Photon impact factor and k_T-factorization for DIS in the next-to-leading order

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JLAB & ODU

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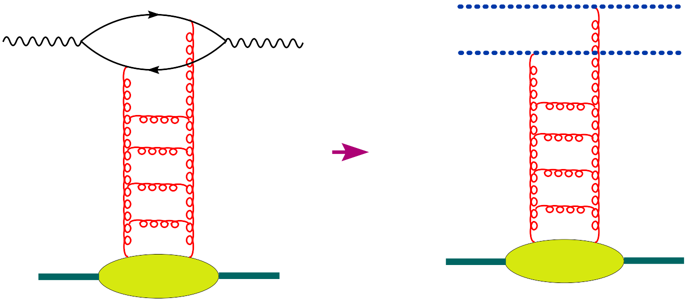
Outline

- High-energy scattering and Wilson lines.
- Evolution equation for color dipoles.
- Leading order: BK equation.
- Conformal composite dipoles and NLO BK kernel in $\mathcal{N} = 4$.
- NLO amplitude in $\mathcal{N} = 4$ SYM
- Photon impact factor.
- NLO BK kernel in QCD.
- k_T-factorization and NLO BFKL.
- Conclusions
- Outlook: color dipoles, gluon light-ray operators and gluon TMDs

DIS at high energy

- At high energies, particles move along straight lines $\Rightarrow$ the amplitude of $\gamma^* A \to \gamma^* A$ scattering reduces to the matrix element of a two-Wilson-line operator (color dipole):



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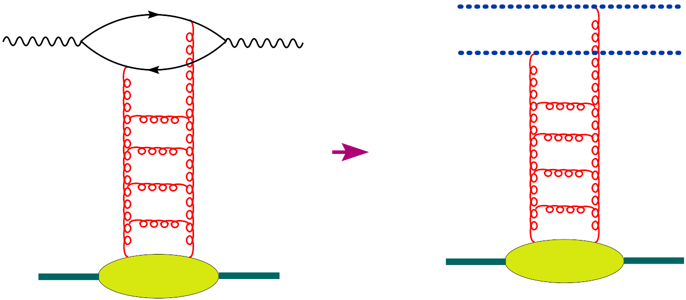
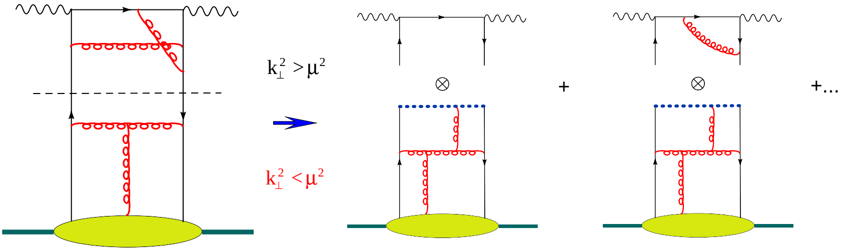


$$A(s) = \int \frac{d^2k_\perp}{4\pi^2} I^A(k_\perp) \langle B|\mathrm{Tr}\{U(k_\perp)U^\dagger(-k_\perp)\}|B\rangle$$

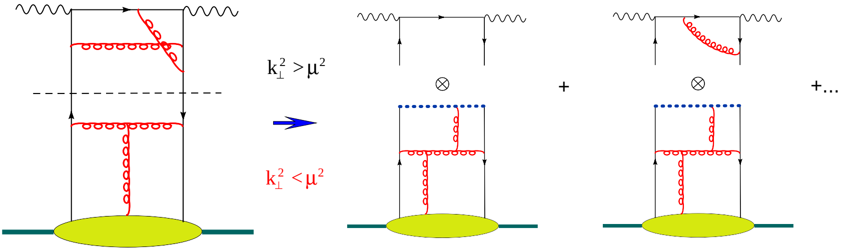
Formally, ➤ means the operator expansion in Wilson lines



μ^2 - factorization scale (normalization point)

$k_\perp^2 \ > \ \mu^2$ - coefficient functions

$k_\perp^2 \ < \ \mu^2$ - matrix elements of light-ray operators (normalized at μ^2)



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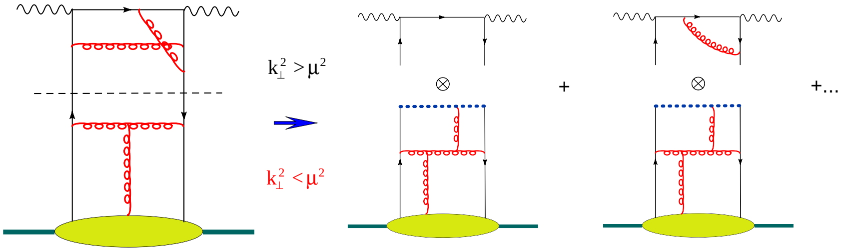
OPE in light-ray operators $(x-y)^2 \to 0$

$$T\{j_\mu(x)j_\nu(y)\} \ = \ \frac{x_\xi}{2\pi^2 x^4}\Big[1+\frac{\alpha_s}{\pi}(\ln x^2\mu^2+C)\Big]\bar\psi(x)\gamma_\mu\gamma^\xi\gamma_\nu[x,y]\psi(y)+O(\frac{1}{x^2})$$

$$[x,y] \ \equiv \ Pe^{ig\int_0^1 du \ (x-y)^\mu A_\mu(ux+(1-u)y)}$$ - gauge link

Light-cone expansion and DGLAP evolution in the NLO



μ^2 - factorization scale (normalization point)

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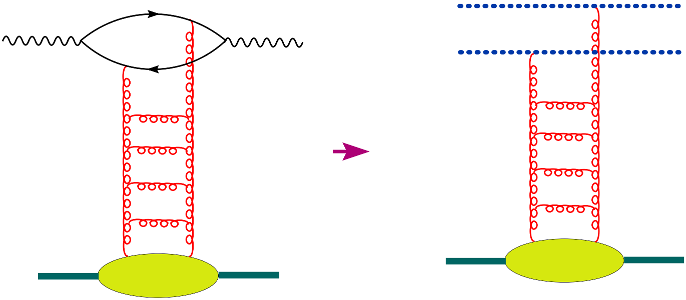
Renorm-group equation for light-ray operators $\Rightarrow$ DGLAP evolution of parton densities $(x-y)^2 = 0$

$$\mu^2 \frac{d}{d\mu^2}\bar\psi(x)[x,y]\psi(y) \ = \ K_{\rm LO}\bar\psi(x)[x,y]\psi(y) + \alpha_s K_{\rm NLO}\bar\psi(x)[x,y]\psi(y)$$

Four steps of an OPE

- Factorize an amplitude into a product of coefficient functions and matrix elements of relevant operators.
- Find the evolution equations of the operators with respect to factorization scale.
- Solve these evolution equations.
- Convolute the solution with the initial conditions for the evolution and get the amplitude

- At high energies, particles move along straight lines $\Rightarrow$ the amplitude of $\gamma^* A \to \gamma^* A$ scattering reduces to the matrix element of a two-Wilson-line operator (color dipole):



$$A(s) = \int \frac{d^2k_\perp}{4\pi^2} I^A(k_\perp) \langle B|\mathrm{Tr}\{U(k_\perp)U^\dagger(-k_\perp)\}|B\rangle$$

$$U(x_\perp) = \mathrm{Pexp}\Big[ig\int_{-\infty}^{\infty} du\, n^\mu A_\mu(un + x_\perp)\Big]$$ Wilson line

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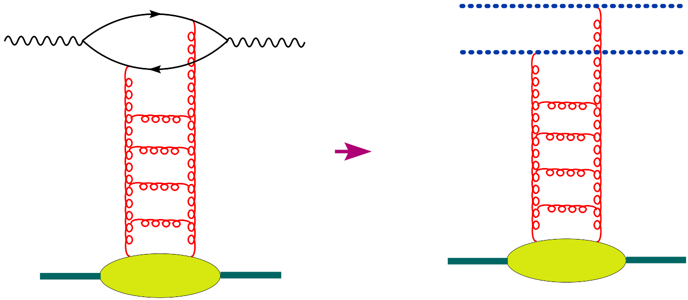

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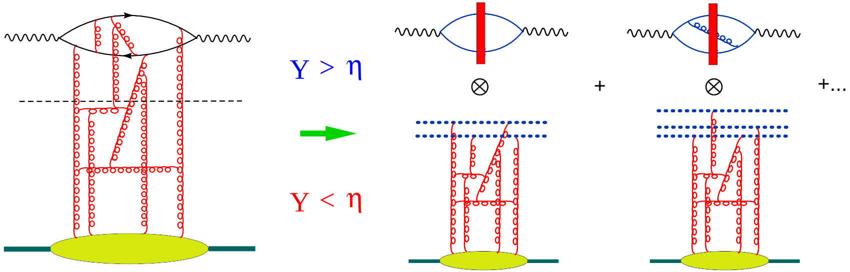
Formally, ➤ means the operator expansion in Wilson lines

Rapidity factorization



η - rapidity factorization scale

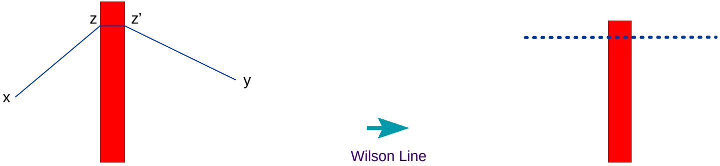
Rapidity Y > η - coefficient function (“impact factor”)
Rapidity Y < η - matrix elements of (light-like) Wilson lines with rapidity divergence cut by η

$$U_x^\eta = \text{Pexp}\Big[ig\int_{-\infty}^{\infty} dx^+ A_+^\eta(x_+, x_\perp)\Big]$$

$$A_\mu^\eta(x) = \int \frac{d^4k}{(2\pi)^4}\theta(e^\eta - |\alpha_k|)e^{-ik\cdot x}A_\mu(k)$$

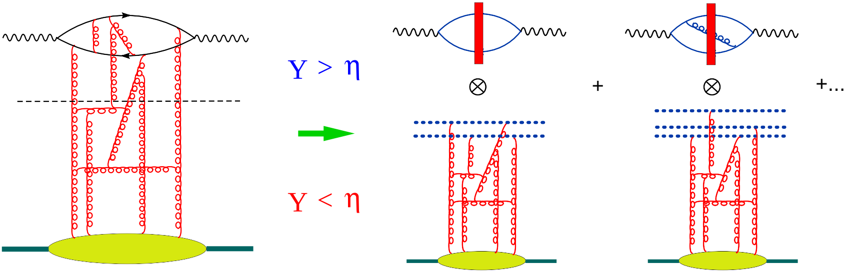
Spectator frame: propagation in the shock-wave background.

Each path is weighted with the gauge factor $Pe^{ig\int dx_\mu A^\mu}$. Quarks and gluons do not have time to deviate in the transverse space $\Rightarrow$ we can replace the gauge factor along the actual path with the one along the straight-line path.



[$x \to z$: free propagation]$\times$
[$U^{ab}(z_\perp)$ - instantaneous interaction with the $\eta < \eta_2$ shock wave]$\times$
[$z \to y$: free propagation]

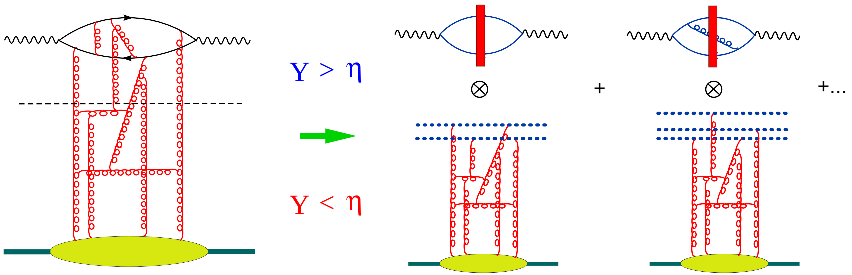
High-energy expansion in color dipoles



The high-energy operator expansion is

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y)\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$
$$+\ \text{NLO contribution}$$

High-energy expansion in color dipoles



η - rapidity factorization scale

Evolution equation for color dipoles

$$\frac{d}{d\eta}\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\} = \frac{\alpha_s}{2\pi^2}\int d^2z \frac{(x-y)^2}{(x-z)^2(y-z)^2}[\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\}\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\}$$
$$- N_c\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\}] + \alpha_s K_{\mathrm{NLO}}\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\} + O(\alpha_s^2)$$

(Linear part of K_{NLO} = $K_{\mathrm{NLO\ BFKL}}$)

Evolution equation for color dipoles

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

BK equation

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) = \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\ (x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z) + \hat{\mathcal{U}}(z,y) - \hat{\mathcal{U}}(x,y) - \hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

I. B. (1996), Yu. Kovchegov (1999)
Alternative approach: JIMWLK (1997-2000)

Non-linear evolution equation

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LLA for DIS in pQCD $\Rightarrow$ BFKL (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$)

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Alternative approach: JIMWLK (1997-2000)

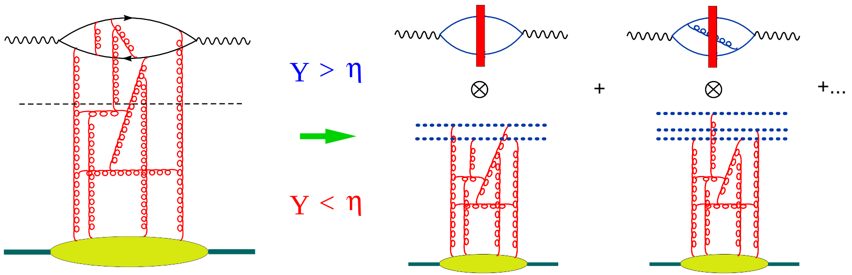
LLA for DIS in pQCD ⇒ BFKL (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$)

LLA for DIS in sQCD ⇒ BK eqn (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1, \ \alpha_s A^{1/3} \sim 1$)

(s for semiclassical)

Why NLO correction?

- To check that high-energy OPE works at the NLO level.
- To check conformal invariance of the NLO BK equation(in $\mathcal{N}$=4 SYM)
- To determine the argument of the coupling constant of the BK equation(in QCD).
- To get the region of application of the leading order evolution equation.



The high-energy operator expansion is $\mathcal{O} \equiv \mathrm{Tr}\{Z^2\}$

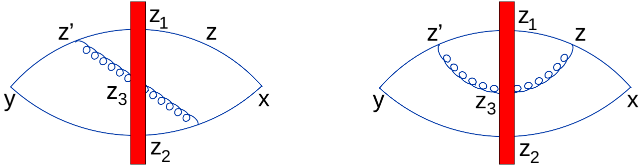
$$T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}(y)\} = \int d^2z_1 d^2z_2\ I^{\mathrm{LO}}(z_1, z_2)\mathrm{Tr}\{\hat{U}^{\eta}_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

$$+ \int d^2z_1 d^2z_2 d^2z_3\ I^{\mathrm{NLO}}(z_1, z_2, z_3)[\frac{1}{N_c}\mathrm{Tr}\{T^n\hat{U}^{\eta}_{z_1}\hat{U}^{\dagger\eta}_{z_3}T^n\hat{U}^{\eta}_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - \mathrm{Tr}\{\hat{U}^{\eta}_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

In the leading order - conf. invariant impact factor

$$I_{\mathrm{LO}} = \frac{x_+^{-2}y_+^{-2}}{\pi^2\mathcal{Z}_1^2\mathcal{Z}_2^2}, \qquad \mathcal{Z}_i \equiv \frac{(x-z_i)_\perp^2}{x_+} - \frac{(y-z_i)_\perp^2}{y_+} \qquad CCP, 2007$$

NLO impact factor (in $\mathcal{N}=4$ SYM



(a) (b)

$$I^{\rm NLO}(x,y;z_1,z_2,z_3;\eta) \;=\; -\,I^{\rm LO}\times\frac{\lambda}{\pi^2}\frac{z_{13}^2}{z_{12}^2z_{23}^2}\Big[\ln\frac{\sigma s}{4}\mathcal{Z}_3-\frac{i\pi}{2}+C\Big]$$

The NLO impact factor is not Möbius invariant $\Rightarrow$ the color dipole with the cutoff η is not invariant

However, if we define a composite operator (a - analog of μ^{-2} for usual OPE)

$$\begin{aligned}&[\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf} \;=\; \mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\\ &+\frac{\lambda}{2\pi^2}\int d^2z_3\;\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\mathrm{Tr}\{T^n\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}T^n\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\}-N_c\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]\ln\frac{az_{12}^2}{z_{13}^2z_{23}^2}\;+\;O(\lambda^2)\end{aligned}$$

the impact factor becomes conformal in the NLO.

Conformal composite dipole

$$[\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf} = \mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$
$$+\frac{\lambda}{2\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\mathrm{Tr}\{T^n\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}T^n\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]\ln\frac{az_{12}^2}{z_{13}^2z_{23}^2} + O(\lambda^2)$$

High-energy OPE:

$$T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}(y)\} = \int d^2z_1d^2z_2\ I^{\rm LO}(z_1,z_2)\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}^{\rm conf}$$
$$+\int d^2z_1d^2z_2d^2z_3\ I^{\rm NLO}(z_1,z_2,z_3)[\frac{1}{N_c}\mathrm{Tr}\{T^n\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}T^n\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - \mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

$I^{\rm LO}$ and $I^{\rm NLO}$ are Möbius invariant.

We think that one can construct the composite conformal dipole operator order by order in perturbation theory.

Analogy: when the UV cutoff does not respect the symmetry of a local operator, the composite local renormalized operator in must be corrected by finite counterterms order by order in perturbaton theory.

NLO BK equation in $\mathcal{N}=4$ SYM

Define

$$\hat{\mathcal{U}}^a_{\rm conf}(z_1,z_2)$$
$$= \hat{\mathcal{U}}^\eta(z_1,z_2) + \frac{\alpha_s N_c}{4\pi^2}\int d^2z \frac{z_{12}^2}{z_{13}^2 z_{23}^2}\ln\frac{ae^{2\eta}z_{12}^2}{z_{13}^2 z_{23}^2}\big[{\rm Tr}\{T^a\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}T^a\hat{U}_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\big]$$

such that $\frac{d}{d\eta}\hat{\mathcal{U}}^a_{\rm conf}(z_1,z_2)=0$.

⇒The evolution rewritten in terms of a is Möbius invariant

$$2a\frac{d}{da}\big[{\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\big]^{\rm conf}$$
$$= \frac{\alpha_s}{\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2 z_{23}^2}\Big[1-\frac{\alpha_s N_c}{4\pi}\frac{\pi^2}{3}\Big]\big[{\rm Tr}\{T^a\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}T^a\hat{U}_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\big]^{\rm conf}$$
$$-\frac{\alpha_s^2}{4\pi^4}\int d^2z_3 d^2z_4\frac{z_{12}^2}{z_{13}^2 z_{24}^2 z_{34}^2}\Big\{2\ln\frac{z_{12}^2 z_{34}^2}{z_{14}^2 z_{23}^2} + \Big[1+\frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\Big]\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}\Big\}$$
$$\times\,{\rm Tr}\{[T^a,T^b]\hat{U}^\eta_{z_1}T^{a'}T^{b'}\hat{U}^{\dagger\eta}_{z_2} + T^bT^a\hat{U}^\eta_{z_1}[T^{b'},T^{a'}]\hat{U}^{\dagger\eta}_{z_2}\}[(\hat{U}^\eta_{z_3})^{aa'}(\hat{U}^\eta_{z_4})^{bb'} - (z_4\to z_3)]$$

NLO Amplitude in $\mathcal{N}$=4 SYM theory

The pomeron contribution to a 4-point correlation function in $\mathcal{N} = 4$ SYM can be represented as $\lambda \equiv g^2 N_c$

$$(x-y)^4(x'-y')^4\langle\mathcal{O}(x)\mathcal{O}^\dagger(y)\mathcal{O}(x')\mathcal{O}^\dagger(y')\rangle$$
$$= \frac{i}{8\pi^2}\int d\nu\, \tilde{f}_+(\nu)\tanh\pi\nu\frac{\sin\nu\rho}{\sinh\rho}F(\nu,\lambda)R^{\frac{1}{2}\omega(\nu,\lambda)} \qquad \text{Cornalba}(2007)$$

$\omega(\nu,\lambda) = \frac{\lambda}{\pi}\chi(\nu) + \lambda^2\omega_1(\nu) + ...$ is the pomeron intercept,
$\tilde{f}_+(\omega) = (e^{i\pi\omega} - 1)/\sin\pi\omega$ is the signature factor.
$F(\nu,\lambda) = F_0(\nu) + \lambda F_1(\nu) + ...$ is the “pomeron residue”.

R and r are two conformal ratios:

$$R = \frac{(x-x')(y-y')^2}{(x-y)^2(x'-y')^2}, \quad r = R\Big[1 - \frac{(x-y')^2(y-x')^2}{(x-x')^2(y-y')^2} + \frac{1}{R}\Big]^2, \quad \cosh\rho = \frac{\sqrt{r}}{2}$$

In the Regge limit $s \to \infty$ the ratio R scales as s while r does not depend on energy.

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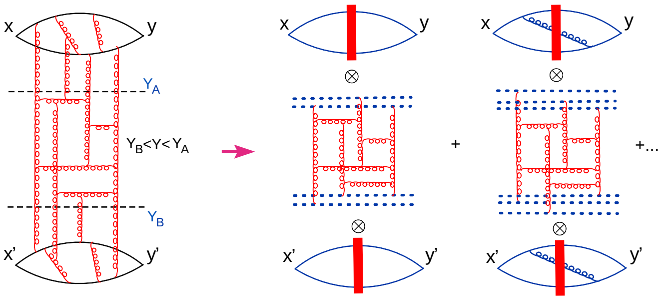
In the Regge limit $s\to\infty$ the ratio R scales as s while r does not depend on energy.

$\omega_0(\nu)$, $\omega_1(\nu)$ and $F_0(\nu)$ were known.

We reproduced $\omega_1(\nu)$ (Lipatov & Kotikov, 2000) and found $F_1(\nu)$

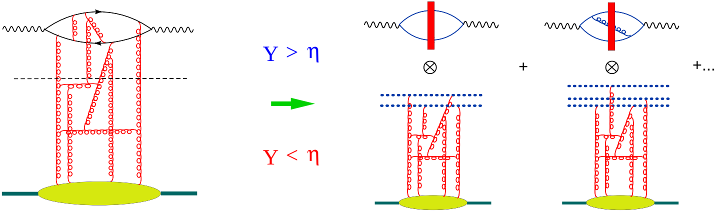


$$(x-y)^4(x'-y')^4\langle T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}^\dagger(y)\hat{\mathcal{O}}(x')\hat{\mathcal{O}}^\dagger(y')\}\rangle$$
$$=\int d^2z_{1\perp}d^2z_{2\perp}d^2z'_{1\perp}d^2z'_{2\perp}\,\mathrm{IF}^{a_0}(x,y;z_1,z_2)[\mathrm{DD}]^{a_0,b_0}(z_1,z_2;z'_1,z'_2)\mathrm{IF}^{b_0}(x',y';z'_1,z'_2)$$

Result : (G.A. Chirilli and I.B.)

$$F(\nu) = \frac{N_c^2}{N_c^2-1}\frac{4\pi^4\alpha_s^2}{\cosh^2\pi\nu}\left\{1+\frac{\alpha_s N_c}{\pi}\left[-\frac{2\pi^2}{\cosh^2\pi\nu}+\frac{\pi^2}{2}-\frac{8}{1+4\nu^2}\right]+O(\alpha_s^2)\right\}$$

NLO high-energy OPE in QCD



DIS structure function $F_2(x)$: photon impact factor + evolution of color dipoles+ initial conditions for the small-x evolution

Photon impact factor in the LO

$$(x-y)^4 T\{\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)\hat{\bar{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \int \frac{d^2z_1 d^2z_2}{z_{12}^4} I^{\rm LO}_{\mu\nu}(z_1,z_2)\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

$$I^{\rm LO}_{\mu\nu}(z_1,z_2) = \frac{\mathcal{R}^2}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)-\frac{1}{2}\kappa^2(\zeta_1\cdot\zeta_2)\Big].$$

$$\kappa \equiv \frac{1}{\sqrt{s}x^+}\Big(\frac{p_1}{s}-x^2p_2+x_\perp\Big)-\frac{1}{\sqrt{s}y^+}\Big(\frac{p_1}{s}-y^2p_2+y_\perp\Big)$$

$$\zeta_i \equiv \Big(\frac{p_1}{s}+z_{i\perp}^2p_2+z_{i\perp}\Big), \qquad \mathcal{R} \equiv \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$$

Composite “conformal” dipole $[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}$ - same as in $\mathcal{N}=4$ case.

$$(x-y)^4 T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}$$
$$= \int\frac{d^2z_1d^2z_2}{z_{12}^4}\,\Big\{I^{\mu\nu}_{\rm LO}(z_1,z_2)\big[1+\frac{\alpha_s}{\pi}\big][\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}$$
$$+\int d^2z_3\Big[\frac{\alpha_s}{4\pi^2}\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big(\ln\frac{\kappa^2(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)}{2(\kappa\cdot\zeta_3)^2(\zeta_1\cdot\zeta_2)}-2C\Big)I^{\mu\nu}_{\rm LO}+\ I^{\mu\nu}_2\Big]$$
$$\times\ [\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\}-N_c\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}\Big\}$$

$$(I_2)_{\mu\nu}(z_1,z_2,z_3)\ =\ \frac{\alpha_s}{16\pi^8}\frac{\mathcal{R}^2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big\{\frac{(\kappa\cdot\zeta_2)}{(\kappa\cdot\zeta_3)}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[-\frac{(\kappa\cdot\zeta_1)^2}{(\zeta_1\cdot\zeta_3)}$$
$$+\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}+\ \frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_3)}-\frac{\kappa^2(\zeta_1\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}\Big]$$
$$+\frac{(\kappa\cdot\zeta_2)^2}{(\kappa\cdot\zeta_3)^2}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)}{(\zeta_2\cdot\zeta_3)}-\frac{\kappa^2(\zeta_1\cdot\zeta_3)}{2(\zeta_2\cdot\zeta_3)}\Big]+(\zeta_1\leftrightarrow\zeta_2)\Big\}$$

With two-gluon (NLO BFKL) accuracy

$$\frac{1}{N_c}(x-y)^4 T\{\hat{\bar\psi}(x)\gamma^\mu\hat\psi(x)\hat{\bar\psi}(y)\gamma^\nu\hat\psi(y)\} = \frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\int\frac{dz_1dz_2}{z_{12}^4}\,\hat{\mathcal{U}}_{a_0}(z_1,z_2)\big[\mathcal{I}^{\rm LO}_{\alpha\beta}\big(1+\frac{\alpha_s}{\pi}\big)+\mathcal{I}^{\rm NLO}_{\alpha\beta}\big]$$

$$\mathcal{I}^{\alpha\beta}_{\rm LO}(x,y;z_1,z_2) = \mathcal{R}^2\frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2)-\zeta_1^\alpha\zeta_2^\beta-\zeta_2^\alpha\zeta_1^\beta}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$$

$$\begin{aligned}\mathcal{I}^{\alpha\beta}_{\rm NLO}(x,y;z_1,z_2) = \frac{\alpha_sN_c}{4\pi^7}\mathcal{R}^2\Big\{&\frac{\zeta_1^\alpha\zeta_2^\beta+\zeta_1\leftrightarrow\zeta_2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[4{\rm Li}_2(1-\mathcal{R})-\frac{2\pi^2}{3}+\frac{2\ln\mathcal{R}}{1-\mathcal{R}}+\frac{\ln\mathcal{R}}{\mathcal{R}}\\&-4\ln\mathcal{R}+\frac{1}{2\mathcal{R}}-2+2(\ln\frac{1}{\mathcal{R}}+\frac{1}{\mathcal{R}}-2)\big(\ln\frac{1}{\mathcal{R}}+2C\big)-4C-\frac{2C}{\mathcal{R}}\Big]\\&+\Big(\frac{\zeta_1^\alpha\zeta_1^\beta}{(\kappa\cdot\zeta_1)^2}+\zeta_1\leftrightarrow\zeta_2\Big)\Big[\frac{\ln\mathcal{R}}{\mathcal{R}}-\frac{2C}{\mathcal{R}}+2\frac{\ln\mathcal{R}}{1-\mathcal{R}}-\frac{1}{2\mathcal{R}}\Big]-\frac{2}{\kappa^2}\Big(g^{\alpha\beta}-2\frac{\kappa^\alpha\kappa^\beta}{\kappa^2}\Big)\\&+\Big[\frac{\zeta_1^\alpha\kappa^\beta+\zeta_1^\beta\kappa^\alpha}{(\kappa\cdot\zeta_1)\kappa^2}+\zeta_1\leftrightarrow\zeta_2\Big]\Big[-2\frac{\ln\mathcal{R}}{1-\mathcal{R}}-\frac{\ln\mathcal{R}}{\mathcal{R}}+\ln\mathcal{R}-\frac{3}{2\mathcal{R}}+\frac{5}{2}+2C+\frac{2C}{\mathcal{R}}\Big]\\&+\frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2)}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[\frac{2\pi^2}{3}-4{\rm Li}_2(1-\mathcal{R})\\&-2\big(\ln\frac{1}{\mathcal{R}}+\frac{1}{\mathcal{R}}+\frac{1}{2\mathcal{R}^2}-3\big)\big(\ln\frac{1}{\mathcal{R}}+2C\big)+6\ln\mathcal{R}-\frac{2}{\mathcal{R}}+2+\frac{3}{2\mathcal{R}^2}\Big]\Big\}\end{aligned}$$

5 tensor structures (CCP, 2009)

Photon Impact Factor at NLO

Reminder

$$\kappa^\mu = \frac{1}{\sqrt{s}x^+}(\frac{p_1^\mu}{s} - x^2 p_2^\mu + x_\perp^\mu) - \frac{1}{\sqrt{s}y^+}(\frac{p_1^\mu}{s} - y^2 p_2^\mu + y_\perp^\mu)$$

$$\zeta_1^\mu = (\frac{p_1^\mu}{s} + z_{1\perp}^2 p_2^\mu + z_{1\perp}^\mu), \qquad \zeta_2^\mu = (\frac{p_1^\mu}{s} + z_{2\perp}^2 p_2^\mu + z_{2\perp}^\mu)$$

DIS photon impact factor is a linear combination of the following tensor basis

$$\mathcal{I}_1^{\mu\nu} = g^{\mu\nu} \qquad \mathcal{I}_2^{\mu\nu} = \frac{\kappa^\mu \kappa^\nu}{\kappa^2}$$

$$\mathcal{I}_3^{\mu\nu} = \frac{\kappa^\mu \zeta_1^\nu + \kappa^\nu \zeta_1^\mu}{\kappa \cdot \zeta_1} + \frac{\kappa^\mu \zeta_2^\nu + \kappa^\nu \zeta_2^\mu}{\kappa \cdot \zeta_2}$$

$$\mathcal{I}_4^{\mu\nu} = \frac{\kappa^2 \zeta_1^\mu \zeta_1^\nu}{(\kappa \cdot \zeta_1)^2} + \frac{\kappa^2 \zeta_2^\mu \zeta_2^\nu}{(\kappa \cdot \zeta_2)^2} \qquad \mathcal{I}_5^{\mu\nu} = \frac{\zeta_1^\mu \zeta_2^\nu + \zeta_2^\mu \zeta_1^\nu}{\zeta_1 \cdot \zeta_2}$$

Cornalba, Costa, Penedones (2010)

Mellin representation of the LO impact factor

$$\int \frac{d^2z_1 d^2z_2}{z_{12}^2} I_{LO}^{\mu\nu}(z_1,z_2)\Big(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\Big)^\gamma = \frac{1}{\pi^4} B(1-\gamma)\Gamma(\gamma+2)\Gamma(3-\gamma)$$
$$\times \Big\{\frac{\gamma(1-\gamma)D_1^{\mu\nu}}{12(1+\gamma)(2-\gamma)} + \frac{D_2^{\mu\nu}}{2(1+\gamma)(2-\gamma)} - \frac{D_3^{\mu\nu}}{8(1+\gamma)(2-\gamma)}$$
$$-\frac{\gamma(1-\gamma)D_4}{16(1+2\gamma)(3-2\gamma)(1+\gamma)(2-\gamma)} - \frac{D_1^{\mu\nu}+D_2^{\mu\nu}}{8}\Big\}_{\mu\nu}\Big(\frac{\kappa^2}{(\kappa\cdot\zeta_0)^2}\Big)^\gamma$$

where

$$(D_1+D_2)^{\mu\nu} = -2\Delta^2 x^+ y^+ \kappa^{-2}\partial_x^\mu \partial_y^\nu \kappa^2$$
$$D_2^{\mu\nu} = -\Delta^2 x^+ y^+ \partial_x^\mu(\ln\kappa^2)\partial_y^\nu \ln\kappa^2$$
$$D_3^{\mu\nu} = 4\gamma\Delta^2 x^+ y^+\big[(\partial_x^\mu \ln\kappa^2)\partial_\nu^y \ln(\kappa\cdot\zeta_0) + (\partial_y^\nu \ln\kappa^2)\partial_\mu^x \ln(\kappa\cdot\zeta_0) - (\partial_x^\mu \ln\kappa^2)\partial_y^\nu \ln\kappa^2\big]$$
$$D_4^{\mu\nu} = 4\gamma(1+2\gamma)\Delta^2 x^+ y^+\big[-\frac{1}{3}\partial_x^\mu\partial_y^\nu \ln\kappa^2 - \partial_x^\mu(\ln\kappa^2)\partial_y^\nu \ln\kappa^2$$
$$+(\partial_x^\mu \ln\kappa^2)\partial_\nu^y \ln(\kappa\cdot\zeta_0) + (\partial_y^\nu \ln\kappa^2)\partial_\mu^x \ln(\kappa\cdot\zeta_0) - 2\partial_\mu^x \ln(\kappa\cdot\zeta_0)\partial_\nu^y \ln(\kappa\cdot\zeta_0)\big]$$

$B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, $C=-\psi(1)$ is the Euler constant, and $\psi'(a)=\frac{d}{da}\ln\Gamma(a)$

Mellin representation of the photon impact factor (I.B. and G. A. C.)

$$\int \frac{d^2z_1 d^2z_2}{z_{12}^4}[I^{\mu\nu}_{LO}(z_1,z_2) +^{\mu\nu}_{NLO}(z_1,z_2)]\Big(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\Big)^\gamma = \frac{N_c}{4\pi^6\Delta^4}\frac{\Gamma(\gamma+1)\Gamma(2-\gamma)}{\Delta^2 x^+ y^+}$$

$$\times\Bigg[\frac{\bar\gamma\gamma D_1}{3}\Big\{1+\frac{\alpha_s N_c}{4\pi}\Big[\frac{\pi^2}{3}-\frac{\pi^2}{\sin^2\pi\gamma}-C\chi_\gamma-\frac{1}{\gamma\bar\gamma}+\frac{1}{2}--\frac{\chi_\gamma}{\gamma\bar\gamma}\Big]\Big\}$$

$$+2D_2\Big\{1+\frac{\alpha_s N_c}{4\pi}\Big[\frac{\pi^2}{3}-\frac{\pi^2}{\sin^2\pi\gamma}C\chi_\gamma-\frac{3}{4\gamma\bar\gamma}+\frac{1}{2}\chi_\gamma+\frac{\chi_\gamma}{2\gamma\bar\gamma}\Big]\Big\}$$

$$-\frac{D_3}{2}\Big\{1+\frac{\alpha_s N_c}{4\pi}\Big[\frac{\pi^2}{3}-\frac{\pi^2}{\sin^2\pi\gamma}-C\chi_\gamma+\frac{1}{2}-\frac{1}{\gamma\bar\gamma}+\frac{\chi_\gamma}{4}+\frac{\chi_\gamma}{2\gamma\bar\gamma}\Big]\Big\}$$

$$+\frac{\bar\gamma\gamma D_4}{4(3+4\bar\gamma\gamma)}\Big\{1+\frac{\alpha_s N_c}{4\pi}\Big[\frac{\pi^2}{3}-\frac{\pi^2}{\sin^2\pi\gamma}-C\chi_\gamma+\frac{1}{2}-\frac{4}{\gamma\bar\gamma}+\frac{3}{2\gamma^2\bar\gamma^2}-\frac{\chi_\gamma}{2\gamma\bar\gamma}\Big]\Big\}$$

$$-\frac{D_1+D_2}{2}(2+\bar\gamma\gamma)\Big\{1+\frac{\alpha_s N_c}{4\pi}\Big[\frac{\pi^2}{3}-\frac{\pi^2}{\sin^2\pi\gamma}-C\chi_\gamma+\frac{1}{2}$$

$$-\frac{4\gamma\bar\gamma+3}{2\gamma\bar\gamma(2+\bar\gamma\gamma)}+\frac{1+2\gamma\bar\gamma}{\gamma\bar\gamma(2+\bar\gamma\gamma)}\chi_\gamma\Big]\Big\}\Bigg]^{\mu\nu}\Big(\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\Big)^\gamma\frac{\Gamma^2(\bar\gamma)}{\Gamma(2\bar\gamma)}$$

$\bar\gamma \equiv 1-\gamma$

Mellin representation of the impact factor for polarized DIS

Contribution of spin 2 in t-channel:

$$\int \frac{d^2z_1 d^2z_2}{z_{12}^4}\left(I_{LO}^{\mu\nu}(z_1,z_2)+I_{NLO}^{\mu\nu}(z_1,z_2)\right)\left(\frac{z_{12}}{z_{10}z_{20}}\right)^{\gamma+1}\left(\frac{\bar z_{12}}{\bar z_{10}\bar z_{20}}\right)^{\gamma+1} = B(2-\gamma)\Gamma(3-\gamma)\Gamma(2+\gamma)$$
$$\times\Big[1+\frac{\alpha_s N_c}{4\pi}\Big\{4\psi'(1+\gamma)+4\psi'(2-\gamma)-8\psi'(3)-\frac{6\chi(2,\gamma)}{(1+\gamma)(2-\gamma)}+6+4C\chi(2,\gamma)$$
$$-\frac{6C}{(2-\gamma)(1+\gamma)}-\frac{6}{(1+\gamma)(2-\gamma)}\Big\}\Big]\left(\frac{\kappa^2}{(\kappa\cdot\zeta_0)^2}\right)^{\gamma}\left(\partial_\mu^x\frac{X^2\bar Y-Y^2\bar X}{x_+y_+(\kappa\cdot\zeta_0)}\right)\left(\partial_\nu^y\frac{X^2\bar Y-Y^2\bar X}{x_+y_+(\kappa\cdot\zeta_0)}\right)$$

$\chi(2,\gamma)=2\psi(1)-\psi(2-\gamma)-\psi(1+\gamma)$ $X\equiv x-z_0,\ Y\equiv y-z_0$

NLO kernel

In general

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} = \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} + O(\alpha_s^3)$$

NLO kernel

In general

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} = \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} + \alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} + O(\alpha_s^3)$$

$$\alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} = \frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} - \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} + O(\alpha_s^3)$$

NLO kernel

In general

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} = \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} + \alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} + O(\alpha_s^3)$$

$$\alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} = \frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} - \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} + O(\alpha_s^3)$$

We calculate the “matrix element” of the r.h.s. in the shock-wave background

$$\langle\alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\}\rangle = \frac{d}{d\eta}\langle\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\}\rangle - \langle\alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\}\rangle + O(\alpha_s^3)$$

NLO kernel

In general

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} = \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} + \alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} + O(\alpha_s^3)$$

$$\alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} = \frac{d}{d\eta}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} - \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} + O(\alpha_s^3)$$

We calculate the "matrix element" of the r.h.s. in the shock-wave background

$$\langle \alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\}\rangle = \frac{d}{d\eta}\langle\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\}\rangle - \langle\alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\}\rangle + O(\alpha_s^3)$$

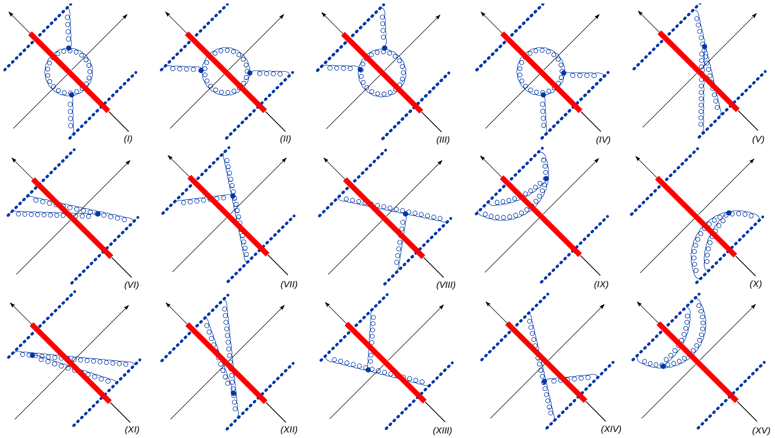
Subtraction of the (LO) contribution (with the rigid rapidity cutoff)

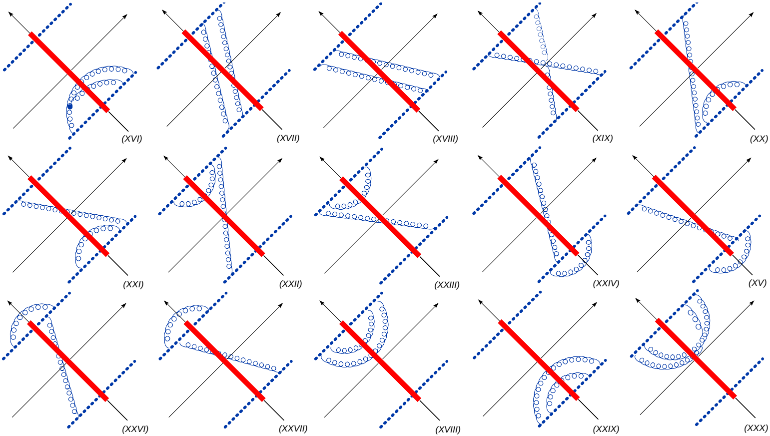
$\Rightarrow$ $\left[\frac{1}{v}\right]_+$ prescription in the integrals over Feynman parameter v

Typical integral

$$\int_0^1 dv\, \frac{1}{(k-p)_\perp^2 v + p_\perp^2(1-v)}\left[\frac{1}{v}\right]_+ = \frac{1}{p_\perp^2}\ln\frac{(k-p)_\perp^2}{p_\perp^2}$$

Gluon part of the NLO BK kernel: diagrams




(XVI)
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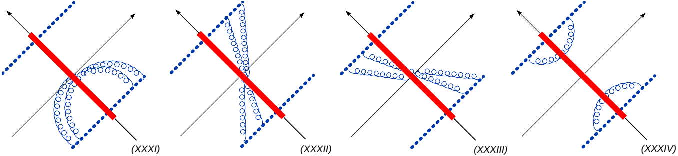
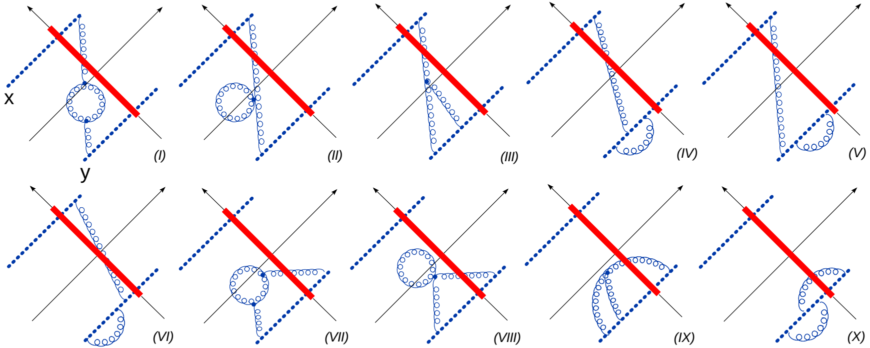
(XXVII)
(XVIII)
(XXIX)
(XXX)

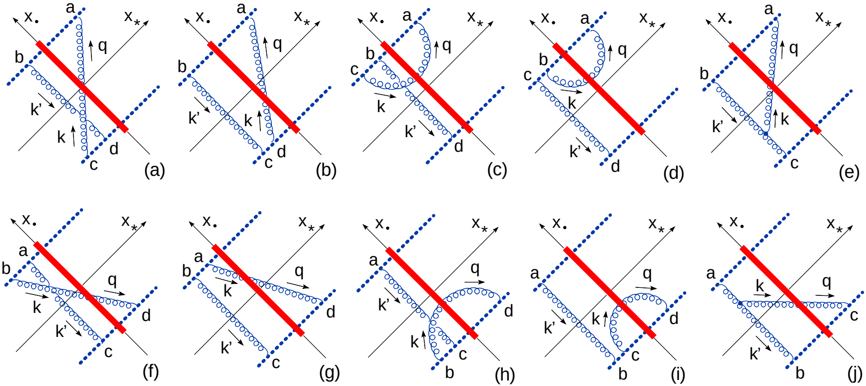
Diagrams for 1→3 dipoles transition



"Running coupling" diagrams



1 → 2 dipole transition diagrams



I. B. and G. Chirilli

$$a\frac{d}{da}[\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}]^{\mathrm{conf}}_a = \frac{\alpha_s}{2\pi^2}\int d^2z_3\,\Big([\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}]^{\mathrm{conf}}_a$$
$$\times\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\Big(b\ln z_{12}^2\mu^2 + b\frac{z_{13}^2-z_{23}^2}{z_{13}^2z_{23}^2}\ln\frac{z_{13}^2}{z_{23}^2}+\frac{67}{9}-\frac{\pi^2}{3}\Big)\Big]$$
$$+\frac{\alpha_s}{4\pi^2}\int\frac{d^2z_4}{z_{34}^4}\Big\{\Big[-2+\frac{{z_{23}}^2z_{23}^2+z_{24}^2z_{13}^2-4z_{12}^2z_{34}^2}{2({z_{23}}^2z_{23}^2-z_{24}^2z_{13}^2)}\ln\frac{{z_{23}}^2z_{23}^2}{z_{24}^2z_{13}^2}\Big]$$
$$\times[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_4}\}\{U_{z_4}U^\dagger_{z_2}\} - \mathrm{tr}\{U_{z_1}U^\dagger_{z_3}U_{z_4}U^\dagger_{z_2}U_{z_3}U^\dagger_{z_4}\} - (z_4\to z_3)]$$
$$+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\Big[2\ln\frac{z_{12}^2z_{34}^2}{{z_{23}}^2z_{23}^2}+\Big(1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-{z_{23}}^2z_{23}^2}\Big)\ln\frac{z_{13}^2z_{24}^2}{{z_{23}}^2z_{23}^2}\Big]$$
$$\times[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_4}\}\mathrm{tr}\{U_{z_4}U^\dagger_{z_2}\} - \mathrm{tr}\{U_{z_1}U^\dagger_{z_4}U_{z_3}U^\dagger_{z_2}U_{z_4}U^\dagger_{z_3}\} - (z_4\to z_3)]\Big\}$$

$$b=\tfrac{11}{3}N_c-\tfrac{2}{3}n_f$$

$K_{\mathrm{NLO\ BK}}$ = Running coupling part + Conformal "non-analytic" (in j) part + Conformal analytic ($\mathcal{N}=4$) part

Linearized $K_{\mathrm{NLO\ BK}}$ reproduces the known result for the forward NLO BFKL kernel.

k_T-factorization in the NLO

With two-gluon (one-dipole) accuracy

$$\int d^4x\, e^{iqx}\int d^4z\; \delta(z_-)\langle p_B|T\{\hat{j}_\mu(x+z)\hat{j}_\nu(z)\}|p_B\rangle = \int d^2k_\perp I_{\mu\nu}(q,k_\perp)\langle\langle p_B|\mathcal{U}(k_\perp)|p_B\rangle\rangle$$

$$\langle p_B|\mathcal{U}(k)|p_B+\beta p_B\rangle \;=\; 2\pi\delta(\beta)\langle\langle p_B|\mathcal{U}(k)|p_B\rangle\rangle$$

$$\langle\langle p_B|\mathcal{U}(k)|p_B\rangle\rangle \;=\; \int d^2z\; e^{-i(k,z)_\perp}\langle\langle p_B|\mathcal{U}(z)|p_B\rangle\rangle, \qquad \mathcal{U}(z)\;\equiv\;1-\frac{1}{N_c}\mathrm{Tr}\{U_zU_0^\dagger\}$$

k_T-factorization in the NLO

With two-gluon (one-dipole) accuracy

$$\int d^4x\, e^{iqx} \int d^4z\, \delta(z_-) \langle p_B | T\{\hat{j}_\mu(x+z)\hat{j}_\nu(z)\} | p_B\rangle \;=\; \int d^2k_\perp I_{\mu\nu}(q,k_\perp) \langle\langle p_B | \mathcal{U}(k_\perp) | p_B\rangle\rangle$$

$$\langle p_B | \mathcal{U}(k) | p_B + \beta p_B\rangle \;=\; 2\pi\delta(\beta) \langle\langle p_B | \mathcal{U}(k) | p_B\rangle\rangle$$

$$\langle\langle p_B | \mathcal{U}(k) | p_B\rangle\rangle \;=\; \int d^2z\, e^{-i(k,z)_\perp} \langle\langle p_B | \mathcal{U}(z) | p_B\rangle\rangle, \qquad \mathcal{U}(z) \;\equiv\; 1 - \frac{1}{N_c}\mathrm{Tr}\{U_z U_0^\dagger\}$$

$$I^{\mu\nu}(q,k_\perp) \;=\; \frac{N_c}{32}\int \frac{d\nu}{\pi\nu} \frac{\sinh \pi\nu}{(1+\nu^2)\cosh^2 \pi\nu} \Big(\frac{k_\perp^2}{Q^2}\Big)^{\frac{1}{2}-i\nu}$$
$$\times \Big\{\Big[\Big(\frac{9}{4}+\nu^2\Big)\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_1(\nu)\Big)P_1^{\mu\nu} + \Big(\frac{11}{4}+3\nu^2\Big)\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_2(\nu)\Big)P_2^{\mu\nu}\Big]$$

$$P_1^{\mu\nu} \;=\; g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \qquad P_2^{\mu\nu} \;=\; \frac{1}{q^2}\Big(q^\mu - \frac{p_2^\mu q^2}{q\cdot p_2}\Big)\Big(q^\nu - \frac{p_2^\nu q^2}{q\cdot p_2}\Big)$$

$$\mathcal{F}_{1(2)}(\nu) \;=\; \Phi_{1(2)}(\nu) + \chi_\gamma \Psi(\nu),$$

$$\Psi(\nu) \;\equiv\; \psi(\bar\gamma) + 2\psi(2-\gamma) - 2\psi(4-2\gamma) - \psi(2+\gamma), \qquad \gamma \equiv \frac{1}{2} + i\nu$$

$$\Phi_1(\nu) = F(\gamma) + \frac{3\chi_\gamma}{2+\bar{\gamma}\gamma} + 1 + \frac{25}{18(2-\gamma)} + \frac{1}{2\bar{\gamma}} - \frac{1}{2\gamma} - \frac{7}{18(1+\gamma)} + \frac{10}{3(1+\gamma)^2}$$

$$\Phi_2(\nu) = F(\gamma) + \frac{3\chi_\gamma}{2+\bar{\gamma}\gamma} + 1 + \frac{1}{2\bar{\gamma}\gamma} - \frac{7}{2(2+3\bar{\gamma}\gamma)} + \frac{\chi_\gamma}{1+\gamma} + \frac{\chi_\gamma(1+3\gamma)}{2+3\bar{\gamma}\gamma}$$

$$F(\gamma) = \frac{2\pi^2}{3} - \frac{2\pi^2}{\sin^2\pi\gamma} - 2C\chi_\gamma + \frac{\chi_\gamma - 2}{\bar{\gamma}\gamma}$$

Evolution equation for color dipole in momentum representation

$\mathcal{V}_a(z) \equiv \partial^2 \mathcal{U}_a(z)$
$\mathcal{V}_a(k) \equiv \int dz\, e^{-i(k,z)_\perp} \mathcal{V}_a(z)$ (sometimes called "unintegrated gluon TMD")

$$
\begin{aligned}
2a\frac{d}{da}\mathcal{V}_a(k) \;=\; & \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2k'}{(k-k')^2}\Big\{\Big(2\mathcal{V}(k') - \frac{k^2}{k'^2}\mathcal{V}_a(k)\Big) \\
& + \frac{\alpha_s b}{4\pi}\Big[\Big(2\mathcal{V}(k') - \frac{k^2}{k'^2}\mathcal{V}_a(k)\Big)\Big(\ln\frac{\mu^2}{k^2} + \Big(\frac{67}{9} - \frac{\pi^2}{3}\Big) - \frac{10n_f}{9N_c}\Big) \\
& - 2\Big(\mathcal{V}_a(k')\ln\frac{(k-k')^2}{k'^2} - \mathcal{V}_a(k)\frac{k^2}{k'^2}\ln\frac{(k-k')^2}{k^2}\Big)\Big]\Big\} \\
& + \frac{\alpha_s^2 N_c^2}{4\pi^3}\int d^2k'\,\Big[-\frac{1}{(k-k')^2}\ln^2\frac{k^2}{k'^2} + F(k,k') + \Phi(k,k')\Big]\,\mathcal{V}_a(k') + 3\frac{\alpha_s^2 N_c^2}{2\pi^2}\zeta(3)\mathcal{V}_a(k)
\end{aligned}
$$

$$
\begin{aligned}
F(k,k') \;=\; & \Big(1+\frac{n_f}{N_c^3}\Big)\frac{3(k,k')^2 - 2k^2k'^2}{16k^2k'^2}\Big(\frac{2}{k^2} + \frac{2}{k'^2} + \frac{k^2-k'^2}{k^2k'^2}\ln\frac{k^2}{k'^2}\Big) \\
& - \Big[3 + \Big(1+\frac{n_f}{N_c^3}\Big)\Big(1 - \frac{(k^2+k'^2)^2}{8k^2k'^2} + \frac{3k^4+3k'^4-2k^2k'^2}{16k^4k'^4}(k,k')^2\Big)\Big]\int_0^\infty \frac{dt}{k^2+t^2k'^2}\ln\frac{1+t}{|1-t|}, \\
\Phi(k,k') \;=\; & \frac{(k^2-k'^2)}{(k-k')^2(k+k')^2}\Big[\ln\frac{k^2}{k'^2}\ln\frac{k^2k'^2(k-k')^4}{(k^2+k'^2)^4} \\
& + 2\mathrm{Li}_2\Big(-\frac{k'^2}{k^2}\Big) - 2\mathrm{Li}_2\Big(-\frac{k^2}{k'^2}\Big)\Big] - \Big(1 - \frac{(k^2-k'^2)^2}{(k-k')^2(k+k')^2}\Big)\Big[\int_0^1 - \int_1^\infty\Big]\frac{du}{(k-k'u)^2}\ln\frac{u^2k'^2}{k^2}
\end{aligned}
$$

Agrees with NLO BFKL

Wilson line has extra g for each gluon $\Rightarrow$ $\mathcal{L}(k) = \frac{1}{g^2(k)}\mathcal{V}(k)$

$$2a\frac{d}{da}\mathcal{L}_a(k) = \frac{\alpha_s(k^2)N_c}{\pi^2}\int d^2k'\Big\{\Big[\frac{\mathcal{V}_a(k')}{(k-k')^2} - \frac{(k,k')\mathcal{V}_a(k)}{k'^2(k-k')^2}\Big]\Big(1+\frac{\alpha_s N_c}{4\pi}\big(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10n_f}{9N_c}\big)\Big) - \frac{b\alpha_s}{4\pi}\Big[\frac{\mathcal{V}_a(k')}{(k-k')^2}\ln\frac{(k-k')^2}{k^2} - \frac{k^2\mathcal{V}_a(k)}{k'^2(k-k')^2}\ln\frac{(k-k')^2}{k^2}\Big] + \frac{\alpha_s N_c}{4\pi}\Big[-\frac{\ln^2(k^2/k'^2)}{(k-k')^2} + F(k,k') + \Phi(k,k')\Big]\mathcal{L}_a(k')\Big\} + 3\frac{\alpha_s^2 N_c^2}{2\pi^2}\zeta(3)\mathcal{L}_a(k)$$

The kernel is exactly $K(q,p)$ of the NLO equation for correlation function of two reggeized gluons

$$\omega G_\omega(q,q') = \delta^{(2)}(q-q') + \int d^2p\, K(q,p)G_\omega(p,q')$$

This is somewhat surprising since the evolution of the composite (in $\mathcal{N}=4$ SYM - conformal) dipole with respect to a gives the evolution of forward reggeized gluon scattering amplitude with respect to rapidity η (of which ω is the Mellin transform).

Conclusions

- High-energy operator expansion in color dipoles works at the NLO level.

Conclusions

- High-energy operator expansion in color dipoles works at the NLO level.
- The NLO BK kernel in for the evolution of conformal composite dipoles in $\mathcal{N}=4$ SYM is Möbius invariant in the transverse plane.
- The NLO BK kernel agrees with NLO BFKL equation.
- NLO photon impact factor is calculated
- The NLO k_T-factorization formula for the contribution of the BFKL pomeron to structure functions of DIS is derived.

Outlook: relation to conformal light-ray operators

Gluon parton density $\mathcal{D}(x_B, \mu^2)$ is proportional to matrix element of the light-ray operator

$$\mathcal{O}(x_B, \mu^2) \; = \; \int d\lambda \; e^{i\lambda x_B} \; \mathrm{Tr}\{G_{+i}(\lambda e^+)[\lambda e^+, 0]G_{+i}(0)[0, \lambda e^+]\}^\mu$$

Conformal light-ray operator $\mathcal{O}_j$ (j - conformal spin in $SL(2,R)$ group)

$$\mathcal{O}_j^\mu \; = \; \int d\lambda \; \lambda^{1-j} \; \mathrm{Tr}\{G_{+i}(\lambda e^+)[\lambda e^+, 0]G_{+i}(0)[0, \lambda e^+]\}^\mu$$

Anomalous dimension

$$\mu \frac{d}{d\mu}\mathcal{O}_j \; = \; \gamma_j(\alpha_s)O_j$$

At $j = n$ γ_n is an anomalous dimension of the local twist-2 operator

$$G^{+i}(D^+)^{n-2}G_i^+$$

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Expansion of conformal dipoles in conformal light-ray operators - ?

Outlook: relation to conformal light-ray operators

In the leading order relation this expansion is trivial: $x_\perp^2$ is the normalization point of gluon light-ray operator and $x_B = e^{-\eta}$:

$$\mathrm{Tr}\{\partial_i U_x \partial^i U_0\}^\eta = \mathcal{D}^{\mu^2 = x_\perp^{-2}}_{x_B = e^{-\eta}} + O(x_\perp^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{dj}{2\pi i} \frac{\Gamma(j-1)}{x_B^{j-1}} (x_\perp^2 \mu^2)^{-\gamma_j} \mathcal{O}_j^{\mu^2}$$

$$= \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{d\omega}{2\pi i} \Gamma(\omega) e^{\omega\eta} (x_\perp^2 \mu^2)^{-\gamma_\omega} \mathcal{O}_\omega^{\mu^2}$$

This should be compared to LO rapidity evolution of color dipole $\omega_{\gamma=\frac{1}{2}+i\nu} = \omega(\nu)$ - pomeron intercept)

$$\mathrm{Tr}\{\partial_i U_x \partial^i U_0\}^\eta = = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} e^{\omega_\gamma(\eta-\eta_0)} (x_\perp^2 \mu^2)^{-\gamma} \int d^2z \, (z_\perp^2)^{1-\gamma} \mathcal{U}(z_\perp)^{\eta_0}$$

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$\Rightarrow$

$$\omega = \omega(\gamma, \alpha_s) \Leftrightarrow \gamma = \gamma(\omega, \alpha_s) \simeq \sum \frac{\alpha_s^n}{\omega^n} = \frac{\alpha_s}{\omega} + \frac{\alpha_s^3}{\omega^3} + \ldots$$

BFKL gives the anomalous dimensions in all orders as $\omega \to 0$ which corresponds to the the non-physical point $j = n = 1$ for γ_n of local operators

Outlook: relation to conformal light-ray operators

In the NLO the expansion of conformal dipoles in conformal light-ray operators is not straightforward due to mismatch of UV and rapidity regularizations.

$$\tilde{\omega}(\alpha_s, \gamma) = \omega(\alpha_s, \gamma + \frac{1}{2}\omega) \quad \Rightarrow \gamma \ = \ \gamma(\tilde{\omega}, \alpha_s)$$

$\omega(\alpha_s, \gamma)$ is the pomeron intercept which enters stands in the formula for the amplitude in terms of conformal ratios.
$\tilde{\omega}(\alpha_s, \gamma)$ determines anomalous dimensions of conformal light-ray operators.

The difficulty is probably due to the fact that conformal dipoles are invariant under $SL(2, C)$ and light-ray operators under $SL(2, R)$

Gluon TMDs may serve as a bridge between these two approaches

$$\text{Gluon TMD (without subtractions)}: \quad D(x_B,\eta,k_\perp,\mu^2) \sim \int d^2k_\perp\, e^{ik_\perp\cdot z_\perp}$$
$$\times \int dudv\, e^{i(u-v)x_B\frac{s}{2}}\langle[-\infty,u]_z G_{+i}(z_\perp+up_1)[u,-\infty]_z[-\infty,u]_0 G_{+i}(vp_1)[u,-\infty]_0\rangle^\eta$$

Two evolutions: η and μ^2 $\Rightarrow$ double logs.

At $x_B=0$ we get the "dipole gluon TMD" ($U_i \equiv U_i^\dagger i\partial_i U$)

$$D(x_B,\eta,k_\perp) = \frac{1}{g^2 x_B}\mathcal{V}^\eta(k) = \int d^2k_\perp\, e^{ik_\perp\cdot z_\perp}\langle \mathrm{Tr}\{U_i(z_\perp)U_i(0_\perp)\}\rangle^\eta$$
$$= \int d^2k_\perp\, e^{ik_\perp\cdot z_\perp}\int dudv\langle[-\infty,u]_z G_{+i}(z_\perp+up_1)[u,-\infty]_z[-\infty,u]_0 G_{+i}(vp_1)[u,-\infty]\rangle^\eta$$

No μ dependence (dipole amplitudes are UV finite) $\Rightarrow$ rapidity evolution only.

Evolution of gluon TMD probably depends on the interplay between x_B and η