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DIFFRACTION 2012

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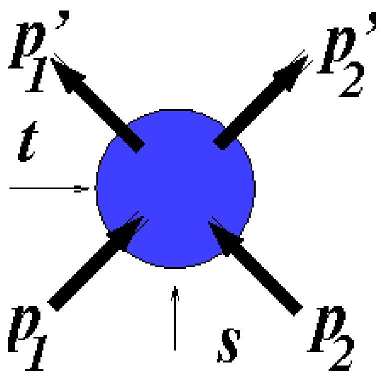
REGGE TRAJECTORIES IN QCD

- Feynman diagrams asymptotic
- Factorization small and long distance
- QCD
- Summation and Regge singularities
- Some comments
- Instead of Conclusion

Some history

AE, I.Ginzburg, Fortsch.Phys. 22(74)575



60's -- Feynman diagrams asymptotic

1968 -- first factorization theorem for ϕ^4

1970 -- factorization theorem for $\bar{q}q\phi$

Sum of ALL Short Distance (SD) logarithms.

Chisholm representation:

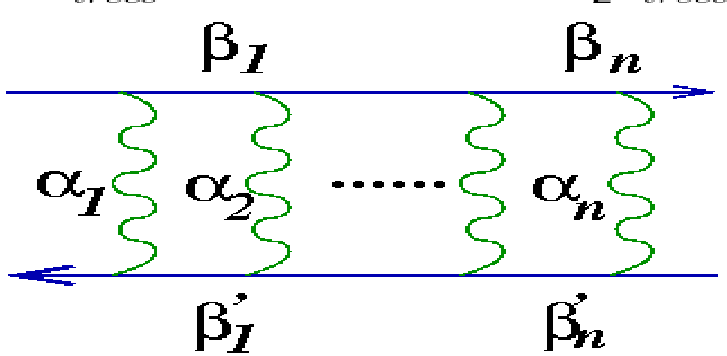
$$\Delta(k^2) \propto i \int_0^\infty d\alpha e^{i(k^2-m^2+i\epsilon)\alpha}$$

$$F(S,t) \propto \int_0^\infty \frac{\prod d\alpha}{D^2(\alpha)} G(S,t,\alpha) \cdot \exp\left\{ i \left[S\frac{A(\alpha)}{D(\alpha)} + tT(\alpha) + \sum m^2\alpha \right] \right\}$$

$J(t,m,\alpha)$

$$\left[D(\alpha) = \sum_{trees} \prod \alpha \; ; \quad A(\alpha) = \sum_{2-trees} \prod \alpha \right]$$

E.g.



$$A(\alpha) = \alpha_1\alpha_2 \cdots \alpha_n$$

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Mellin transform

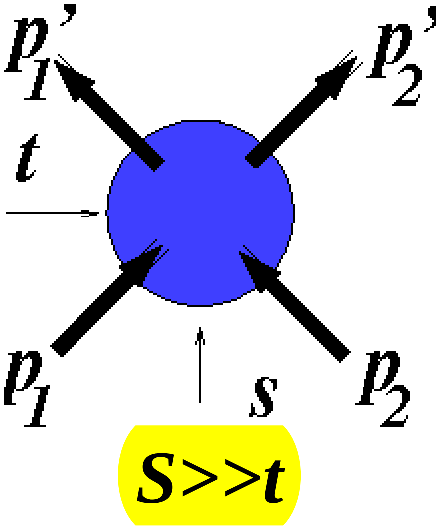
$$F^{\pm}(S,t) = \frac{1}{2i}\int_{-i\infty}^{i\infty} dj \frac{|S|^j (\mathrm{e}^{i\pi j} \pm 1)}{\Gamma(j+1)\sin \pi j} f^{\pm}(j,t)$$

$$f^{\pm}(j,t) \propto \int_0^{\infty} \frac{\prod d\alpha}{D^2(\alpha)} g(j,t,\alpha) \cdot \left|\frac{A(\alpha)}{D(\alpha)}\right|^j [\theta(A) \pm \theta(-A)]\, \mathrm{e}^{iJ(t,m,\alpha)}$$

Integration over $\lambda_V = \Sigma_V \alpha < 1/\mu^2$ generates pole
e.g. *f(j, t) ~ 1/j* for each V.

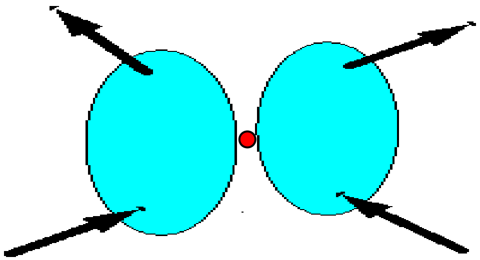
Power of pole = number of independent SD-subgraphs. Residue determines by graph which rest after contraction of all SD Vs.

Feynman diagrams asymptotic



At $S_1 \cdots S_k \gg t_1 \cdots t_l, m^2$ comes from

1a) Short distant (SD - small α's) contribution of a sub-graphs which "kills" all S_k*-dependence*, being contracted into a point.

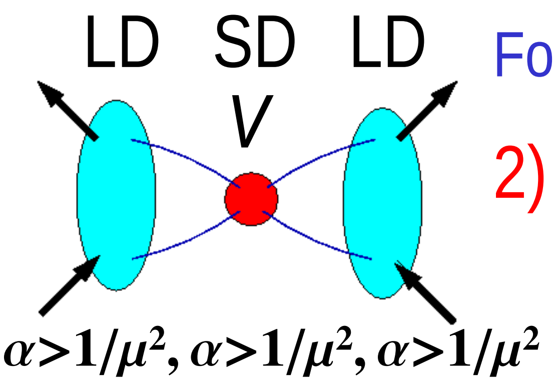


1b) Asymptotic in *S*: $\propto S^{\frac{1}{2}(4-\sum(\text{twist of ext.lines}))}$

1c) Leading poles:j=4$-\Sigma$(twist of ext. lines) =0 (4 external lines of twist 1).

For even j - only in C-odd signature,

For odd j - only in C-even signature.

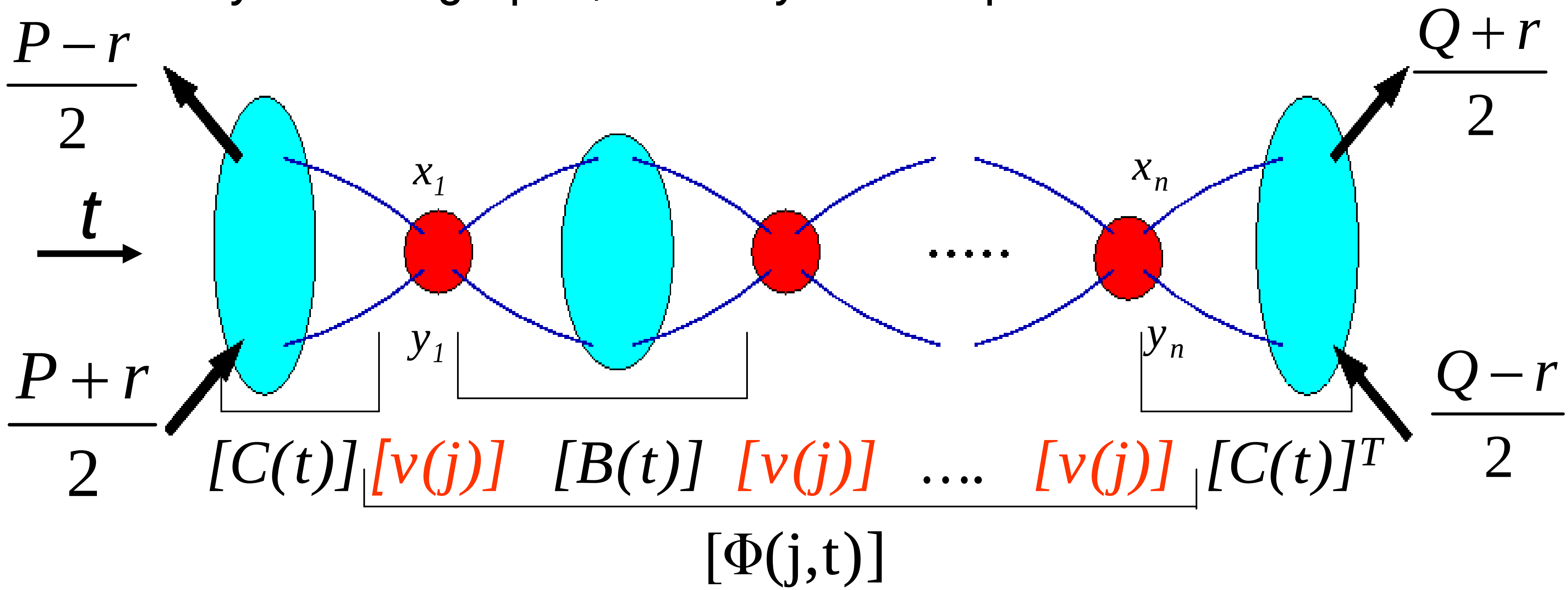


2) In opposite signatures also pinch singularities (cancellations in middle α regions) are contribute

We will consider SD contribution in nonsinglet channel

Factorization (S>>t)

Many SD-subgraphs, contrary to hard processes



$$f(j,t) = C(t)\Phi(j,t)C^T(t) + \text{high twist}$$

$S=2PQ$, $t = r^2$, $C(t)$, $v(j)$, $B(t)$ matrices [2x2] or [3x3] in $\bar{q}q$ spin space.

More details:

$$(2\pi)^4\delta^{(4)}(p_1+p'_1-p_2+p'_2)F(S,t) =$$

$$\int\int[dxdy]\tilde{C}_a(p_1,p'_1;x_1,y_1)\tilde{\Phi}_{ab}(x_1,y_1;x_n,y_n)\tilde{C}_b(p_2,p'_2;x_n,y_n)$$

$$\tilde{C}_a = \langle p'_1| : \bar{q}(x_1)\Gamma_a q(x_2) : |p_1\rangle_{R_\mu} \;,\;\; \Gamma = 1,\gamma_\mu,\sigma_{\mu\nu},\gamma_\mu\gamma_5,\gamma_5$$

$$\tilde{\Phi}_{ab} = \langle 0| : \bar{\eta}(x_1)\Gamma_a\eta(y_2) :: \bar{\eta}(x_n)\Gamma_a\eta(y_n) : |0\rangle,\;\; \eta = \frac{\delta S}{\delta q}S^\dagger$$

Taylor decomposition over $\xi_1 = x_1 - y_1$ and $\xi_n = x_n - y_n$

$$\tilde{C}_a = \mathrm{e}^{irX_1}\sum_j \frac{1}{j!}\langle p'_1|O_{a,\nu_1\cdots\nu_j}(0)|p_1\rangle_{R_\mu}\xi_1^{\nu_1}\cdots\xi_1^{\nu_j};\;\; (X_1 = \frac{x_1+y_1}{2})$$

$$O_{a,\nu_1\cdots\nu_j} = \bar{q}(0)\Gamma_a\partial_{\nu_1}\cdots\partial_{\nu_j}q(0);\;\;\; C(j,t)\{P_{\nu_1}\cdots P_{\nu_j}\} + \mathcal{O}(r)$$

$$P = p_1+p'_1,\; Q = p_2+p'_2,\; r = p_1-p'_1 = p'_2-p_2,\; t = r^2,\; S = 2(PQ)$$

Integration of Φ over $[dxdy]$ with $\xi_1^{\nu_1}\cdots\xi_1^{\nu_j}\cdot\xi_n^{\mu_1}\cdots\xi_n^{\mu_j}$ gives
$\int\int\tilde{\Phi} \propto j!\delta_a^b\delta_{\{\mu_1\cdots\mu_j\}}^{\{\nu_1\cdots\nu_j\}}\Phi(j,t) + \mathcal{O}(r)$ and

$$F(S,t) = \sum_{j=0}^{\infty}\frac{S^j}{j!}C(t)_a\Phi_a(j,t)C_a^T(t) + \text{high twist}$$

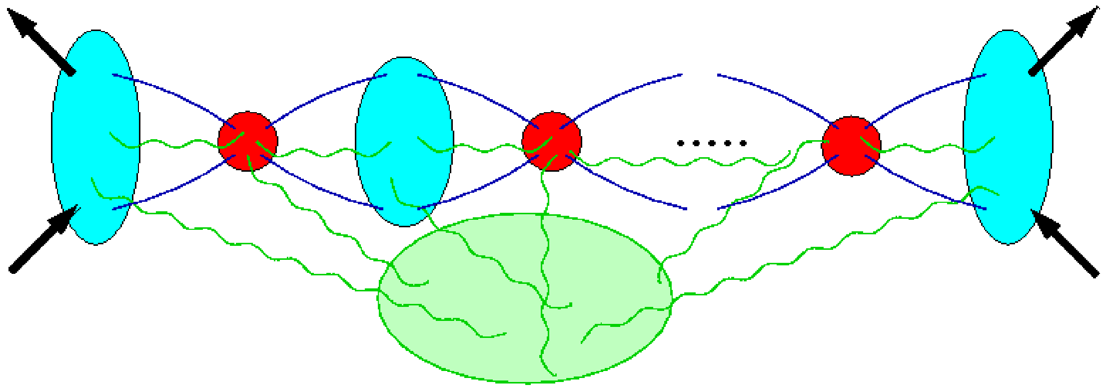
QCD

$$\phi \longrightarrow \begin{cases} A_\mu & - \text{twist-0, gauge dependent} \\ G_{\mu\nu} & - \text{twist-1, gauge invariant} \end{cases}$$

For non-singlet *t*-channel," +" signature, planar.

If initial and final states are color singlets, twist-0 parts sums

AE, Radyushkin JINR E2-80-521



AE, Radyushkin Revista NC3(80)1; TMF42(80)147

$$\bar{q}(x)\Gamma_a q(y) \longrightarrow \bar{q}(x)\Gamma_a e^{ig\int_y^x dz_\mu \hat{A}_\mu(z)} q(y)$$

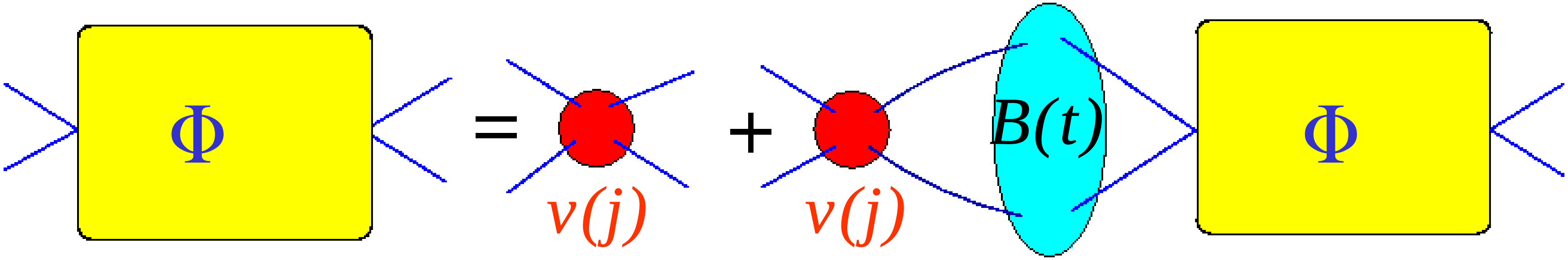
and $\partial_\nu \longrightarrow D_\nu = \partial_\nu - ig\hat{A}_\nu$

(Just the same as for hard exclusive processes, like pion form-factor.)

Factorization persists!

Summation and Regge singularities

$$\Phi(j,t) = v(j) + v(j)B(t)\Phi(j,t)$$



Solution:

$$f(j,t) = C(t)v(j)\left[1 - v(j)B(t)\right]^{-1} C^{T}(t)$$

Fixed singularity: $v(j)$ – corrections to LLA from CDs

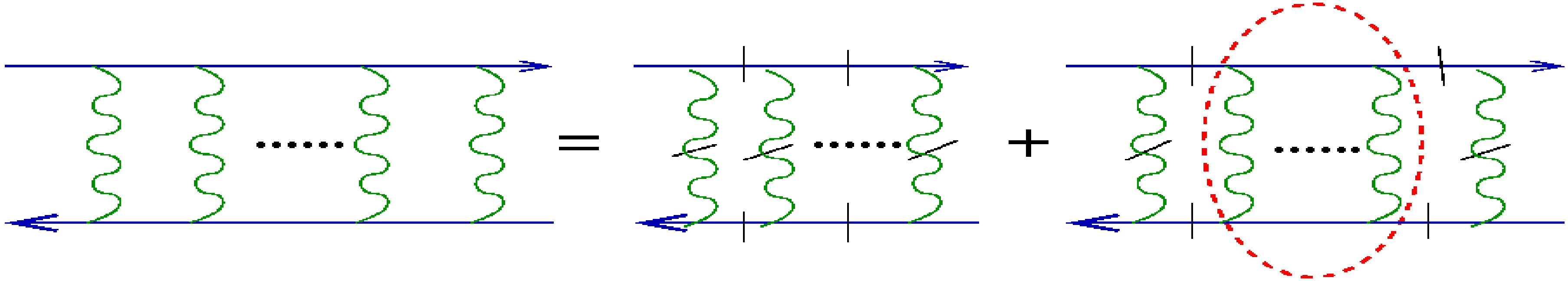
Moving Regge poles: $\text{Det}\left[1 - v(j)B(t)\right] = 0;\ j = \alpha\left(t, \mu^2, g_s^2\right)$

due to LD contributions

Which are dominant is unknown

Consider now SD-subgraph $v(j)$

$$jv(j) = h + c\psi(j)c - v(j);\ \psi(j) = v(j) + v(j)b\psi(j)$$



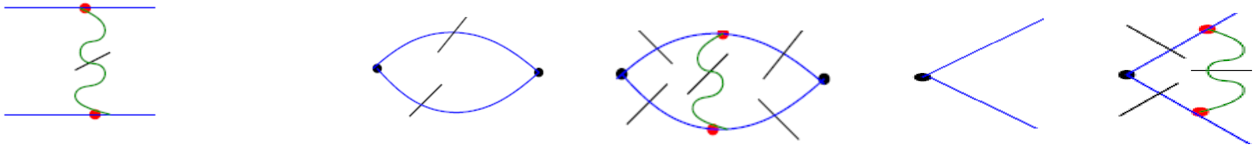
$$(j+1)v(j) = h + cv(j)\left[1 - v(j)b\right]^{-1} c;\ \ v(j) = \frac{h}{u(j)}$$

$$u(j) = \frac{1}{2}\left[(j + hb + 1 - c^2) + \sqrt{(j - hb + 1 - c^2)^2 - 4hbc^2}\ \right]$$

Fixed branch points in j-plane:

$$j = \pm 2c\sqrt{hb} + hb - (1 - c^2) \approx \pm g$$

In PQCD $h \approx g^2 + \cdots,\ b \approx 1 + g^2 + \cdots,\ c \approx 1 + g^2 + \cdots$



UV-divergences

Inside SD-subgraphs generates additional poles $\frac{1}{j}$. Regularization with μ_R, gives RG-equation for $v(j, \mu_R, \mu, g)$

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g) \frac{\partial v}{\partial g} - 4\gamma_q\right) v(j, \mu_R, \mu, g) = 0$$

Put $\mu_R = \mu$. RG-equation

$$\beta(g) \frac{\partial v}{\partial g} = (j + 1 + 4\gamma_q)\, v - \frac{cvc}{1 - vb} - h$$

Fixed condensed poles for asymptoticaly free $g_s^2(\mu)$.

Ginzburg, Serbryakov YaF3(66)164; Gizburg,Vasjev YaF5(67)669; 22(74)575 Berger, de Calan PRD20(79)2047

Regge Trajectories

$$\text{Det}\,[u(j) - hB(t)] = 0; \;\; j = \alpha\left(t, \mu^2, g_s^2(\mu)\right)$$

where

$$\int d^4X \mathrm{e}^{irX} \langle p_1' | O_{a,\nu_1\cdots\nu_j}(X) O_{a,\mu_1\cdots\mu_j}(0) | p_1 \rangle_{R_\mu}$$

$$= j!\,\delta^{\{\nu_1\cdots\nu_j\}}_{\{\mu_1\cdots\mu_j\}} B_a(t,j) + \mathcal{O}(r)$$

Regge trajectories are RG-invariant

$$\alpha\left(t, \mu^2, g_s^2(\mu)\right) = \alpha\left(\frac{m^2}{t}, \frac{\Lambda^2}{t}, g_s^2(t)\right)$$

Some comments

• *If vacuum expectation values of some products of field operators are non-zero (vacuum condensate) only equation for B and C are changed (by adding of diagrams with insertion of condensate blobs)*

• *We cannot say what of the contributions is dominant, fixed singularity in j-plane (LLA type) or Regge pole. In this sense improvements of LLA, similar to NLLA, … could be misleading!*

• *In the gauge theory (QED and QCD) for singlet channel $j_0 = 1$ ("Azimov shift" due to possibility of double gluon exchange), and asymptotics of Pomeron amplitude (positive signature) is determined by complex mixtrure of small distance and pinch singularities. LL results were obtained,*
[Kuraev,Lipatov,Fadin,Sov.Phys.JETP45(1977)199],

but *NLLA results seem misleading!*

Odderon

• *Vice versa, the asymptotics of odderon (singlet channel, negative signature) has much simpler structure. It is determined by only SD singularities of diagrams at j=1.The corresponding direct analysis, starting from diagrams, was not done till now.*

• *In this sense the representation*

odderon = colored Pomeron + gluon

Looks for us misleading!

Instead of Conclusion

How to calculate *B(t)* and *C(t)* ???

- *Effective chiral models?*
- *QCD Sum Rule?*
- *Instanton models?*
- *Perturbative QCD (large* t)?

- *How all the things looks like in singlet channel with positive (Pomeron) and negative signature (odderon)??*

A lot of interesting and important things are before us!

Thank you for attention!