Mueller-Navelet jets in high-energy hadron collisions

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Outline

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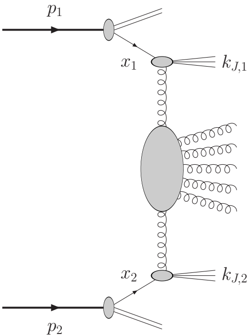
Mueller-Navelet jets: kinematics



- $\vec{k}_{J,1}^{\,2} \sim \vec{k}_{J,2}^{\,2} \gg \Lambda^2_{\rm QCD} \quad \longrightarrow \quad$ pQCD applicable
- $Y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|} \gg 1, \qquad \longleftrightarrow \qquad s = 2p_1 \cdot p_2 \gg \vec{k}_{J,1\,2}^{\,2}$

$\longrightarrow$ BFKL resummation: $\sum_n c_n \; \alpha_s^n \ln^n s + d_n \; \alpha_s^n \ln^{n-1} s$

Mueller-Navelet jets: factorization

- QCD collinear factorization

$$\frac{d\sigma}{dx_{J1}dx_{J2}d^2\vec{k}_{J1}d^2\vec{k}_{J2}} = \sum_{i,j=q,\bar{q},g}\int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1,\mu) f_j(x_2,\mu)\frac{d\hat{\sigma}_{i,j}(x_1 x_2 s,\mu)}{dx_{J1}dx_{J2}d^2\vec{k}_{J1}d^2\vec{k}_{J2}}$$

- Cross section of the hard process in the BFKL approach

$$\frac{d\hat{\sigma}_{i,j}(x_1 x_2 s,\mu)}{dx_{J1}dx_{J2}d^2\vec{k}_{J1}d^2\vec{k}_{J2}} = \frac{s}{(2\pi)^2}\int\frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_i(\vec{q}_1, s_0, x_1; \vec{k}_{J1}, x_{J1})\int\frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_j(-\vec{q}_2, s_0, x_2; \vec{k}_{J2}, x_{J2})$$

$$\times \int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1,\vec{q}_2)$$

$\Rightarrow$ Valid both in
LLA (resummation of all terms $(\alpha_s \ln s)^n$)
NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

Outline

State of the art

- Observables of interest:
  - Differential cross section
  - Moments of the azimuthal decorrelation of the jets
- Convolution of the NLA Green's function with the LO jet vertices

  [A. Sabio Vera (2006)]

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- Full NLO calculation

  [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

Our work

Convolution of the NLA Green's function with the NLO jet vertices in the small cone approximation (SCA)

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Our work

Convolution of the NLA Green's function with the NLO jet vertices in the small cone approximation (SCA)

Novelties

- We use the jet vertex in the SCA $\longrightarrow$ small jet cone aperture in the rapidity-azimuthal angle plane : [D.Yu. Ivanov, A. Papa (2012)]
  - Easily implementable in numerical calculations
  - Particularly suitable for a semi-analytical cross-check of the numerical approaches which treat the cone size exactly

- The parameters s_0 and μ_R that enters the terms beyond NLA are not fixed a priori $\longrightarrow$ we determine their values with an optimization method

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Solution of the BFKL equation

- The Green's function obeys the BFKL equation

$$\delta^2(\vec{q}_1-\vec{q}_2)=\omega\, G_\omega(\vec{q}_1,\vec{q}_2)-\int d^2\vec{q}\, K(\vec{q}_1,\vec{q})\, G_\omega(\vec{q},\vec{q}_2)$$

- Transverse momentum space definition

$$\hat{\vec{q}}\,|\vec{q}_i\rangle=\vec{q}_i|\vec{q}_i\rangle\,,\quad \langle\vec{q}_1|\vec{q}_2\rangle=\delta^{(2)}(\vec{q}_1-\vec{q}_2)\,,\quad \langle A|B\rangle=\langle A|\vec{k}\rangle\langle\vec{k}|B\rangle=\int d^2k A(\vec{k})B(\vec{k})$$

- The kernel operator $\hat{K}$ is

$$K(\vec{q}_2,\vec{q}_1)=\langle\vec{q}_2|\hat{K}|\vec{q}_1\rangle$$

- Straightforward solution in the transverse momentum space

$$\hat{1}=(\omega-\hat{K})\hat{G}_\omega \quad\longrightarrow\quad \hat{G}_\omega=(\omega-\hat{K})^{-1}\,,\qquad \hat{K}=\bar{\alpha}_s\hat{K}^0+\bar{\alpha}_s^2\hat{K}^1\,,\quad \bar{\alpha}_s=\frac{\alpha_s N_c}{\pi}$$

- Solution for the $\hat{G}_\omega$ operator with NLA accuracy

$$\hat{G}_\omega=(\omega-\bar{\alpha}_s\hat{K}^0)^{-1}+(\omega-\bar{\alpha}_s\hat{K}^0)^{-1}\left(\bar{\alpha}_s^2\hat{K}^1\right)(\omega-\bar{\alpha}_s\hat{K}^0)^{-1}+\mathcal{O}\left[\left(\bar{\alpha}_s^2\hat{K}^1\right)^2\right]$$

- Basis of eigenfunctions of the LLA kernel

$$\hat{K}^0|n,\nu\rangle = \chi(n,\nu)|n,\nu\rangle\,, \qquad \chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

$$\langle\vec{q}\,|n,\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\;2}\right)^{i\nu-\frac{1}{2}} e^{in\theta}\,, \qquad \cos\theta \equiv q_x$$

$$\langle n',\nu'|n,\nu\rangle = \int \frac{d^2\vec{q}}{2\pi^2}\left(\vec{q}^{\;2}\right)^{i\nu-i\nu'-1} e^{i(n-n')\theta} = \delta(\nu-\nu')\,\delta_{nn'}$$

- The action of the full NLA BFKL kernel on these functions may be expressed as follows:

$$\begin{aligned}\hat{K}|n,\nu\rangle \quad &= \quad \bar{\alpha}_s(\mu_R)\chi(n,\nu)|n,\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(n,\nu) + \frac{\beta_0}{4N_c}\chi(n,\nu)\ln(\mu_R^2)\right)|n,\nu\rangle \\ &+ \quad \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(n,\nu)\left(i\frac{\partial}{\partial\nu}\right)|n,\nu\rangle\,,\end{aligned}$$

Impact factors

- Projection of the impact factors $\phi_i(\vec{q})$ onto the eigenfunctions of LO BFKL kernel, i.e. the transfer to the (ν, n)-representation

$$c_i(\nu,n) = \int d^2\vec{q}\,\frac{\phi_i(\vec{q})}{\vec{q}^{\;2}}\frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\;2}\right)^{i\nu-\frac{1}{2}} e^{in\phi}\,.$$

Here ϕ is the azimuthal angle of the vector $\vec{q}$ counted from some fixed direction in the transverse space.

- Basis of eigenfunctions of the LLA kernel

$$\hat{K}^0|n,\nu\rangle = \chi(n,\nu)|n,\nu\rangle\,, \qquad \chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

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Final expressions

- Differential cross section in the exponentiated representation

$$\frac{d\sigma_{\rm exp}}{dy_{J1}dy_{J2}d|\vec{k}_{J1}|\,d|\vec{k}_{J2}|} = \frac{x_{J1}x_{J2}}{|\vec{k}_{J1}||\vec{k}_{J2}|}\int_{-\infty}^{+\infty} d\nu\ \exp\Bigg[\ln\left(\frac{x_{J1}x_{J2}s}{s_0}\right)\Bigg(\bar{\alpha}_s(\mu_R)\chi(0,\nu)$$
$$+\bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(0,\nu)+\frac{\beta_0}{8C_A}\chi(0,\nu)\left(-\chi(0,\nu)+\frac{10}{3}+\ln\frac{\mu_R^4}{\vec{k}_{J1}^2\vec{k}_{J2}^2}\right)\right)\Bigg)\Bigg]$$
$$\times\alpha_s^2(\mu_R)c_1(0,\nu,|\vec{k}_{J1}|,\phi_{J1},x_{J1},\mu_F)c_2(0,\nu,|\vec{k}_{J2}|,\phi_{J2},x_{J2},\mu_F)$$
$$\Bigg[1+\frac{\alpha_s(\mu_R)}{2\pi}\left(-\frac{\beta_0}{2}\ln\frac{\vec{k}_{J1}^2\vec{k}_{J2}^2}{\mu_R^4}+2C_A\chi(0,\nu)\ln\frac{s_0}{|\vec{k}_{J1}||\vec{k}_{J2}|}\right)$$
$$+\alpha_s(\mu_R)\left(\frac{c_1^{(1)}(0,\nu,|\vec{k}_{J1}|,\phi_{J1}=0,x_{J1},\mu_F)}{c_1(0,\nu,|\vec{k}_{J1}|,\phi_{J1},x_{J1},\mu_F)}+\frac{c_2^{(1)}(0,\nu,|\vec{k}_{J2}|,\phi_{J2}=0,x_{J2},\mu_F)}{c_2(0,\nu,|\vec{k}_{J2}|,\phi_{J2},x_{J2},\mu_F)}\right)\Bigg]$$

- The moments of the azimuthal decorrelations are given by

$$\langle\cos[m(\phi_{J1}-\phi_{J2}-\pi)]\rangle \equiv \langle\cos(m\varphi)\rangle = \frac{\mathcal{C}_m}{\mathcal{C}_0}$$

where

$$\mathcal{C}_m = \int_0^{2\pi} d\phi_{J1}\int_0^{2\pi} d\phi_{J2}\ \cos[m(\phi_{J1}-\phi_{J2}-\pi)]\,\frac{d\sigma}{dy_{J1}dy_{J2}\,d|\vec{k}_{J1}|\,d|\vec{k}_{J2}|d\phi_{J1}d\phi_{J2}}$$

Outline

Kinematical settings

- In order to compare our predictions with the forthcoming LHC data we fix

$$\sqrt{s} = 14\mathrm{TeV}\ , \qquad R = 0.5$$

- Following a quite recent CMS study [S. Cerci, D. dEnterria (2009)] we restrict the rapidities of the jets to the region

$$3 \le |y_J| \le 5\ , \qquad \text{with steps } \Delta y_J = 0.5$$

- Symmetric case:
  - $|\vec{k}_{J1}| = |\vec{k}_{J2}| = 35$ GeV $\longrightarrow$ comparison with previous results [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]
  - $|\vec{k}_{J1}| = |\vec{k}_{J2}| = 20$ GeV $\longrightarrow$ more undetected gluons radiated in the final state

- Asymmetric case: $|\vec{k}_{J1}| = 20$ GeV , $|\vec{k}_{J2}| = 35$ GeV

Outline

Our analysis

- We determine the dependence of the observables under study on $Y = y_{J,1} - y_{J,2}$, in particular we consider the interval from 6 to 10.
- Large NLA corrections $\longrightarrow$ optimization needed!
- If we keep just the NLA terms we don't have any dependence on the scales μ_R and s_0
- Unavoidable terms beyond NLA depend on μ_R and s_0, whose numerical impact can be non-negligible

PMS

We take as optimal choices for μ_R and s_0 those values for which the physical observable under exam exhibits the minimal sensitivity to changes of both these scales [P.M. Stevenson (1981)]

The complete resummation of the perturbative series would not depend on μ_R and s_0
$\Longrightarrow$ PMS is supposed to mimic the effect of the most relevant unknown subleading terms

$$Y_0 = \ln \frac{s_0}{|\vec{k}_{J1}||\vec{k}_{J2}|} = 0, 1, 2, \ldots \qquad \mu_R = N\sqrt{|\vec{k}_{J1}||\vec{k}_{J2}|} \qquad N = 1, 2, 3, \ldots$$

$\Longrightarrow$ For each fixed value of Y we look for a stationary point in the $N - Y_0$ plane

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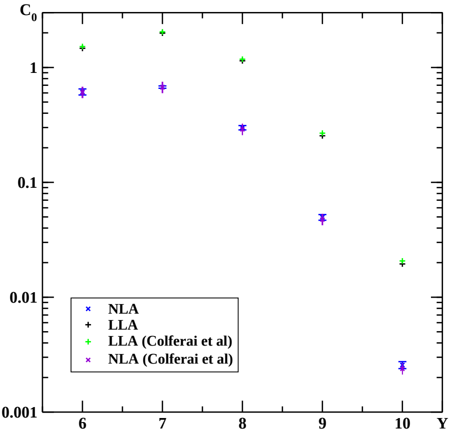
Results: $|\vec{k}_{J1}| = |\vec{k}_{J2}| = 35$ GeV

Y	Y_0	$\mu_R/\sqrt{|\vec{k}_{J1}||\vec{k}_{J2}|}$
6	1	6
7	1	4
8	2	3
9	3	3
10	4	3

Values of the parameters corresponding to our "optimal" values

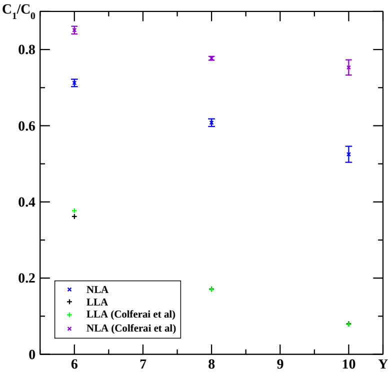
$\longrightarrow$ We have found wide regions of stability

LLA: $\mu_R^2 = s_0 = |\vec{k}_{J1}||\vec{k}_{J2}|$

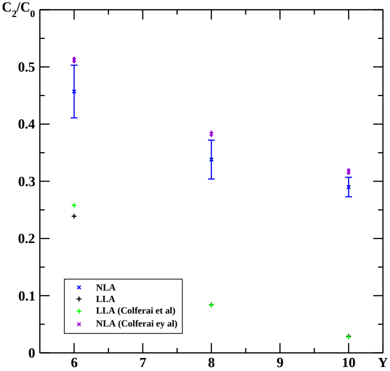


- With our approach we reproduced the same results of previous calculations
- We extimate a systematic error as the standard deviation of the optimal value from the determinations in neighbouring points

Results: $|\vec{k}_{J1}| = |\vec{k}_{J2}| = 35$ GeV



Y	Y_0	$\mu_R/\sqrt{\|\vec{k}_{J1}\|\|\vec{k}_{J2}\|}$
6	1	7
8	2	5
10	2	5



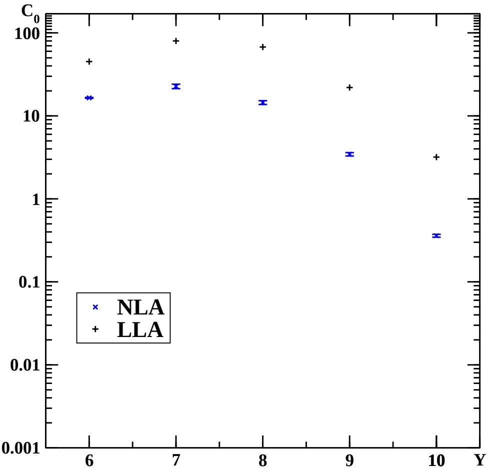
Y	Y_0	$\mu_R/\sqrt{\|\vec{k}_{J1}\|\|\vec{k}_{J2}\|}$
6	1	4
8	2	3
10	4	6

- The stationarity regions for these observables are less evident than the case of C_0, thus making our analysis more complicated

Results: $|\vec{k}_{J1}| = |\vec{k}_{J2}| = 20$ GeV

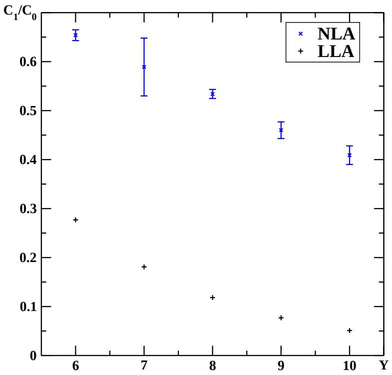
Y	Y_0	$\mu_R/\sqrt{|\vec{k}_{J1}||\vec{k}_{J2}|}$
6	1	6
7	1	3
8	2	3
9	2	3
10	3	3

- We have found wide regions of stability
- The same scales for the "optimal" values

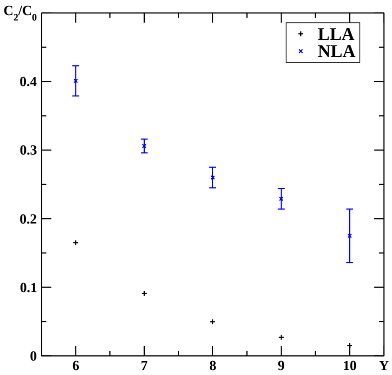


- Smaller transverse momentum $\longrightarrow$ more undetected gluons $\longrightarrow$ possible improved discrimination between BFKL and DGLAP

Results: $|\vec{k}_{J1}| = |\vec{k}_{J2}| = 20$ GeV

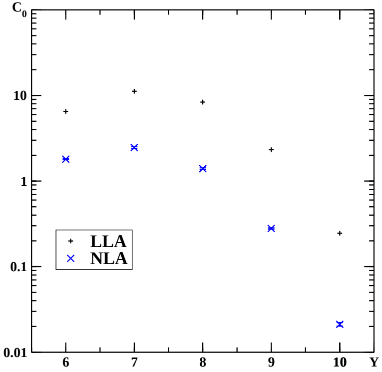


Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J1}\rvert\lvert\vec{k}_{J2}\rvert}$
6	1	10
7	2	6
8	2	5
9	1	5
10	1	5



Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J1}\rvert\lvert\vec{k}_{J2}\rvert}$
6	1	12
7	1	3
8	2	6
9	3	8
10	3	4

Results: $|\vec{k}_{J1}| = 20$ GeV, $|\vec{k}_{J2}| = 35$ GeV



Y	Y_0	$\mu_R/\sqrt{|\vec{k}_{J1}||\vec{k}_{J2}|}$
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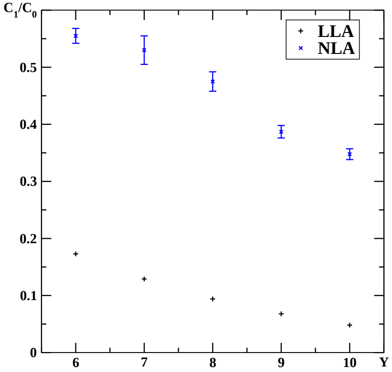
7	1	5
8	2	5
9	1	8
10	2	4



Y	Y_0	$\mu_R/\sqrt{|\vec{k}_{J1}||\vec{k}_{J2}|}$
6	1	8
7	1	5
8	2	5
9	1	8
10	1	9

- For C_2/C_0 we cannot find stability regions

Conclusions

- We have studied the Y-dependence of the differential cross section and of the azimuthal decorrelation of the Mueller-Navelet jets in full NLO BFKL using the jet vertex in the small cone approximation
- We did not fix the energy scale and the argument of the running coupling $\longrightarrow$ we used an optimization method
- Our results for $|\vec{k}_{J1}| = |\vec{k}_{J2}| = 35$ GeV are compatible with those performed by Colferai et al.
  $\longrightarrow$ Confirmation of their results
  $\longrightarrow$ Reliability of the small cone approximation
- We considered also smaller trasverse momenta w.r.t. Colferai et al. $\longrightarrow$ we expect more discrimination between BFKL and DGLAP dynamics

Outlook

- Study of C_2/C_1 $\longrightarrow$ a preliminary analysis using PMS does not show clear regions of stability
- Comparison with the DGLAP approach for the case $|\vec{k}_{J1}| = |\vec{k}_{J2}| = 20$ GeV
- Study od azimuthal decorrelation in the region of even smaller transverse momenta

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Thank you!

$$\frac{d\hat{\sigma}_{i,j}(x_1 x_2 s, \mu)}{dx_{J1} dx_{J2} d^2\vec{k}_{J1} d^2\vec{k}_{J2}} = \frac{s}{(2\pi)^2} \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_i(\vec{q}_1, s_0, x_1; \vec{k}_{J1}, x_{J1}) \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_j(-\vec{q}_2, s_0, x_2; \vec{k}_{J2}, x_{J2})$$
$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- Colferai et al:

$$\frac{x_1 x_2 s}{s_0} \longrightarrow \frac{x_1 x_2 s}{s_0'}$$

$$s_0' = \frac{x_1 x_2 |\vec{k}_{J1}||\vec{k}_{J2}|}{x_{J1} x_{J2}}$$