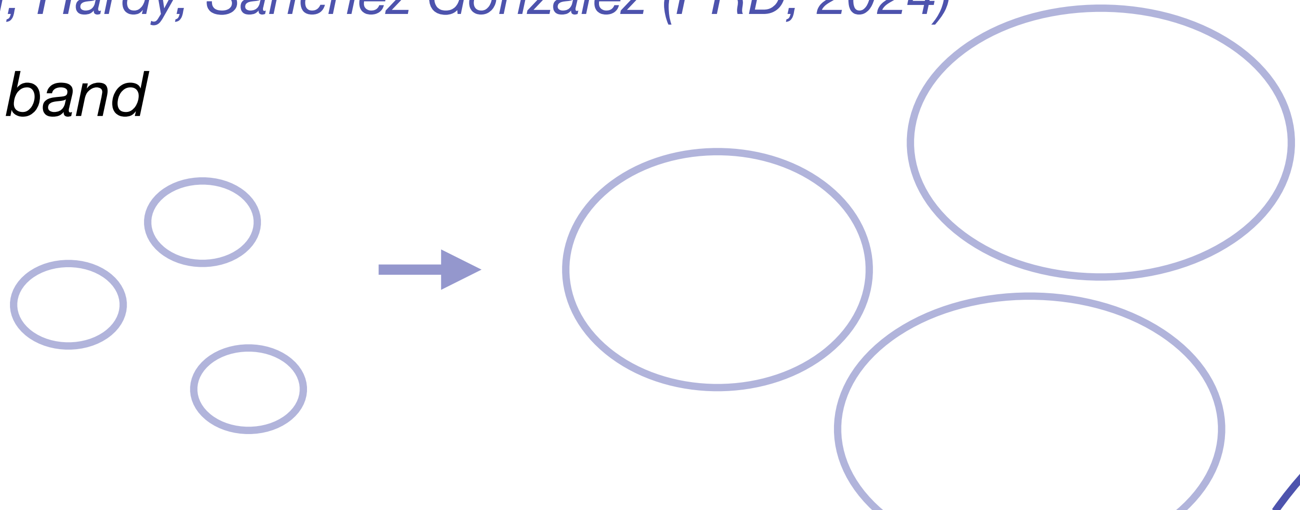
  … like a rubber band

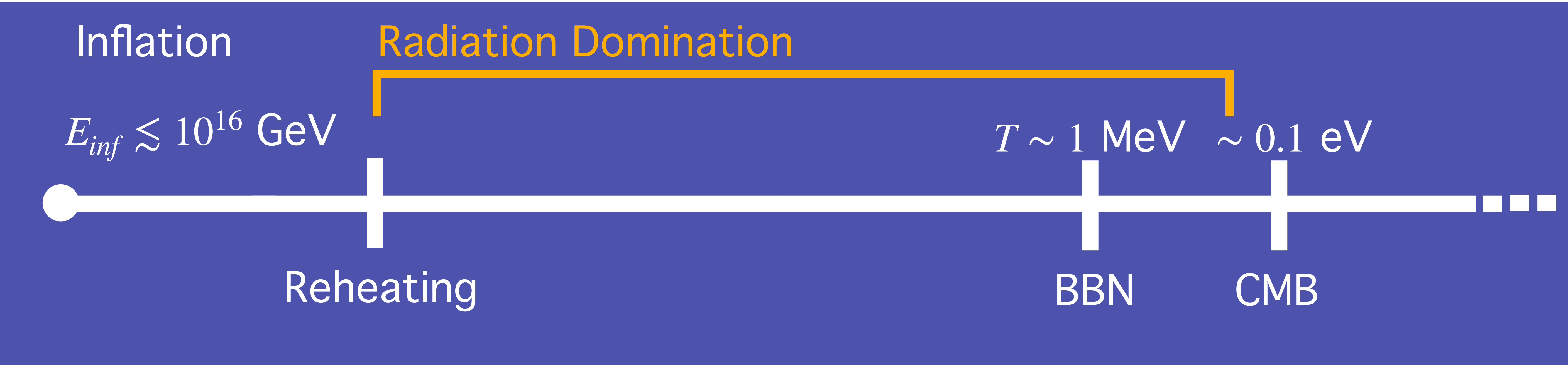


1. Introduction

1.3 Early Universe

- Observational constraints from **BBN** and **CMB**

- Standard cosmological history: inflation followed by *radiation domination*



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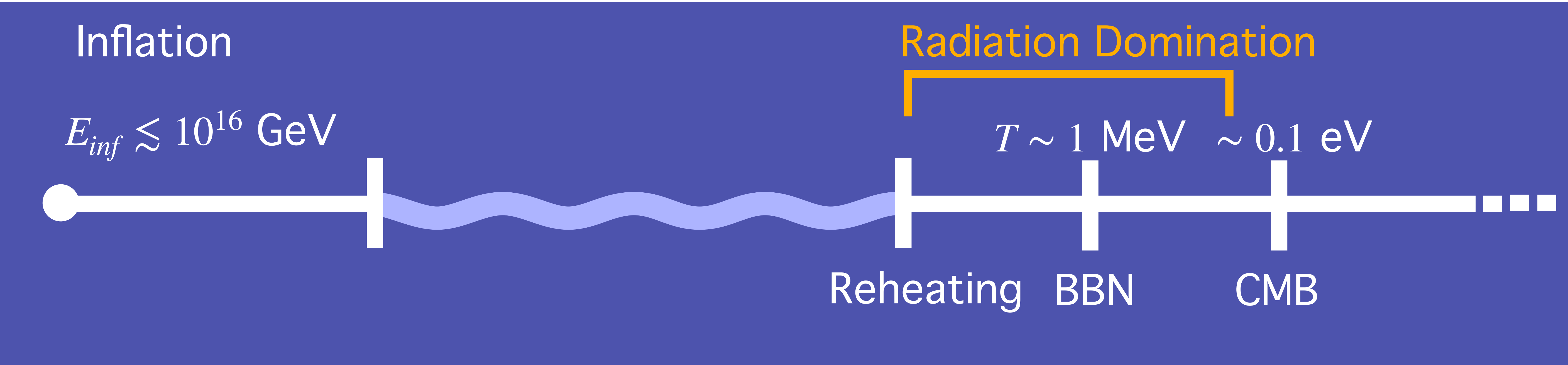

1. Introduction

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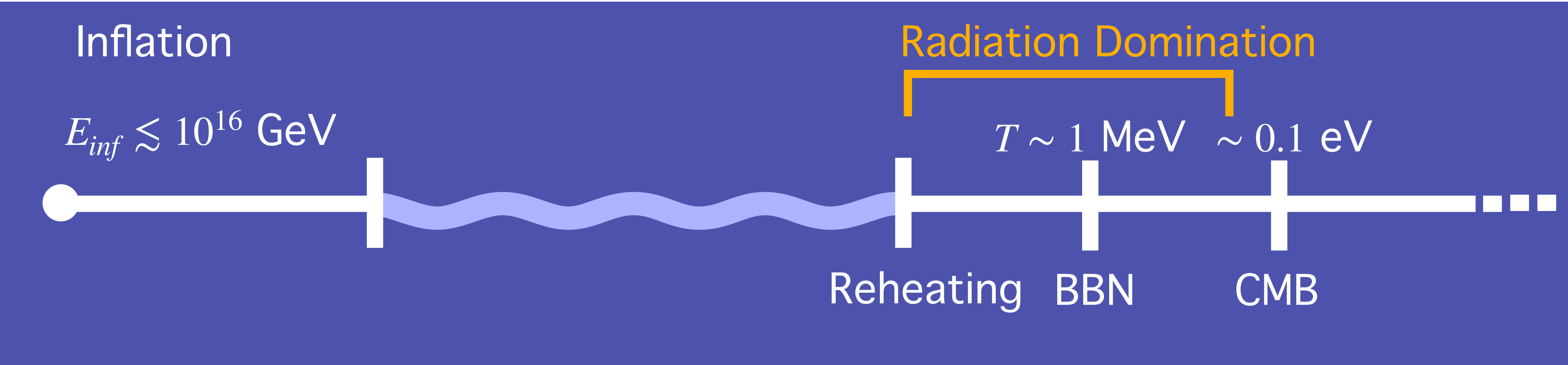
- Observational constraints from **BBN** and **CMB**

✦ Goal: ***String Phenomenology***

Apers, Conlon, Copeland, Mosny, Revello (JCAP, 2024)



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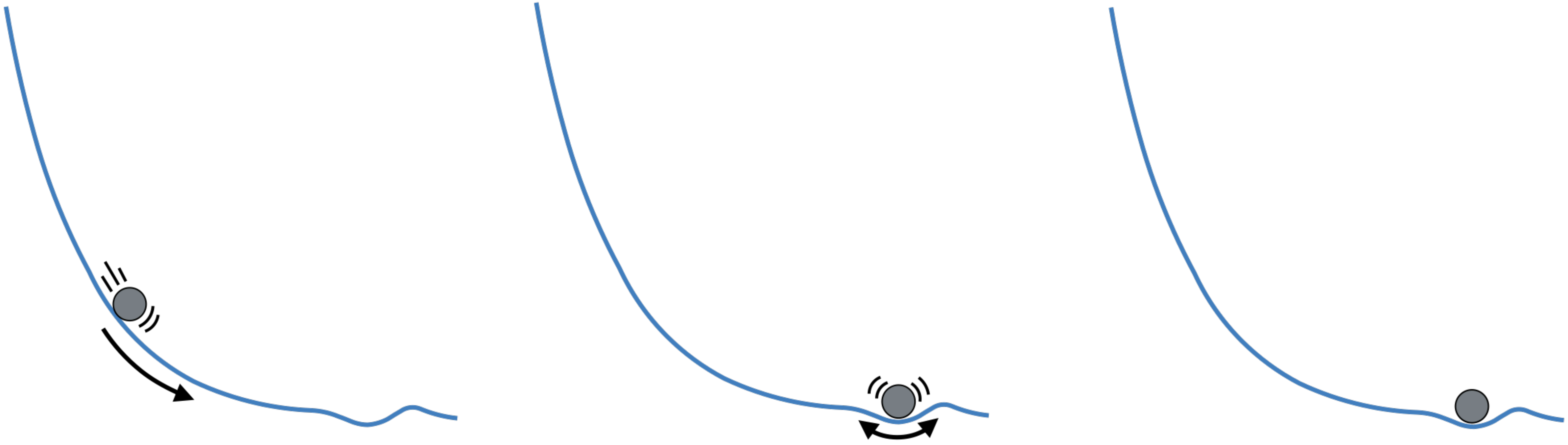
1.3 Early Universe

…What 'stringy' physics can we expect?



1. Rolling moduli

2. Moduli domination

3. Moduli decay and reheating

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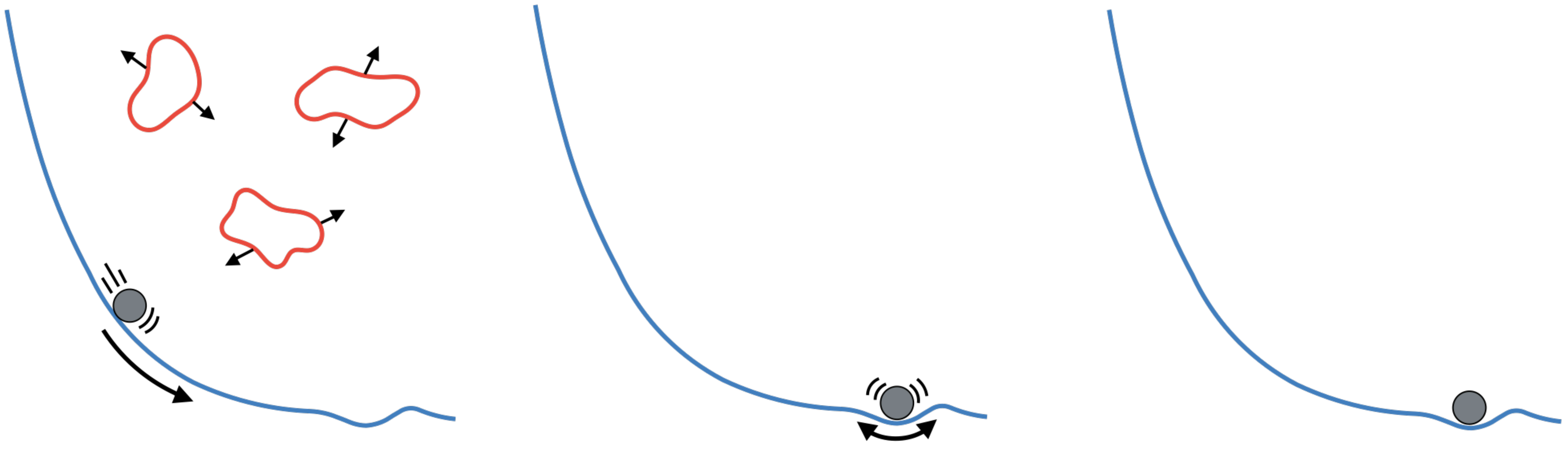

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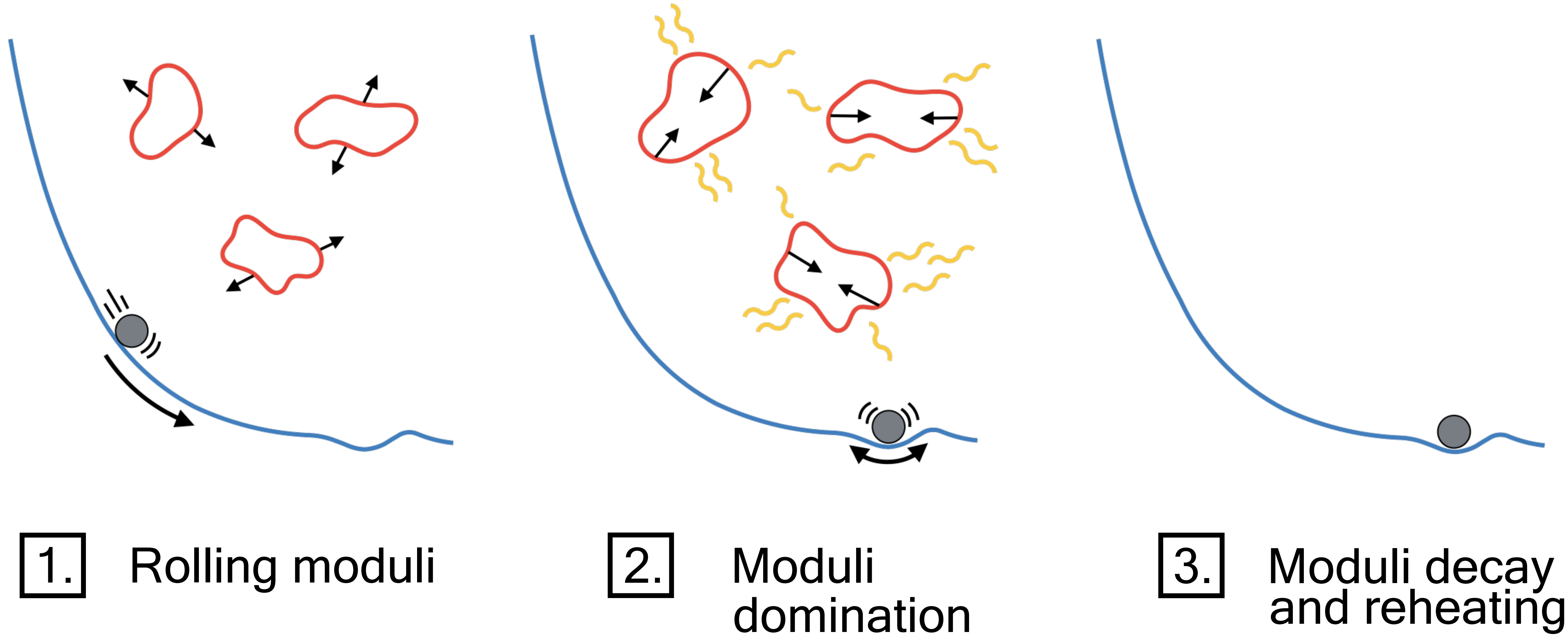

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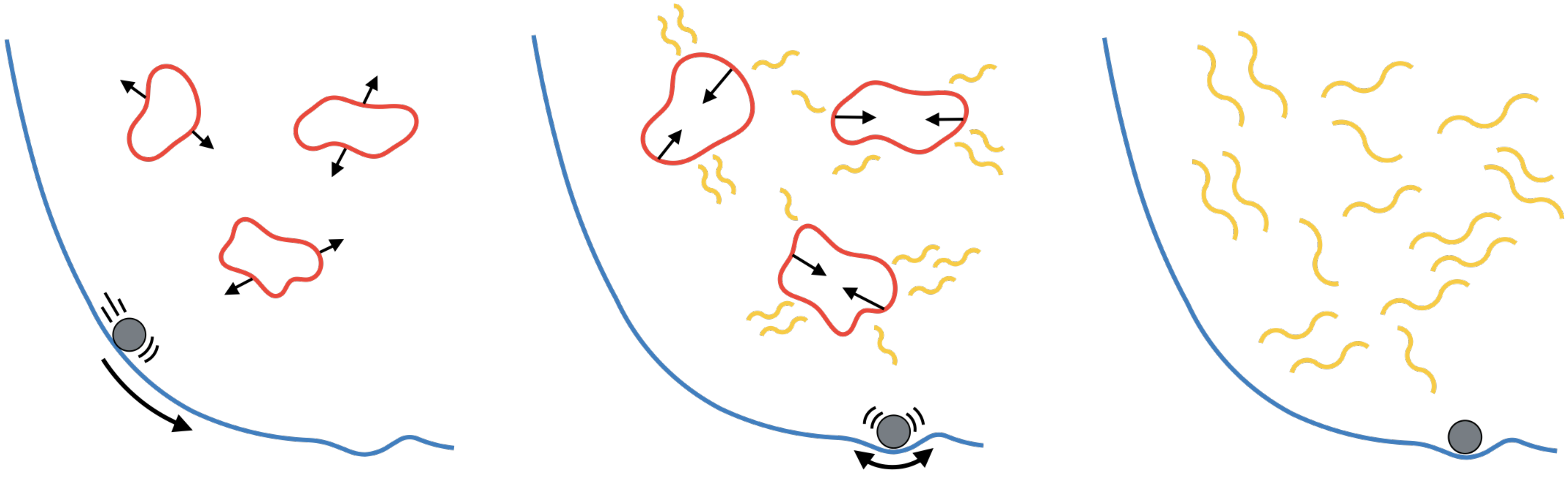

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2. Dynamical systems analysis

2.1 Set-up

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$$\rho_{loops}(a, \Phi) \sim a^{-3} \cdot e^{-\beta\, \Phi/M_P}$$

For **F-strings,** $\beta = \sqrt{3/8}$

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1. Large Volume Scenario (LVS) $\lambda = \sqrt{27/2}$

2. Canonical $V \sim m_s^4$ $\lambda = \sqrt{6}$

2. Dynamical systems analysis

2.2 Equations of motion

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- Friedmann equation $\longrightarrow$ $$3M_P^2H^2 = \dot{\Phi}^2/2 + V(\Phi) + \rho_{loops}(a,\Phi) \quad (1)$$

- Continuity equations $\longrightarrow$ $$\begin{cases} \ddot{\Phi} + 3H\dot{\Phi} + \dfrac{\partial V}{\partial \Phi} + \dfrac{\partial \rho_{loops}}{\partial \Phi} = 0 & (2) \\ \dot{\rho}_{loops} + 3H\rho_{loops} - \dfrac{\partial \rho_{loops}}{\partial \Phi}\dot{\Phi} = 0 & (3) \end{cases}$$

2. Dynamical systems analysis

2.2 Equations of motion

change of variables

Friedmann
+
continuity
equations

(a, Φ)

$$\longrightarrow \boxed{\begin{aligned} x^2 &= \Omega_{\dot{\Phi}} \\ y^2 &= \Omega_V \\ z^2 &= \Omega_{loops} \end{aligned}}$$

s.t $x^2 + y^2 + z^2 = 1$

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Autonomous system

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_1(x, y) \\ f_2(x, y) \end{pmatrix}$$

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✦ **Goal:** find the fixed points, i.e. $x' = y' = z' = 0$

2. Dynamical systems analysis

2.3 Results

F-strings in Large Volume Scenario

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$x^2 = \Omega_{\dot{\Phi}}$

$y^2 = \Omega_V$

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F-strings in Large Volume Scenario

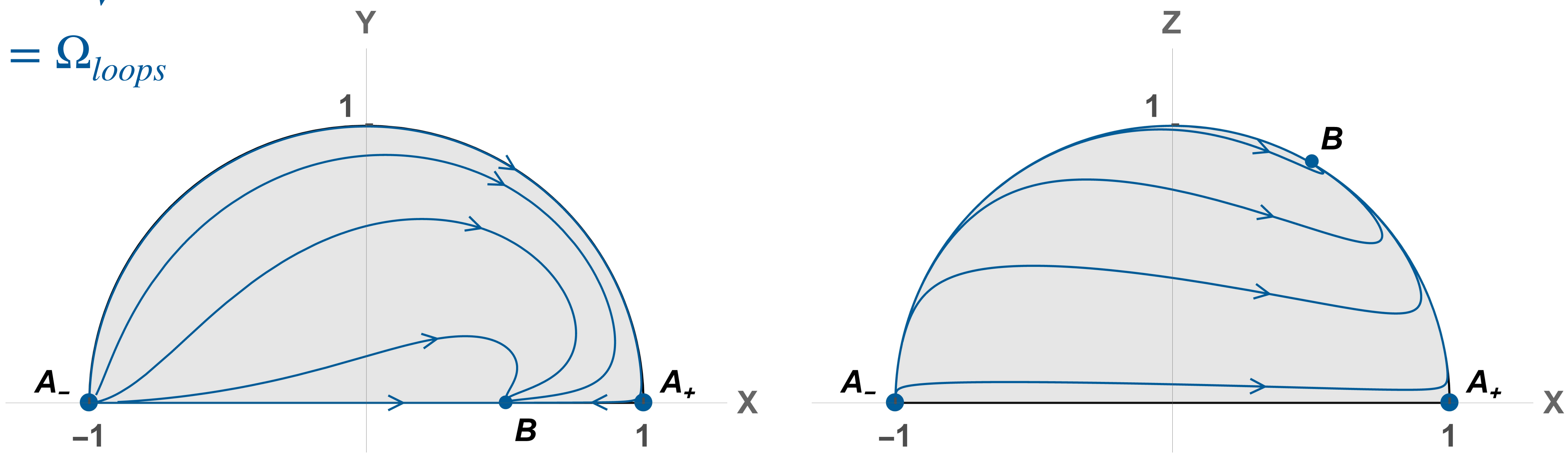

Fig. 1 Evolution of the energy fractions in phase space (LVS)

2. Dynamical systems analysis

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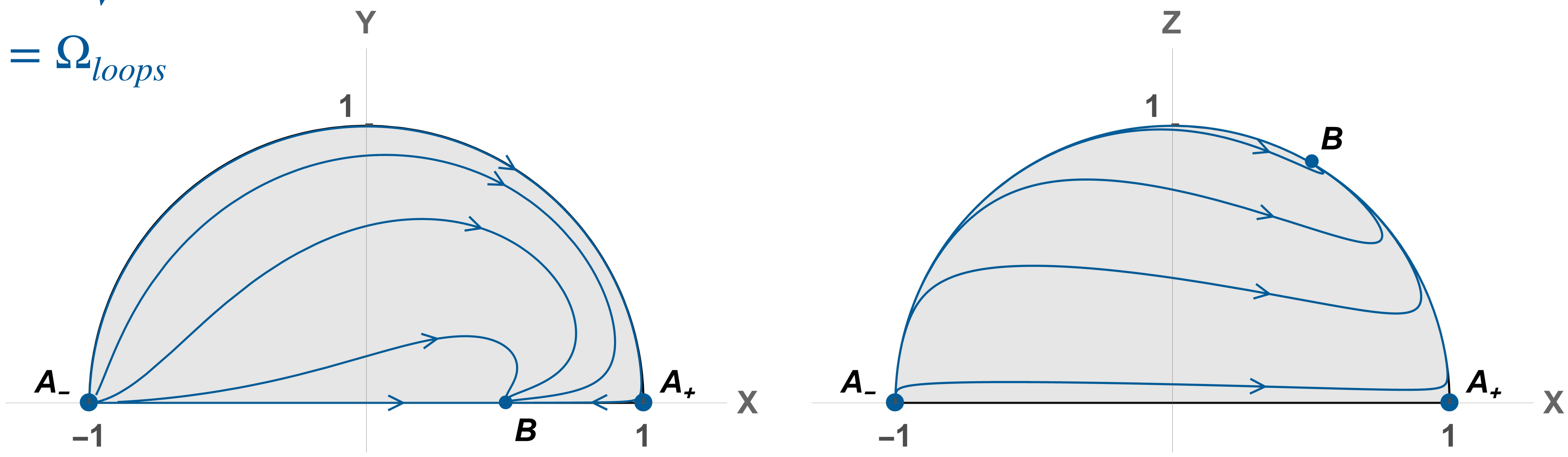
F-strings in Large Volume Scenario



NEW!

(B) String Loop Tracker $(\Omega_{\dot{\phi}} = 1/4,\ \Omega_{loops} = 3/4)$

2. Dynamical systems analysis

2.3 Results

F-strings in Large Volume Scenario

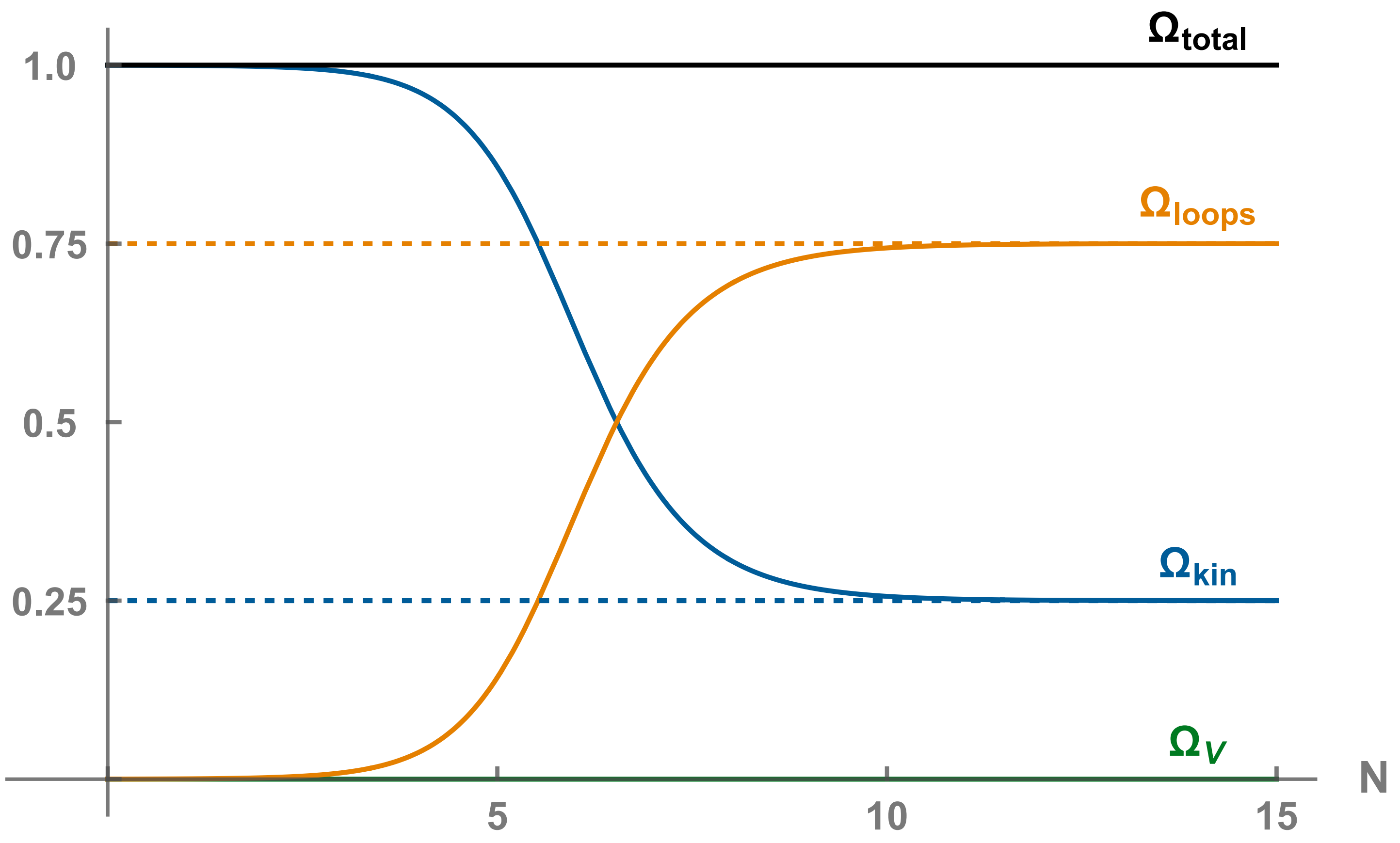


Fig. 2 Evolution of the energy fractions in time (LVS)

2. Dynamical systems analysis

2.3 Results

$x^2 = \Omega_{\dot{\Phi}}$

$y^2 = \Omega_V$

$z^2 = \Omega_{loops}$

vs. in a canonical potential

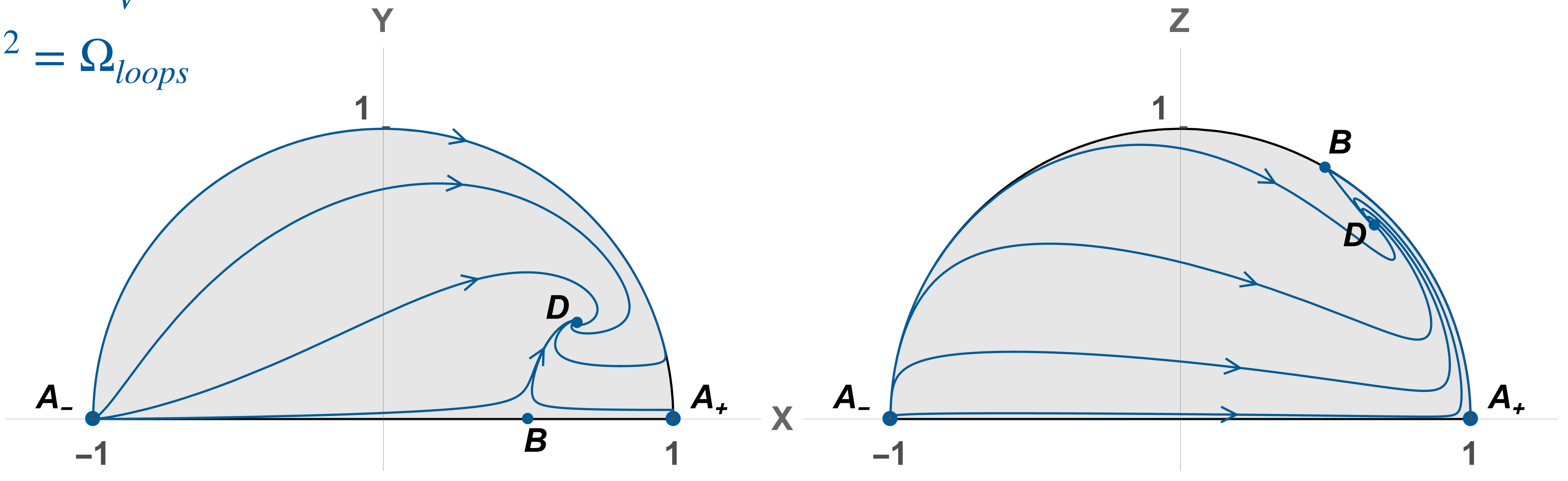


Fig. 3 Evolution of the energy fractions in phase space (Canonical)

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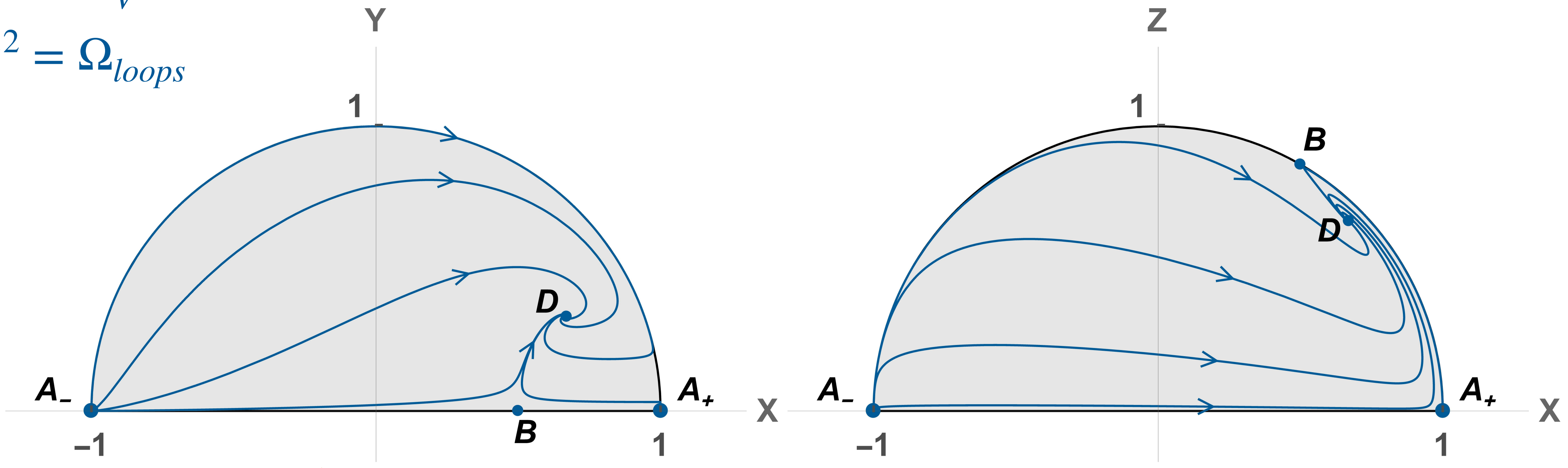


NEW!

(D) Mixed Tracker $(\Omega_{\dot{\phi}} = \Omega_{loops} = 4/9, \Omega_V = 1/9)$

2. Dynamical systems analysis

2.4 Adding radiation

2. Dynamical systems analysis

Brunelli, Cicoli, Gil Pedro (2025)

2.4 Adding radiation

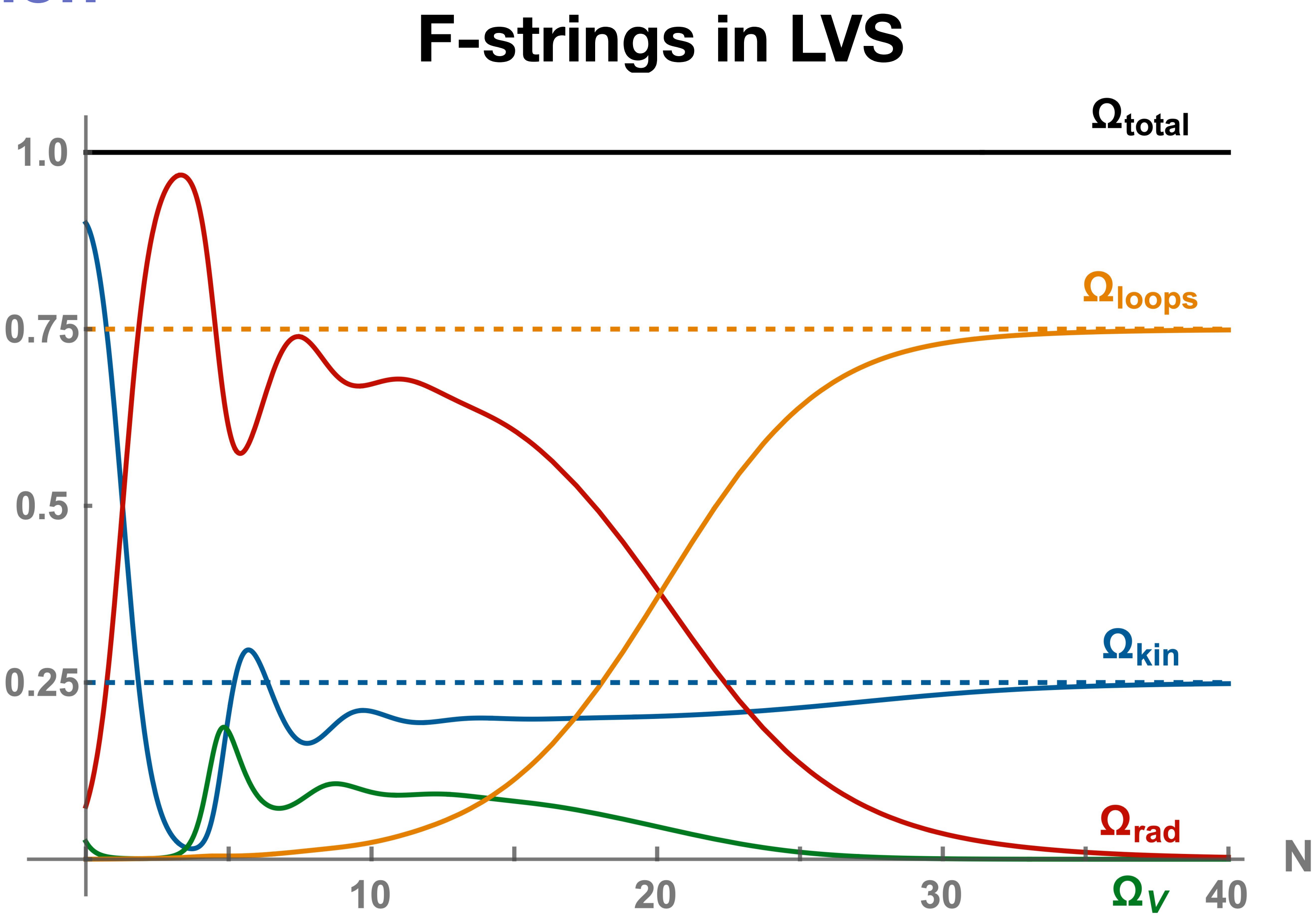
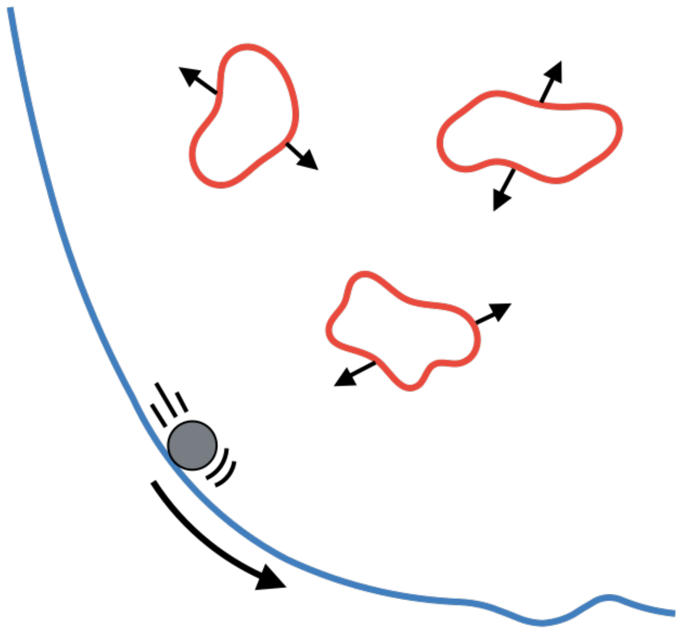


Fig. 4 Evolution of the energy fractions in time

3. Gravitational wave signal

3.1 Summary of the physics



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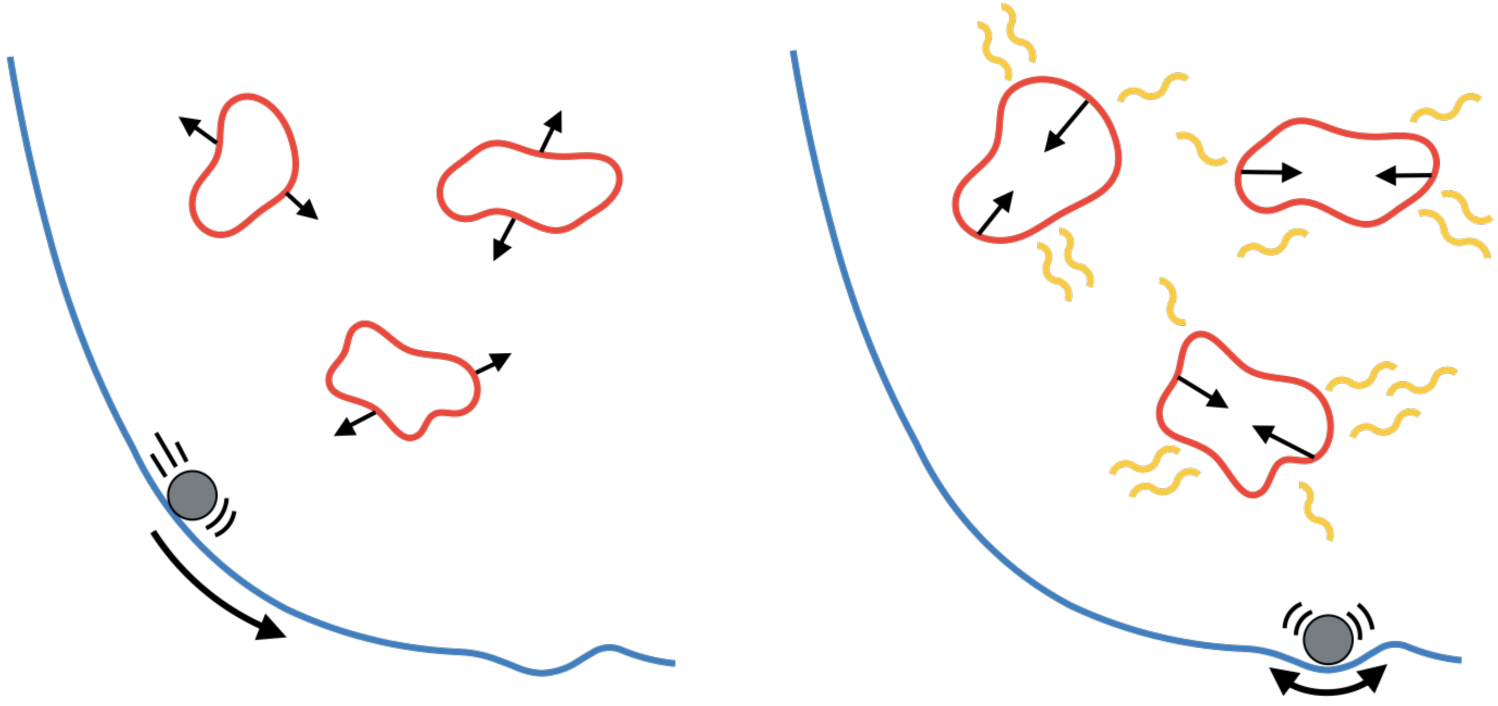

1\. String Loop Tracker

2\. Moduli domination

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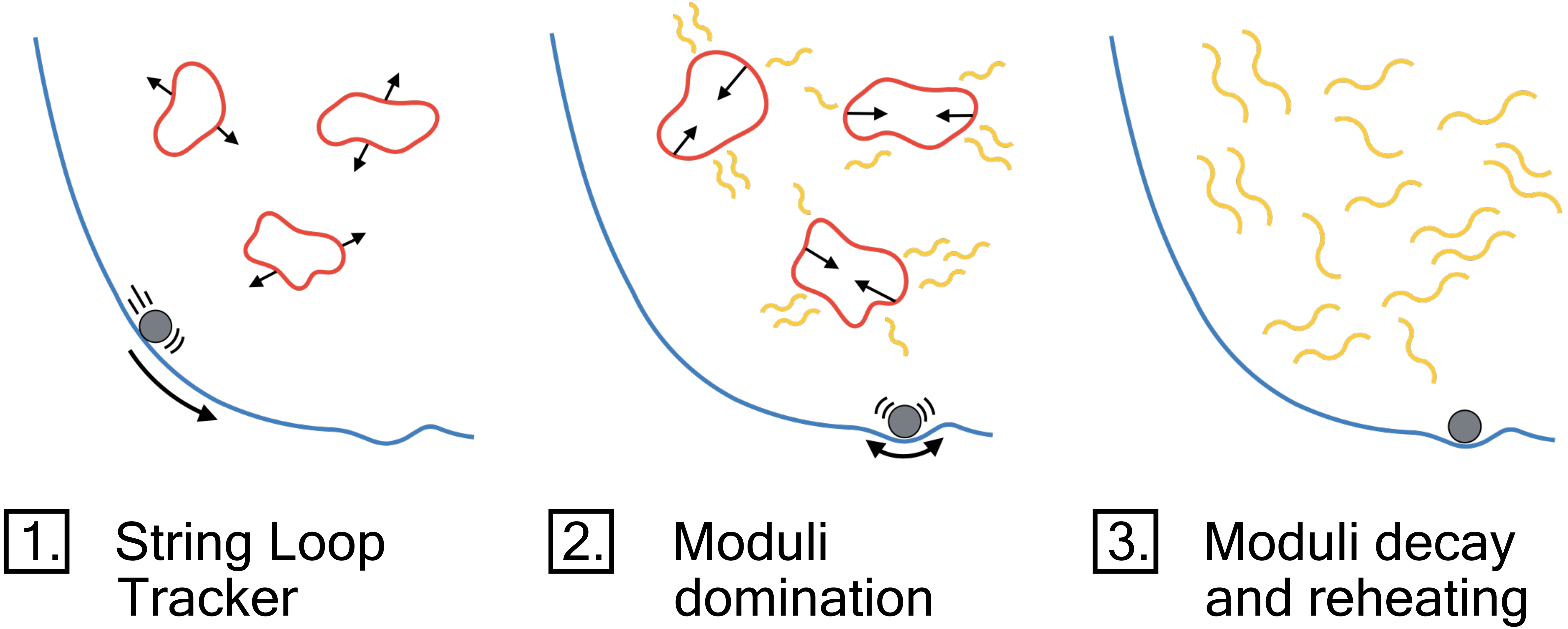

3. Gravitational wave signal

3.2 GW emission from loops

- Frequency of emission $f^{\mathrm{em}} \sim \ell^{-1}$
- Lifetime due to gravitational radiation $\tau = \mu\ell/\dot{E}_{GW} \sim \ell/G\mu$

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Main idea

Sub-horizon loops $\propto a^{-3}$ vs. Gravitational waves $\propto a^{-4}$

with $\Omega^{i}_{\mathrm{loops}} \sim \mathcal{O}(1)$ redshift during moduli domination

3. Gravitational wave signal

3.2 GW emission from loops

- **Faster moduli decay** $\longrightarrow$ shorter MD $\longrightarrow$ **larger amplitudes** Ω^0_{GW}

  Cicoli, Hebecker, Jaeckel, Wittner (JHEP, 2022)

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- In **LVS**,
$$\left.\begin{aligned} m_\phi / M_P &\sim {Vol_{CY}}^{-3/2} \\ G\mu &\sim {Vol_{CY}}^{-1} \end{aligned}\right\} \text{ not independent!}$$

Ghoshal, Revello, Villa (JCAP, 2025)

3. Gravitational wave signal

3.3 GW Power Spectrum

- Integral over **duration of the emission** $(t_i, t_i + \tau)$

$$\boxed{\Omega^0_{\mathrm{GW}}(f^{\mathrm{obs}}) = \frac{f^{\mathrm{obs}}}{\rho_{c,0}} \cdot \int_{t_i}^{t_i+\tau} dt^{\mathrm{em}} \, \frac{d\dot{\rho}^{\mathrm{em}}_{GW}}{df^{\mathrm{em}}} \cdot \left(\frac{a(t^{\mathrm{em}})}{a_0}\right)^3}$$

- The **peak of the spectrum** comes from the moment **when the loops decay**

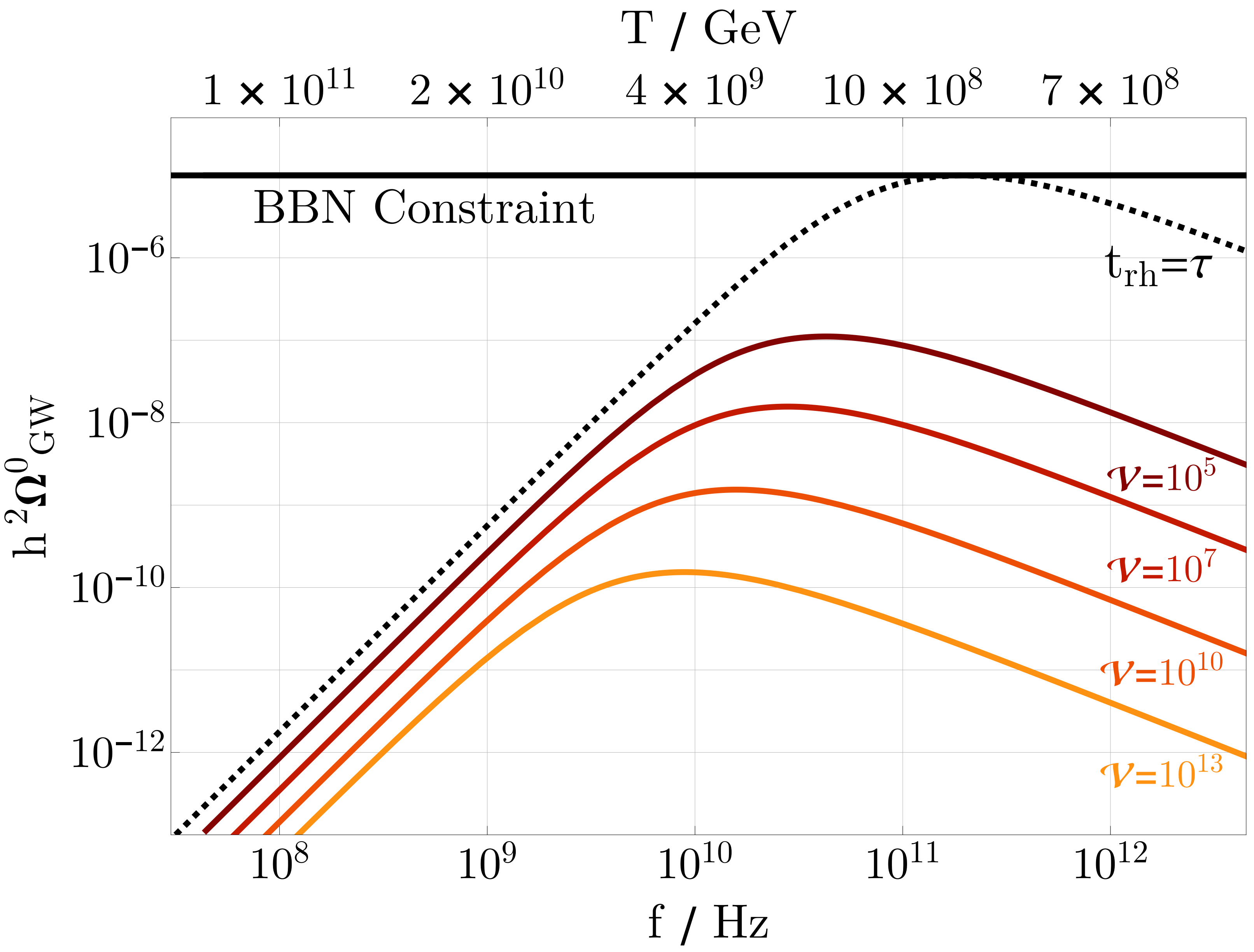
$$\boxed{t^{\mathrm{em}}_{\mathrm{peak}} \sim \tau}$$

3. Gravitational wave signal

3.3 GW Power Spectrum

- **Sub-H isolated loops** with

$\Omega^i_{\text{loops}} \sim 1$



3. Gravitational wave signal

3.3 GW Power Spectrum

- **Sub-H isolated loops** with

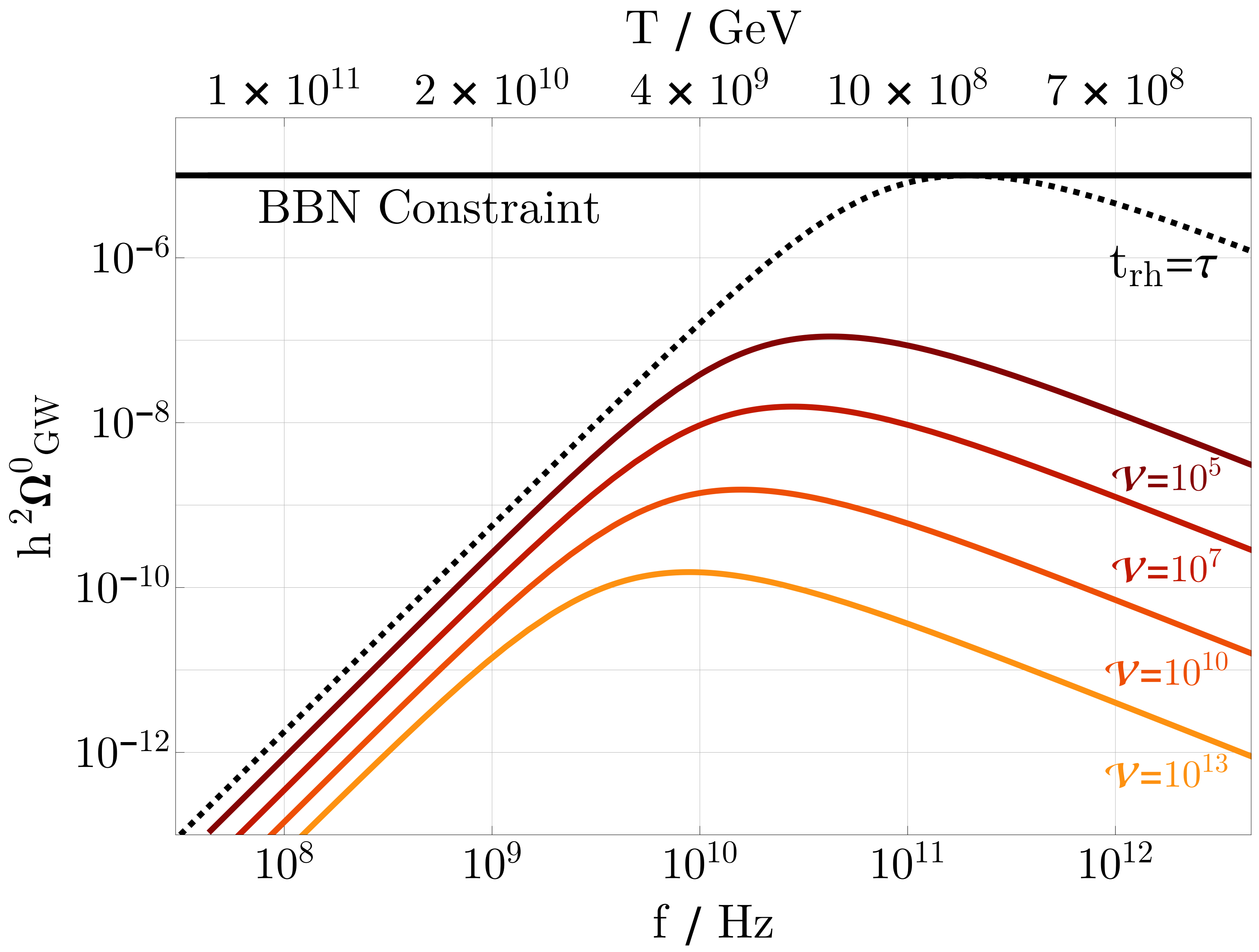
  $\Omega^{i}_{\text{loops}} \sim 1$

- **LVS with a fast decay**

  $\Gamma_\phi \sim (M_P/m_\phi)^{4/3}(m_\phi^3/M_P^2)$

- If faster decay $t_{rh} = \tau$,

  it can reach **BBN bounds**



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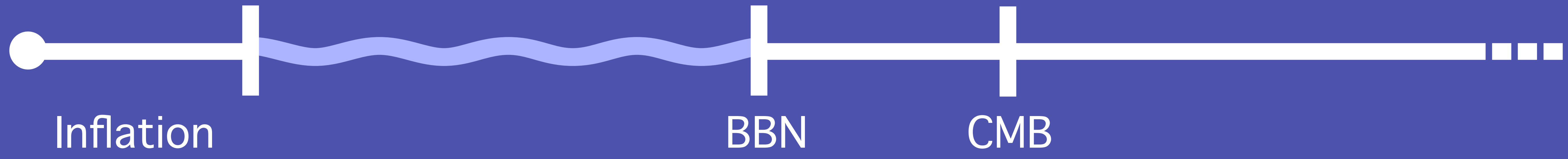
  - Moduli evolution
  - Cosmic strings with time-dependent tensions



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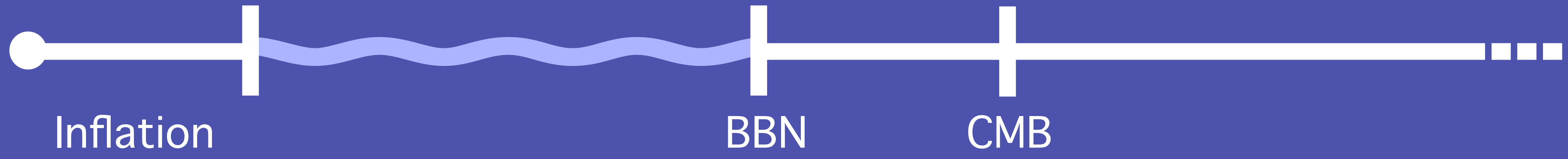

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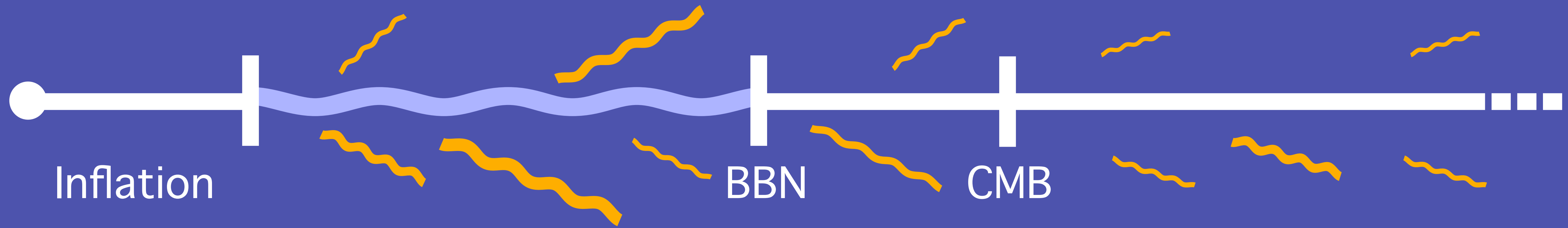

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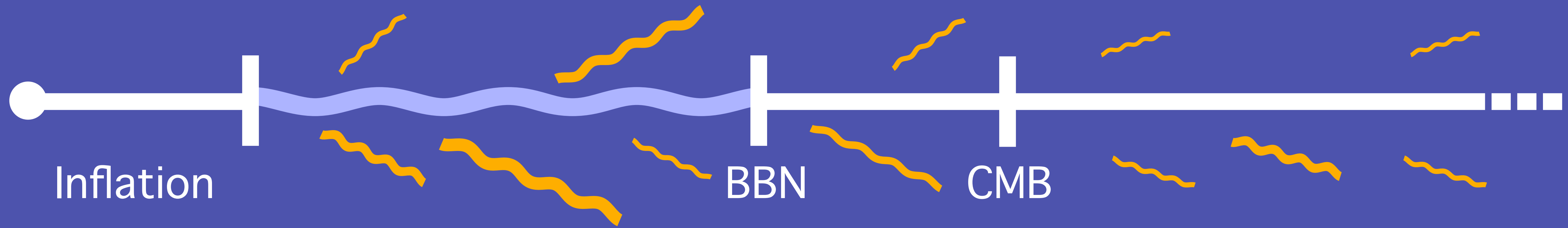
Adding interactions,

PBH formation,

enhanced moduli decays,

evolving couplings…



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