BFKL EVOLUTION AS A COMMUNICATOR BETWEEN HIGH AND LOW ENERGY SCALES

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Thresholds for new physics below such scales will affect the quality of the fit of HERA data to the results of such an analysis.
This could be an indirect signal for such BSM physics

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  which is solved in terms of a set of eigenfunctions subject to given boundary conditions.
  No kinematic constraints are imposed of the eigenfunctions in the determination of the eigenvalues.
- The BFKL equation is supplemented by an infrared boundary condition, which consists of imposing a given phase for the oscillations of the eigenfunctions at low transverse momentum.
  This is in contrast to the more usual approach in which a hard "cut" is placed on the eigenfunctions at some low transverse momentum.

L.O. BFKL equation with (L.O.) running coupling

$$\beta\omega \ln\left(k^2/\Lambda^2\right) f_\omega(k) \;=\; \int \frac{dk'}{k'}\,\mathcal{K}(k,k')f_\omega(k')$$

$$f_\omega(k) \;=\; \int_{\mathcal{C}} d\nu \left(k^2\right)^{i\nu} \exp\left\{ \frac{i}{\beta\omega}\int^{\nu} \chi_0(\nu')d\nu' \right\}$$

For $k > k_{crit}$, steepest descent contour $\mathcal{C}$ runs parallel to imaginary axis leading to exponentially decaying function as $k \to \infty$

$$f_\omega(k) \;\sim\; (k^2)^{-\kappa(k,\omega)}, \;\; (k > k_{crit})$$

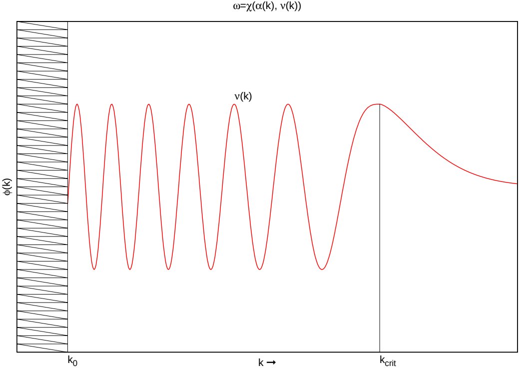
This must be matched to the phase of the oscillatory behaviour for $k < k_{crit}$

$$f_\omega \;=\; e^{i\phi}(k^2)^{i\nu(k,\omega)} \;\; (k < k_{crit})$$

$$k_{crit} \;=\; \Lambda e^{\chi_0(0)/(2\beta\omega)}$$

(rapidly increases as $\omega \to 0$)

Assume that at some low-$k = k_0$, IR (non-perturbative) features of QCD fix the phase, η of eigenfunctions.
(Lipatov '86)



Leads to a discrete spectrum of Regge poles

$$\sum_i f_{\omega_i}(k^2) s^{\omega_i}$$

An example of how the infrared phase may be imposed can be seen in a model in which infrared physics generates an effective "mass", m for the gluons - thereby breaking the conformal invariance in the transverse space.
In the conformal limit, we may write the BFKL equation (in impact parameter, ρ, space) in the form

$$\omega f_\omega(\rho) \ = \ -\alpha_s \left[\ln(\hat{p}^2) + \ln(\rho^2) + 2\gamma_E \right] f_\omega(\rho), \qquad \hat{p} = -i \frac{d}{d\rho}$$

If a mass is introduced this becomes

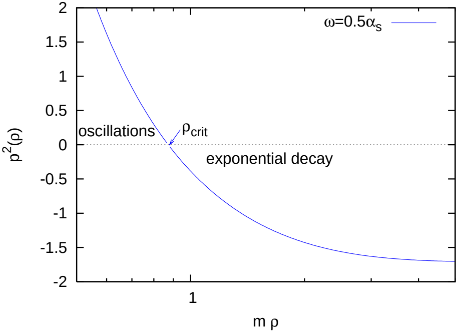
$$\omega f_\omega(\rho) \ = \ \alpha_s \left[2\omega_g(\hat{p}^2) f_\omega(\rho) - 2K_0(m\rho) f_\omega(\rho) \right]$$

$$\omega_g(p^2) \ = \ -\frac{(p^2+m^2)}{p\sqrt{(}p^2+4m^2)} \ln\left(\frac{\sqrt{p^2+4m^2}+p}{4m^2} \right)$$

(Gluon Regge trajectory)
In the semi-classical approach, we solve for $p(\rho)$ to obtain

$$f(\rho) \ \sim \ e^{i\int^\rho p(\rho')d\rho'}$$

For any ω, there exists a value, ρ_{crit} where $p^2 = 0$



$$\text{For } p^2(\rho) > 0,\ (\rho < \rho_{crit}),\quad f_\omega(\rho)\ \sim\ \sin(\nu_\rho \rho + \eta)$$

$$\text{For } p^2(\rho) < 0,\ (\rho > \rho_{crit}),\quad f_\omega(\rho)\ \sim\ e^{-\kappa_\rho \rho}$$

Matching the solutions at $\rho = \rho_{crit}$ determines the phase, η of the oscillations.

This is not a particularly realistic model and so we allow ourselves to fit the infrared phase as a function of eigenvalue, $\eta(\omega)$ to HERA data, within a given parameterization

Infrared phase at $k = k_0$ for eigenfunction n

$$\eta_n(k_0) = \eta_2(n-1)^{\kappa}, \quad \text{[3 parameters } ,k_0, \eta_2, \kappa]$$

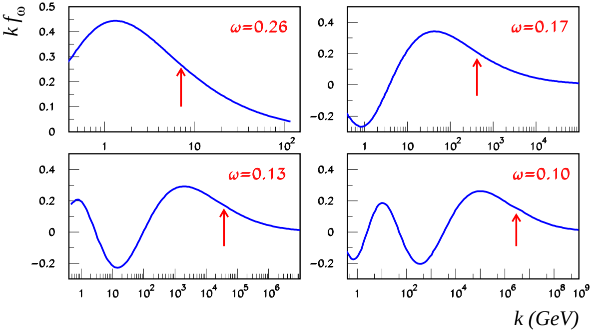
Also need the Proton impact factor (coupling of proton to QCD pomeron)

$$\Phi_p(k) = Ak^2 e^{-bk^2} \quad \text{[2 parameters } ,A, b]$$

108 data points with $x < 0.01$ and $Q^2 > 8\,\mathrm{GeV}^2$ (to avoid saturation)

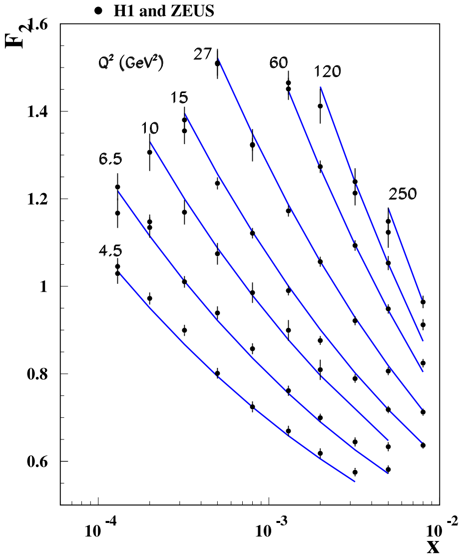
103 DoF

First Four Eigenfunctions

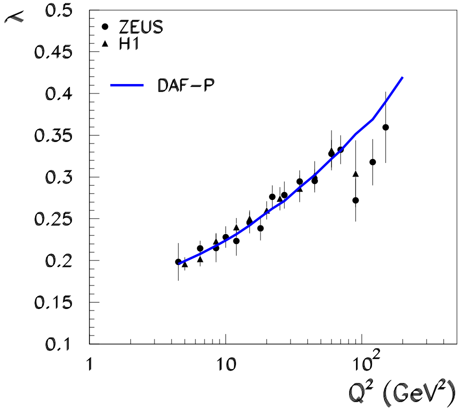


For large n

$$\omega_n \approx \frac{a}{n+\eta(\omega_n)} \quad (0 \leq \eta \leq \pi)$$


H1 and ZEUS
F2
Q² (GeV²)
4.5
6.5
10
15
27
60
120
250
1.6
1.4
1.2
1
0.8
0.6
10 -4
10 -3
10 -2
x

$$F_2(x,Q^2) \overset{x\to 0}{\to} \left(\frac{1}{x}\right)^{\lambda(Q^2)} \quad \left(= \sum_n a_n(Q^2)x^{-\omega_n}\right)$$



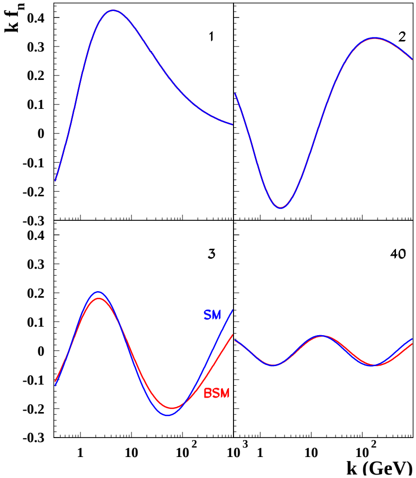
What happens if there is new physics (e.g. MSSM SUSY) below $k = k_{crit}$?

β-function is affected by loops of SUSY particles at 1 and 2-loop levels.
The value of k_{crit} (where oscillatory behaviour becomes and exponential suppression) is strongly affected.

$$k_{crit} \propto e^{\sim 4n}, \;\; > \;\; E_{BSM} \;\; \text{for} \;\; n > \sim 3$$

This, in turn, has a very significant effect on the allowed eigenvalues, ω
BFKL kernel (at NLO) also acquires corrections due to loops of SUSY particles
Affects the eigenvalues of the BFKL kernel, and amends the phases of the oscillatory part of the eigenfunctions.

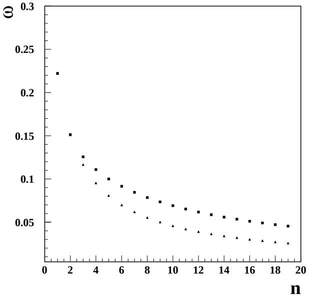
If $k_{crit} > M_{SUSY}$ eigenfunctions of BFKL kernel are affected



which in turn affects the quality of the fit to HERA data.

	SM	SUSY at 10 TeV
k_0 (MeV)	294	275
η_2	0.16	0.2
κ	0.6	0.56
A	813	693
$b\,(GeV^{-2})$	18.2	17.4

But there are significant changes to the eigenvalues



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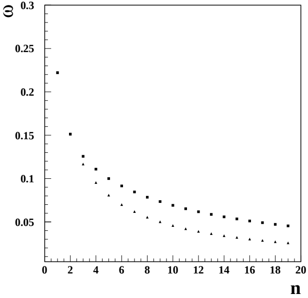
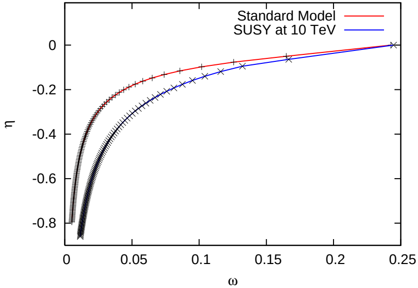
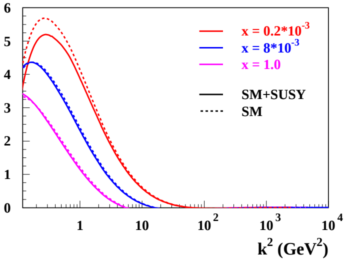
Dependence of infrared phase η on eigenvalue ω is significantly different

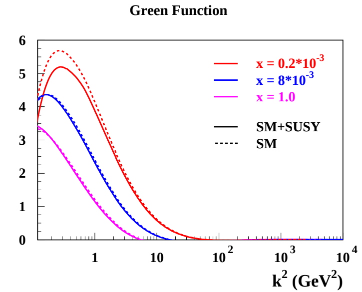




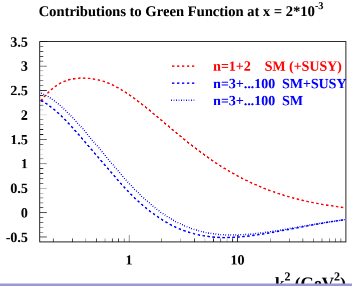
$$\omega_n \approx \frac{a}{n+\eta(\omega_n)}$$

Green Function





Green Function
6
5
4
3
2
1
0
x = 0.2*10^-3
x = 8*10^-3
x = 1.0
SM+SUSY
SM
1
10
10^2
10^3
10^4
k^2 (GeV^2)



Contributions to Green Function at x = 2*10^-3
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
n=1+2 SM (+SUSY)
n=3+...100 SM+SUSY
n=3+...100 SM
1
10
k^2 (GeV^2)

Change in quality of fit for different SUSY thresholds (103 DoF)

SUSY SCALE (TeV)	χ^2
3	126
6	114
10	110
30	118
∞ (SM)	123

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- The fit does NOT improve if we assume MSSM within the LHC region.
- The fit DOES improve considerably if we assume MSSM with a threshold ~ 10 TeV
- This type of analysis is a useful tool for the investigation of signals for BSM physics from experiments far below the threshold for such BSM physics