BOUNDS ON SOFT SCATTERING IN THE TEV-SCALE

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INTRODUCTION

In this talk I shall re-examine the approach of p-p scattering toward s and t unitarity saturation in the TeV-scale. This issue was re-kindled recently by Block and Halzen (BH), utilizing the AUGER σ_{inel} and σ_{tot} data at 57 TeV.

An indication of p-p approaching saturation, is obtained from the s dependence of $\sigma_{inel}/\sigma_{tot} = 1 - \sigma_{el}/\sigma_{tot}$.

The p-p cross section data base of interest, is confined to accelerator data (TEVATRON and LHC) and p-Air Cosmic Rays data, from which partial p-p features are obtained through model dependent calculations.

Enforcing s and t unitarity is not a unique procedure, resulting in a market rich with models based on varying levels of rigor.

As we shall see, the TEVATRON(1.8)-LHC(7)-AUGER(57) TeV-scale data consistently indicate that soft scattering amplitudes populate only a slow growing fraction of the available phase space confined by s and t unitarity.

The importance of unitarity screenings in soft scatterings has become evident long ago, dating to the ISR epoch. Recall the remarkable success of the DL Regge Pomeron ($I\!\!P$) model which reproduce the total and elastic cross sections in the GeV-scale. The DL $I\!\!P$ parameters are:

$$\alpha_{I\!\!P}(t) = 1 + \Delta_{I\!\!P} + \alpha'_{I\!\!P}t, \quad \Delta_{I\!\!P} = 0.08 \ and \ \alpha'_{I\!\!P} = 0.25 GeV^{-2}.$$

The DL parametrization, as well as other models confined to only the elastic amplitude with out or with screening, suffer from built-in deficiencies:

Unscreened super critical $\Delta_{I\!\!P} > 0$, initiates $\sigma_{el} > \sigma_{tot}$ at high enough energies.

Unscreened models do not offer an interpretation of the very mild s dependence of σ_{sd} and σ_{dd}, which is very different from σ_{el} dependence on s.

Unitarity screening initiates the peripheral profile of diffractive b-amplitudes.

Ignoring the diffractive sector implies a neglect of the impact of Good-Walker (GW) "low mass" diffraction and Mueller's "high mass" (non GW) $3I\!\!P$ diffraction on the elastic amplitude.

SINGLE CHANNEL UNITARITY

Enforcing s-channel unitarity is model dependent.

Assume a single channel unitarity equation in b-space

$$2Im a_{el}(s,b) = \mid a_{el}(s,b)\mid^2 + G^{in}(s,b).$$

It is no more than stating that $\sigma_{tot} = \sigma_{el} + \sigma_{inel}$.

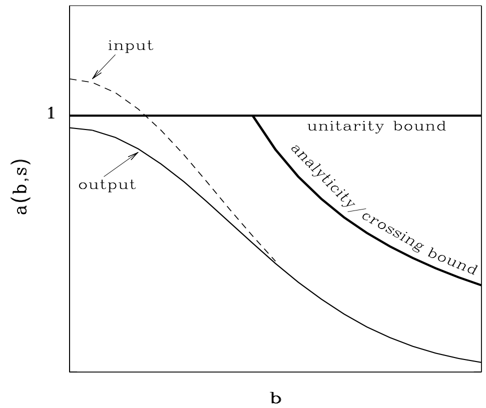
Its general solution can be written as

$$a_{el}(s,b) = i\left(1 - e^{-\Omega(s,b)/2}\right) \ \textbf{and} \ \ G^{in}(s,b) = 1 - e^{-\Omega(s,b)},$$

where $\Omega(s,b)$ **is arbitrary. It induces a unitarity bound of** $\mid a_{el}(s,b)\mid \leq 2$.

In a Glauber type eikonal approximation the input opacity $\Omega(s,b)$ **is real, i.e.** $a_{el}(s,b)$ **is imaginary. The opacity equals the imaginary part of the input Born term, a single** $I\!\!P$ **exchange in a** $I\!\!P$ **model.**

The initiated bound is $\mid a_{el}(s,b)\mid \leq 1$, **which is the black disc bound.**



FROISSART-MARTIN BOUND

In a single channel eikonal model, the screened cross sections are:

$$\sigma_{tot} = 2\int d^2b \left(1 - e^{-\Omega(s,b)/2}\right), \quad \sigma_{el} = \int d^2b \left(1 - e^{-\Omega(s,b)/2}\right)^2, \quad \sigma_{inel} = \int d^2b \left(1 - e^{-\Omega(s,b)}\right).$$

The figure above shows the effect of s-channel screening, securing that the screened elastic amplitude is bounded by unity. The figure illustrates, also, the bound implied by analyticity/crossing on the expanding b-amplitude. Saturating s-channel unitarity and analyticity/crossing bounds, we get the Froissart-Martin bound, $\sigma_{tot} \leq C ln^2(s/s_0)$**.** $s_0 = 1GeV^2$, $C = \pi/2m_\pi^2 \simeq 30mb$.

C is too large to be of use. At W=100 TeV the bound is $\simeq 1.6 \cdot 10^4 mb$.

Coupled to Froissart-Martin is MacDowell-Martin bound $\frac{\sigma_{tot}}{B_{el}} \leq 18\,\pi\,\frac{\sigma_{el}}{\sigma_{tot}}$.

The Froissart-Martin $ln^2 s$ **behavior relates to the bound, NOT to the total cross section, which can grow faster or slower than** $ln^2 s$**, below the bound.**

Recall that, in t-space, σ_{tot} **is proportional to a single point,** $d\sigma_{el}/dt(t=0)$**.**

σ_{tot} **in b-space is obtained from a** b^2 **integration over** $2(1 - e^{-\frac{1}{2}\Omega(s,b)})$.

Consequently, saturation in b-space is a differential feature attained first at b=0 and then expands very slowly with energy.

In a non GW single channel representation, $\sigma_{el} \leq \frac{1}{2}\sigma_{tot}$ **and** $\sigma_{inel} \geq \frac{1}{2}\sigma_{tot}$.

At saturation, regardless at what energy it is attained, $\sigma_{el} = \sigma_{inel} = \frac{1}{2}\sigma_{tot}$.

An intriguing issue remains opened for investigation:

Are the bounds, just presented, significantly different in a GW representation?

Are GW features maintained at asymptotic energies?

What are the bounds on diffractive cross sections?

UNITARITY BOUNDS IN A MULTI CHANNEL GW MODEL

In the following I shall use the GLM notation in which the diffractive states are presented by a single state with an unknown mass.

The elastic, SD and DD amplitudes in a 2 channel GW model are:

$a_{el}(s,b) = i\{\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\}$,

$a_{sd}(s,b) = i\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2 - \beta^2) A_{1,2} + \beta^2 A_{2,2}\}$,

$a_{dd}(s,b) = i\alpha^2\beta^2\{A_{1,1} - 2A_{1,2} + A_{2,2}\}$.

$A_{i,k}(s,b) = \left(1 - e^{\frac{1}{2}\Omega_{i,k}(s,b)}\right) \leq 1$. $\Omega_{i,k}(s,b) = \nu_{i,k}(s)\,\Gamma_{i,k}(s,b)$. $\Gamma_{i,k}(s,b)$ **is the b-profile.**

$\nu_{i,k}(s) = g_i g_k (\frac{s}{s_0})^{\Delta_{I\!P}}$. $\alpha^2 + \beta^2 = 1$. $a_{el}(s,b)$ **reaches its bound at (s,b), when and only when,** $A_{1,1}(s,b) = A_{1,2}(s,b) = A_{2,2}(s,b) = 1$, **independent of** β, **the GW phase.**

In GW multi channel models, we distinguish between GW and non GW diffraction. We obtain the Pumplin bound: $(\sigma_{el} + \sigma^{GW}_{diff}) \leq \frac{1}{2}\sigma_{tot}$, $\sigma^{GW}_{diff} = \sigma^{GW}_{sd} + \sigma^{GW}_{dd}$.

Consequently, $\sigma_{el} \leq \frac{1}{2}\sigma_{tot} - \sigma^{GW}_{diff}$, $\sigma_{inel} \geq \frac{1}{2}\sigma_{tot} + \sigma^{GW}_{diff}$,

At saturation, $a_{el}(s,b) = 1$, $a_{sd}(s,b) = a_{dd}(s,b) = 0$. $\sigma^{GW}_{diff} = 0$, $\sigma_{el} = \sigma_{inel} = \frac{1}{2}\sigma_{tot}$.

CROSSED CHANNEL UNITARITY

Mueller(1971) applied 3 body unitarity to equate the cross section of

$a + b \to M^2 + b$ **to the triple Regge diagram** $a + b + \bar{b} \to a + b + \bar{b}$.

The signature of this presentation is a triple vertex with a leading $3I\!\!P$ term.

The $3I\!\!P$ approximation is valid when $\frac{m_p^2}{M^2} << 1$ and $\frac{M^2}{s} << 1$.

The leading energy/mass dependences are $\frac{d\sigma^{3I\!\!P}}{dt\,dM^2} \propto s^{2\Delta_{I\!\!P}} (\frac{1}{M^2})^{1+\Delta_{I\!\!P}}$.

Mueller's $3I\!\!P$ approximation for non GW diffraction is the lowest order of multi $I\!\!P$ t-channel interactions, which are compatible with t-channel unitarity.

Recall that unitarity screening of GW ("low mass") diffraction is carried out in GLM and KMR by eikonalization, while the unitarity screening of non GW ("high mass") diffraction is carried out by the survival probability.

In GLM multi $I\!\!P$ interactions are summed by the MPSI procedure.

Other $I\!\!P$ models use different procedures.

DIFFRACTION AT THE TEV-SCALE

The investigation of soft diffraction at the TeV-scale is expected to indicate if we are approaching unitarity saturation of the elastic amplitude which implies the diminishing of diffraction.

Early signals of unitarity initiated correlation between the b structure of the elastic and diffractive amplitudes are well known:

The b-space centrality of the elastic amplitude initiates, through s-channel unitarity, the peripherality of the diffractive amplitudes.

The significant difference between the energy dependences of the elastic and diffractive cross sections is explained as a product of eikonalization (GW diffraction) and survival probability (non GW diffraction).

The above implies that a systemic reduction in the diffractive cross sections, as a function of energy, signals that a_{el} is getting close to saturation.

IS SATURATION ATTAINABLE?

Our partial information on p-p cross sections in the TeV-scale is limited to 2 experiments. I am omitting the Tevatron 1.8TeV which has conflicting results.

TOTEM(7 TeV): $\sigma_{tot} = 98.3 \pm 0.2(stat) \pm 2.8(sys)mb,$

$\sigma_{el} = 24.8 \pm 0.2(stat) \pm 1.2(sys)$ **mb,** $B_{el} = 20.1 \pm 0.2(stat) \pm 0.3(sys)GeV^{-2}.$

AUGER(57 TeV): $\sigma_{tot} = 133 \pm 13(stat) \pm^{17}_{20}(sys) \pm 16(Glauber)mb,$

$\sigma_{inel} = 92 \pm 7(stat) \pm^{9}_{11}(sys) \pm 16(Glauber)mb.$

$\sigma_{inel}/\sigma_{tot}(TOTEM) = 0.75, \ \ \sigma_{inel}/\sigma_{tot}(AUGER) = 0.69, \ \ \sigma_{tot}/B_{el}(TOTEM) = 12.6 < 14.1.$

The ratios above suggest that saturation of the elastic amplitude has not been attained up to 57 TeV. Note though, that the margin of AUGER errors is large.

In the following I shall try to offer a reasonable guess of saturation attainability based on an investigation of the outputs of GLM and KMR, which are multi channel $I\!P$ models, and BH, which is single channeled based on a logarithmic parametrization.

	$7\,TeV$			$14\,TeV$			$57\,TeV$		$100\,TeV$		
	GLM	KMR	BH	GLM	KMR	BH	GLM	BH	GLM	KMR	BH
σ_{tot}	98.6	97.4	95.4	109.0	107.5	107.3	130.0	134.8	139.0	138.8	147.1
σ_{inel}	74.0	73.6	69.0	81.1	80.3	76.3	95.2	92.9	101.5	100.7	100.0
$\frac{\sigma_{inel}}{\sigma_{tot}}$	0.75	0.76	0.72	0.74	0.75	0.71	0.73	0.70	0.73	0.73	0.68

A) Total and Inelastic Cross Sections

The Table above, compares σ_{tot} and σ_{inel} outputs of GLM, KMR and BH in the energy range of 7-100 TeV.

GLM and KMR have a bound of validity implied by the approximations they take. Their 100 TeV results may be under estimated.

As can be easily seen, the 3 models have compatible outputs up to 100 TeV.

The critical observation is that $\frac{\sigma_{inel}}{\sigma_{tot}} > 0.5$, over 7-100 TeV.

The BH model, which is much simpler than GLM and KMR, can be run at arbitrary high energies. The prediction of BH at the Planck-scale (1.22$\cdot 10^{16} TeV$) is: $\sigma_{inel}/\sigma_{tot} = 1131mb/2067mb = 0.55$, which is below A_{el} saturation.

TeV	$1.8 \to 7.0$	$7.0 \to 14.0$	$7.0 \to 57.0$	$57.0 \to 100.0$	$14.0 \to 100.0$
$\Delta_{eff}(GLM)$	0.081	0.072	0.066	0.060	0.062
$\Delta_{eff}(KMR)$	0.076	0.071			0.065
$\Delta_{eff}(BH)$	0.088	0.085	0.082	0.078	0.080

B) $\Delta_{I\!P}^{eff}$ Dependence on Energy

$\Delta_{I\!P}^{eff}$ serves as a simple measure of the rate of cross section growth. When compared with input $\Delta_{I\!P}$, we can assess the strength of the applied screening.

The screenings of $\sigma_{tot}, \sigma_{el}, \sigma_{sd}$, σ_{dd} and M^2_{diff} are not identical. Hence, their $\Delta_{I\!P}^{eff}$ values are different.

The cleanest determination of $\Delta_{I\!P}^{eff}$ is from the energy dependence of σ_{tot}.

All other options require also a determination of $\alpha'_{I\!P}$.

The table above compares $\Delta_{I\!P}^{eff}$ values obtained by GLM, KMR and BH.

The continuous reduction of $\Delta_{I\!P}^{eff}$, in the models considered, is a consequence of unitarity screenings, disregarding its different modelings.

The rate of the screened σ_{tot} increase with s, is lower than $ln^2 s$.

	$7\,TeV$		$14\,TeV$		$57\,TeV$	$100\,TeV$	
	GLM	KMR	GLM	KMR	GLM	GLM	KMR
σ_{tot}	98.6	97.4	109.0	107.5	130.0	134.0	138.8
σ_{el}	24.6	23.8	27.9	27.2	34.8	37.5	38.1
σ_{sd}^{GW}	10.7	7.3	11.5	8.1	13.0	13.6	10.4
σ_{sd}	14.88		17.31		21.68		
σ_{dd}^{GW}	6.21	0.9	6.79	1.1	7.95	8.39	1.6
σ_{dd}	7.45		8.38		18.14		
$\frac{\sigma_{el}+\sigma_{dif}^{GW}}{\sigma_{tot}}$	0.42	0.33	0.42	0.34	0.43	0.43	0.36

C) Diffractive Cross Sections

GLM and KMR total, elastic and GW diffractive cross sections are presented.

As seen in the table above, GLM GW σ_{sd} and, in particular σ_{dd}, are larger than KMR. σ_{tot} and σ_{el} outputs of GLM and KMR are compatible.

The non vanishing of the diffractive cross sections indicates that they are below saturation. The GW components are compatible with the Pumplin bound.

Warning: the analysis of soft diffraction, is hindered by the lack of uniform experimental and theoretical definitions of its signatures and bounds!

D) MacDowel-Martin Bound

MacDowel-Martin Bound is $R_{el}^{tot} = \frac{\sigma_{tot}}{B_{el}} \leq 18\pi \frac{\sigma_{el}}{\sigma_{tot}}$.

GLM and KMR ratios and bounds are:

$7\,TeV: \ R_{el}^{tot} = 12.5 \leq 14.1(GLM), \ \ R_{el}^{tot} = 12.3 \leq 13.8(KMR).$

$14\,TeV: \ R_{el}^{tot} = 13.0 \leq 14.5(GLM), \ \ R_{el}^{tot} = 12.8 \leq 14.3(KMR).$

$100\,TeV: \ R_{el}^{tot} = 13.8 \leq 15.3(GLM), \ \ R_{el}^{tot} = 13.8 \leq 15.5(KMR).$

As seen, the numbers above are compatible with a non saturated A_{el}.

CONCLUSIONS

Both the limited experimental data in the TeV-scale and the output of GLM KMR and BH, consistently indicate that the p-p elastic amplitude does not saturate up to 100 TeV and possibly (BH) up to the Planck-scale.

This conclusion does not rule out the possibility that $A_{el}(s,b)$ **has a black core.**