NLO forward jet vertex

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in collaboration with
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Outline

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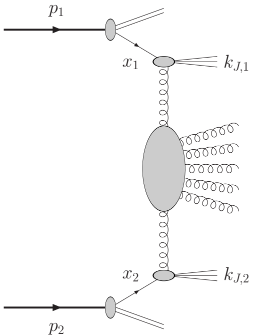
1. Introduction
   - The Mueller-Navelet jet production process
   - Theoretical setup: BFKL and collinear factorization
   - Jet definition
2. The Impact Factor in the LLA
3. The Impact Factor in the NLA
   - Collinear and QCD coupling counterterms
   - Quark-initiated subprocess
   - Gluon-initiated subprocess
4. Conclusions

Mueller-Navelet jets



$$\text{proton}(\text{p}_1) + \text{proton}(\text{p}_2) \to \text{jet}_1(\text{k}_1) + \text{jet}_2(\text{k}_2) + \text{X}$$

Sudakov decomposition:

$$k_{J,1} = x_{J,1} p_1 + \frac{\vec{k}_{J,1}^{\,2}}{x_{J,1} s} p_2 + k_{J,1\perp}\,, \quad k_{J,1\perp}^2 = -\vec{k}_{J,1}^{\,2}\,, \quad s = 2 p_1 \cdot p_2$$

$$k_{J,2} = x_{J,2} p_2 + \frac{\vec{k}_{J,2}^{\,2}}{x_{J,2} s} p_1 + k_{J,2\perp}\,, \quad k_{J,2\perp}^2 = -\vec{k}_{J,2}^{\,2}$$

- large jet transverse momenta: $\vec{k}_{J,1}^{\,2} \sim \vec{k}_{J,2}^{\,2} \gg \Lambda_{\rm QCD}^2$ $\longrightarrow$ perturbative QCD applicable
- large rapidity gap between jets, $\Delta y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$, $\longrightarrow$ $s = 2 p_1 \cdot p_2 \gg \vec{k}_{J,1\,2}^{\,2}$

$\longrightarrow$ BFKL resummation: $\sum_n c_n\, \alpha_s^n \ln^n s + d_n\, \alpha_s^n \ln^{n-1} s$

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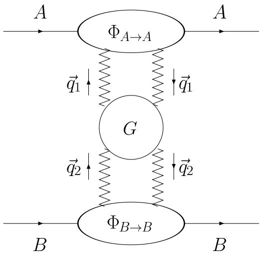
BFKL "survival kit"

Total cross section $A + B \to X$, via the optical theorem, $\sigma = \frac{\mathrm{Im_s}\mathcal{A}}{s}$

- **Regge limit** ($s \to \infty$)
  $\Rightarrow$ BFKL factorization for $\mathrm{Im}_s\mathcal{A}$:
  convolution of the **Green's function** of two interacting Reggeized gluons and of the **impact factors** of the colliding particles.



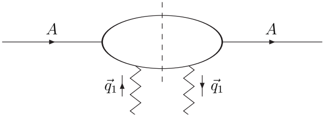
- Valid both in
  LLA (resummation of all terms $(\alpha_s \ln s)^n$)
  NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

$$\mathrm{Im}_s\mathcal{A} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- The **Green's function** is **process-independent** and is determined through the **BFKL equation**. [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)\,.$$

- **Impact factors** are **process-dependent**;



only very few have been calculated in the NLA:

- forward jet production [J. Bartels, D. Colferai, G.P. Vacca (2003)]

  (small-cone approximation) [D.Yu. Ivanov, A. Papa (2012)]
- $\gamma^* \longrightarrow \gamma^*$ [Bartels et al (2001) $\rightarrow$]

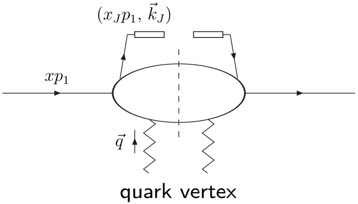
  (coordinate representation) [I. Balitsky, G.A. Chirilli (2011)]
- $\gamma^* \longrightarrow V$, with $V = \rho^0, \omega, \phi$, forward case

  [D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]
- colliding partons

  [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]

  [M. Ciafaloni and G. Rodrigo (2000)]

- **Step 1**: "open" one of the integrations over the phase space of the intermediate state to allow one parton to form a jet



- **Step 2**: take the convolution with the PDFs and the jet function

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes S_J \qquad + \qquad f_g \otimes (\text{gluon vertex}) \otimes S_J$$

- **Step 3**: project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer to the (ν, n)-representation

$$\Phi(\nu, n) = \int d^2\vec{q}\, \frac{\Phi(\vec{q})}{\vec{q}^{\,2}} \frac{1}{\pi\sqrt{2}} \left(\vec{q}^{\,2}\right)^{\gamma - \frac{n}{2}} \left(\vec{q}\cdot\vec{l}\,\right)^n , \qquad \gamma = i\nu - \frac{1}{2}\,, \qquad \vec{l}^{\,2} = 0$$

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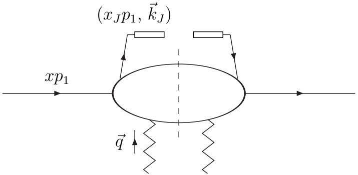
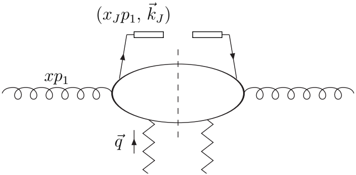


quark vertex



gluon vertex

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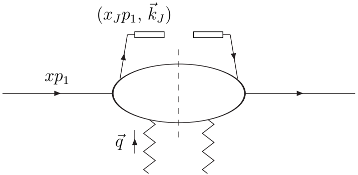
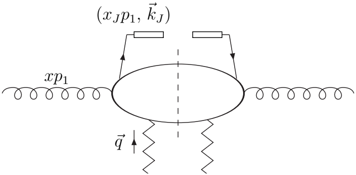


quark vertex



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Outline

Jet definition

LO: one-particle intermediate state

The kinematics of the produced parton *a* is completely fixed by the jet kinematics

NLO, virtual corrections: one-particle intermediate state (see Fig. 1)

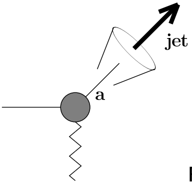


Fig. 1

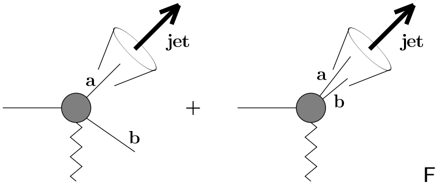


Fig. 2

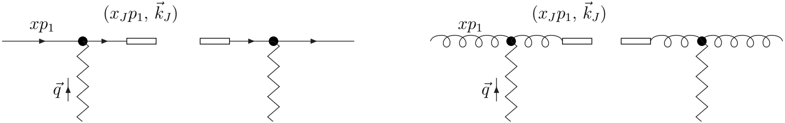
NLO, real corrections: two-particle intermediate state (see Fig. 2)

The Impact Factor in the LLA

The starting point is given by "inclusive" LO parton impact factors:

$$\Phi_q = g^2 \frac{\sqrt{N^2-1}}{2N} \,, \qquad \Phi_g = \frac{C_A}{C_F} \Phi_q$$

- Step 1: "open" the integration over the one-particle intermediate state, i.e. introduce a delta function



- Step 2: take the convolution with the PDFs and the jet function ($D = 4 + 2\varepsilon$)

$$\frac{d\Phi_J^{(0)}(\vec{q})}{dJ} = \Phi_q \int_0^1 dx \int d^{D-2}\vec{k}\, \delta^{(D-2)}\left(\vec{k}-\vec{q}\right) S_J^{(2)}(\vec{q};\; x)$$

$$\times \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x) \right)$$

Outline

Collinear and QCD coupling counterterms

The collinear singularities which will arise in the NLA calculation are to be removed by the **renormalization of PDFs and QCD charge**:

$$f_q(x) = f_q(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\left[P_{qq}(z) f_q(\tfrac{x}{z},\mu_F) + P_{qg}(z) f_g(\tfrac{x}{z},\mu_F)\right]$$

$$f_g(x) = f_g(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\left[P_{gq}(z) f_q(\tfrac{x}{z},\mu_F) + P_{gg}(z) f_g(\tfrac{x}{z},\mu_F)\right]$$

$$\alpha_s = \alpha_s(\mu_R)\left[1 + \frac{\alpha_s(\mu_R)}{4\pi}\left(\frac{11C_A}{3} - \frac{2N_F}{3}\right)\left(\frac{1}{\hat\epsilon} + \ln\frac{\mu_R^2}{\mu^2}\right)\right]$$

$$\frac{1}{\hat\epsilon} = \frac{1}{\epsilon} + \gamma_E - \ln(4\pi) \approx \frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^\epsilon}$$

- **Collinear counterterm**:

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int\limits_0^1 d\beta \int\limits_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)\right) + \frac{C_A}{C_F}\left(P_{gg}(\beta)f_g(x) + P_{gq}(\beta)\sum_{a=q,\bar{q}}f_a(x)\right)\right]$$

- **QCD renormalization counterterm**:

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{charge c.t}}}{dJ} = \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_R^2}{\mu^2}\right)\left(\frac{11C_A}{6} - \frac{N_F}{3}\right)\Phi_q^{(0)}$$

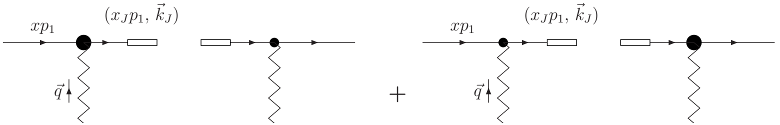
$$\times\int_0^1 dx\left(\frac{C_A}{C_F}f_g(x) + \sum_{a=q,\bar{q}}f_a(x)\right)S_J^{(2)}(\vec{q};x)$$

In the following

$$\frac{d\Phi_J^{(1)}(\vec{q}\,)}{dJ} \equiv \frac{\alpha_s}{2\pi}\Phi_q^{(0)}\,\mathbf{V}(\vec{\mathbf{q}}\,)$$

Outline

Virtual corrections



$$V_q^{(V)}(\vec{q}\,) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\;2}\right)^\varepsilon \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\, S_J^{(2)}(\vec{q}\,;x)$$

$$\times\left[C_F\left(\frac{2}{\varepsilon}-3\right)+\left(\frac{11}{6}C_A-\frac{N_F}{3}\right)+C_A \ln\frac{s_0}{\vec{q}^{\;2}}\right] + \text{finite terms}$$

QCD renormalization counterterm

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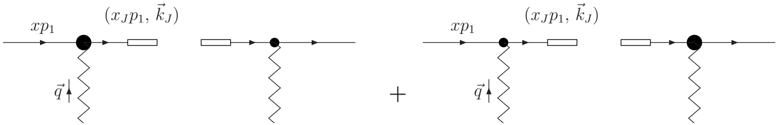
$$\times \int_0^1 dx \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x)\right) S_J^{(2)}(\vec{q};x)$$

Virtual corrections



$$V_q^{(V)}(\vec{q}\,) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^{\varepsilon}}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\;2}\right)^{\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\, S_J^{(2)}(\vec{q}\,;x)$$

$$\times\left[C_F\left(\frac{2}{\varepsilon}-3\right)+\left(\frac{11}{6}C_A-\frac{N_F}{3}\right)+C_A\ln\frac{s_0}{\vec{q}^{\;2}}\right]+\text{finite terms}$$

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$$\times\int_0^1 dx\left(\frac{C_A}{C_F}f_g(x)+\sum_{a=q,\bar{q}}f_a(x)\right)S_J^{(2)}(\vec{q};x)$$

"Real" corrections: quark-gluon intermediate state

Starting point

$$V_q^{(R)}(\vec{q}) = \frac{1}{(4\pi)^{\varepsilon}} \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^1 d\beta\, \mathcal{P}_{gq}(\varepsilon,\beta) \frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2\left(\vec{k}-\beta\vec{q}\right)^2}$$

$$\times\left\{C_F\beta^2\left(\vec{q}-\vec{k}\right)^2 + C_A(1-\beta)\,\vec{k}\cdot(\vec{k}-\beta\vec{q})\right\} S_J^{(3)}\left(\vec{q}-\vec{k},\vec{k},x\beta;x\right)$$

β, $1-\beta$ and $\vec{k}$, $\vec{q}-\vec{k}$: relative longitudinal and transverse momenta of the gluon, quark

- **term proportional to** C_F

$$V_q^{(R)(C_F)}(\vec{q}) = C_F \frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^{\varepsilon}} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \left[\left(\frac{2}{\varepsilon}-3\right) S_J^{(2)}(\vec{q};x)\right.$$

$$\left. + \int_0^1 d\beta\, P_{qq}(\beta)\, S_J^{(2)}(\vec{q};x\beta)\right] + \text{finite terms}$$

Virtual corrections

$$V_q^{(V)}(\vec{q}) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^{\varepsilon}}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\,2}\right)^{\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\, S_J^{(2)}(\vec{q};x)$$

$$\times\left[C_F\left(\frac{2}{\varepsilon}-3\right) + \left(\cancel{\frac{11}{6}}C_A - \cancel{\frac{N_f}{3}}\right) + C_A \ln\frac{s_0}{\vec{q}^{\,2}}\right] + \text{finite terms}$$

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$$\times \left[C_F\left(\frac{2}{\varepsilon}-3\right) + \left(\cancel{\frac{11}{6}C_A} - \cancel{\frac{N_F}{3}}\right) + C_A \ln\frac{s_0}{\vec{q}^{\,2}} \right] + \text{finite terms}$$

Collinear counterterm

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int_0^1 d\beta\int_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

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$$\left.+\int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2}\ln\frac{s_0}{\left(|\vec{k}|+|\vec{q}-\vec{k}|\right)^2}S_J^{(2)}\left(\vec{q}-\vec{k};x\right)\right] + \text{finite terms}$$

After the **projection** onto the eigenfunctions of the LO BFKL kernel

Virtual corrections

$$V_q^{(V)}(\vec{q}) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\,2}\right)^\varepsilon\int_0^1 dx\sum_{a=q,\bar{q}}f_a(x)\,S_J^{(2)}(\vec{q}\,;x)$$

$$\times\left[C_F\left(\cancel{\frac{2}{\varepsilon}}-\cancel{3}\right)+\left(\cancel{\frac{11}{6}}C_A-\cancel{\frac{N_F}{3}}\right)+C_A\ln\frac{s_0}{\vec{q}^{\,2}}\right]+\text{finite terms}$$

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$$\left.+\int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2}\ln\frac{s_0}{\left(|\vec{k}|+|\vec{q}-\vec{k}|\right)^2}S_J^{(2)}\left(\vec{q}-\vec{k};x\right)\right] + \text{finite terms}$$

After the **projection** onto the eigenfunctions of the LO BFKL kernel

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$$V_q^{(V)}(\vec{q}) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^{\varepsilon}}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\,2}\right)^{\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\,S_J^{(2)}(\vec{q}\,;x)$$

$$\times\left[C_F\left(\cancel{\frac{2}{\varepsilon}}-\cancel{3}\right)+\left(\cancel{\frac{11}{6}}C_A-\cancel{\frac{N_F}{3}}\right)+C_A\ln\frac{s_0}{\vec{q}^{\,2}}\right] + \text{finite terms}$$

Collinear counterterm

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int\limits_0^1 d\beta \int\limits_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)\right) + \frac{C_A}{C_F}\left(P_{gg}(\beta)f_g(x) + P_{gq}(\beta)\sum_{a=q,\bar{q}} f_a(x)\right)\right]$$

- **term proportional to C_A**

$$V_q^{(R)(C_A)}(\vec{q}) = \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\left[\int_0^1 d\beta\,\frac{\Gamma[1-\varepsilon]}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\frac{P_{gq}(\beta)}{C_F}\,S_J^{(2)}(\vec{q};x\beta)\right.$$

$$\left. + \int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\,\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2}\ln\frac{s_0}{\left(|\vec{k}|+|\vec{q}-\vec{k}|\right)^2}\,S_J^{(2)}\left(\vec{q}-\vec{k};x\right)\right] + \text{finite terms}$$

After the **projection** onto the eigenfunctions of the LO BFKL kernel

Virtual corrections

$$V_q^{(V)}(\vec{q}) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\,2}\right)^\varepsilon\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\,S_J^{(2)}(\vec{q}\,;x)$$

$$\times\left[C_F\left(\cancel{\frac{2}{\varepsilon}}-\cancel{3}\right) + \left(\cancel{\frac{11}{6}}C_A - \cancel{\frac{N_F}{3}}\right) + C_A\ln\frac{s_0}{\vec{q}^{\,2}}\right] + \text{finite terms}$$

Collinear counterterm

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int\limits_0^1 d\beta \int\limits_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta) f_a(x) + P_{qg}(\beta) f_g(x)\right) + \frac{C_A}{C_F}\left(P_{gg}(\beta) f_g(x) + P_{gq}(\beta)\sum_{a=q,\bar{q}} f_a(x)\right)\right]$$

- **term proportional to** C_A

$$V_q^{(R)(C_A)}(\vec{q}) = \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\left[\int_0^1 d\beta\,\frac{\Gamma[1-\varepsilon]}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\frac{P_{gq}(\beta)}{C_F}\,S_J^{(2)}(\vec{q};x\beta)\right.$$

$$\left. + \int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\frac{\vec{q}^{\;2}}{\vec{k}^{\;2}\left(\vec{q}-\vec{k}\right)^2}\ln\frac{s_0}{\left(|\vec{k}|+|\vec{q}-\vec{k}|\right)^2}\,S_J^{(2)}\left(\vec{q}-\vec{k};x\right)\right] + \text{finite terms}$$

After the **projection** onto the eigenfunctions of the LO BFKL kernel

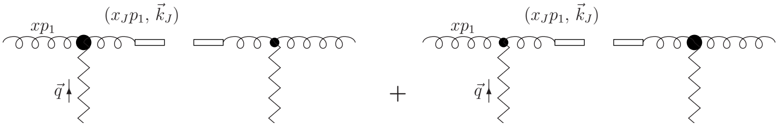
Virtual corrections

$$V_q^{(V)}(\vec{q}\,) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\;2}\right)^\varepsilon\int_0^1 dx\sum_{a=q,\bar{q}} f_a(x)\,S_J^{(2)}(\vec{q}\,;x)$$

$$\times\left[C_F\left(\cancel{\frac{2}{\varepsilon}}-\cancel{3}\right)+\left(\cancel{\frac{11}{6}}C_A-\cancel{\frac{N_F}{3}}\right)+C_A\ln\frac{s_0}{\vec{q}^{\;2}}\right]+\text{finite terms}$$

Outline

Virtual corrections



$$V_g^{(V)}(\vec{q}) = -\frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^{\varepsilon}} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} \left(\vec{q}^{\,2}\right)^{\varepsilon} \int_0^1 dx\, \frac{C_A}{C_F}\, f_g(x)\, S_J^{(2)}(\vec{q}\,;x)$$
$$\times \left[\left(\frac{N_F}{3} - \frac{11}{6} C_A \right) + C_A \left(\ln\left(\frac{s_0}{\vec{q}^{\,2}} \right) + \frac{2}{\varepsilon} \right) \right] + \text{finite terms}$$

“Real” corrections

Starting point ($T_R = 1/2$)

- **quark-antiquark intermediate state**:

$$V_g^{(R_{q\bar{q}})}(\vec{q}\,) = \frac{N_F}{(4\pi)^\varepsilon} \int_0^1 dx\, \frac{C_A}{C_F} f_g(x) \int_0^1 d\beta \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\;2}}{\vec{k}^{\,2}(\vec{k}-\vec{q}\,)^2} \times T_R \left(1 - \frac{2\beta\,(1-\beta)}{1+\varepsilon}\right)$$

$$\times \left[\frac{C_F}{C_A} + \frac{\beta(1-\beta)\vec{k}\cdot(\vec{q}-\vec{k})}{(\vec{k}-\beta\vec{q}\,)^2}\right] S_J^{(3)}(\vec{q}-\vec{k}, \vec{k}, x\beta; x)\,,$$

β, $1-\beta$ and $\vec{k}$, $\vec{q}-\vec{k}$: relative longitudinal and transverse momenta of the quark, antiquark

- **two-gluon intermediate state**:

$$V_g^{(R_{gg})}(\vec{q}\,) = \frac{C_A}{(4\pi)^\varepsilon} \int_0^1 dx\, \frac{C_A}{C_F} f_g(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^{1-\beta_0} d\beta\, \frac{\vec{q}^{\;2}\,\mathcal{P}_{gg}(\beta)}{(\vec{k}-\beta\vec{q}\,)^2\vec{k}^2(\vec{k}-\vec{q}\,)^2}$$

$$\times \left\{\beta^2(\vec{k}-\vec{q}\,)^2 + (1-\beta)^2\vec{k}^2 - \beta(1-\beta)\vec{k}\cdot(\vec{q}-\vec{k})\right\} S_J^{(3)}(\vec{q}-\vec{k}, \vec{k}, x\beta; x)$$

where $\mathcal{P}_{gg}(\beta) = P\,(\beta) + P\,(1-\beta)$, with $P\,(\beta) = \left(\frac{1}{\beta} + \frac{\beta}{2}\right)(1-\beta)$

β, $1-\beta$ and $\vec{k}$, $\vec{q}-\vec{k}$: relative longitudinal and transverse momenta of the two gluons

- **term proportional to N_F**

$$V_g^{(R)\,(N_F)}(\vec{q}\,) = N_F\,\frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^\varepsilon}\,\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\int_0^1 dx\, f_g(x)\left[\frac{2}{3}\,\frac{C_A}{C_F}\,S_J^{(2)}\,(\vec{q};x)\right.$$

$$\left.+2\int_0^1 d\beta\, P_{qg}(\beta)\, S_J^{(2)}(\vec{q};x\beta)\right] + \text{finite terms}$$

- **term proportional to C_A**

$$V_g^{(R)\,(C_A)}(\vec{q}\,) = \frac{C_A}{C_F}\int_0^1 dx\, f_g(x)\left\{\frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^\varepsilon}\,\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left[C_A\left(\frac{2}{\varepsilon}-\frac{11}{3}\right)\,S_J^{(2)}\,(\vec{q};x) + \int_0^1 d\beta\right.\right.$$

$$\left.\left.\times\, P_{gg}\,(\beta)\, S_J^{(2)}\,(\vec{q};x\beta)\right] + \frac{C_A}{(4\pi)^\varepsilon}\int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\,\frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q}\,)^2}\,\ln\frac{s_0}{(|\vec{k}|+|\vec{q}-\vec{k}|)^2}\;S_J^{(2)}(\vec{q}-\vec{k};x)\right\} + \text{f. t.}$$

After the **projection** onto the eigenfunctions of the LO BFKL kernel there is a complete cancellation of divergences.

- **term proportional to N_F**

$$V_g^{(R)\,(N_F)}(\vec{q}\,) = N_F \, \frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} \int_0^1 dx\, f_g(x) \left[\frac{2}{3} \, \frac{C_A}{C_F} \, S_J^{(2)}\,(\vec{q};x) \right.$$

$$\left. + 2 \int_0^1 d\beta \, P_{qg}(\beta)\, S_J^{(2)}(\vec{q}; x\beta) \right] + \text{finite terms}$$

- **term proportional to C_A**

$$V_g^{(R)\,(C_A)}(\vec{q}\,) = \frac{C_A}{C_F} \int_0^1 dx\, f_g(x) \left\{ \frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} \left[C_A \left(\frac{2}{\varepsilon} - \frac{11}{3} \right) S_J^{(2)}\,(\vec{q};x) + \int_0^1 d\beta \right.\right.$$

$$\left.\left. \times\, P_{gg}\,(\beta)\, S_J^{(2)}\,(\vec{q}; x\beta) \right] + \frac{C_A}{(4\pi)^\varepsilon} \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q}\,)^2} \ln \frac{s_0}{(|\vec{k}|+|\vec{q}-\vec{k}|)^2} \; S_J^{(2)}(\vec{q}-\vec{k};x) \right\} + \text{f. t.}$$

After the **projection** onto the eigenfunctions of the LO BFKL kernel there is a complete cancellation of divergences.

Conclusions

- We have recalculated, completely within the original BFKL approach at the next-to-leading order, the jet vertex relevant for the production of Mueller-Navelet jets in proton collisions.

- It has been explicitly shown that the result is free of IR and UV divergences.

  An essential role in the cancellation of IR divergences has been played by the projection onto LO BFKL eigenfunctions.

- The energy scale s_0 remains untouched and need not be fixed at any definite value.

- Fixing the s_0 scale at the value suggested by J. Bartels, D. Colferai, G.P. Vacca (2003), we find a complete agreement before the projection.