Low-mass Drell-Yan production at the LHC; and treatment of infrared region in pQCD

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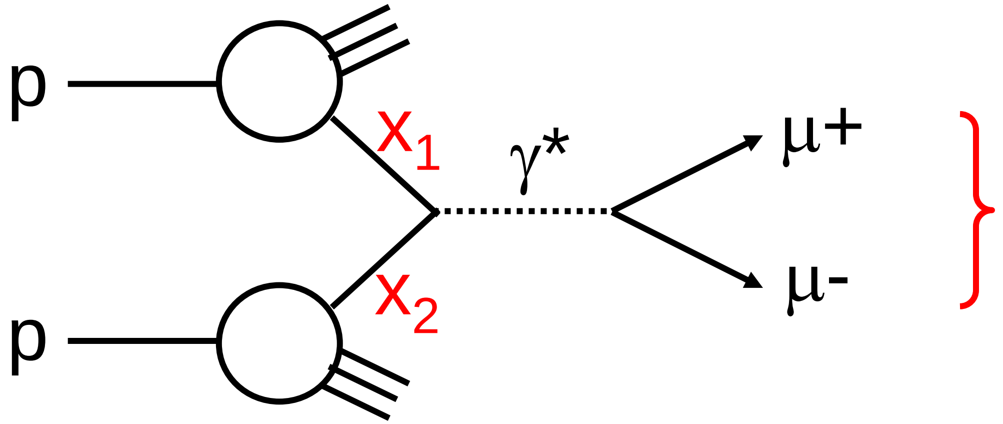


low M, high Y
very low x_2

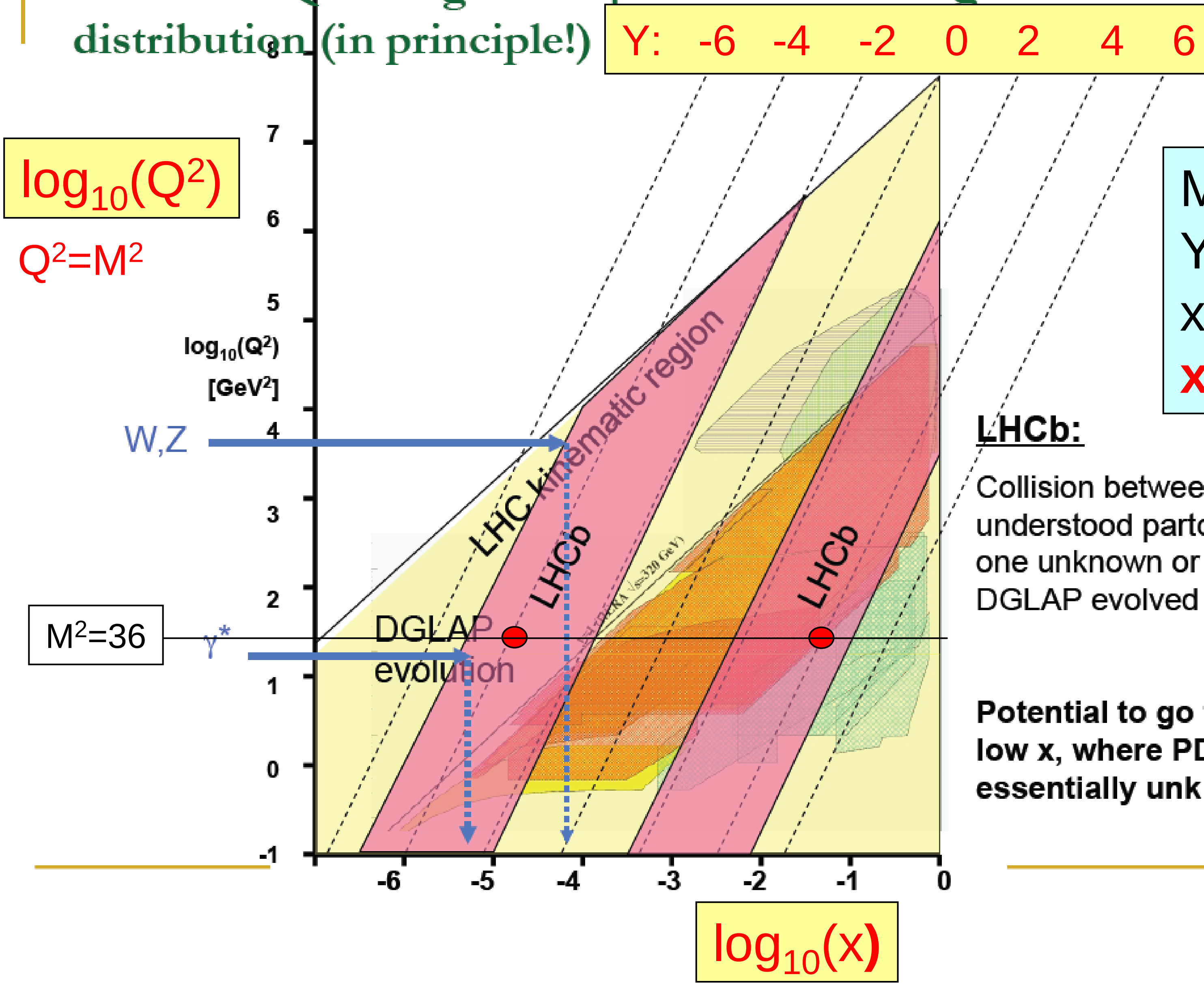
Diffraction 2012
Lanzarote, Sept.10-15

$$x_{1,2} = \frac{M}{\sqrt{s}} \exp(\pm Y)$$

From Ronan McNulty (LHCb at 7 TeV)

$$x_{1,2} = \frac{M}{\sqrt{s}} \exp(\pm Y)$$

DY at low Q^2 is of great important for low-x gluon distribution (in principle!)



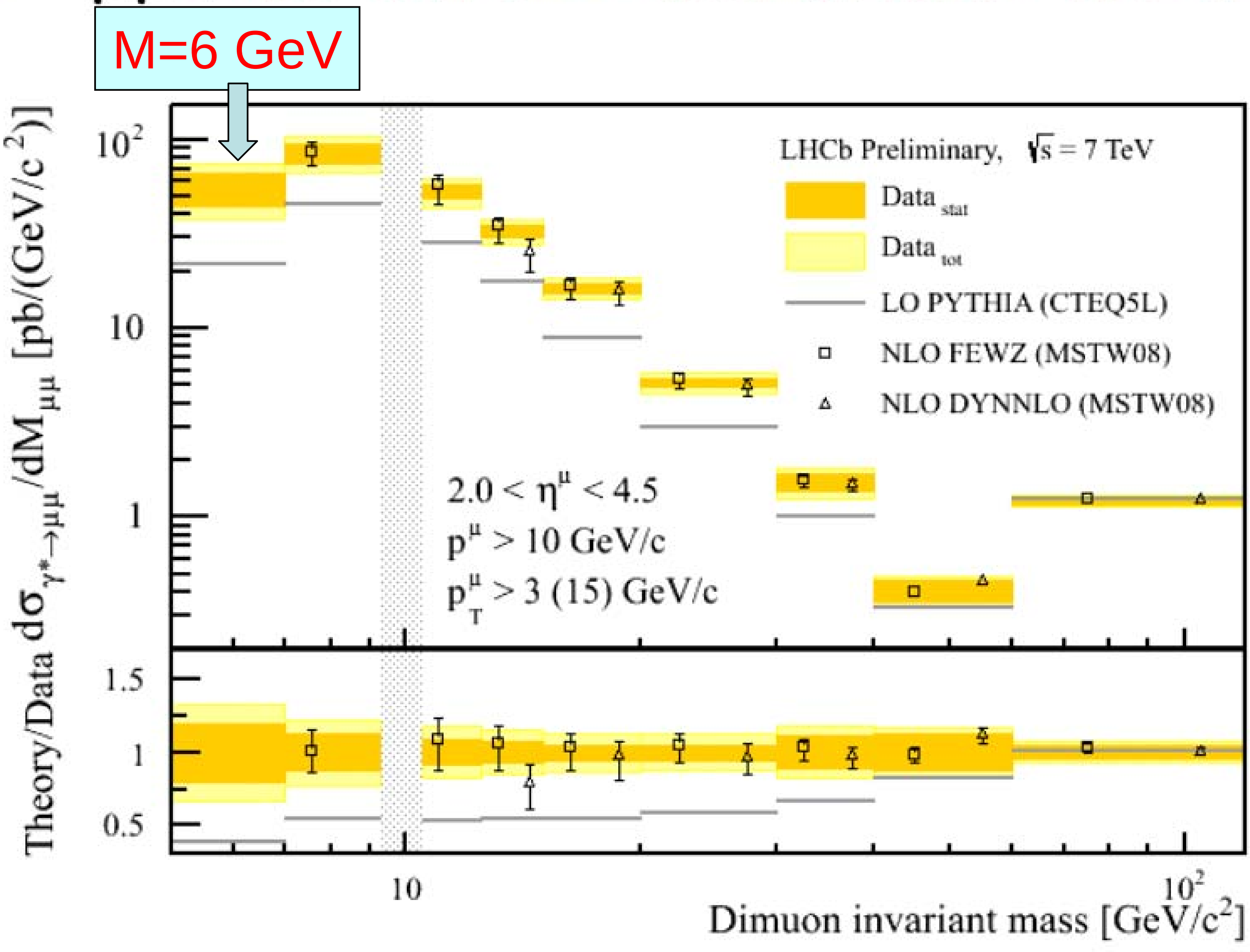
$M_{\mu+\mu-}$=6GeV
Y=4
$x_1=4.7\times10^{-2}$
$x_2=1.6\times10^{-5}$

LHCb:

Collision between one well understood parton and one unknown or large DGLAP evolved parton.

Potential to go to very low x, where PDFs essentially unknown

DY→μμ cross-section measured down to 5 GeV.



The factorization scale μ_F

$$d\sigma/d^3p \;=\; \int dx_1 dx_2 \;\mathrm{PDF}(x_1,\mu_F)\;|\mathcal{M}(p;\mu_F,\mu_R)|^2\;\mathrm{PDF}(x_2,\mu_F)$$

parton virtuality $q^2 < \mu_F^2$ $\qquad q^2 > \mu_F^2$

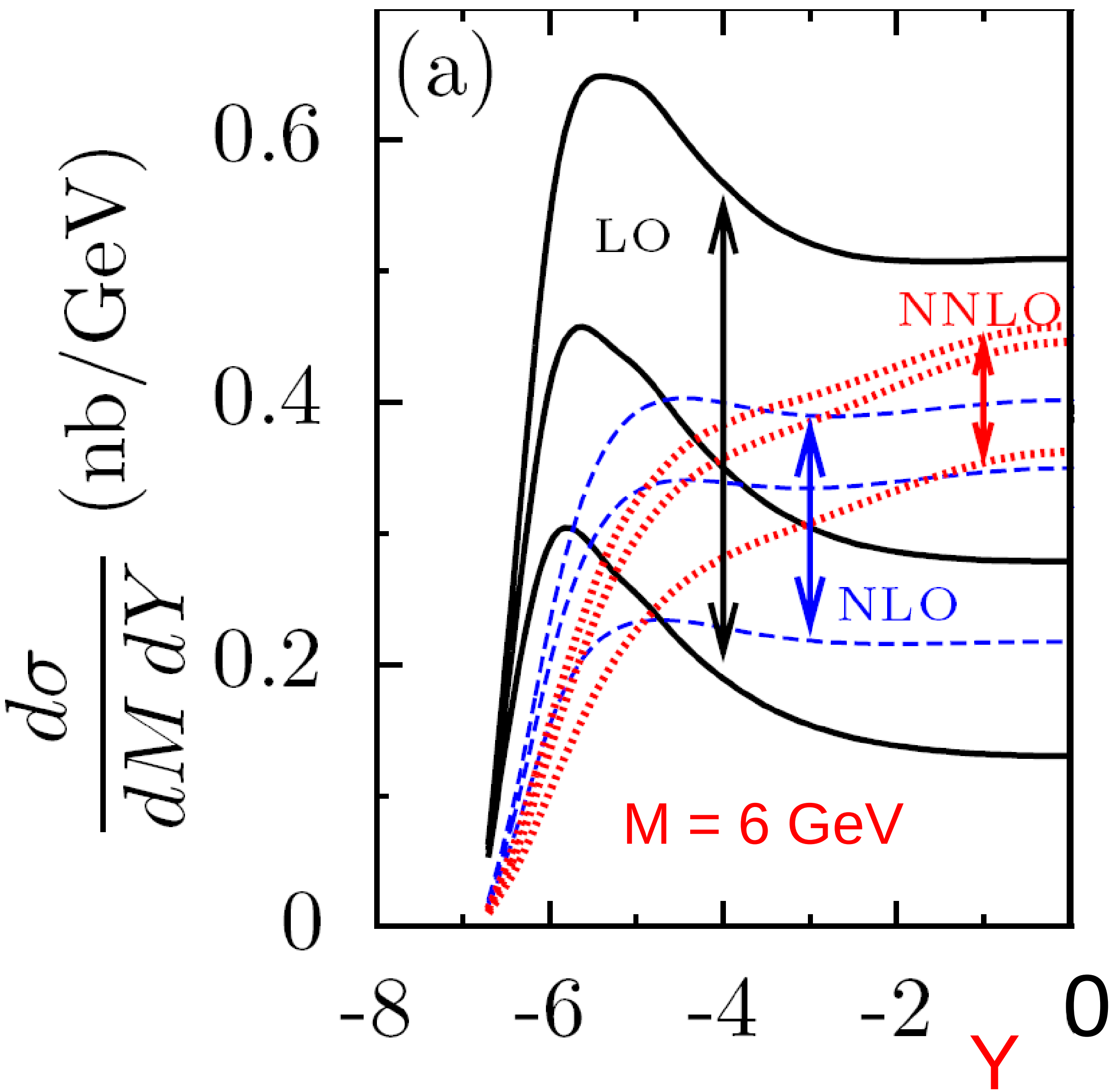
At low x, the PDFs strongly depend on choice of μ_F.
Worse, dominance of g at low x (i.e. low M) means
LO $q\bar{q}\to\gamma^*$ overshadowed by NLO $gq\to q\gamma^*$ subproc.

At low x, probability to emit new parton in $\Delta\mu_F$ enhanced:

mean number emitted if ln(1/x)~8, $0.5\mu<\mu_F<2\mu$ } $\langle n\rangle \simeq \frac{\alpha_s N_C}{\pi}\ln(1/x)\,\Delta\ln\mu_F^2$ ~ 8

but $|\mathcal{M}^{NLO}|^2$ can emit only one → so no compensation

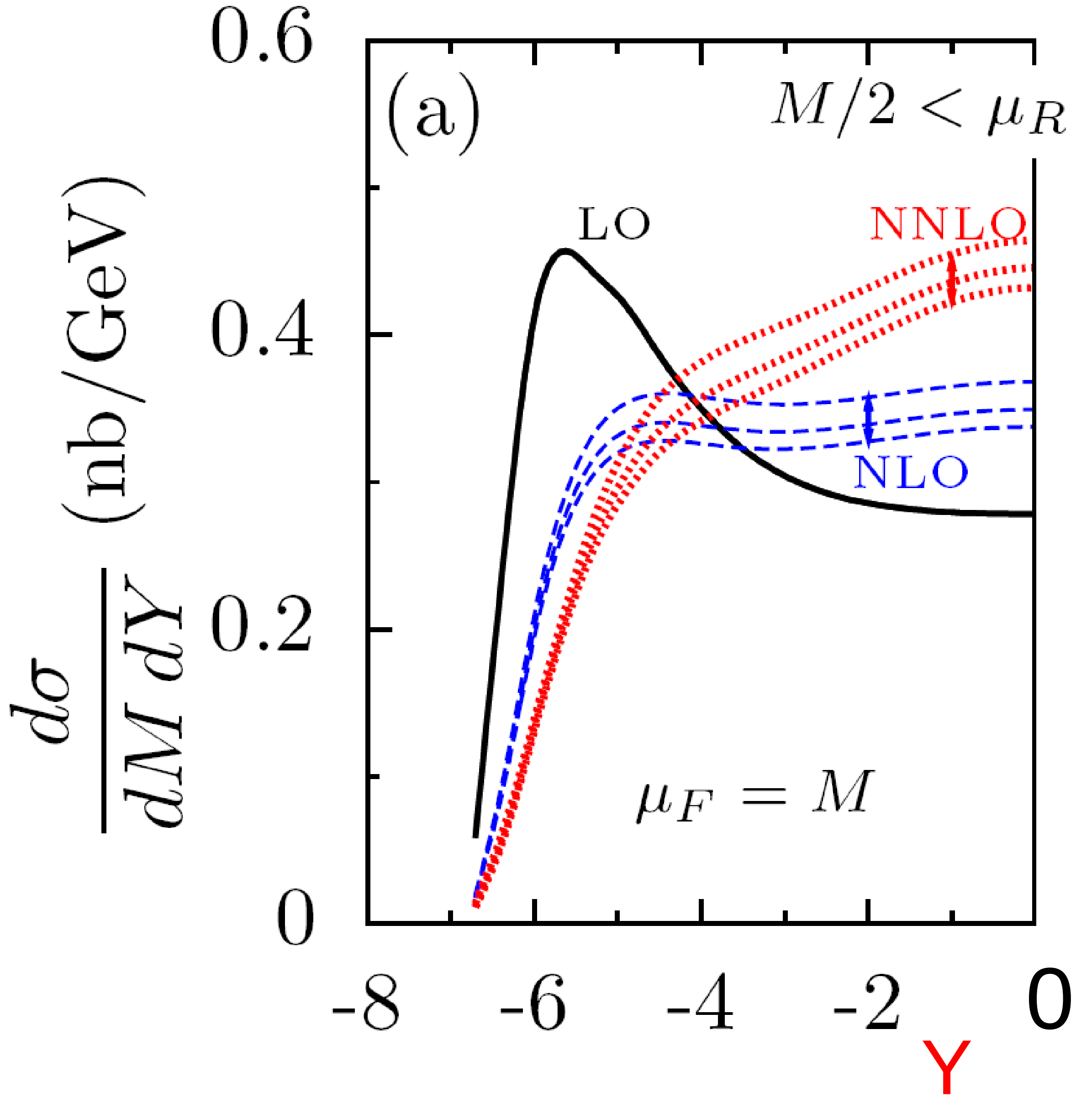
Factorization scale μ_F dependence



Large μ_F dependence
μ_F = M/2, M, 2M

huge at LO
large at NLO
sizeable at NNLO

Renormalization scale μ_R dependence



M = 6 GeV

Idea: use NLO to fix μ_F for LO part, and to show results stable to variations of μ_F in remaining NLO part

Start with LO:

$$\sigma(\mu_F) \; = \; \mathrm{PDF}(\mu_F) \otimes C^{\mathrm{LO}} \otimes \mathrm{PDF}(\mu_F)$$

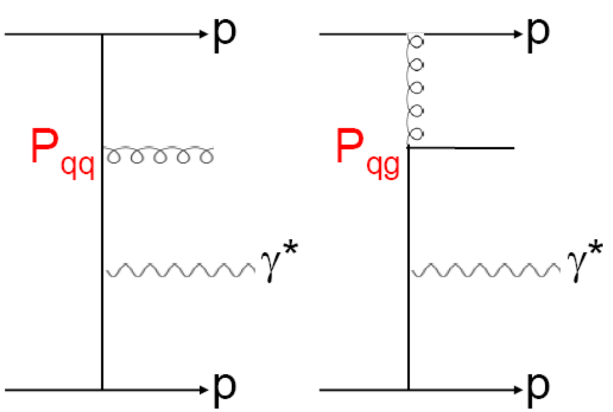
Changing scale from m to μ_F

$$\sigma(\mu_F) = \mathrm{PDF}(m) \otimes \left(C^{\mathrm{LO}} \; + \; \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_F^2}{m^2}\right) (P_{\mathrm{left}} C^{\mathrm{LO}} + C^{\mathrm{LO}} P_{\mathrm{right}}) \right) \otimes \mathrm{PDF}(m)$$

$$P_{\mathrm{left}} = P_{\bar{q}\bar{q}} + P_{\bar{q}g} \qquad\qquad P_{\mathrm{right}} = P_{qq} + P_{qg}$$

This is α_s corr^n in LO DGLAP collinear approach, Leading Log Approx (LLA)

$$\int_{m^2}^{\mu_F^2} \frac{dk_T^2}{k_T^2} \; = \; \ln\left(\frac{\mu_F^2}{m^2}\right)$$



Now NLO expression:

$$\sigma(\mu_F) = \mathrm{PDF}(\mu_F) \otimes (C^{\mathrm{LO}} + \underline{\alpha_s C^{\mathrm{NLO}}_{\mathrm{corr}}}) \otimes \mathrm{PDF}(\mu_F)$$

C^{NLO} means $q\bar{q} \rightarrow g\gamma^*$ and $gq \rightarrow q\gamma^*$ calc better than LLA accuracy, but part already included to LLA accuracy --- subtract it off.
At this stage C^{NLO} becomes dependent on μ_F --- $C^{\mathrm{NLO}}_{\mathrm{rem}}(\mu_F)$

Changing μ_F redistributes α_s contribution between two terms

$$(\mathrm{PDF} \otimes C^{\mathrm{LO}} \otimes \mathrm{PDF}) \Longleftrightarrow (\mathrm{PDF} \otimes \alpha_s C^{\mathrm{NLO}}_{\mathrm{rem}} \otimes \mathrm{PDF})$$

Trick is to choose $\mu_F=\mu_0$ in LO part so as to minimize $C^{\mathrm{NLO}}_{\mathrm{rem}}(\mu_F)$

Choose μ_F so as much as possible of “real” NLO ladder-like form is in LO part (where large $\alpha_s\ln(1/x)$ terms are collected in PDFs)

α_S term from LO DGLAP

main NLO subprocess

$$\int_{Q_0^2}^{\mu_F^2} dt\ \frac{d\sigma}{dt}(\alpha_S, \mathrm{LLA}) \;=\; \int_{Q_0^2}^{\mu_F^2} dt\ \frac{d\sigma}{dt}(gq \to q\gamma^*)_{\mathrm{exact}}$$

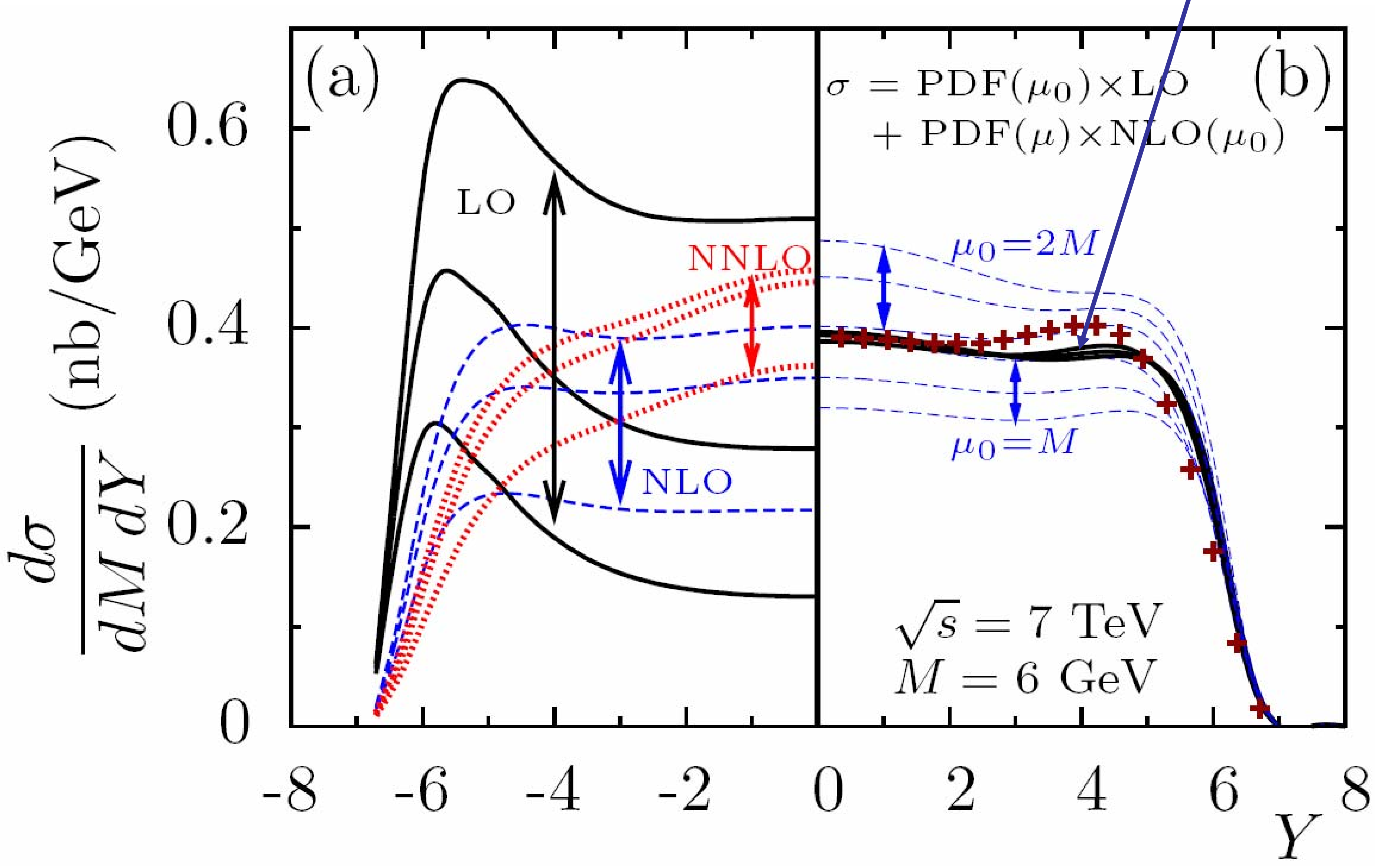
adjust μ_F until equality achieved

$$\mu_F \equiv \mu_0 = 1.4M$$

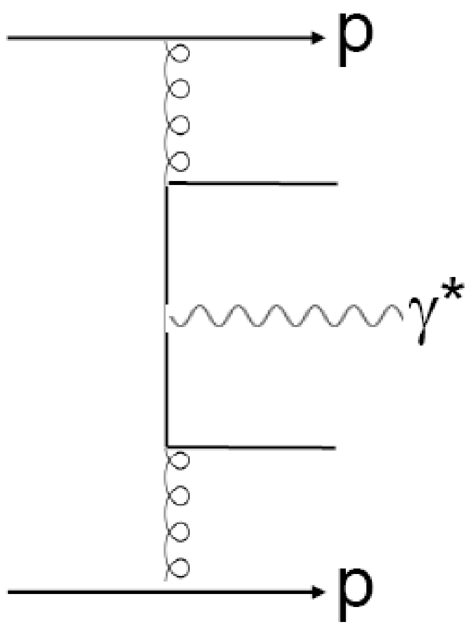
so (LO DGLAP $\otimes$ C^{LO}) well reproduces NLO term

minimizes $C_{\mathrm{rem}}^{\mathrm{NLO}}(\mu_F)$ for $\mu_F = 1.4M$

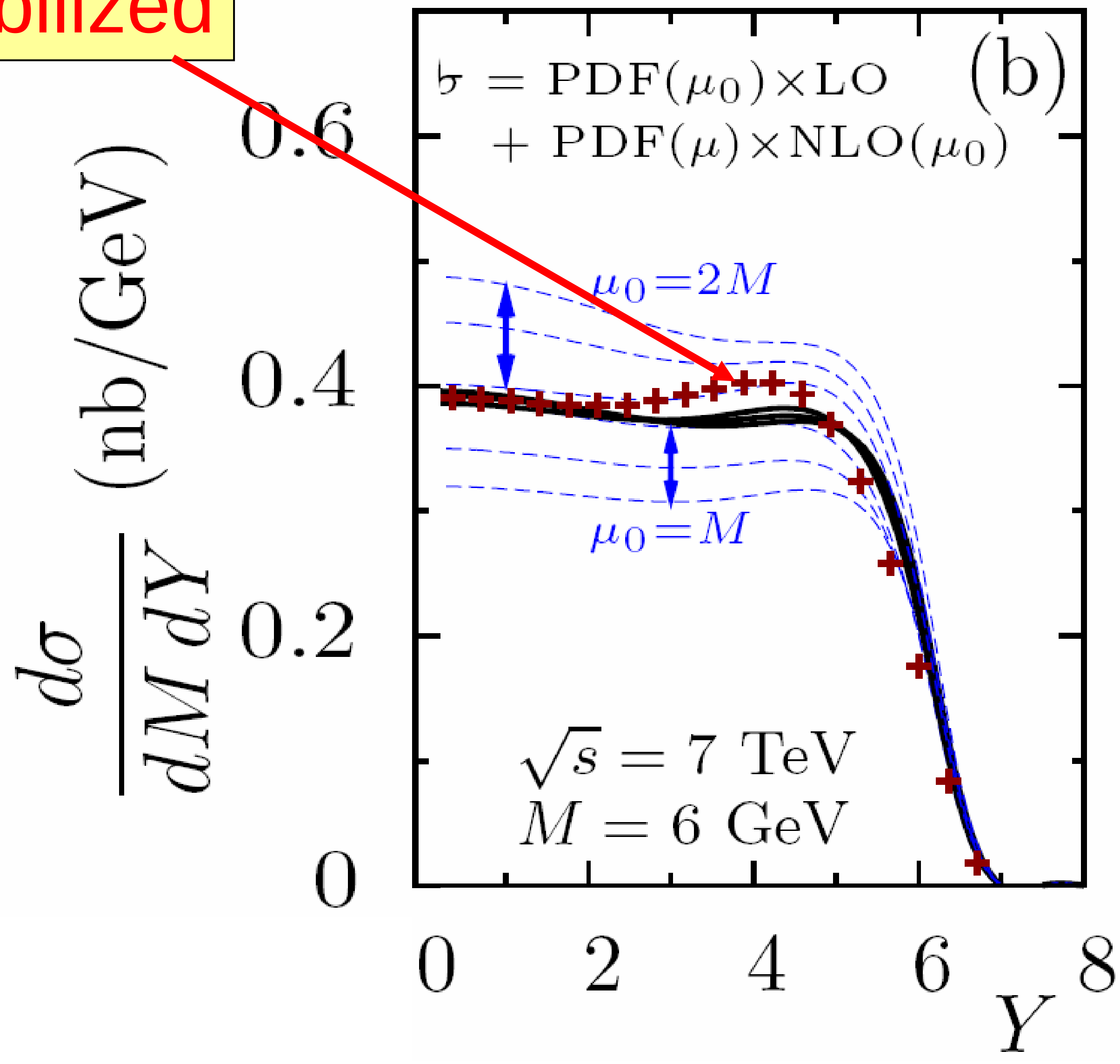
choice $\mu_F=\mu_0=1.4M$ in LO part,
no $\mu_F=M/2,M,2M$ dep. at NLO

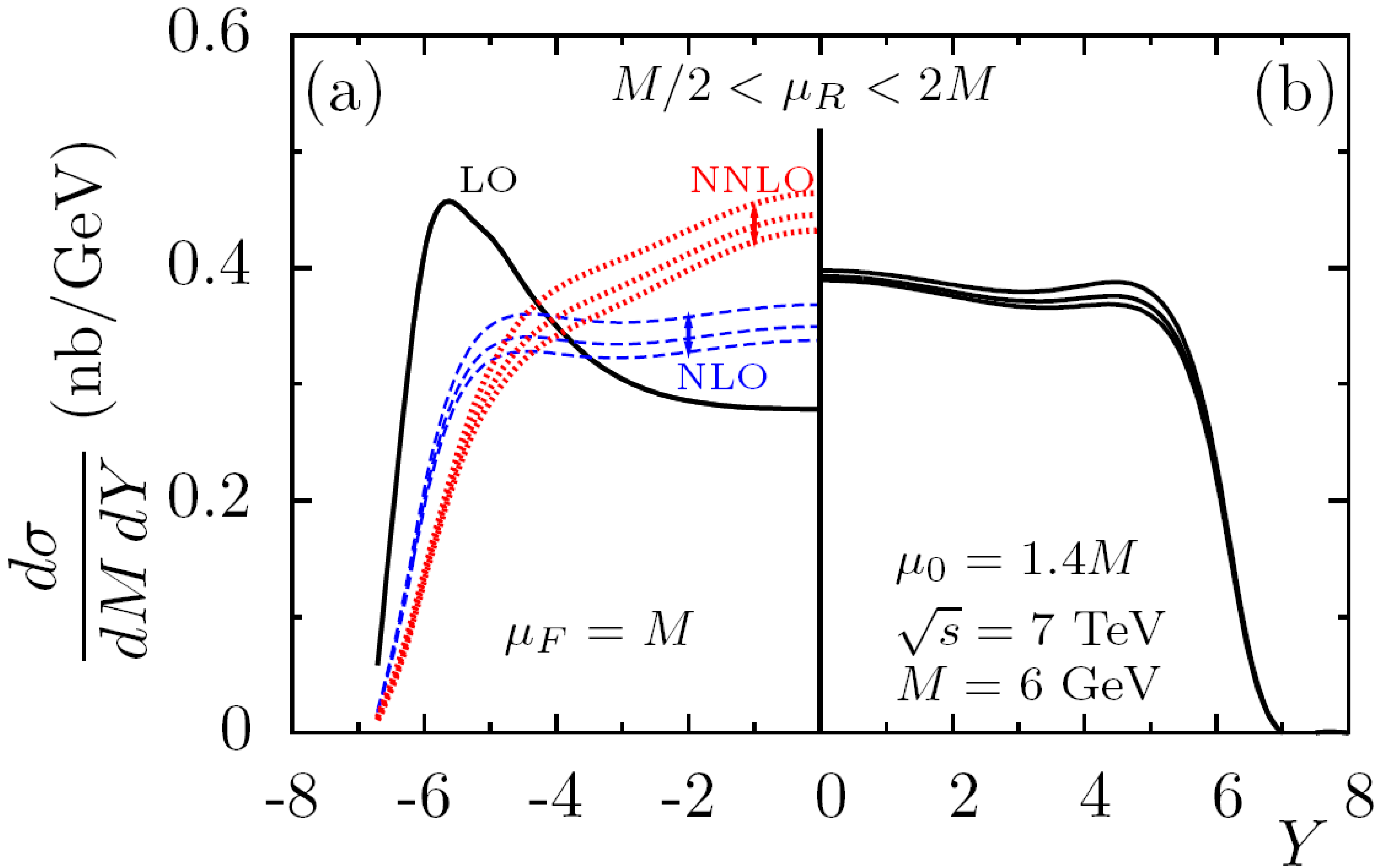


NNLO also stabilized

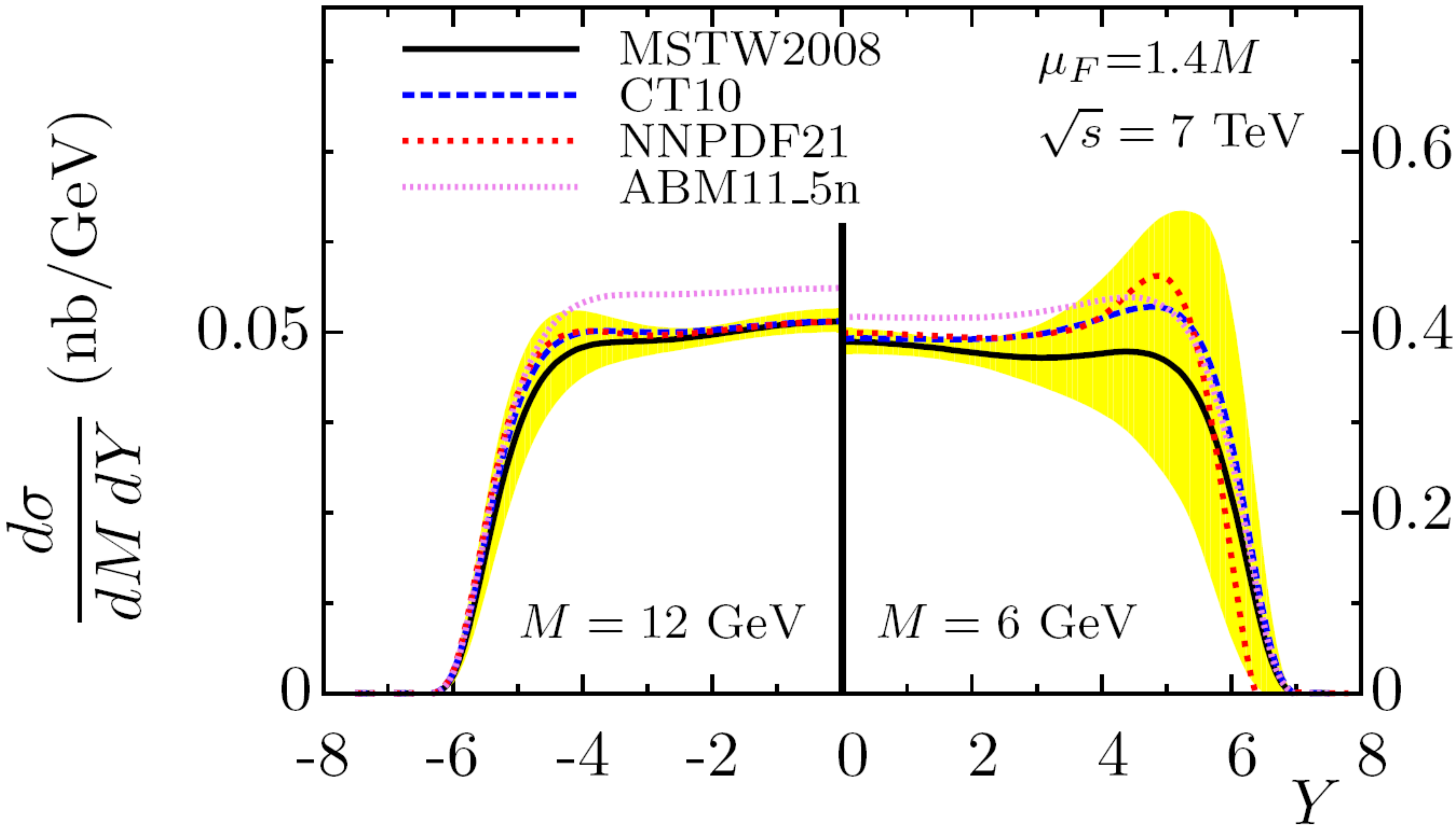


dominant diagram





0.6
0.4
0.2
0
-8
-6
-4
-2
0
2
4
6
8
Y
$\frac{d\sigma}{dM\,dY}$ (nb/GeV)
(a)
(b)
$M/2 < \mu_R < 2M$
LO
NNLO
NLO
$\mu_F = M$
$\mu_0 = 1.4M$
$\sqrt{s} = 7$ TeV
$M = 6$ GeV



For Y > 3, pure DGLAP PDF extrapolations become unreliable due to absence of absorptive, ln(1/x),…modifications

LHCb data provide direct measure of PDFs in this low x domain

Treatment of infrared region in pQCD

1. Physical treatment

work in 4 dimensions

(i) All physics below $Q_0 \gg \Lambda_{QCD}$ is in input PDFs
(ii) To get correct NLO result (and avoid double counting) must subtract contribution generated by LO DGLAP evol.
(iii) Produces unique infrared-convergent integral

2. Conventional treatment

work in $4+2\varepsilon$ dimensions

(i) $1/\varepsilon$ term in NLO result compensated by $1/\varepsilon$ term in LO DGLAP-generated contribution integrated in the same $4+2\varepsilon$ scheme as used to calculate NLO result.
(ii) Leaves ε/ε term which is not $\mathcal{O}(Q_0^2/\mu_F^2)$
(iii) Danger: uses pQCD expressions in confinement region;
(iv) Appears some double counting remains.

Take Drell-Yan as example:

main NLO subprocess

$$\frac{d\hat{\sigma}(gq \to q\gamma^*)}{d|t|} = \frac{\alpha^2 \alpha_s z}{9M^2} \frac{1}{|t|} \left[((1-z)^2 + z^2) + z^2 \frac{t^2}{M^4} - 2z^2 \frac{t}{M^2} \right]$$

To calculate $d\sigma/dM^2$ need to integrate over t from t=0

To avoid double counting, subtract the LO DGLAP $\alpha_s P_{qg}$ term, which exactly removes infrared divergence

explicitly →

$$\frac{d\hat{\sigma}(gq \to q\gamma^*)}{d|t|} = \frac{\alpha^2\alpha_s z}{9M^2}\frac{1}{|t|}\left[((1-z)^2+z^2) + z^2\frac{t^2}{M^4} - 2z^2\frac{t}{M^2}\right]$$

$$= \frac{d\hat{\sigma}^{\rm NLO}_{\rm rem}}{dt} + \underbrace{\frac{d\hat{\sigma}^{\rm LO}_{q\bar{q}}}{dt} \otimes \frac{\alpha_s}{2\pi}P^{\rm LO}_{\bar{q}g}}$$

$$\frac{d\hat{\sigma}^{\rm LO}}{d|t|} \otimes \frac{\alpha_s}{2\pi}P^{\rm LO}_{\bar{q}g} = \frac{\alpha^2\alpha_s z}{9M^2}\frac{1}{|t|}[(1-z)^2+z^2]\,\underline{\Theta(\mu_F^2 - |t|)}$$

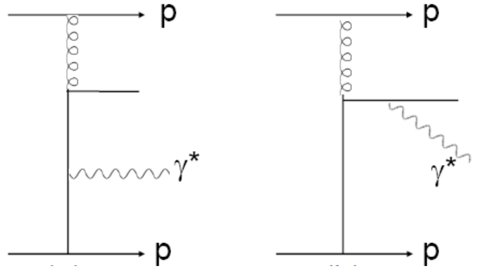
DGLAP α_s term accounts for all virtualities $|t| < \mu_F^2$, where $|t| < Q_0^2$ is hidden in input PDF

After subtraction of this LO generated term

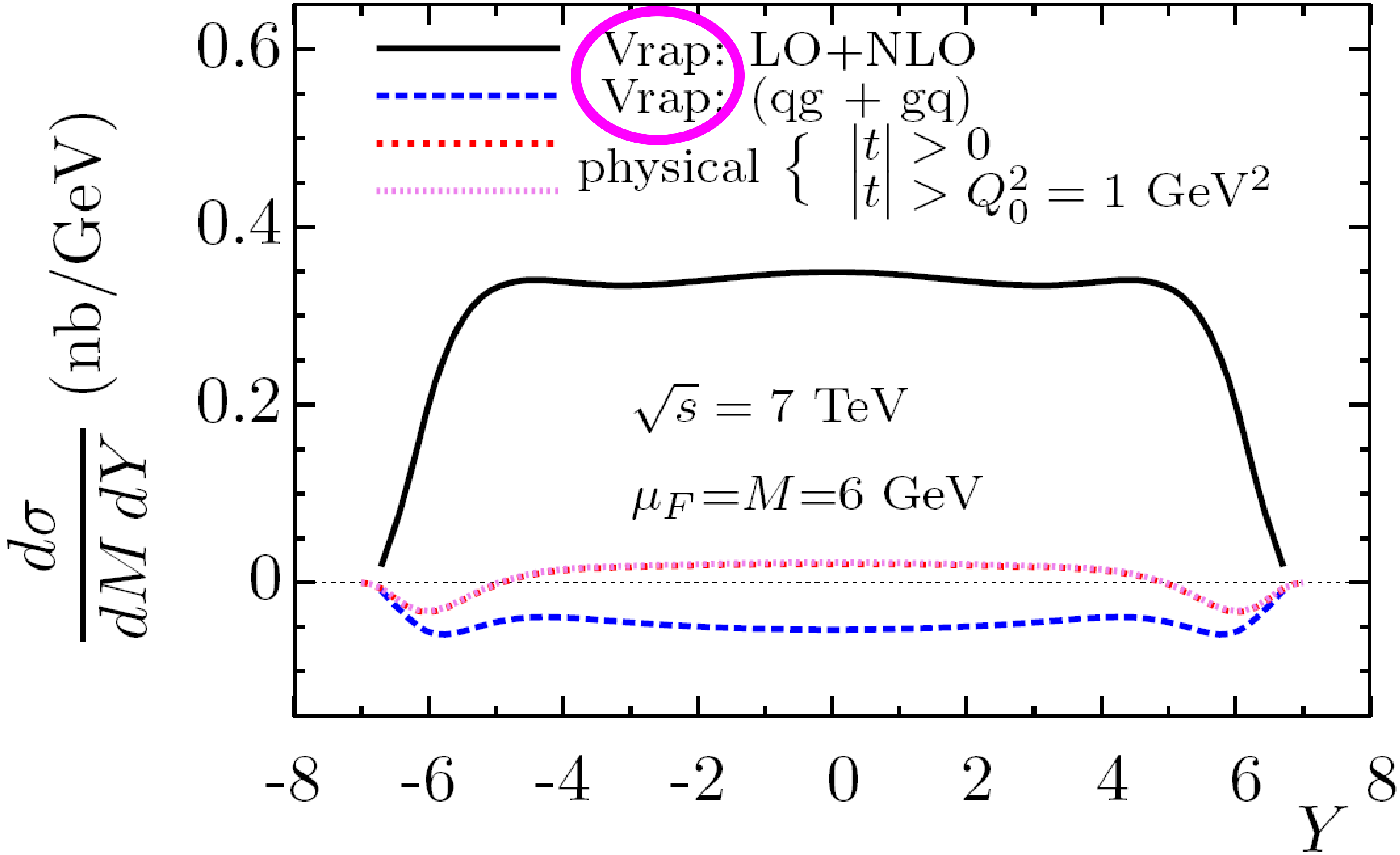
$$\frac{d\hat{\sigma}^{\rm NLO}_{\rm rem}}{d|t|} = \frac{\alpha^2\alpha_s z}{9M^2}\frac{1}{|t|}\left[[(1-z)^2+z^2]\,\underline{\Theta(|t| - \mu_F^2)} + z^2\frac{t^2}{M^4} - 2z^2\frac{t}{M^2}\right]$$

which has no singularity as $t \to 0$.

Non-singular terms vanish as Q_0^2/μ_F^2.



conventional



(i) physical – conventional = pink – blue (~15%)

(ii) note physical is essentially independent of Q_0

(i) Similar discrepancy between conventional and physical treatments of IR region for coeff. fn. C_g in DIS.

(ii) To see the effects on global PDF analyses we need a complete set of physically corrected coeff. and splitting fns. (Expect main effect to be on gluon at low x and low scales.)

(iii) The discrepancy cannot be attributed to a factorization scheme change