# *Do we understand elastic scattering up to LHC energies ?*

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## General considerations

1 - Elastic scattering is a simple but important reaction

2 - The complex scattering amplitude $a(s,t)$ contains several basic dynamical features (one if spin is neglected)

3 - Need to explain them at $t = 0$, $t \neq 0$ and large $t$, as well as their energy dependence

4 - Many theoretical approaches, but only a few are dealing with the full kinematic range

5 - What about the TeV energy domain ?

## BSW since 1979

Collaboration with Claude Bourrely and Tai Tsun Wu

About 20 papers on the subject

Also with N.N. Khuri and André Martin on the importance of the real part at LHC. (See XIth Blois Workshop EDS-2005)

# The BSW model

In the impact picture approach we define the scattering amplitude as

$$a(s,t) = \frac{is}{2\pi} \int e^{-i\mathbf{q}\cdot\mathbf{b}}(1 - e^{-\Omega_0(s,\mathbf{b})})d\mathbf{b} \ ,$$

where $\mathbf{q}$ is the momentum transfer ($t = -\mathbf{q}^2$) and $\Omega_0(s, \mathbf{b})$ is the opaqueness at impact parameter $\mathbf{b}$ and at a given energy $s$. We take

$$\Omega_0(s, \mathbf{b}) = S_0(s)F(\mathbf{b}^2) + R_0(s, \mathbf{b}) \ ,$$

the first term is associated with the "Pomeron" exchange, which generates the diffractive component of the scattering and the second term is the Regge background which is negligible at high energy. The Pomeron energy dependence is given by the complex crossing symmetric expression

$$S_0(s) = \frac{s^c}{(\ln s)^{c'}} + \frac{u^c}{(\ln u)^{c'}} \ ,$$

where $u$ is the third Mandelstam variable.

# The BSW model

The $c$ and $c'$ are two constants given below. For the asymptotic behavior at high energy and small momentum transfer, we have to a good approximation $\ln u = \ln s - i\pi$, so that

$$S_0(s) = \frac{s^c}{(\ln s)^{c'}} + \frac{s^c e^{-i\pi c}}{(\ln s - i\pi)^{c'}}.$$

The choice one makes for $F(\mathbf{b}^2)$ is essential and we take the Bessel transform of

$$\tilde{F}(t) = f[G(t)]^2 \frac{a^2 + t}{a^2 - t} \,,$$

where $G(t)$ stands for the proton "nuclear form factor", parametrized like the electromagnetic form factor, as having two poles,

$$G(t) = \frac{1}{(1 - t/m_1^2)(1 - t/m_2^2)} \,.$$

# Schematic representation of expanding protons

- Diffractive scattering goes like $lns$ (gray region)
- Elastic scattering goes like $(lns)^2$ (black region)

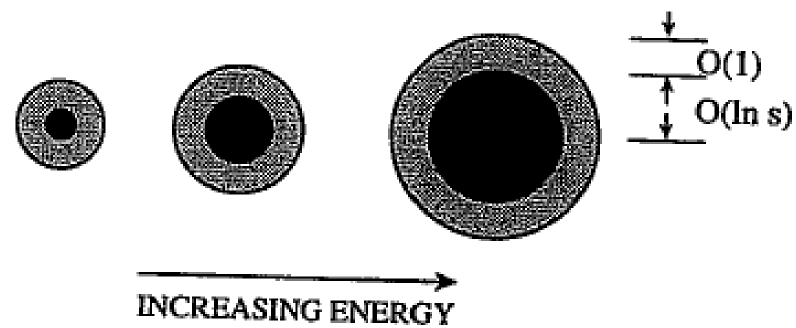



FIG. 4: Schematic representation of the appearance of a high-energy particle [$s = E^2$

When $\sqrt{s}$ increases the proton becomes larger and darker

# The BSW model parameters

At high energies the Regge background is irrelevant and the model depends only on six parameters

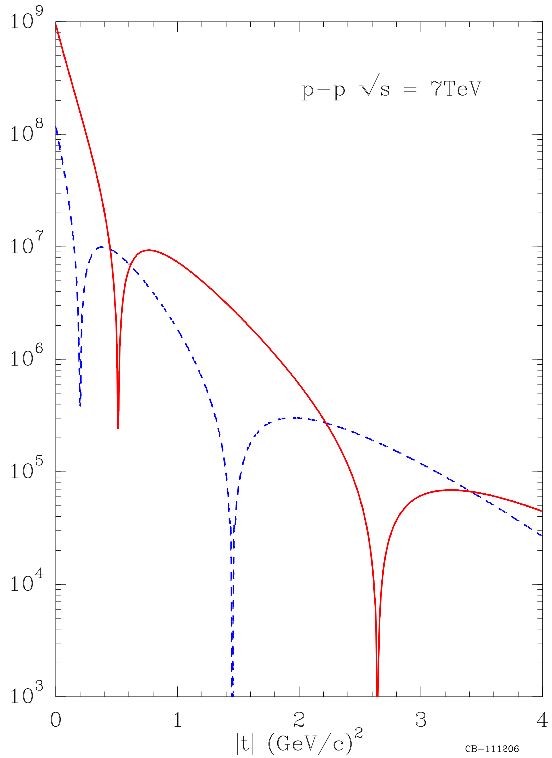
| | | | | | |
|---|---|---|---|---|---|
| $c$ | = | 0.167, | $c'$ | = | 0.748 |
| $m_1$ | = | 0.5779 GeV, | $m_2$ | = | 1.7240 GeV |
| $a$ | = | 1.9312 GeV, | $f$ | = | 7.0932 GeV$^{-2}$ |

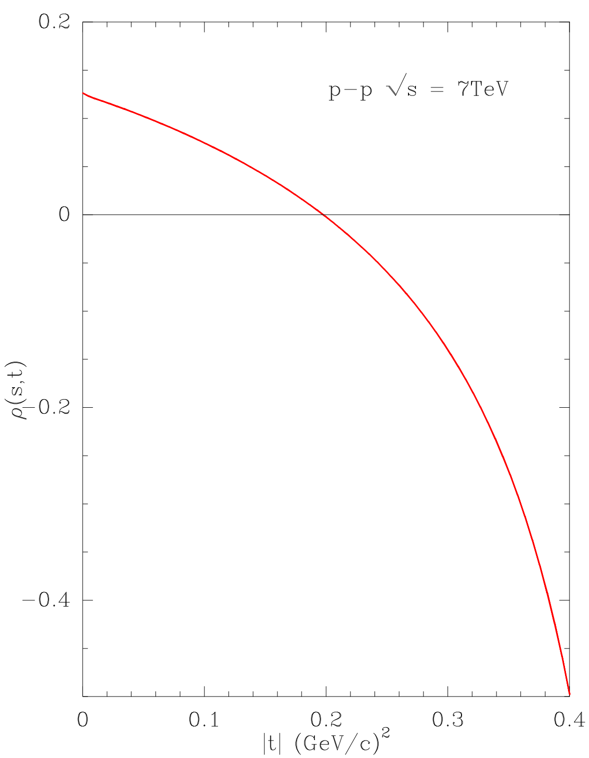
NOTE

In the Abelian case one finds $c' = 3/2$ and it was conjectured that in Yang-Mills non-Abelian gauge theory one would get $c' = 3/4$

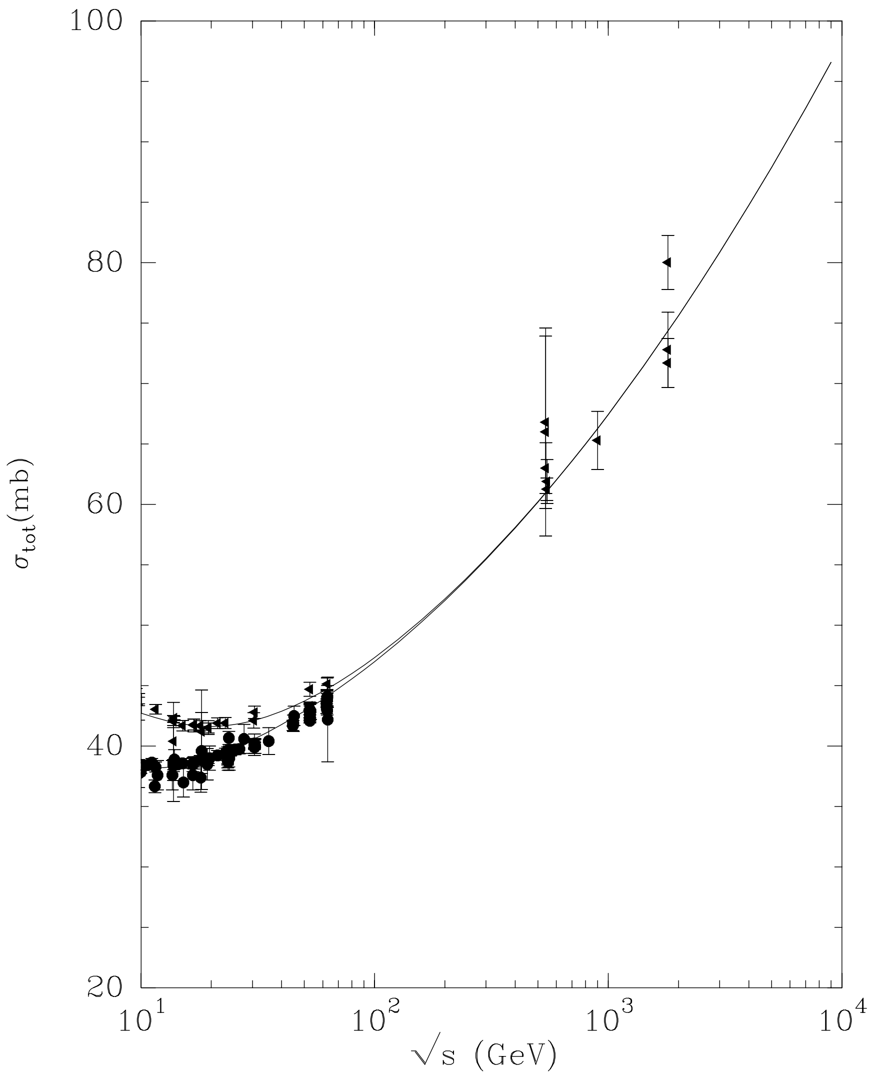
# Real and Imaginary parts of the amplitude

- The phase is built in. They cannot be independent
- They have both some zeroes but at different $t$ values

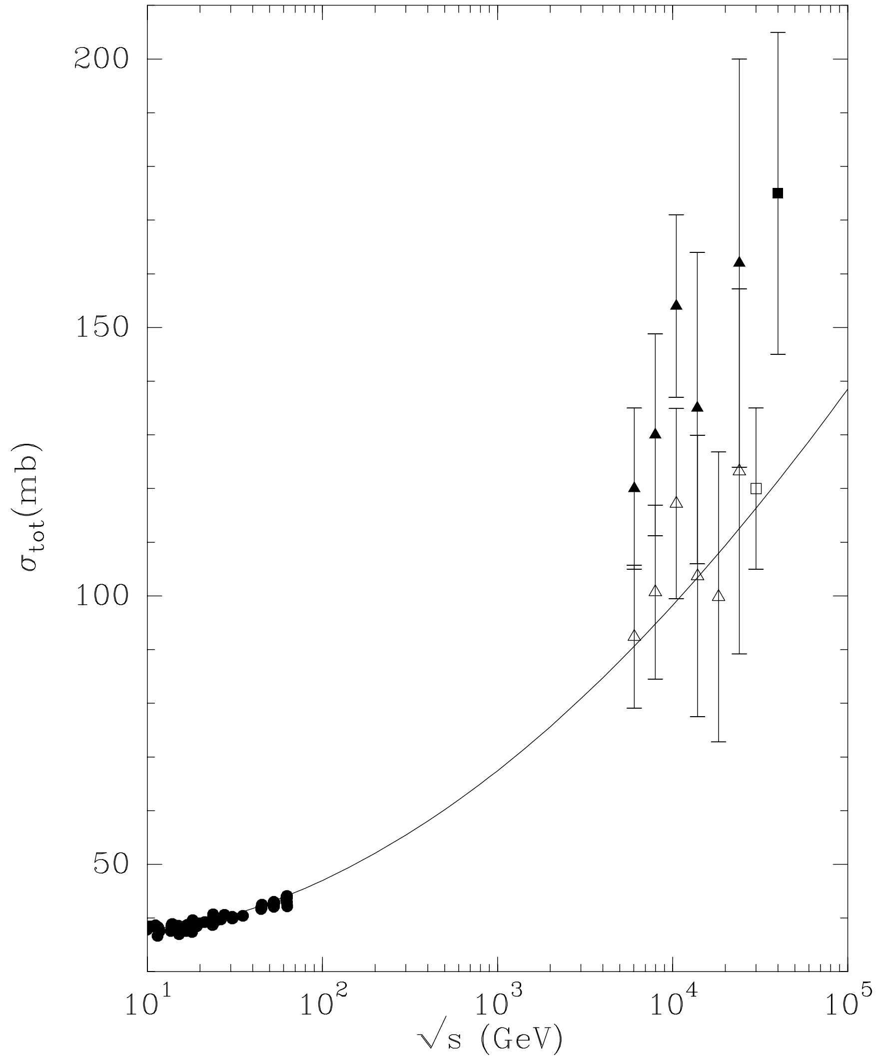


# Total cross section before LHC

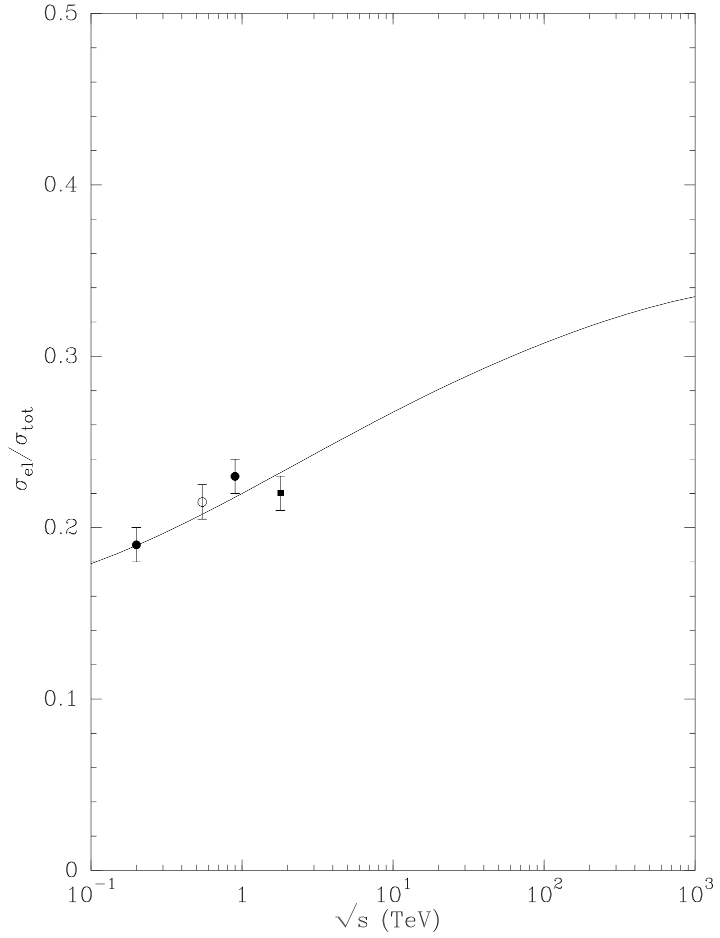


# Total cross section up to cosmic rays region



# Ratio $\sigma_{el}/\sigma_{tot}$

It is predicted to rise with increasing $\sqrt{s}$ up to 1/2, the black disk limit



# Cross sections at LHC and above

The BSW approach predicts at 7 TeV

$\sigma_{tot} = 93.6 \pm 1mb$,

$\sigma_{el} = 24.8 \pm 0.3mb$,

$\sigma_{inel} = 68.8 \pm 1mb$

To be compared with TOTEM

$\sigma_{tot} = 98.3 \pm 0.2(stat) \pm 2.7(syst)mb$,

$\sigma_{el} = 24.8 \pm 0.2(stat) \pm 1.2(syst)mb$,

$\sigma_{inel} = 73.5 \pm 0.6(stat) + 1.8(-1.3)(syst)mb$,

ATLAS

$\sigma_{inel} = 69.1 \pm 2.4(expt) \pm 6.9(extra)mb$,

CMS

$\sigma_{inel} = 68.0 \pm 2.0(syst) \pm 2.4(lum) \pm 4.0(extra)mb$,

ALICE

$\sigma_{inel} = 72.7 \pm 1.1(MonteCarlo) \pm 5.1(lum)mb$,

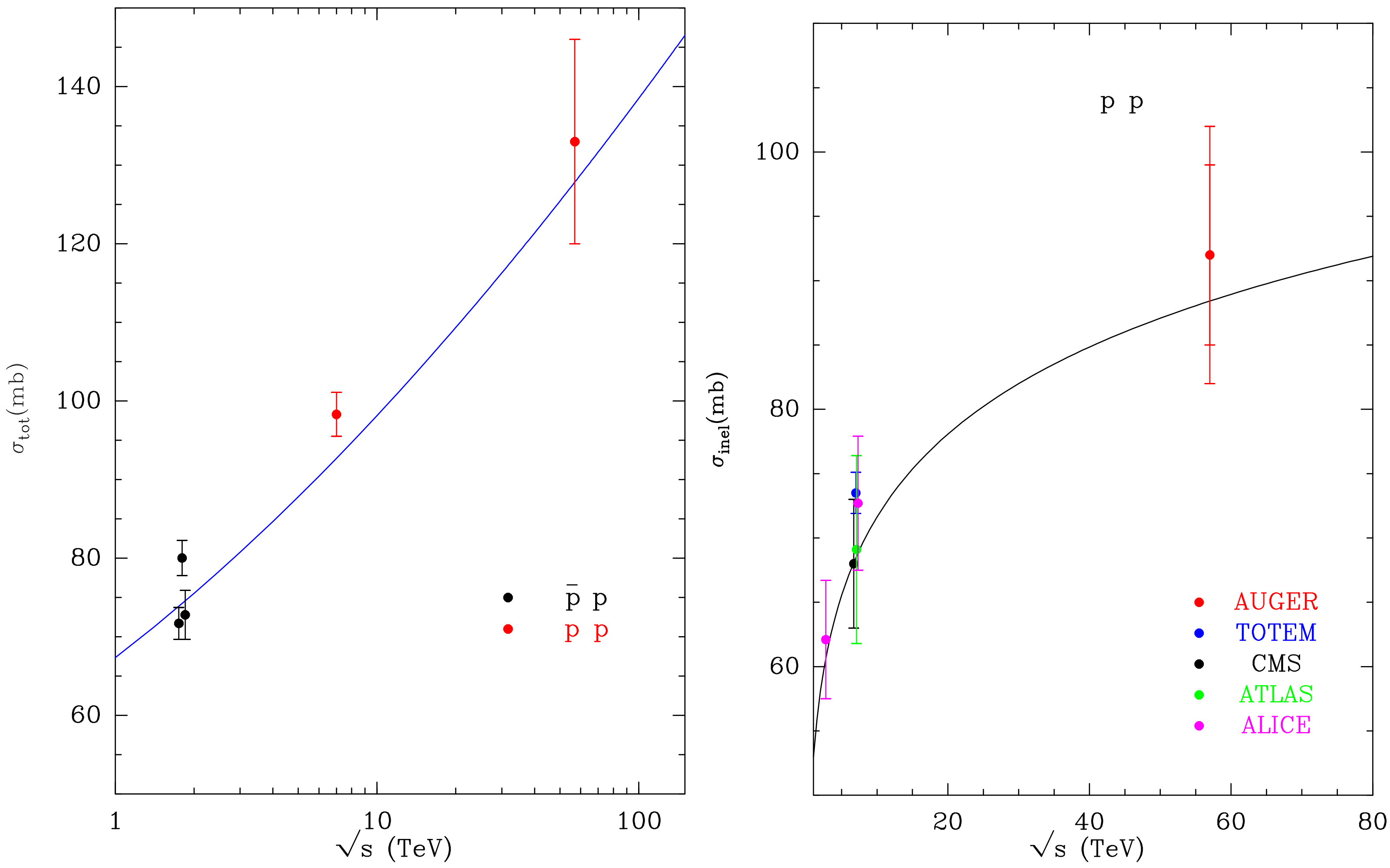
The BSW approach predicts at 57 TeV $\sigma_{tot} = 128.6 \pm 1mb$, and $\sigma_{inel} = 88.4 \pm 1mb$

To be compared with AUGER

$\sigma_{tot} = 133 \pm 13(stat) + 17(-20)(syst) \pm 16(Glauber)mb$,

$\sigma_{inel} = 92 \pm 7(stat) + 9(-11)(syst) \pm 7(Glauber)mb$,

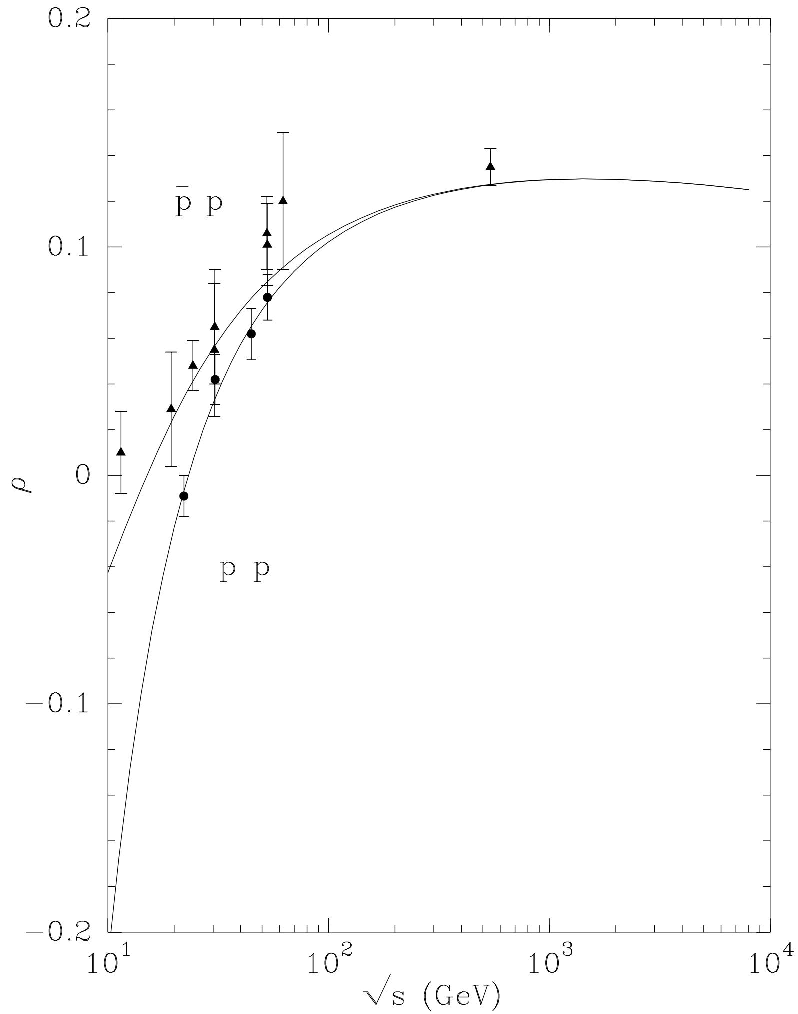
# Cross sections at LHC and above



# Real part of the forward amplitude

$$\rho(s) = \textbf{Re}\ a(s, t=0)/\textbf{Im}\ a(s, t=0)$$

We expect $\rho(s) \to 0$ for $s \to \infty$



# CNI region from UA4 1992

Excellent agreement with BSW prediction and $\rho = 0.13$

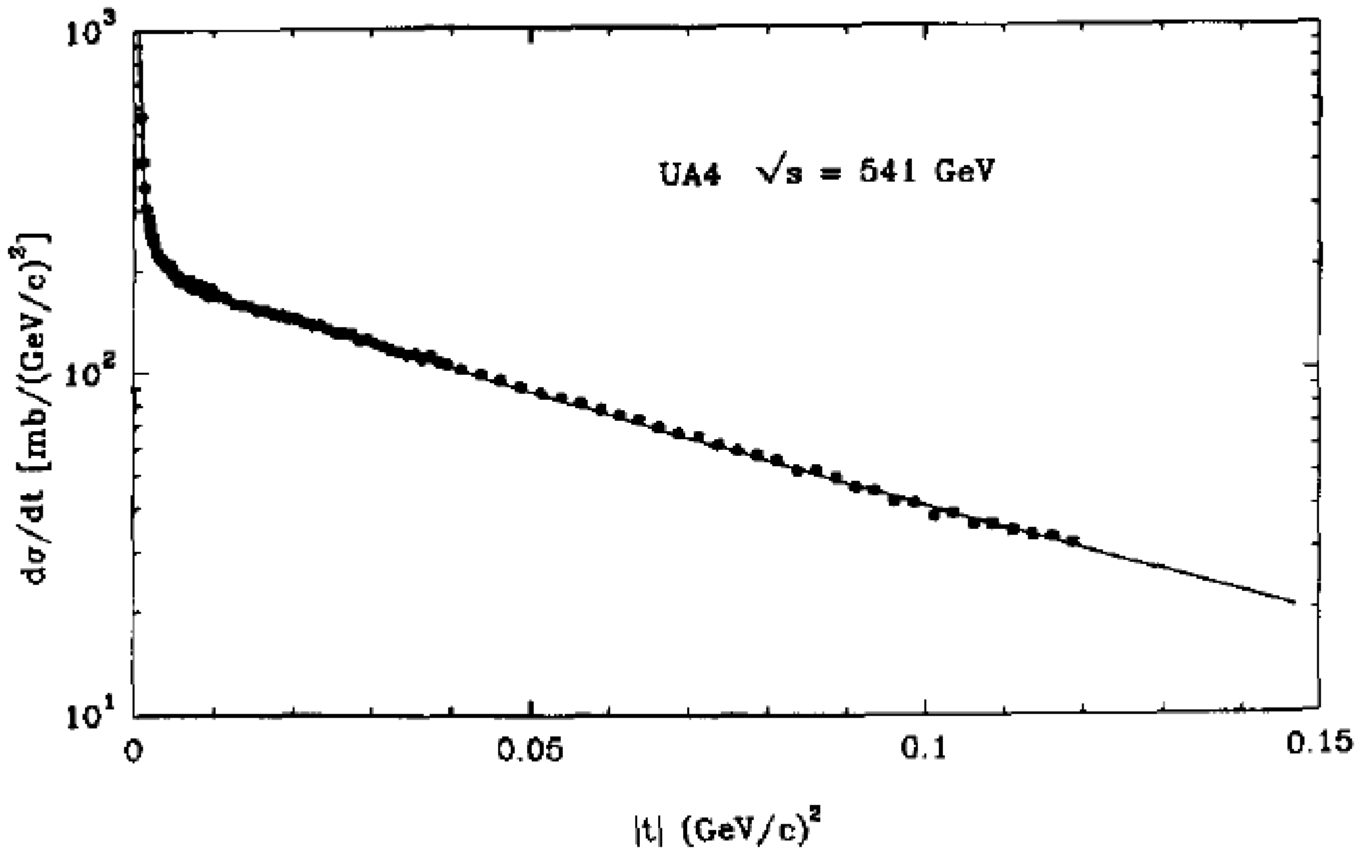
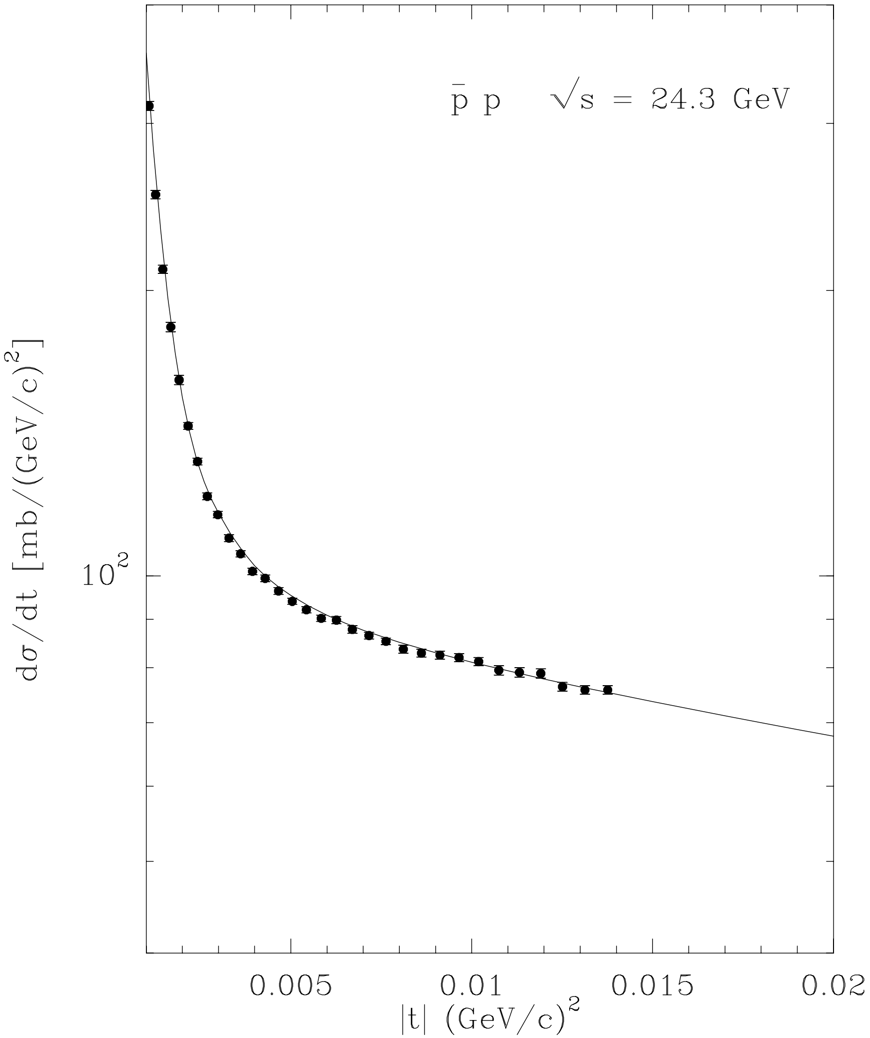


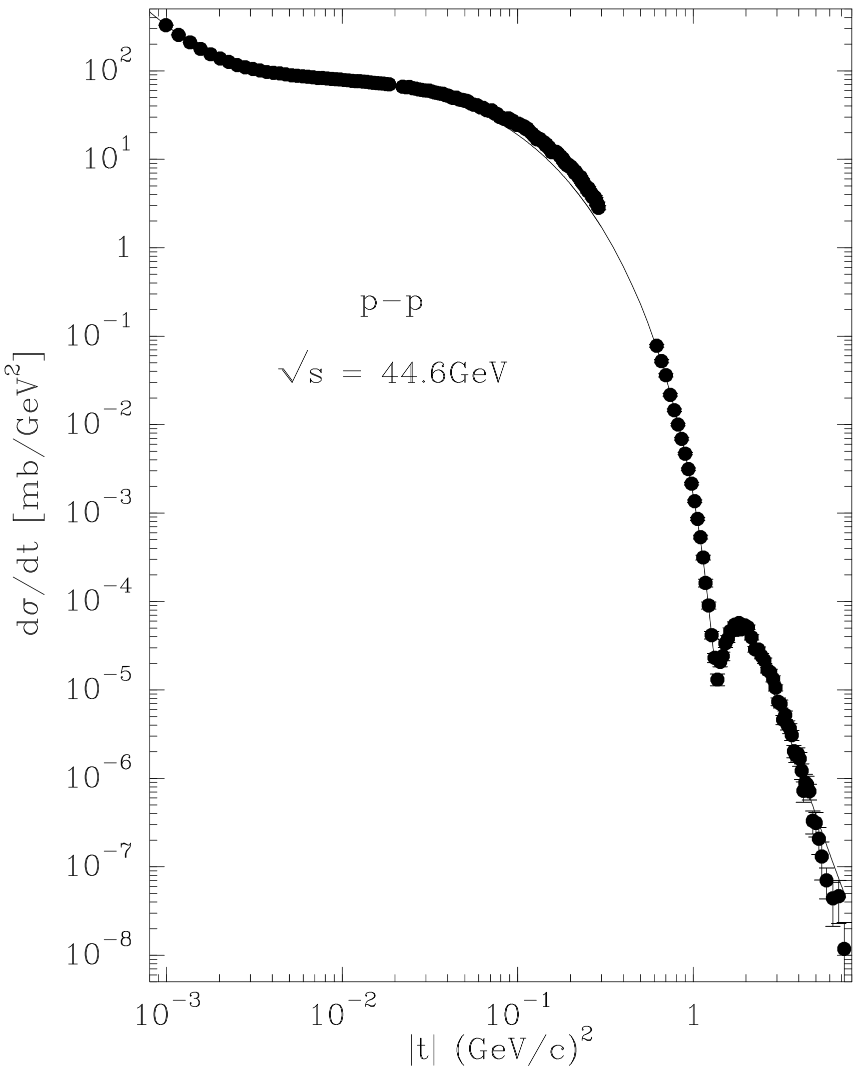

Fig. 1. Comparison of the theoretical prediction of ref. [4] and the new experimental measurement by UA4 [11] at $\sqrt{s} = 541$ GeV in the small $t$-region.

# Elastic cross section in the CNI region

UA6 CERN data agree perfectly with BSW prediction

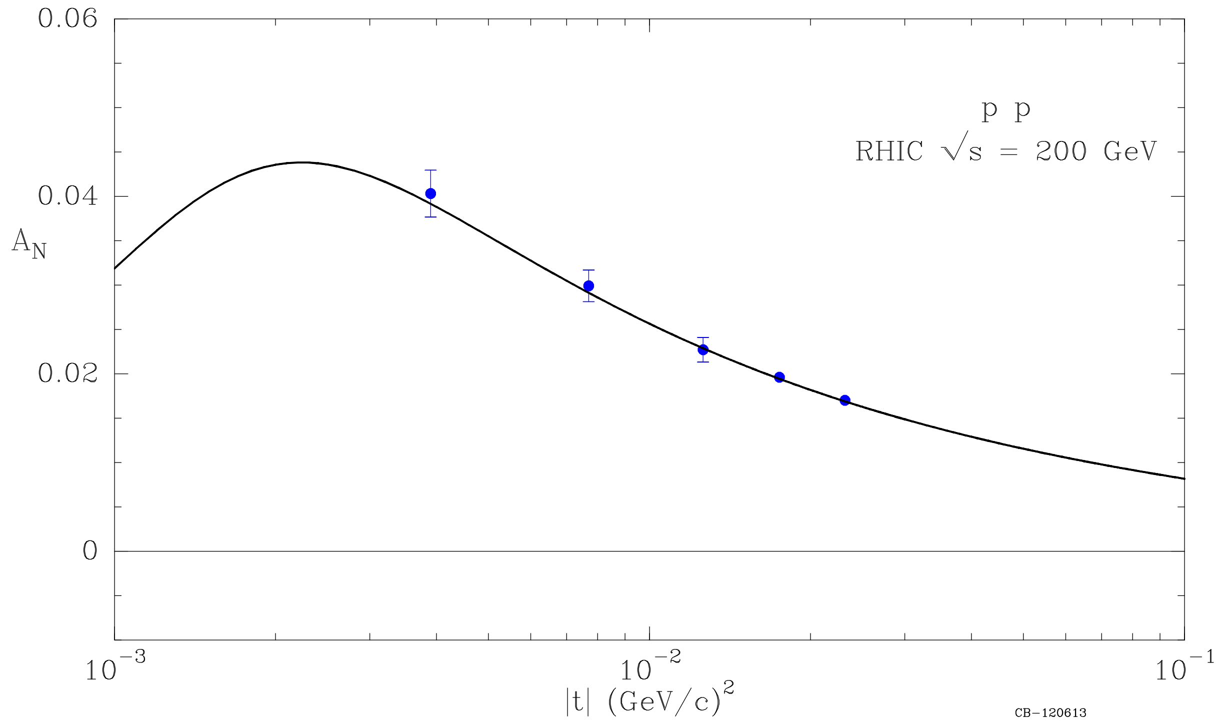


# Elastic cross section including the CNI region at ISR



# Single spin asymmetry in the CNI region at RHIC

Sensitive to an eventual hadronic flip amplitude (See new STAR results)



# Why should $\rho$ be measured at the LHC ?

- Real and Imaginary parts of the scattering amplitude must obey dispersion relations according to local quantum field theory
- In string theory extra dimensions could generate observable non-local effects and therefore a violation of dispersion relations
- Can make a simple model to break polynomial boundness in some regions of the analyticity domain, leading for example to $\rho = 0.21$ at 14TeV
- According to BSW, which satisfy dispersion relations, one should find instead $\rho = 0.122$
- Dispersion relations could be also violated if $\sigma_{tot}$ beyond the LHC energy, behaves very differently, due to some new physics.
- The highest energy where one has a reliable value of $\rho$ is at $\sqrt{s} = 546$GeV, $\rho = 0.135 \pm 0.007$, since the Tevatron value $\rho = 0.140 \pm 0.069$ is useless

- **For all these reasons one needs an accurate value of $\rho$ at LHC**

# The two-scale model proposed in 2000

Model based on the two-scale hadronic structure:

- A **SOFT** one related to the confinement radius $R_c \sim 1/\Lambda_{QCD} \sim 1 fm$
- A **SEMI-HARD** one which originates from the non-perturbative interactions of gluons $r_0 \sim 0.3 fm$ (small size of gluonic spots)

These two parameters lead to a specific form of the Pomeron
The cross section contains two terms

- One energy independent
- One rising with energy

Real part obtained by derivative analyticity relation

**Can only calculate forward slope of $d\sigma/dt$ not high $t$**

# The two-scale model predictions( Kopeliovich et al. hep-ph 1208.5446)

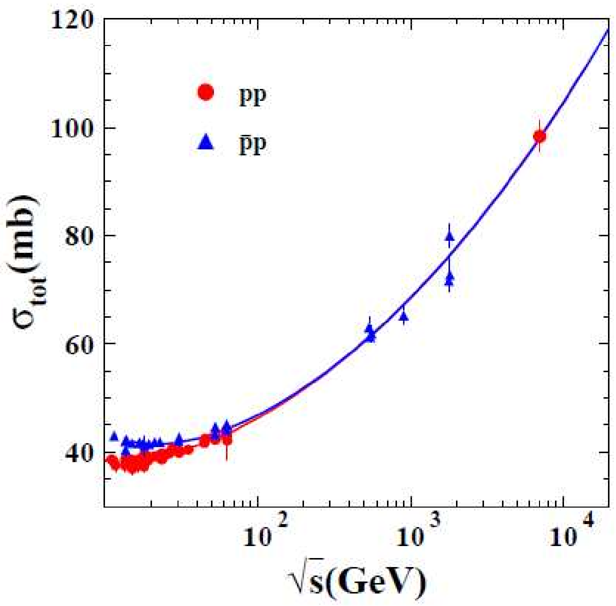



FIG. 2: (Color online) Total cross section predicted in [3, 4] and measured by TOTEM [1, 2].

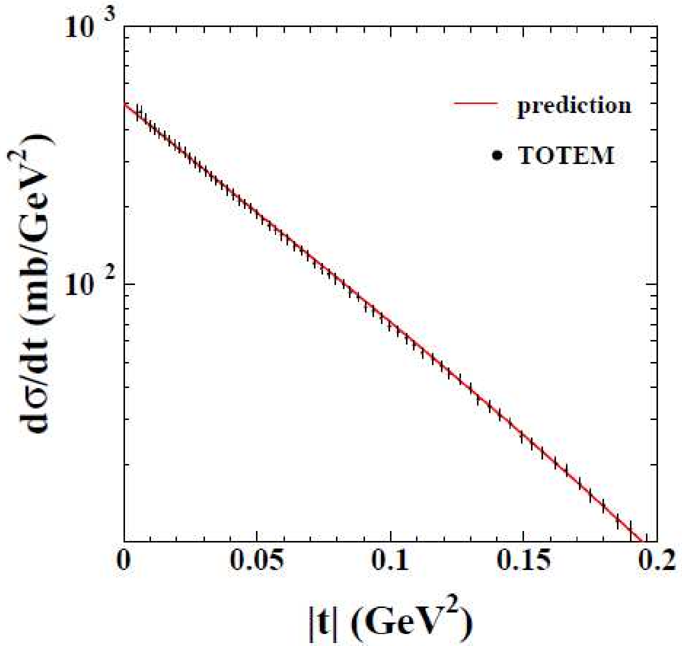
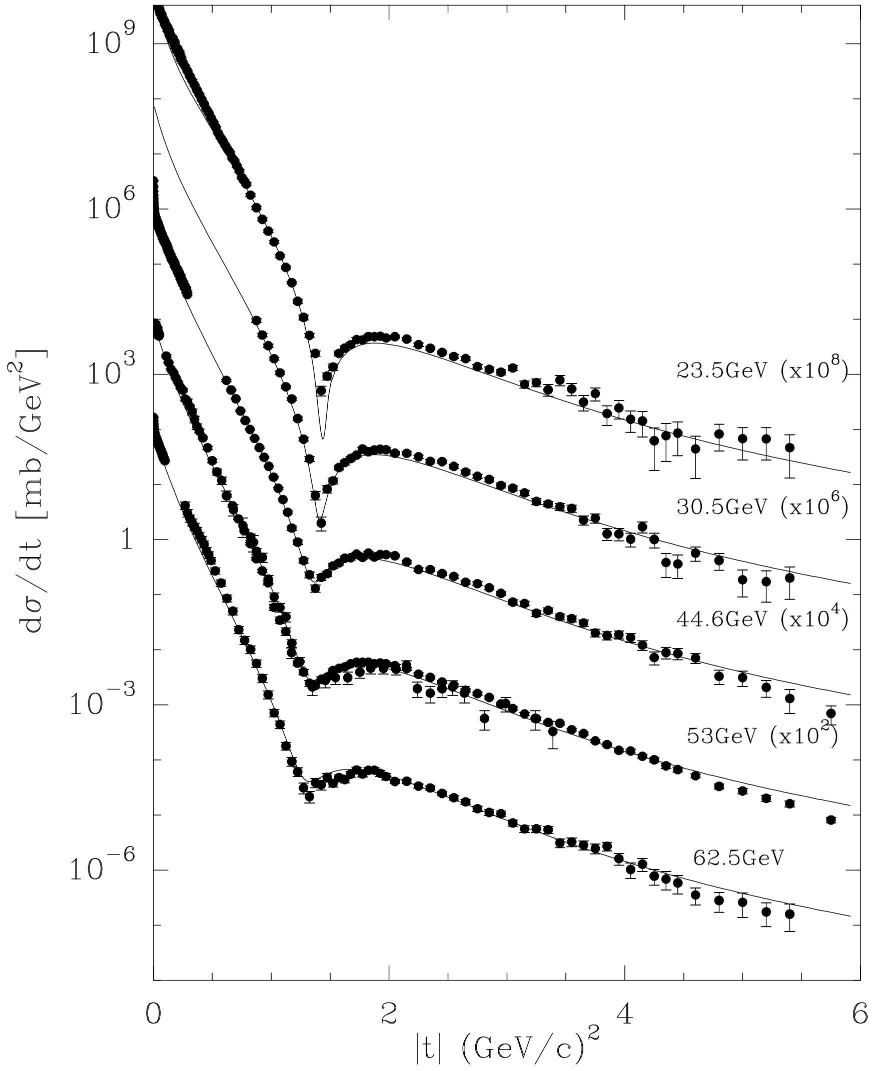
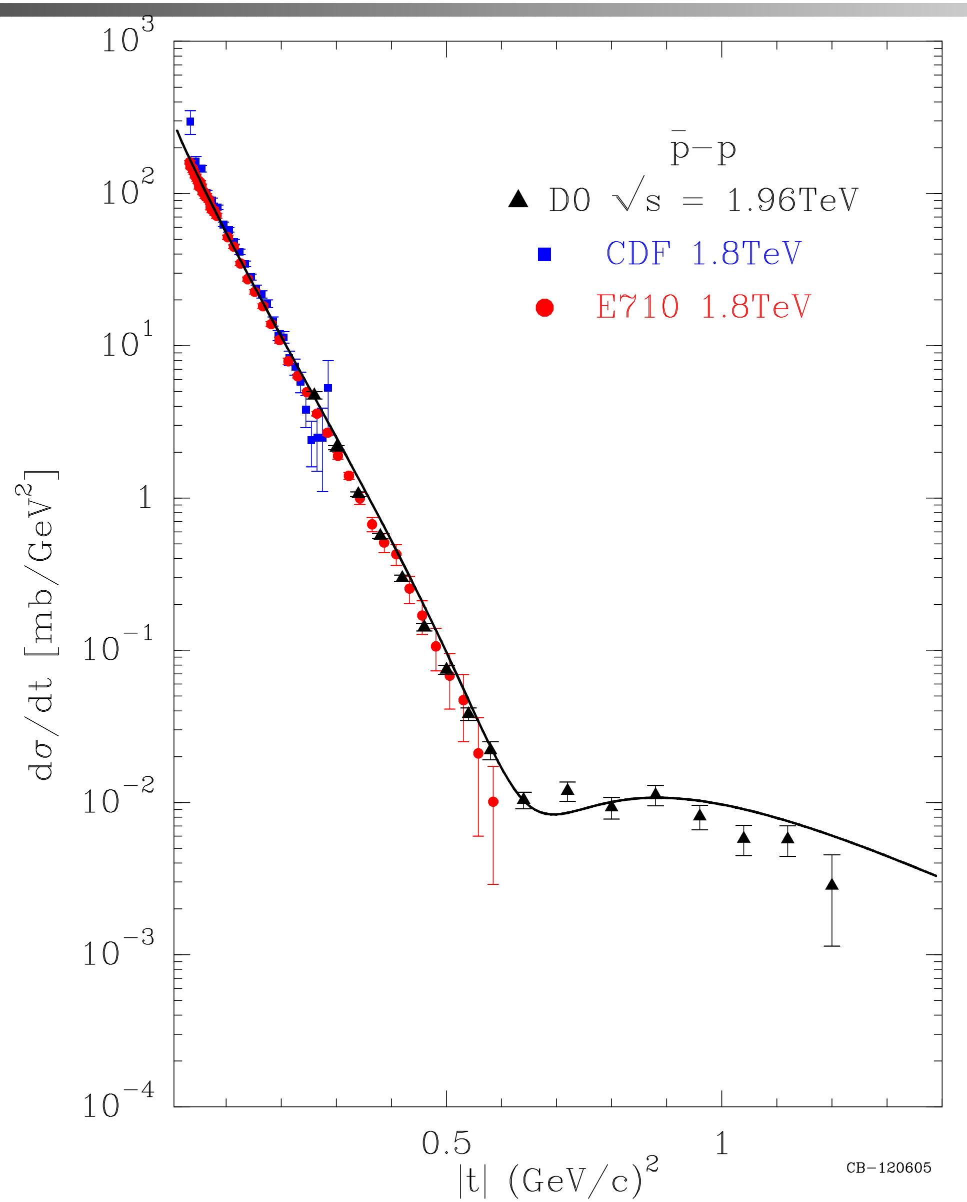



FIG. 1: (Color online) The differential elastic cross section predicted in [3, 4] and measured by TOTEM [1, 2].

# Differential elastic cross sections at ISR from BSW



# Predicted elastic cross section at Tevatron



# New TOTEM data

The TOTEM Collaboration (G. Antchev *et al.*)

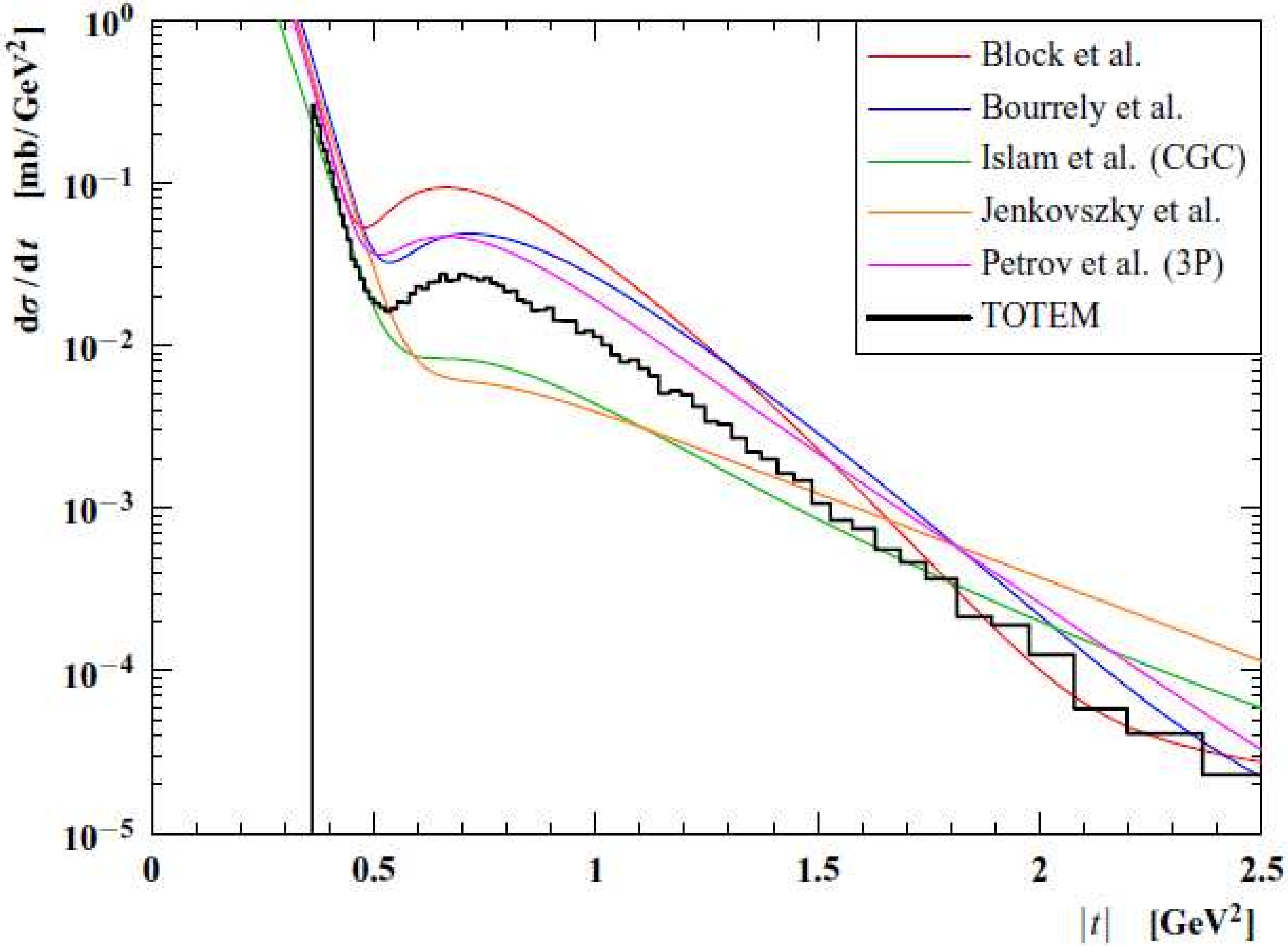
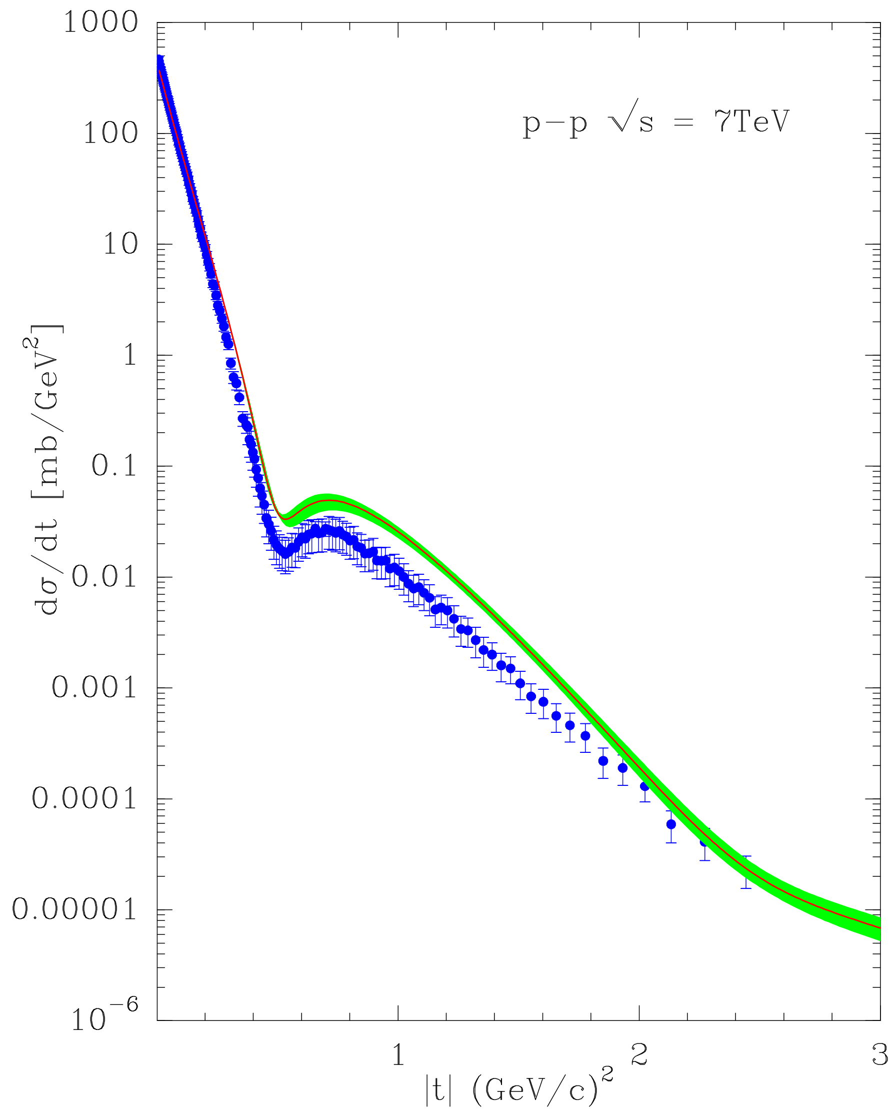



Fig. 4: The measured $d\sigma/dt$ compared to the predictions of several models (see table 4).

# New TOTEM data versus BSW



# Concluding remarks

- LHC is opening up a new area for $pp$ elastic scattering
- TOTEM has confirmed the following basic features expected at LHC
  - $\sigma_{tot}$ increases
  - $\sigma_{el}/\sigma_{tot}$ increases
  - shrinkage of the diffraction peak
  - moving in of the dip position
  - moving up of the second maximum
- So far only partial quantitative agreement with BSW

**More data are needed for confirmation (ALFA ???)**

**Don't forget $\rho$ at LHC**

**Thank you !**