

*D*iffractive neutrino interactions

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Conserved currents

Conservation of vector current is rather obvious:

$$q_\mu j_\mu^V = q_\mu \bar{p}(k') \gamma_\mu n(k) = (m_n - m_p) \bar{p} n = 0 \quad (\text{up to QED corrections})$$

Conservation of axial current looks more problematic

$$q_\mu j_\mu^A = q_\mu \bar{p}(k') \gamma_\mu \gamma_5 n(k) = (m_n + m_p) \bar{p} \gamma_5 n \neq 0$$

Nevertheless, in the general form,

$$j_\mu^A = \bar{p}(k') [g_A \gamma_\mu \gamma_5 - g_P q_\mu \gamma_5] n(k)$$

the axial current can be conserved if

$$g_P(Q^2) = g_A(Q^2) \frac{2m_N}{Q^2}$$

The pole behavior shows presence of a massless Goldstone particle

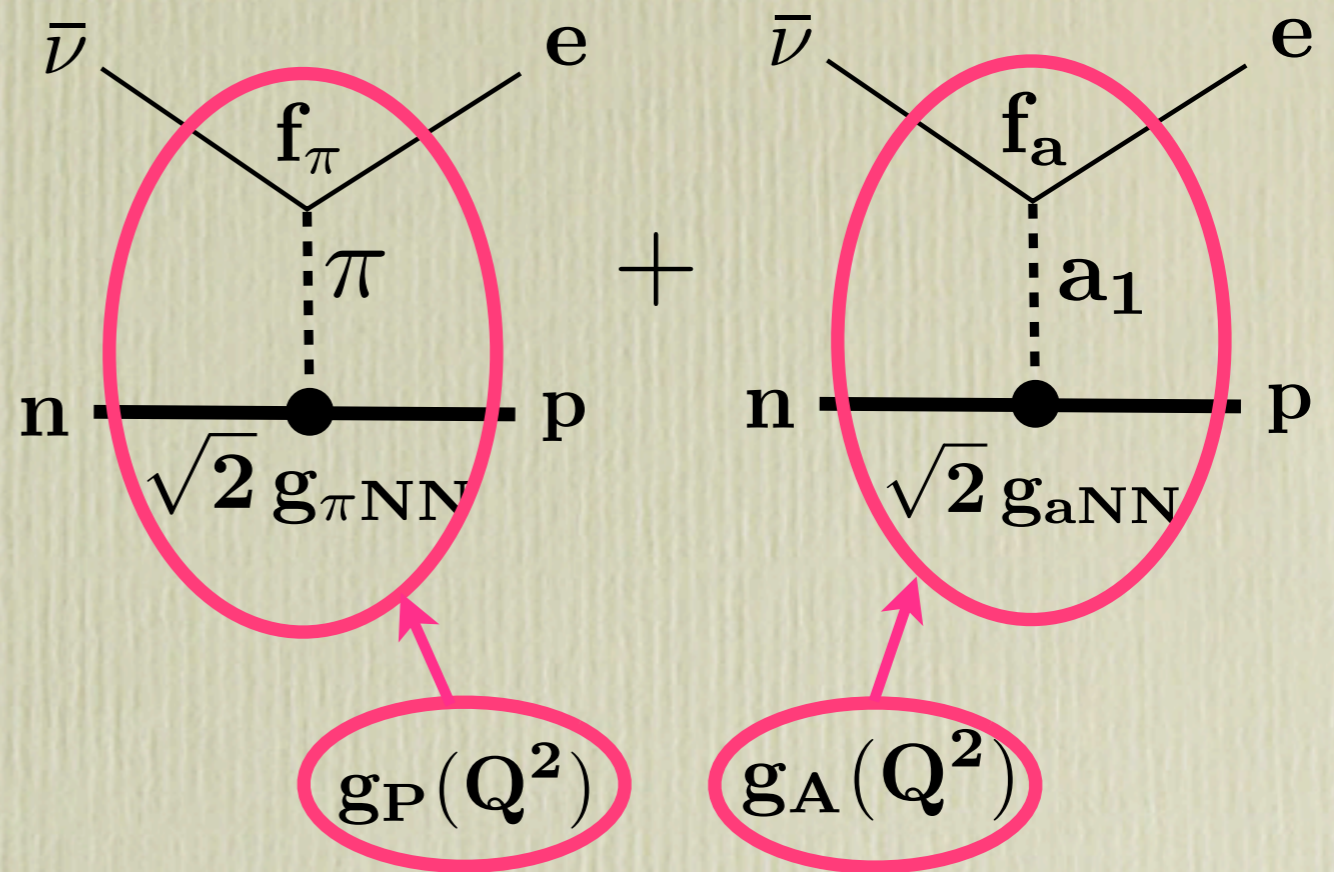
This proves the Goldstone theorem: spontaneous breaking of chiral symmetry generates massless particles identified with pions.

Goldberger-Treiman conspiracy

PCAC leads to a **miraculous** relation between the quantities having very different origin

$$\sqrt{2}m_N g_A(0) = f_\pi g_{\pi NN}$$

It is tempting to interpret the Goldberger-Treiman relation in terms of pion pole dominance.



However, the pion pole does not contribute to the β -decay, because the lepton current is conserved (up to the electron mass).

$$\Gamma(\pi \rightarrow \bar{\nu}e) \propto m_e^2$$

The axial-vector formfactor $g_A(Q^2)$ represents the contribution of heavy states, which are related to the pion term via PCAC.

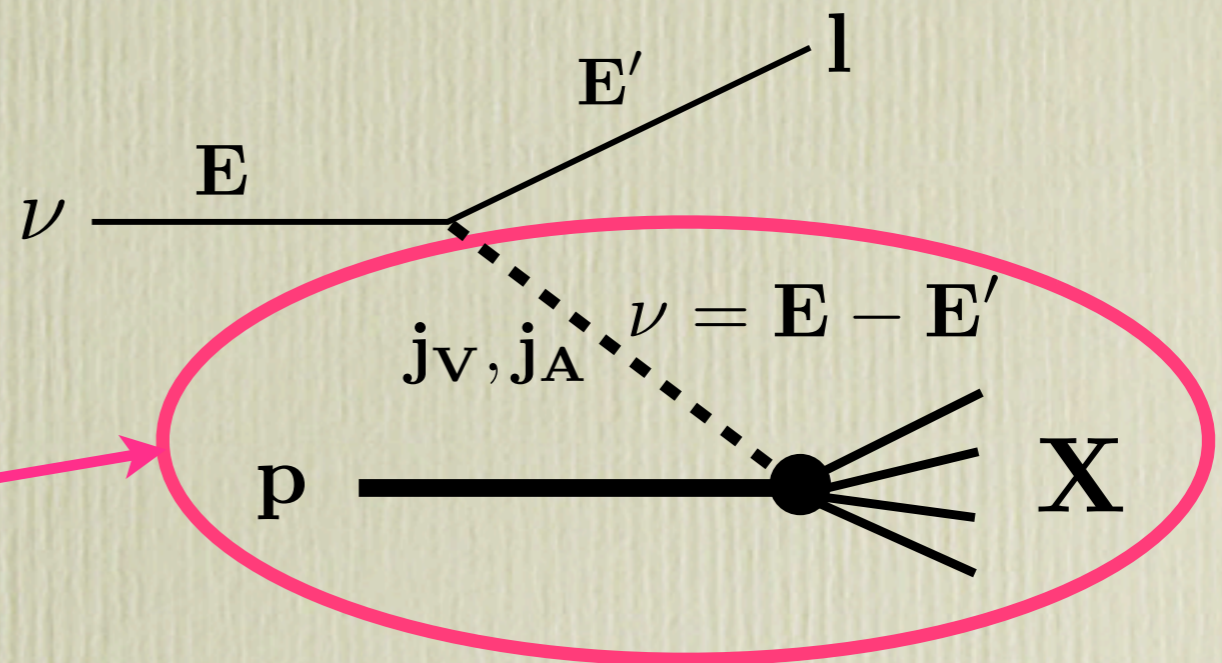
Hadronic properties of neutrinos

Although the non-trivial GT relation is well confirmed by data on neutron decay and muon capture, the PCAC hypothesis should be tested thoroughly in other processes.

The Fock components of a high-energy neutrino at low scale are dominated by the axial-vector hadronic fluctuations, since the vector term vanishes at $Q^2 \rightarrow 0$ due to CVC.

$$\nu + \mathbf{p} \rightarrow \mathbf{l} + \mathbf{X}$$

$$M = \frac{G}{\sqrt{2}} \mathbf{l}_\mu (\mathbf{V}_\mu + \mathbf{A}_\mu)$$



$$\mathbf{l}_\mu = \bar{\mathbf{l}}(\mathbf{k}') \gamma_\mu (\mathbf{1} + \gamma_5) \nu(\mathbf{k})$$

Adler relation

$$|\overline{\mathbf{M}}|^2_{Q^2 \rightarrow 0} = \frac{G^2}{2} \mathbf{L}_{\mu\nu} \mathbf{A}_{\mu\nu}; \quad \mathbf{L}_{\mu\nu}(Q^2 \rightarrow 0) = 2 \frac{\mathbf{E}_\nu(\mathbf{E}_\nu - \nu)}{\nu^2} \mathbf{q}_\mu \mathbf{q}_\nu$$

At $Q^2 \rightarrow 0$ the vector current contribution and the transverse part of the axial term vanish, only $\sigma_{\mathbf{L}}^{\mathbf{A}}$ survives, and

PCAC: $\mathbf{q}_\mu \mathbf{j}_\mu^{\mathbf{A}} = m_\pi^2 \phi_\pi$

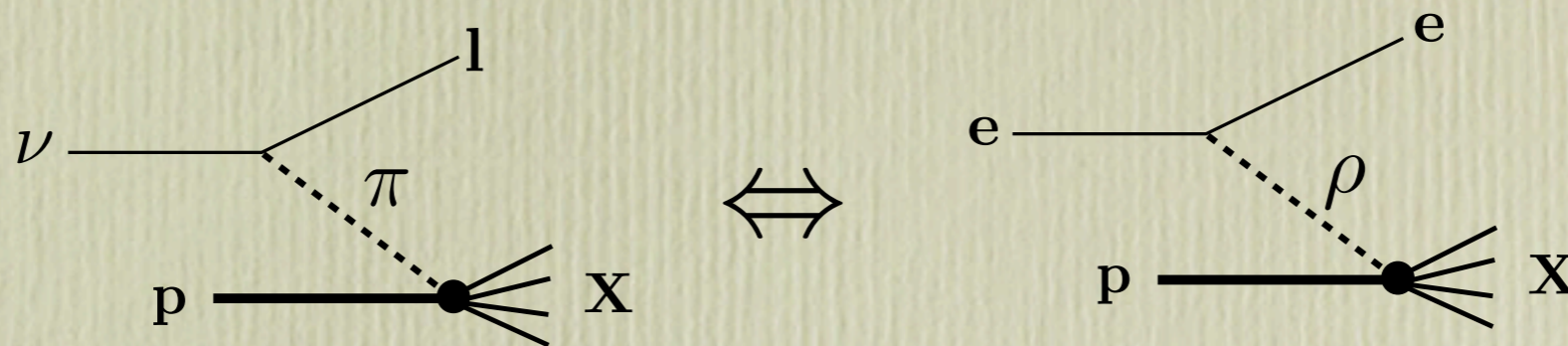
$$\mathbf{q}_\mu \mathbf{A}_{\mu\nu} \mathbf{q}_\nu = \frac{1}{\nu} \mathbf{f}_\pi \sigma(\pi \mathbf{p} \rightarrow \mathbf{X})$$

leading to the **Adler relation**:

$$\left. \frac{d^2 \sigma(\nu \mathbf{p} \rightarrow \mathbf{1} \mathbf{X})}{dQ^2 d\nu} \right|_{Q^2=0} = \frac{G^2}{2\pi^2} \mathbf{f}_\pi^2 \frac{\mathbf{E} - \nu}{\mathbf{E}_\nu} \sigma(\pi \mathbf{p} \rightarrow \mathbf{X})$$

Pion dominance?

In analogy to the vector dominance model it is tempting to interpret the Adler relation as a manifestation of pion dominance.



However, **neutrinos do not fluctuate to pions** because of conservation of the lepton current $q_\mu l_\mu = 0$

$$A_\mu = \frac{f_\pi q_\mu}{Q^2 + m_\pi^2} T(\pi p \rightarrow X) + \frac{f_{a_1}}{Q^2 + m_{a_1}^2} T_\mu(a_1 p \rightarrow X) + \dots$$

The pion pole contains factor q_μ and does not contribute

The contribution of heavy states

A_μ can be conserved only if these two terms are related and cancel each other in $q_\mu A_\mu$

Absorptive corrections to diffraction

Even if this relation $\sigma_{\text{diff}}(\pi p \rightarrow X p) \approx \sigma_{\text{el}}(\pi p \rightarrow \pi p)$

were accurate, it cannot hold for ever.

E.g. at very high energies in the Froissart regime

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \rightarrow \frac{1}{2}$$

while

$$\frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \rightarrow \frac{\text{const}}{\ln(s)}$$

Diffraction is suppressed by absorptive corrections, while elastic cross section is enhanced.

Nuclear targets

Most of neutrino experiments have been and will be performed on nuclear targets

The absorptive corrections to neutrino-nucleus interactions are tremendously enhanced.

On heavy nuclei the PCAC (Adler) condition, $\sigma_{\text{diff}}^{\pi A} \approx \sigma_{\text{el}}^{\pi A}$, is severely broken: diffraction vanishes, while the elastic cross section saturates at the maximal value allowed the unitarity bound.

$$\sigma_{\text{diff}}^{\pi A} \propto A^{1/3} \iff \sigma_{\text{el}}^{\pi A} \propto A^{2/3}$$

- Absorption enhances elastic, but suppresses inelastic diffraction.

Thus, the Adler relation is **incurable**: diffractive diagonal and off-diagonal amplitudes cannot be universally related, since they are affected by absorptive corrections differently.

Characteristic time scales

$$t_c^\pi = \frac{2\nu}{m_\pi^2 + Q^2} \gg t_c^A = \frac{2\nu}{Q^2 + m_A^2}$$

Controls the interference between pions produced in different points.

The \tilde{a}_1 -fluctuation lifetime

III

Correspondingly, there are **three** energy regimes

$$\nu > (Q^2 + m_A^2)R_A$$

$$\nu > 40\text{GeV}$$

II

$$(Q^2 + m_\pi^2)R_A < \nu < (Q^2 + m_A^2)R_A$$

Maximal shadowing

$$0.5 < \nu < 40\text{GeV}$$

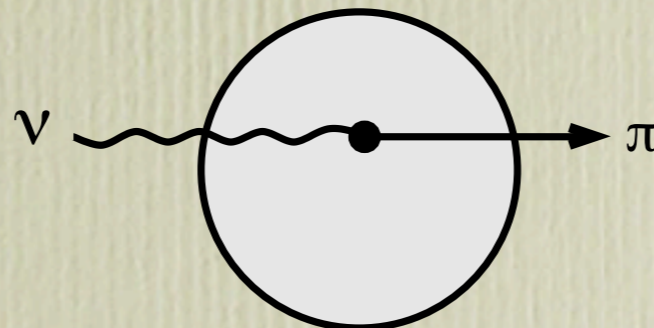
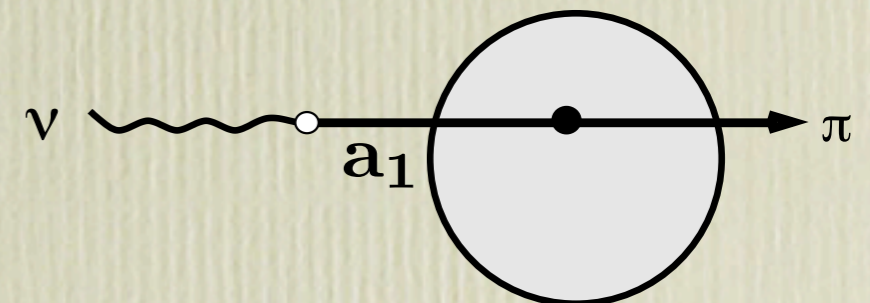
I

$$\nu < (Q^2 + m_\pi^2)R_A$$

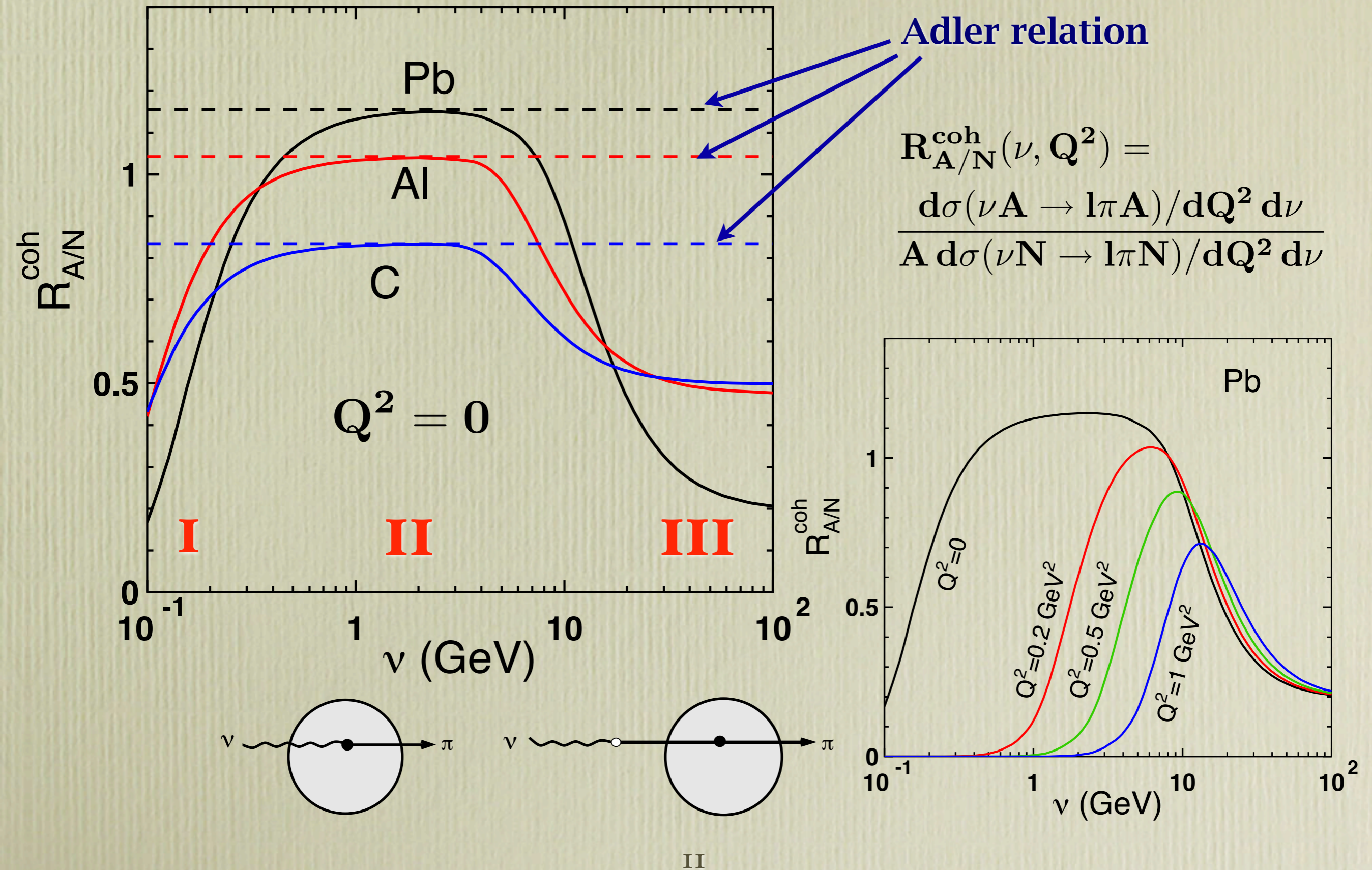
$$\nu < 500\text{MeV}$$

Coherent production is suppressed

Production is coherent, but without shadowing

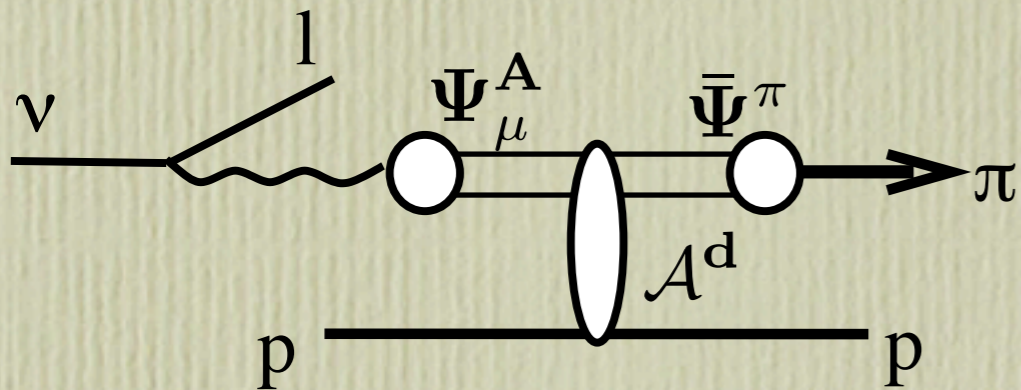


Coherent production of pions: $\nu A \rightarrow l\pi A$



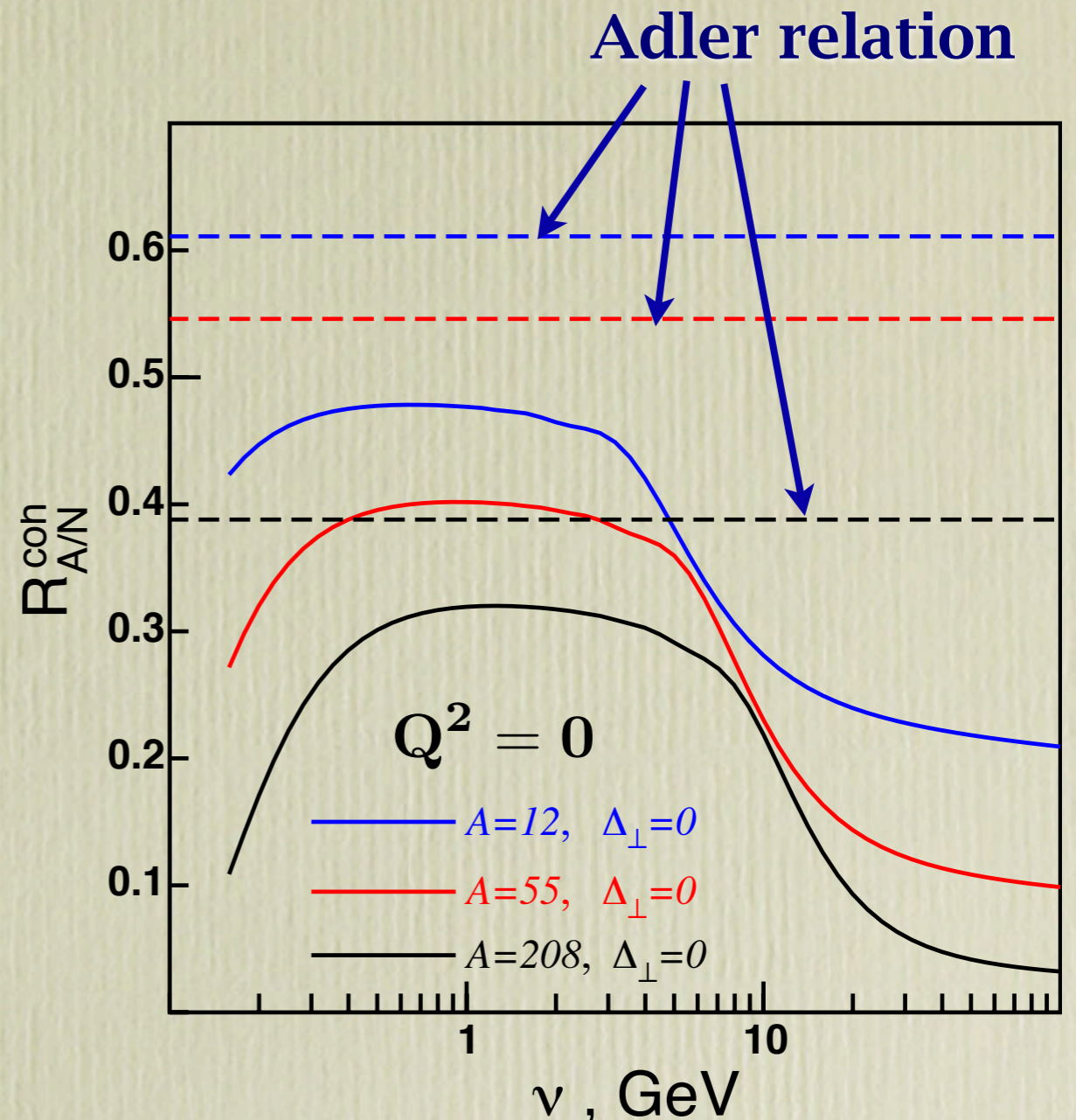
Color dipoles: more of PCAC breaking

$$\nu p \rightarrow l(\bar{q}q)p \rightarrow l\pi p$$



$$\nu \frac{d^3 \sigma_{\nu p \rightarrow \mu \pi p}}{d\nu dt dQ^2} = \frac{G_F^2 L_{\mu\nu} (W_\mu^A)^* W_\nu^A}{32\pi^3 m_N^2 E_\nu^2 \sqrt{1 + Q^2/\nu^2}}$$

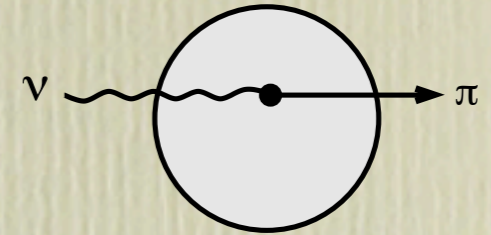
$$W_\mu^A(s, \Delta, Q^2) = \int_0^1 d\beta d^2r \bar{\Psi}^\pi(\beta, r) \mathcal{A}^d(\beta, r; \Delta) \Psi_\mu^A(\beta, r)$$



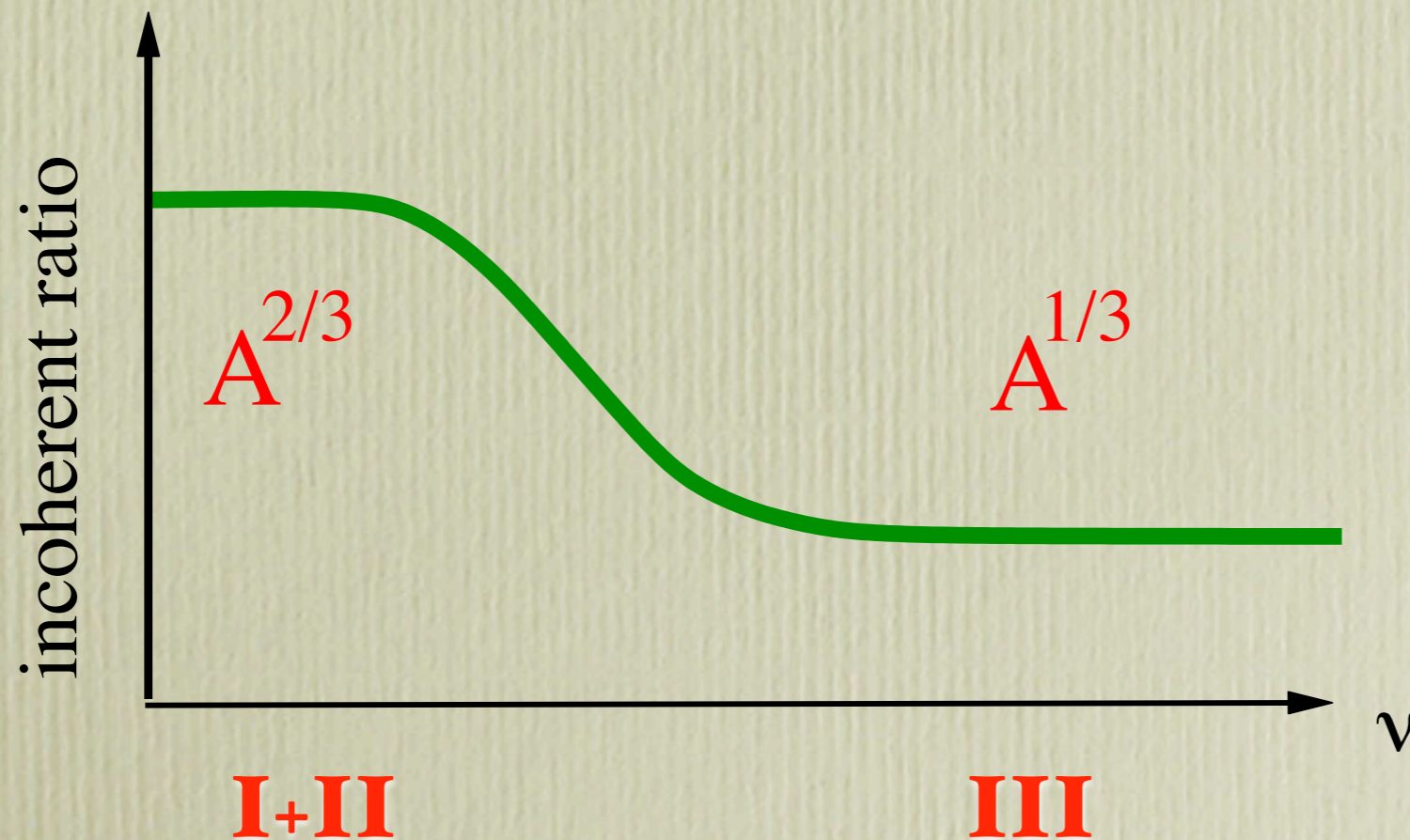
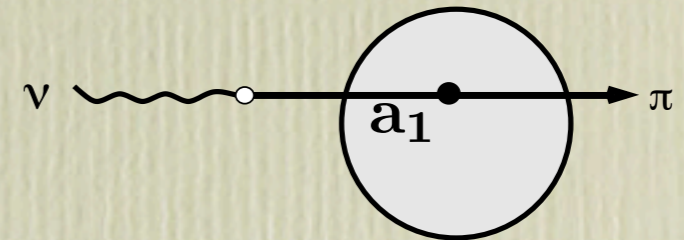
- The dipole amplitude \mathcal{A}^d is fitted to photoproduction and DIS data.
- The light-cone $\bar{q}q$ distribution amplitudes Ψ_μ^A , Ψ^π are calculated in the instanton vacuum model

Incoherent production of pions

Regime **I+II**, final state attenuation: $A^{2/3}$

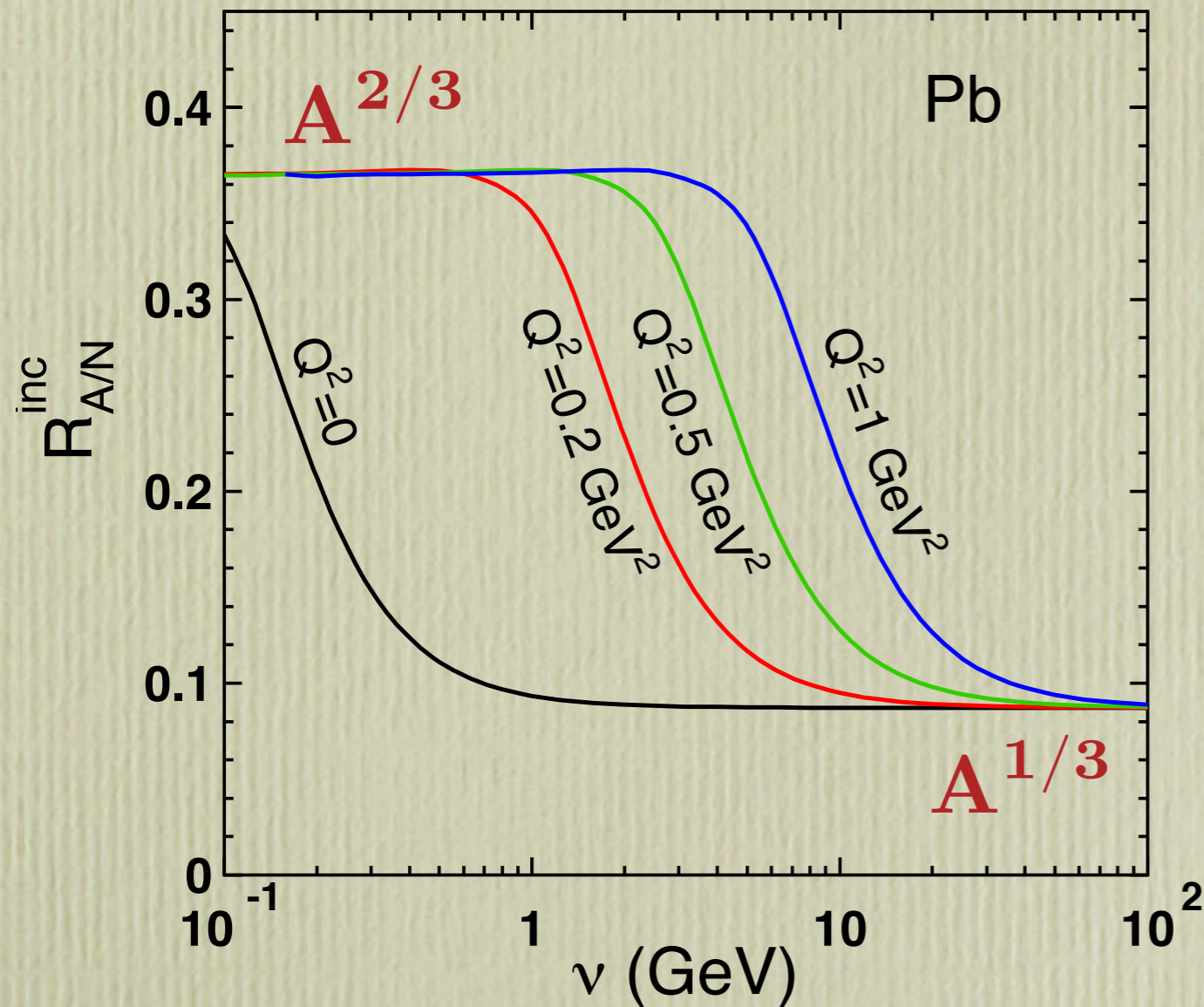


Regimes **III** both initial and final state attenuation: $A^{1/3}$

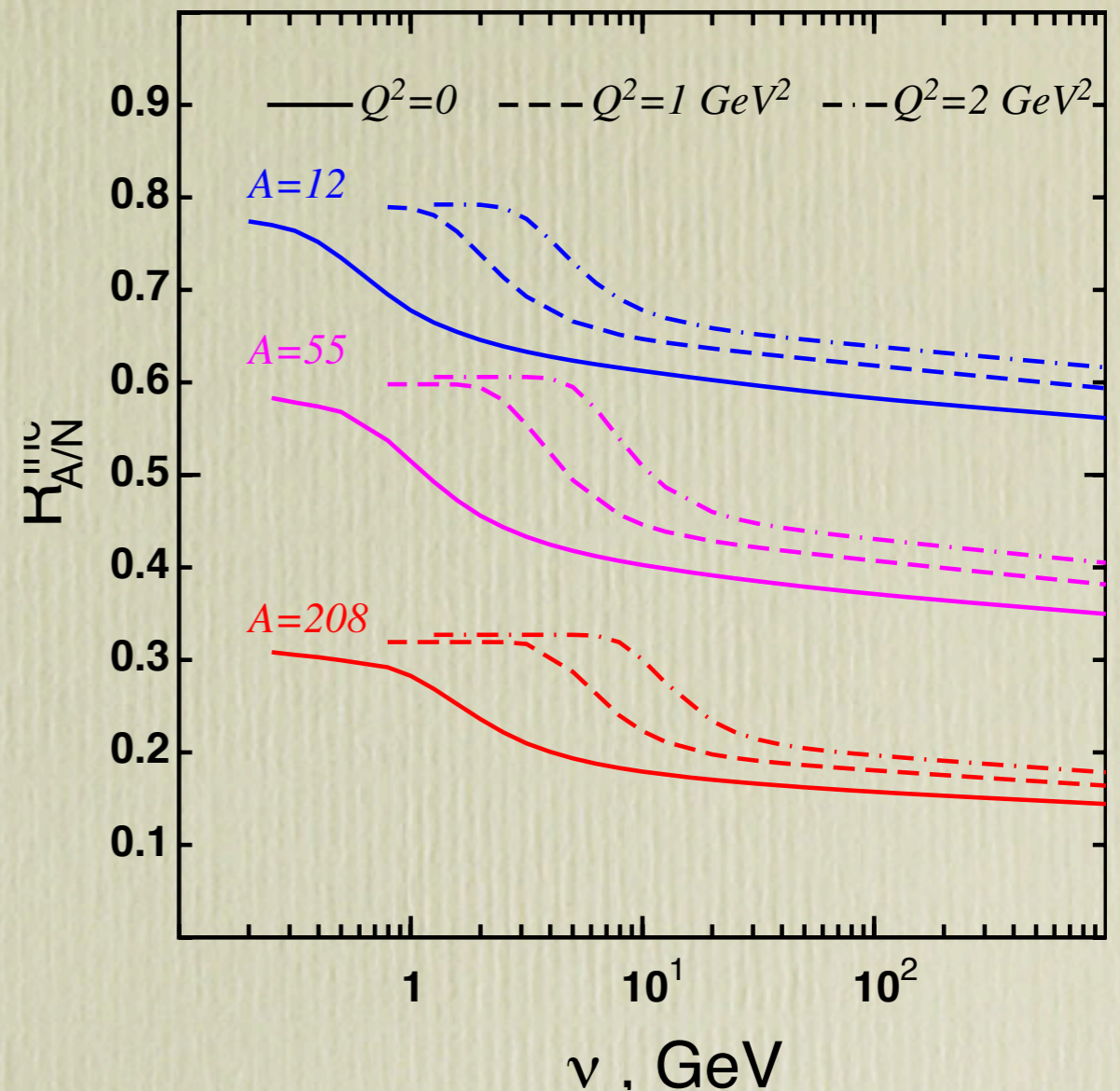


Incoherent production of pions: $\nu A \rightarrow l\pi A^*$

2-channel model



Dipole description



As function of energy $\sigma(\nu A \rightarrow l\pi A^*) \propto A^{2/3} \Rightarrow A^{1/3}$

Adler relation: $\sigma(\nu A \rightarrow l\pi A^*) \propto A^{1/3}$

is restored at high energies !

Summary

- ★ **The Goldberger-Treiman relation is not a result of pion exchange, which is suppressed in β -decay and muon capture. This is a result of a miraculous link between light and heavy states.**
- ★ **In the diffractive neutrino-production of pions PCAC establishes a link between diagonal and off-diagonal amplitudes, which cannot be correct, because both are strongly and differently affected by the absorption.**
- ★ **The Adler relation for coherent neutrino-production of pions is always broken, but especially at high energies. On the contrary, in incoherent pion production the Adler relation is broken at low, but is restored at high energies.**

Piketty-Stodolsky paradox

The first failure of PCAC

The Adler relation says that the combined contribution of all heavy axial states at $Q^2 \rightarrow 0$ should miraculously reproduce the pion pole.

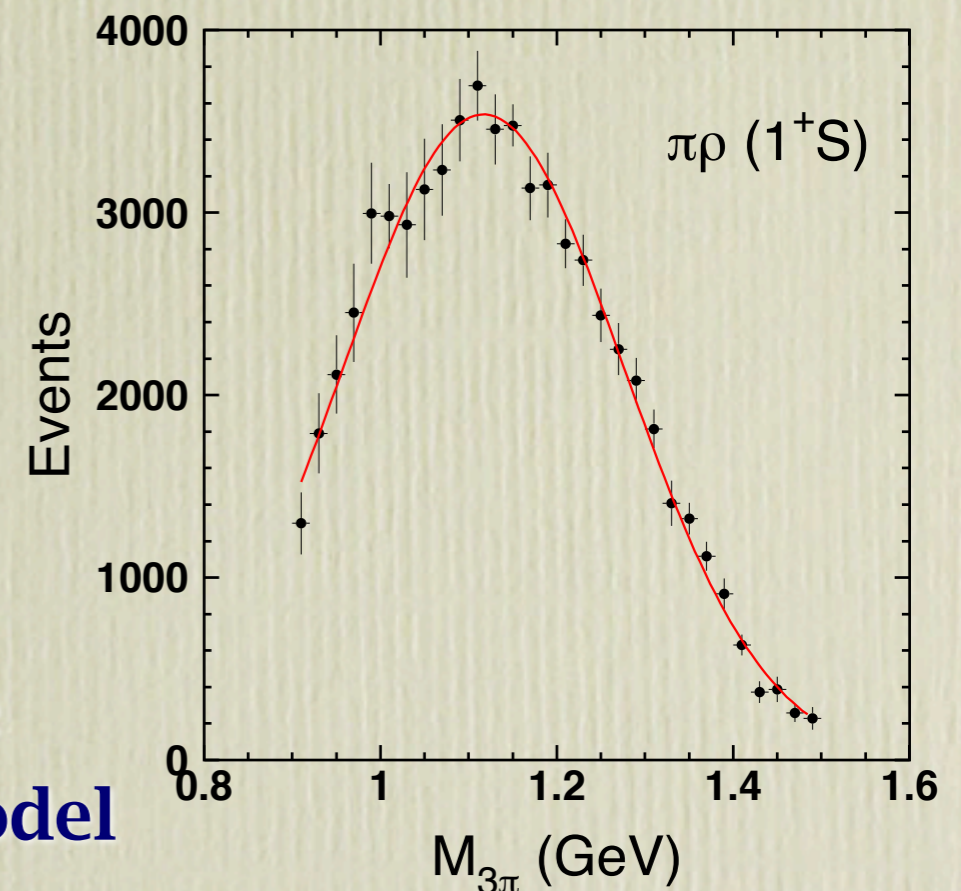
This unusual connection was challenged by Piketty & Stodolsky, who assumed a_1 pole dominance. Then PCAC leads to the

relation $\sigma_{\text{diff}}(\pi p \rightarrow a_1 p) \approx \sigma_{e1}(\pi p \rightarrow \pi p)$

which contradicts data by factor ~ 20 (!)

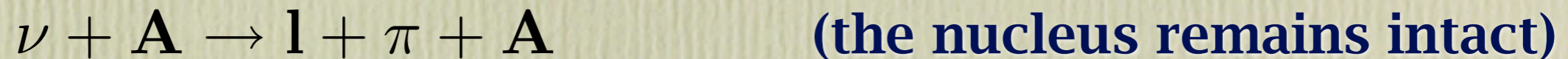
The problem is relaxed after inclusion of the $\rho\pi$ cut and other diffractive excitations into the dispersion relation. Indeed, the relation $\sigma_{\text{diff}}(\pi p \rightarrow X p) \approx \sigma_{e1}(\pi p \rightarrow \pi p)$ does not contradict data.

The $\rho\pi$ cut can be represented by an effective pole \tilde{a}_1 , so we arrive at a two-pole $(\pi + \tilde{a}_1)$ model



Diffraction neutrino-production of pions

Diffraction pion production on a nucleus may be **coherent**

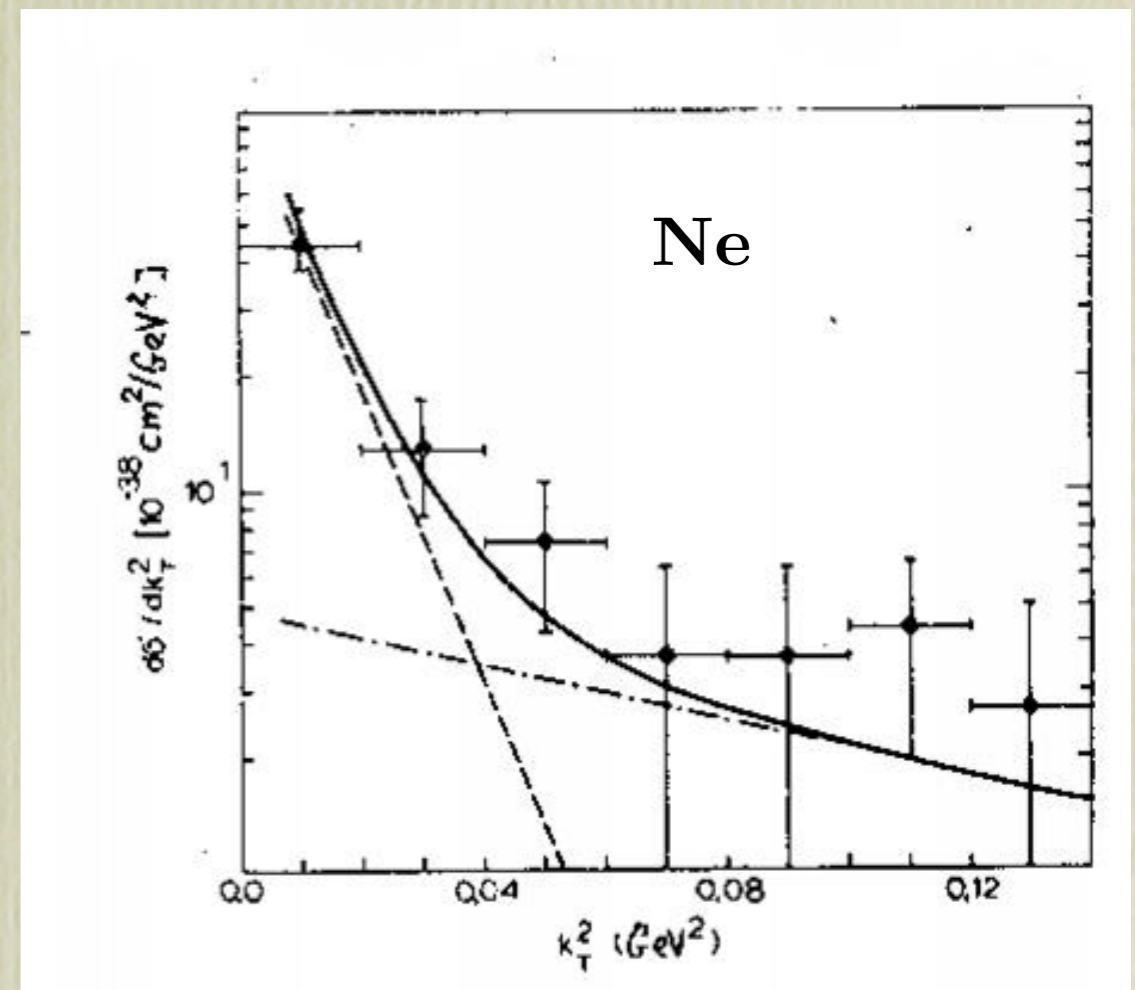


or **incoherent**



The two processes have very different p_T distributions, which help to separate them (statistically)

They also have different energy and Q dependences. Much can be learned from our experience with nuclear effects for vector current.



Absorption effects in $\nu p \rightarrow l\pi p$

Absorptive factor for the diffractive amplitude at impact parameter \mathbf{b}

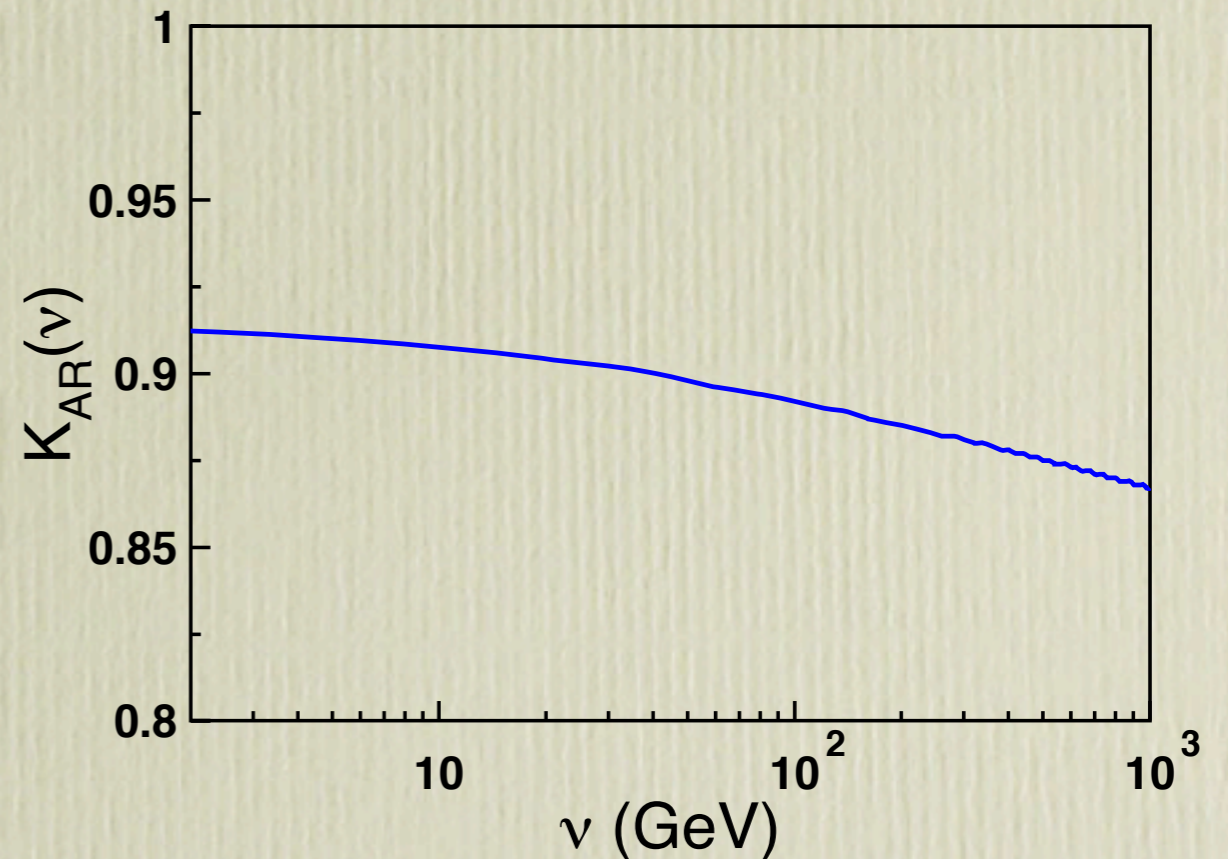
has the eikonal form $e^{-\text{Im} f^{\pi P}(\mathbf{b})}$, where $\text{Im} f^{\pi P}(\mathbf{b}) = \frac{\sigma_{\text{tot}}^{\pi P}}{4\pi B_{\text{el}}^{\pi P}} e^{-b^2/2B_{\text{el}}^{\pi P}}$

$$\sigma_{\text{diff}}^{\pi P}(\mathbf{b}) = [\sigma_{\text{diff}}^{\pi P}(\mathbf{b})]_0 e^{-2\text{Im} f^{\pi P}(\mathbf{b})} = \sigma_{\text{el}}^{\pi P}(\mathbf{b}) e^{-2\text{Im} f^{\pi P}(\mathbf{b})}$$

absorption Adler relation

$$\sigma_{\text{el}}(\mathbf{b}) \equiv \frac{d\sigma_{\text{el}}}{d^2\mathbf{b}} = \left| 1 - e^{-\text{Im} f(\mathbf{b})} \right|^2$$

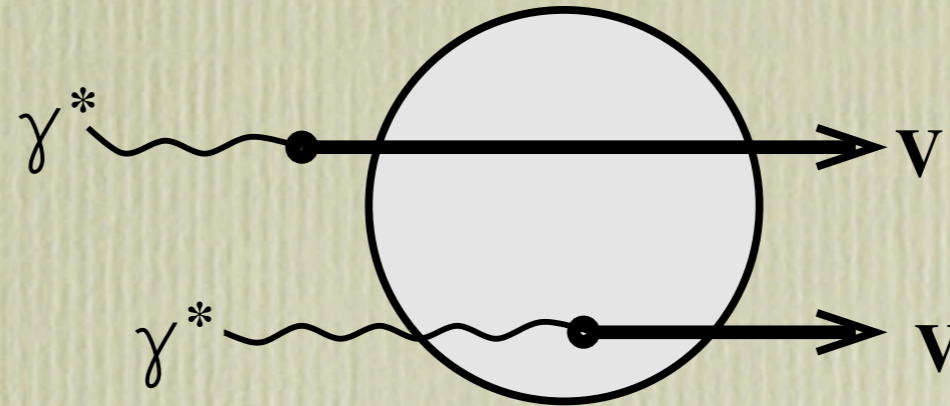
$$\begin{aligned} K_{\text{AR}} &\equiv \frac{\sigma(\nu p \rightarrow l\pi p)}{\sigma_{\text{AR}}(\nu p \rightarrow l\pi p)} = \frac{\sigma_{\text{diff}}^{\pi P}}{\sigma_{\text{el}}^{\pi P}} \\ &= \frac{\int d^2\mathbf{b} \left| 1 - e^{-\text{Im} f^{\pi P}(\mathbf{b})} \right|^2 e^{-2\text{Im} f^{\pi P}(\mathbf{b})}}{\int d^2\mathbf{b} \left| 1 - e^{-\text{Im} f^{\pi P}(\mathbf{b})} \right|^2} \end{aligned}$$



Diffractive electro-production

The important time scale,

$$t_c^\rho = \frac{2\nu}{m_\rho^2 + Q^2}$$



$t_c \gg R_A$ $A^{1/3}$

$t_c \ll R_A$ $A^{2/3}$

