“Reflections” on the Geometry control problem

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Systematic errors in a RLG

The physical mechanisms which are well known to limit the performance of lase gyros are:

1. Variations of the area vector (modulus + orientation)

2. Variations of the side lengths

3. Variations of the backscattering amplitudes and phases

4. Non reciprocities in the plasma and in the optical cavity.

Cavity Geometry deformations

From R. Graham PHD thesis

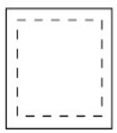
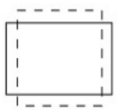
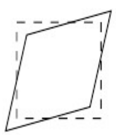
Deformation	Illustration	Strain Field	Discernible?
Rotation		$\frac{\partial \mathbf{u}_x}{\partial y} - \frac{\partial \mathbf{u}_y}{\partial x}$	✓
Dilation		$\frac{\partial \mathbf{u}_x}{\partial x} + \frac{\partial \mathbf{u}_y}{\partial y}$	✓
Rectangular shear		$\frac{\partial \mathbf{u}_x}{\partial x} - \frac{\partial \mathbf{u}_y}{\partial y}$	✓
Diagonal shear		$\frac{\partial \mathbf{u}_x}{\partial y} + \frac{\partial \mathbf{u}_y}{\partial x}$	✗

Table 4.1. Two-dimensional deformation components for a rectangular ring laser cavity

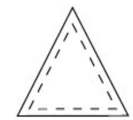
Deformation	Illustration	Strain Field	Discernible?
Rotation		$\frac{\partial \mathbf{u}_x}{\partial y} - \frac{\partial \mathbf{u}_y}{\partial x}$	✓
Dilation		$\frac{\partial \mathbf{u}_x}{\partial x} + \frac{\partial \mathbf{u}_y}{\partial y}$	✓
Rectangular stretch		$\frac{\partial \mathbf{u}_x}{\partial x} - \frac{\partial \mathbf{u}_y}{\partial y}$	✓
Shear		$\frac{\partial \mathbf{u}_x}{\partial y} + \frac{\partial \mathbf{u}_y}{\partial x}$	✓

Table 4.2. Two-dimensional deformation components for a triangular ring laser cavity.

Need to measure:

-Sides lengths

-Diagonals lengths

-Planarity condition

-Orientation

Need to measure:

-Sides lengths

-Orientation (two angles)

$$f_s = \frac{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}{a+b+c}\,\frac{\Omega}{\lambda},$$

Our Choice: the “perfect” square

- A straightforward absolute measurement is the cavity **PERIMETER** (FSR)
- In a **square** one can measure also the **TWO DIAGONALS**
- Fixing the diagonals lenghts, in the same plane, the PERFECT SQUARE GEOMETRY (**minimum perimeter**)
- **Plasma dispersion** affect the real length measurement

There are 6 d.o.f and 3 observable...

PERFECT SQUARE achievement

Consider a system of four mirrors controllable in 3 d.o.f each

1) Arrange the four mirros curvature centers within few microns um from the Perfect square condition

2)Inject an external laser in the diagonals

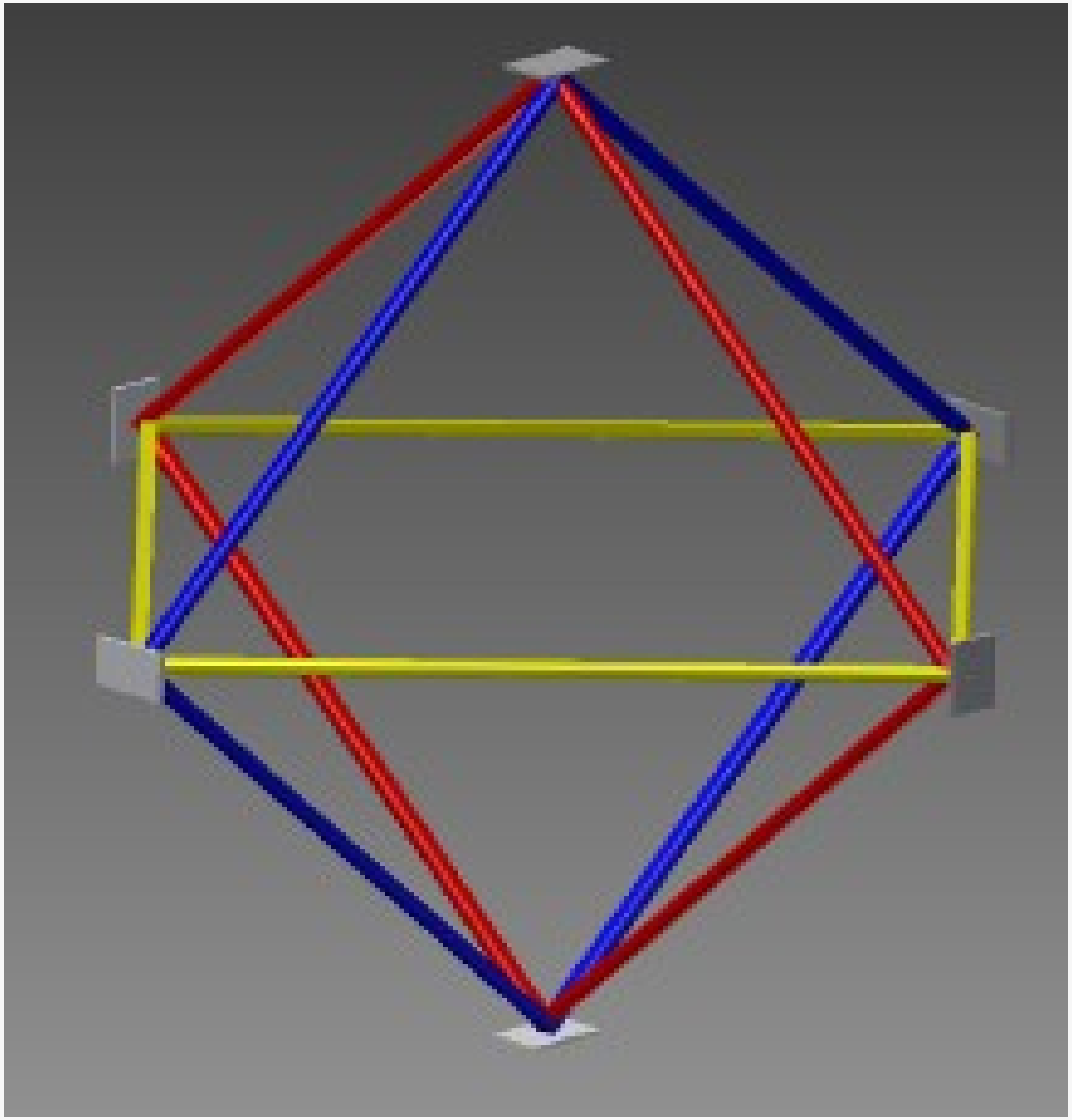
3)Adjust the mirrors in order to fix the absolute lenghts of the diagonals

3)Measure the perimeter by injecting an external laser in the ring (the spot on the mirrors must overlap)

5)Adjust the position of the mirros only by means of rigid movements of the diagonals to the minimum perimeter

When PLANE and REGULAR Geometry is obtained
TURN ON THE RING LASER...

Iterating the procedure will eventually bring to the **“PERFECT OCTAHEDRON”**



Side Lengths measurements

Next generation gyros MUST include a diagnostic of **side lengths** and **relative angles**

Once the absolute lengths are obtained it is necessary to FREEZE the geometry

ABSOLUTE (accuracy down to 10 nm):
-Multi-wavelength
-frequency modulation
-noise modulation

RELATIVE:
-Standard interferometry,
-multiplexing possibilities with DI

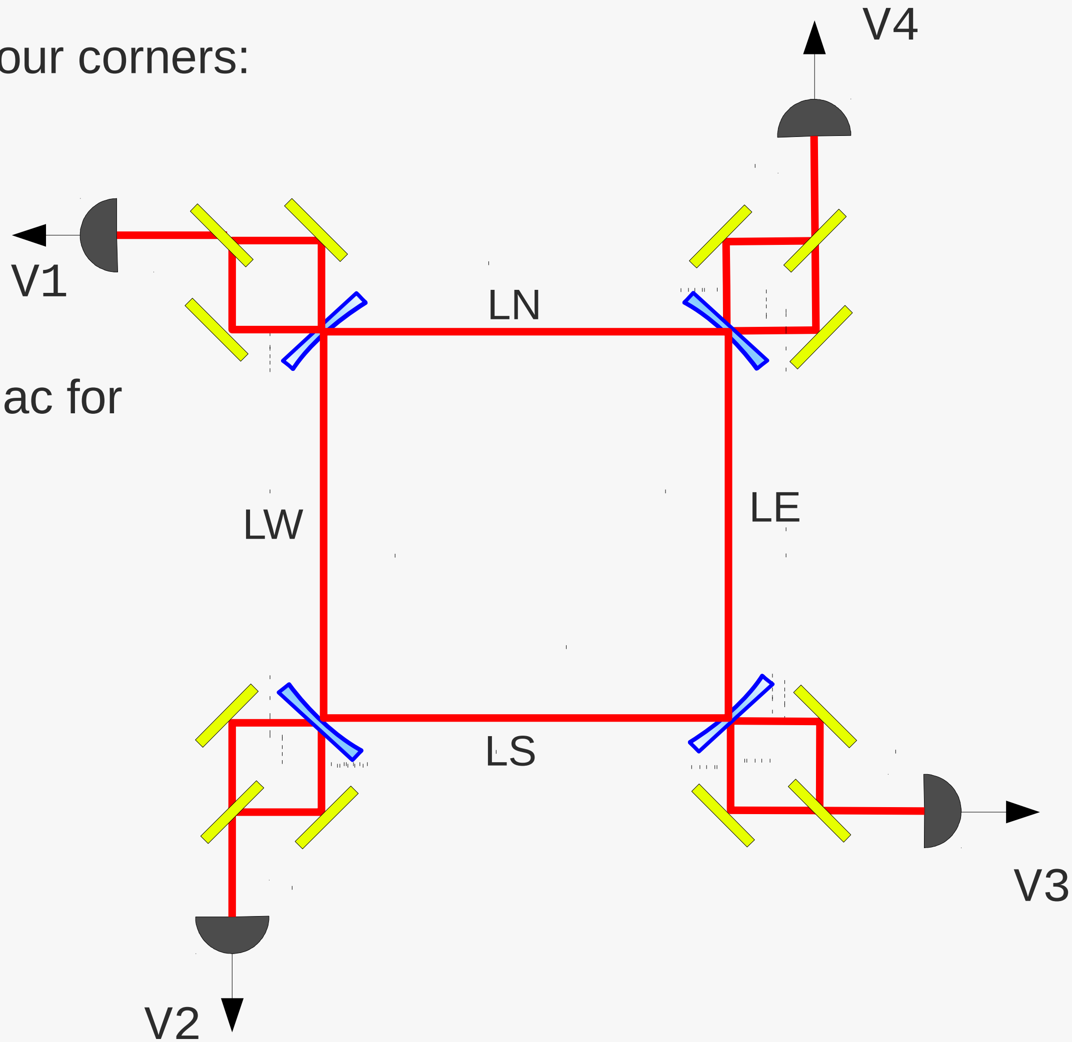
A secondary laser non resonant with the plasma will be suitable

Relative lengths monitoring

(1)All corner Sagnac phase-tracking

-Pick the Sagnac interferogram on the four corners:

$$V_i = V_{0i}\left[1 - 2\sin\left(\omega_S t + \varphi_i\right)\right]$$



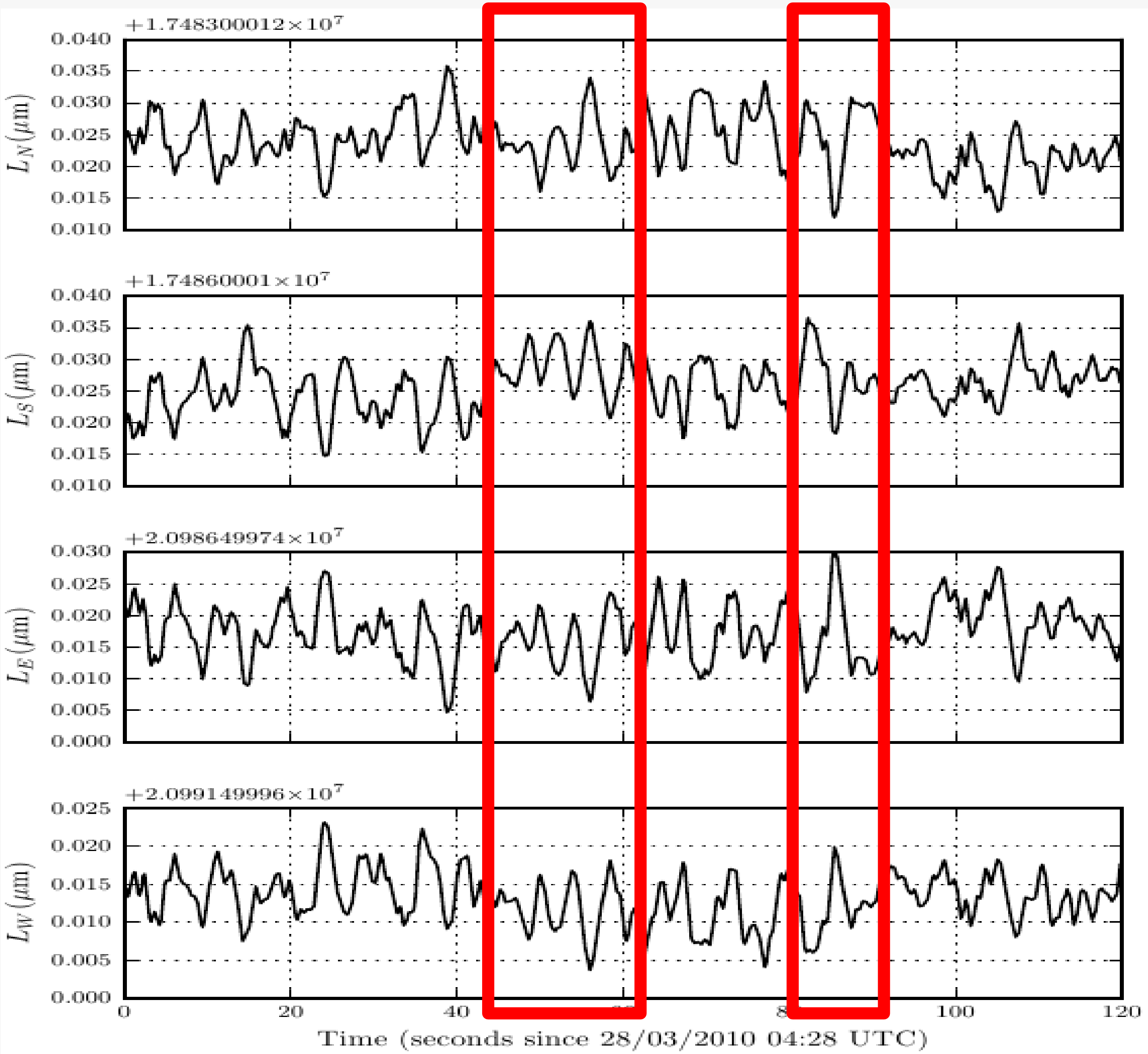
-Use a GPS referenced synthetized Sagnac for demodulation

$$\delta\left(\varphi_1 - \varphi_4\right) = k\,\delta L_N$$
$$\delta\left(\varphi_4 - \varphi_3\right) = k\,\delta L_E$$
$$\delta\left(\varphi_3 - \varphi_2\right) = k\,\delta L_S$$
$$\delta\left(\varphi_2 - \varphi_1\right) = k\,\delta L_W$$

(2) Digital interferometry (Gabriele will explain how..)

4 corners PT for UG II and the “cavern effect”

R.B. Hurst et al. ***Experiments with a 834 m2 ring laser interferometer****. In Journal of Applied Physics, 105 113115, June 2009.*



Correction is very small

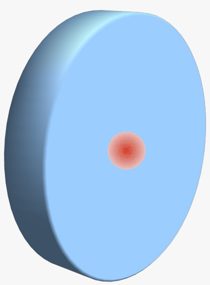
$$f'_s = f_s \frac{A P_0}{A_0 P}.$$

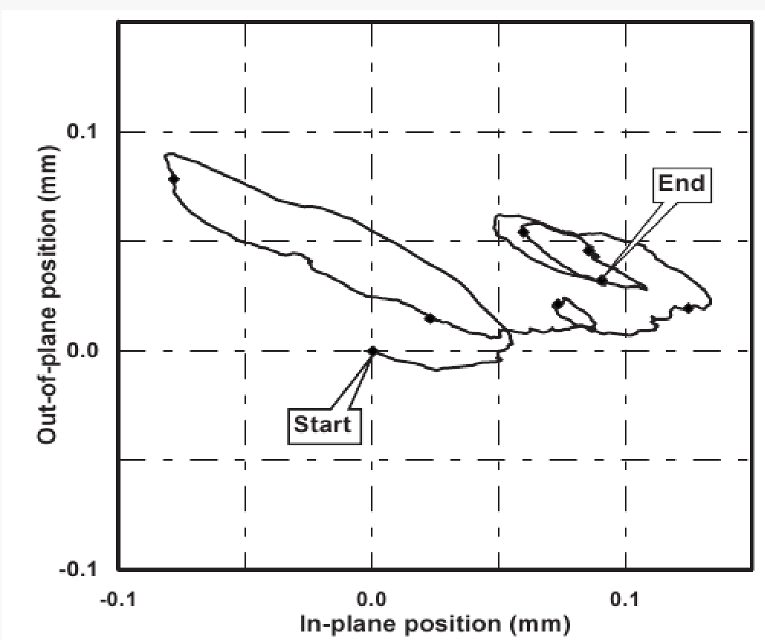
Only 5 uHz

...but

Angular stability

The beam spot positions
Have been observed with a CCD





B. Pritsch et al., **Scale-factor corrections in large ring lasers,**
APPLIED PHYSICS LETTERS 91, 061115 (2007)

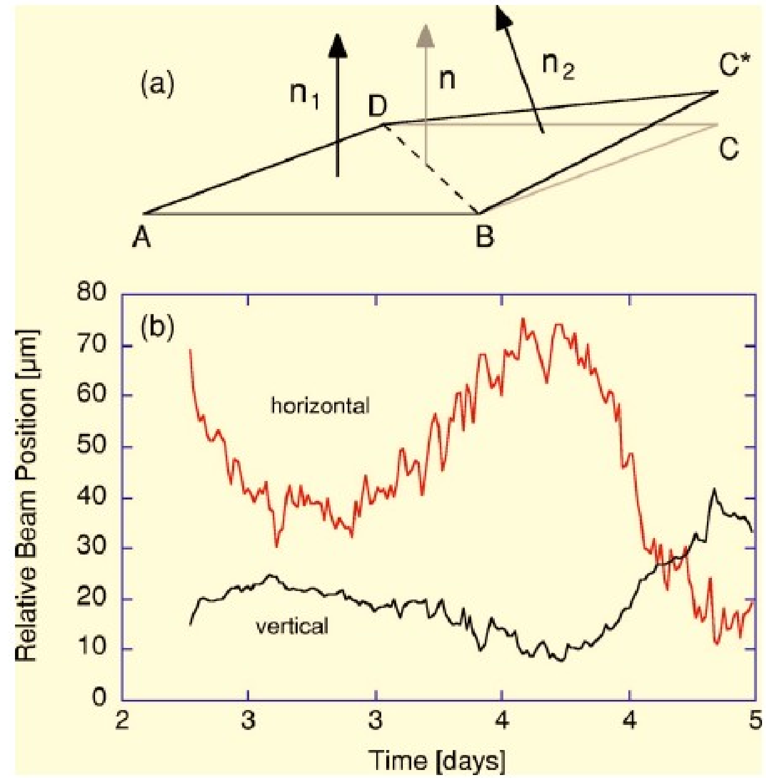


FIG. 1. Sum over the projection of the triangular subareas onto the axis of rotation define the effective ring laser normal (a). The diagram shown in (b) represents the observed laser beam wander in horizontal and vertical direction over a measurement sequence of several days.

Napoli's group work promise much better resolution
By studing cavity modes

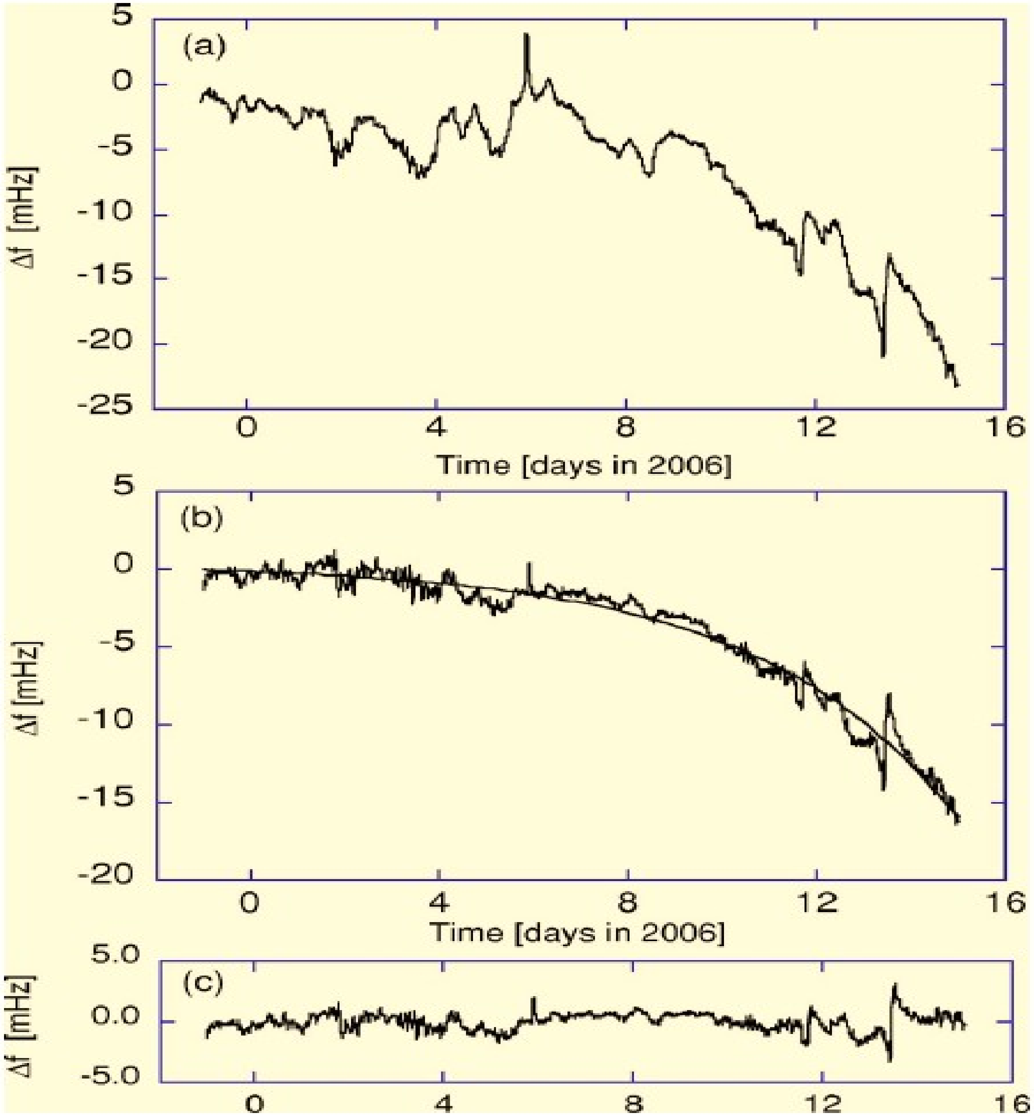
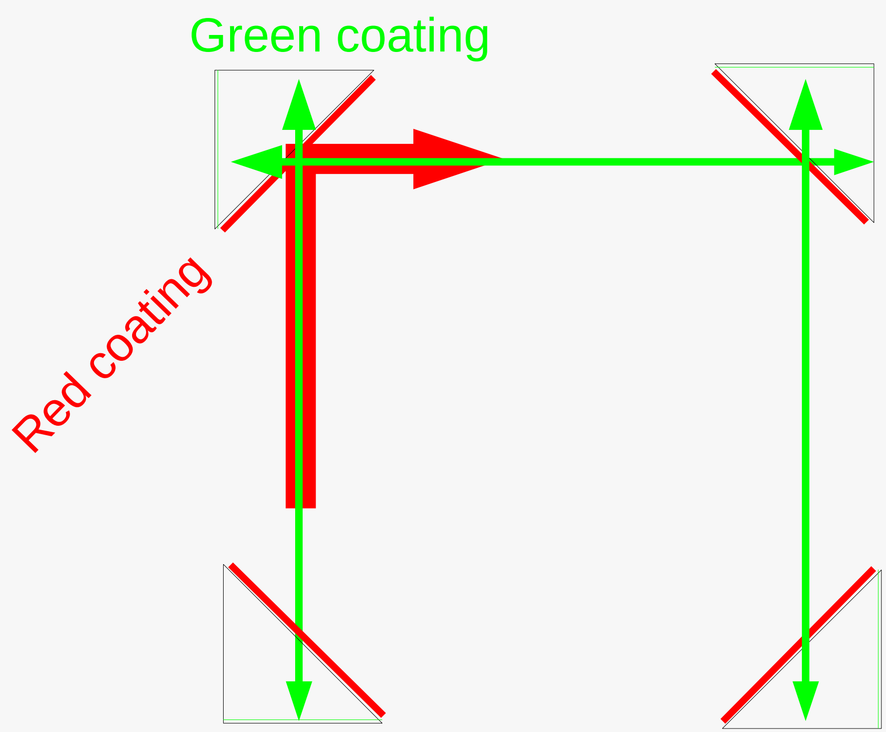


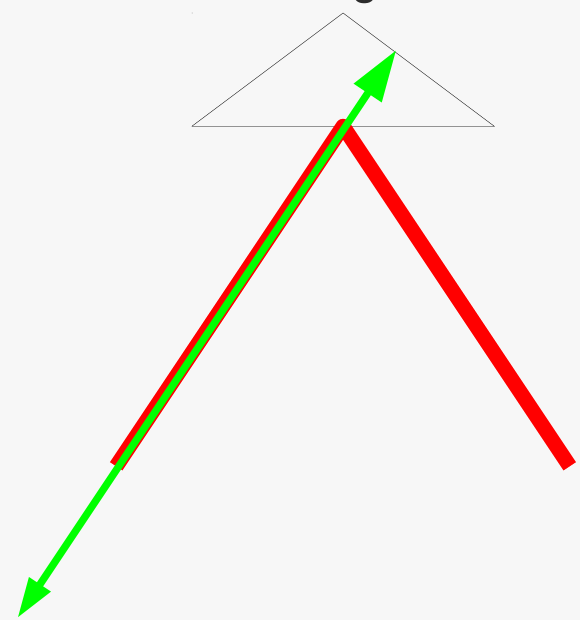
FIG. 2. Time series of the Sagnac beat frequency caused by Earth rotation and monitored with the UG2 gyroscope. The raw data are shown in (a), while the improvement with applied geometric scale factor corrections are given in (b). Finally the reduction of the gain medium contributions reduce the drift as shown in (c).

A proposal for the **absolute** measurement of the side lengths:

Replace mirrors with prisms or rigid ultrastable optical elements arrays

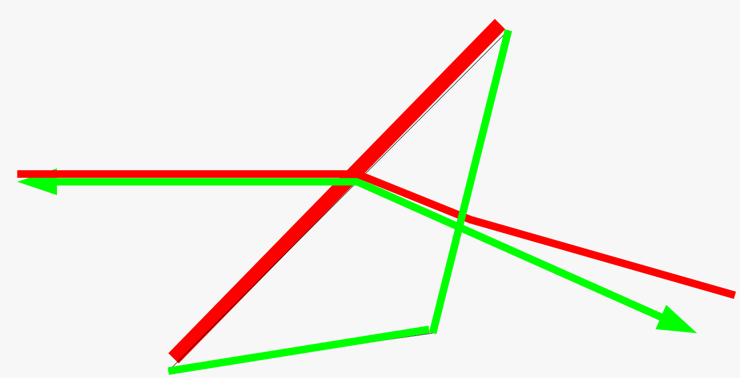


Good also for triagles



Excite the four sides linear **green** fabry-Pérot

Drawback: dispersion from the prisms
must be taken into account!



Summary

The Geometric stability of ring laser is the main issue to tackle in the next future

Geometry affects both bakscattering (small rings) and scale factor (very large rings)

The perfect square is a promising solution but some aspects require further investigations

We saw some experimental results on tilt monitoring and side-lengths measurement

Let's profit of the experience gained by the Ringlaser group in order to reach target stability in the geometry