Momentum space dipole amplitude for DIS and inclusive hadron production

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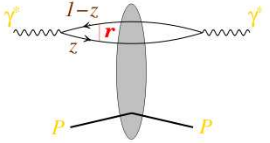
Outline

- Motivation
  - ⇒ Saturation and geometric scaling
  - ⇒ Dipole formalism
- Modeling scattering amplitudes
  - ⇒ Traveling wave method
  - ⇒ Momentum space AGBS model
- Simultaneous fit of AGBS to HERA and RHIC data: hybrid formalism
- LHC predictions: k_t factorization
- Summary

Saturation and Dipole Frame

- e-P scattering at HERA: strong rise of the gluon dist. function for small-x
  - Untamed rising should violate unitarity
  - For $Q^2 \leq Q_s^2(x)$, parton recombination should happen
  - Semihard scale from pQCD $\mathbf{Q_s(x)}$: Saturation scale
- **Dipole frame** is convenient to investigations on small-x



z: longitudinal photon momentum fraction carried by the quark
$\mathbf{r}$: transverse size of the pair $q\bar{q}$

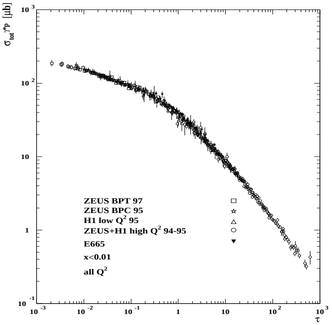
  - γ^* splits into the $q\bar{q}$ pair and the cross section factorizes as [Nikolaev & Zakharov '90; Mueller '94]

$$\sigma_{L,T}^{\gamma^* P}(x, Q^2) = \int_0^1 dz \int d^2\mathbf{r} |\Psi_{L,T}(z, r; Q^2)|^2 \sigma_{dip}(r = |\mathbf{r}|, x) \quad (1)$$

σ_{dip} and geometric scaling

- σ_{dip} includes the target non-perturbative terms (proton) and satisfies the pQCD properties
  - Color transparency: $\sigma_{dip}(r,x) \propto r^2$
  - Black disc limit: $r \leq 1/Q_s(x)$, $\sigma_{dip}(r,x) \sim \pi R_h^2$
- σ_{dip} models describe the geometric scaling observed in the DIS at HERA

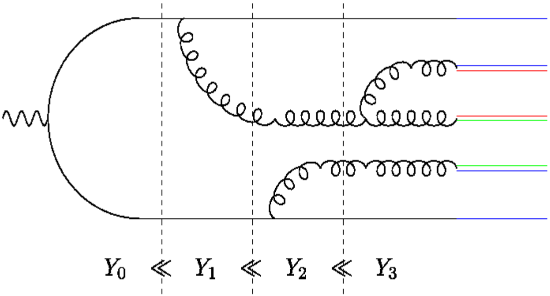
[Stasto, Golec-Biernat and Kwiecinski '2001]



- $\tau = Q^2/Q_s^2(x)$
- $\sigma_{dip}(x, Q^2) \Rightarrow \sigma_{dip}(\tau)$
- Scaling behavior is quite model independent
- Stands as a strong evidence of the saturation phenomena
- Also holds outside the saturation region (geometric scaling window)

Dipole formalism

- Considers the energy evolution ($Y = \ln(1/x)$) of the pair $q\bar{q}$



- Large N_c limit: gluon $\equiv q\bar{q}$
- $\sigma_{dip}(r, Y) = \int d^2\mathbf{b}\mathcal{N}(r, \mathbf{b}, Y) \approx \pi R^2_{target}\mathcal{N}(r, Y)$
- $\mathcal{N}(r, Y)$ describes the interaction through gluon cascade ($\equiv$ non abelian phases $\rightarrow$ Wilson lines)

Dipole evolution

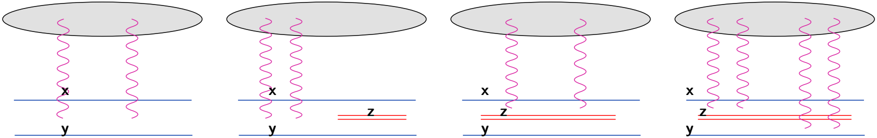
- Balitsky-JIMWLK hierarchy $\mathcal{M}(\mathbf{x},\mathbf{y},\mathbf{z}) = \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2}$ $\bar{\alpha} = \alpha_s N_c/\pi$

$$\frac{\partial}{\partial Y}\langle\mathcal{N}(\mathbf{x},\mathbf{y})\rangle_Y = \frac{\bar{\alpha}}{2\pi}\int d^2\mathbf{z}\mathcal{M}(\mathbf{x},\mathbf{y},\mathbf{z}) \times\left\{-\langle\mathcal{N}(\mathbf{x},\mathbf{y})\rangle_Y + \langle\mathcal{N}(\mathbf{x},\mathbf{z})\rangle_Y + \langle\mathcal{N}(\mathbf{z},\mathbf{y})\rangle_Y - \langle\mathcal{N}(\mathbf{x},\mathbf{z})\mathcal{N}(\mathbf{z},\mathbf{y})\rangle_Y\right\}$$

$$\vdots$$

- Mean field: $\langle\mathcal{N}(\mathbf{x},\mathbf{z})\mathcal{N}(\mathbf{y},\mathbf{z})\rangle_Y \approx \langle\mathcal{N}(\mathbf{x},\mathbf{z})\rangle_Y\langle\mathcal{N}(\mathbf{y},\mathbf{z})\rangle_Y \Rightarrow$ Balitsky-Kovchegov (BK) equation

$$\frac{\partial}{\partial Y}\langle\mathcal{N}(\mathbf{x},\mathbf{y})\rangle_Y = \frac{\bar{\alpha}}{2\pi}\int d^2\mathbf{z}\mathcal{M}(\mathbf{x},\mathbf{y},\mathbf{z}) \times\left\{-\langle\mathcal{N}(\mathbf{x},\mathbf{y})\rangle_Y + \langle\mathcal{N}(\mathbf{x},\mathbf{z})\rangle_Y + \langle\mathcal{N}(\mathbf{z},\mathbf{y})\rangle_Y - \langle\mathcal{N}(\mathbf{x},\mathbf{z})\rangle_Y\langle\mathcal{N}(\mathbf{z},\mathbf{y})\rangle_Y\right\}$$



HEQCD amplitudes and traveling waves

- There is no analytical solution to BK equation
  Asymptotic forms obtained through the mapping:
  QCD $\Rightarrow$ Reaction-diffusion processes
- Geometric scaling of the BK amplitudes seen as traveling waves
- In momentum space BK equation reads

$$\partial_Y N_Y = \bar{\alpha}\chi(-\partial_L) N_Y - \bar{\alpha} {N_Y}^2,$$

  where

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

  is the **BFKL** kernel and $L = \log(k^2/k_0^2)$, with k_0 denoting a soft scale.
- change of variables: $t \sim \bar{\alpha} Y$, $x \sim L$ e $u \sim N$, BK $\rightarrow$ FKPP [Munier e Peschanski '2004]

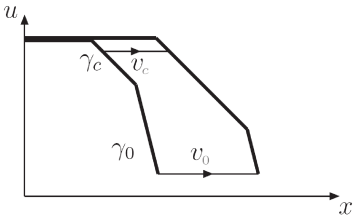
$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)$$

- Dynamics essentially determined by the linear part (BFKL)
- FKPP admits *traveling wave solutions*: $u(x,t) \overset{t\to\infty}{\sim} u(x - v_c t)$ ("geometric scaling")

HEQCD amplitudes and traveling waves

- TW conects $u = 0$ unstable and $u = 1$ stable fixed points.
- $\gamma_0 > \gamma_c$
- $u(x,t) = \int_{c-\imath\infty}^{c+\imath\infty} \frac{d\gamma}{2\imath\pi} u_0(\gamma) e^{-\gamma(x_{wf}+vt)+\omega(\gamma)t}$

$$v_c = \frac{\omega(\gamma_c)}{\gamma_c} = \partial_\gamma \omega(\gamma)|_{\gamma_c}$$



- In QCD variables : $\omega(\gamma_c) = \chi(\gamma_c) \Rightarrow \gamma_c = 0.6275$
- TW translates into geometric scaling of BK amplitudes [Munier e Peschanski '2004]

$$N_Y(k) \overset{k \gg Q_s}{\approx} \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c} \log\left(\frac{k^2}{Q_s^2(Y)}\right) \exp\left[-\frac{\log^2\left(k^2/Q_s^2(Y)\right)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right],$$

$$Q_s^2(Y) = Q_0^2 \exp\left(\lambda Y - \frac{3}{2\gamma_c}\log Y\right), \qquad \lambda = \bar{\alpha} v_c = \bar{\alpha}\chi(\gamma_c)/\gamma_c$$

- Geometric scaling window

$$\log\left(k^2/Q_s^2(Y)\right) \lesssim \sqrt{2\bar{\alpha}\chi''(\gamma_c)Y}$$

AGBS model for σ_{dip}

- Parametrization in momentum space for the dipole-proton scattering amplitude [Amaral, MBGD, Betemps and Soyez '2007]
  - The model uses the traveling wave BK solutions for the large L (dilute) region
  - A Fourier transform of a Theta function models the saturated region

$$N(k) \overset{k \ll Q_s}{=} c - \log\left(\frac{k}{Q_s(Y)}\right)$$

  - The AGBS model interpolates between the two behaviors through ($\rho \equiv \ln(k^2/k_0^2)$ and $\rho_s \equiv \ln(k_0^2/Q_s^2)$):

$$N^{\mathrm{AGBS}}(\rho, Y) = L_F \left(1 - e^{-N_{\text{dil}}}\right),$$

where

$$N_{\text{dil}} = \exp\left[-\gamma_c(\rho - \rho_s) - \frac{\mathcal{L}^2 - \log^2(2)}{2\bar{\alpha}\chi''(\gamma_c)Y}\right],$$

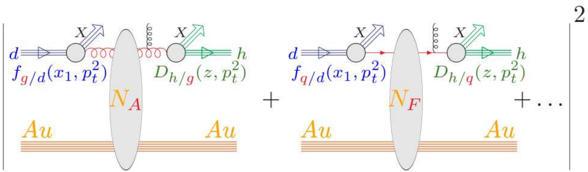
$$\mathcal{L} = \ln\left[1 + e^{(\rho - \rho_s)}\right] \qquad \text{with} \quad Q_s^2(Y) = k_0^2\, e^{\lambda Y},$$

and

$$L_F = 1 + \ln\left[e^{\frac{1}{2}(\rho - \rho_s)} + e^{-\frac{1}{2}(\rho - \rho_s)}\right]$$

Hadron Production from the CGC

- DGLAP evolved projectile and target as a CGC ("Hybrid formalism")
- d-Au scattering at LO accuracy ($d + Au \to h + X$)



- Amplitude: sums diagrams of parton-nucleus (CGC) interaction
- |Amplitude|^2: only 2-point functions (**dipoles**) $N_{A,F}$ enter the cross section [Dumitru, Hayashigaki and Jalilian-Marian '2006]

$$\frac{dN_h(d\,Au \to h(p_t, y_h)\,X)}{dy_h d^2 p_t} = \frac{K(y_h)}{(2\pi)^2} \int_{x_F}^{1} dx_1 \, \frac{x_1}{x_F} \left[f_{q/p}(x_1, p_t^2)\, N_F(q_t, x_2)\, D_{h/q}\left(x_F/x_1, p_t^2\right) \right.$$

$$\left. + f_{g/p}(x_1, p_t^2)\, N_A(q_t, x_2)\, D_{h/g}\left(x_F/x_1, p_t^2\right) \right]$$

Simultaneous fit to HERA and RHIC

[Basso, MBGD and de Oliveira '2011]

- The AGBS model was simultaneously fitted to the last HERA (combined H1 and ZEUS) and RHIC minimum-bias (BRAHMS and STAR) data.
  - DIS was investigated through the proton structure function in momentum space

$$F_2(x, Q^2) = \frac{Q^2 R_p^2 N_c}{4\pi^2} \int_0^\infty \frac{dk}{k} \int_0^1 dz \, |\tilde{\Psi}_{L,T}(z, k; Q^2)|^2 N(k, Y) \qquad (2)$$

  where the photon wave function is now expressed in momentum space

  - Hadron collisions were described by AGBS through the inclusive hadron yield

$$\frac{dN_h(d\,Au \to h(p_t, y_h)\,X)}{dy_h d^2 p_t} = \frac{K(y_h)}{(2\pi)^2} \int_{x_F}^1 dx_1 \, \frac{x_1}{x_F} \Big[f_{q/p}(x_1, p_t^2)\, N_F\,(q_t, x_2)\; D_{h/q}\,(x_F/x_1, p_t^2) + f_{g/p}(x_1, p_t^2)\, N_A\,(q_t, x_2)\; D_{h/g}\,(x_F/x_1, p_t^2) \Big] \qquad (3)$$

Results: fit to HERA

- Before proceed with the simultaneous fit, the AGBS model was fitted to the last H1 and ZEUS combined data [JHEP **0110** 109 (2010)]
  - Fixed parameters: $\gamma_c = 0.6275$ from the LO BFKL and $\bar{\alpha} = 0.2$
  - Free parameters: k_0^2, $\chi''(\gamma_c)$, λ and R_p
  - Kinematic range:

$$\begin{cases} x \leq 0.01, \ \text{small-}x \\ 0.1 \leq Q^2 \leq 150\,\mathrm{GeV}^2 \end{cases}$$

  - Only light quarks were considered, with mass $m_{u,d,s} = 140$ MeV

$\chi^2/\text{d.o.f}$	k_0^2 ($\times 10^{-3}$)	λ	$\chi''(\gamma_c)$	$R(\text{GeV}^{-1})$
0.903	1.129 ± 0.024	0.165 ± 0.002	7.488 ± 0.081	5.491 ± 0.039

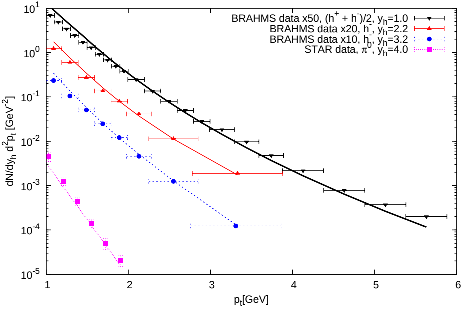
Results: simultaneous fit to HERA and RHIC

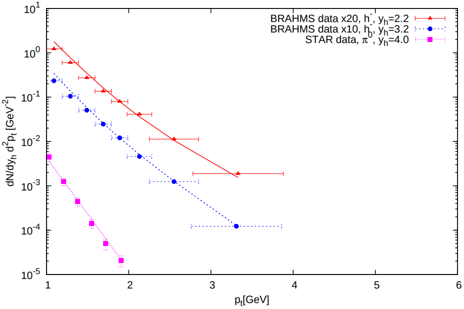
- Simultaneous fit: In addition to the H1 and ZEUS combined data, the model was fitted simultaneously to the BRAHMS [Phys. Rev. Lett. **93**, 242303 (2004)] and STAR [Phys. Rev. Lett. **97**, 152302 (2006)] data on inclusive hadron production
  - Fixed parameters: $\gamma_c = 0.6275$ from the LO BFKL and $\bar{\alpha} = 0.2$
  - Also fixed: $K_{y_h=4} = 0.7$ from DHJ and BUW LO models
  - Free parameters: k_0^2, $\chi''(\gamma_c)$, λ, R_p and $K(y_h)$
  - **CTEQ06** PDF and **KKP** fragmentation functions at scale of $P_t \geq 1$ GeV
  - Kinematic range:

$$HERA \begin{cases} x \leq 0.01, \ (\text{small-}x) \\ 0.1 \leq Q^2 \leq 150\,\text{GeV}^2 \end{cases} \qquad RHIC \begin{cases} P_t \geq 1\,\text{GeV}, \\ 2.2 \leq y_h \leq 4.0\ (\text{small-}x) \\ 1.0 \leq y_h \leq 4.0\ (\text{mid-rapidity test}) \end{cases}$$

  - Only light quarks were considered, with masses $m_{u,d,s} = 140$ MeV
  - $Q^2_{s,A} = A_{eff}^{1/3} Q^2_{s,p}$
  - $A_{eff} = 18.5$ for d-Au collisions

Results: simultaneous fit to HERA and RHIC ($d + Au$)

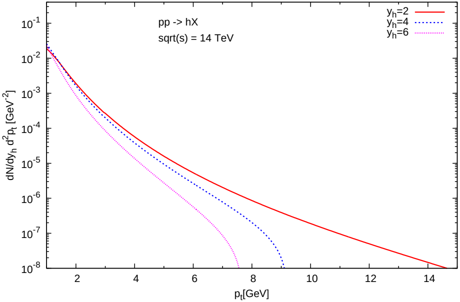


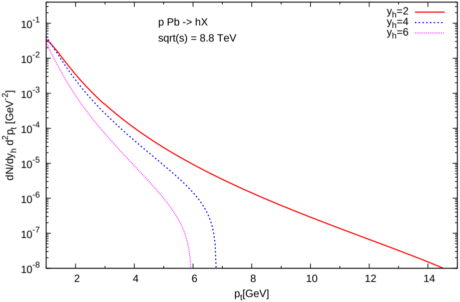


χ^2/d.o.f.	k_0^2 $(\times 10^{-3})$	λ	$\chi''(\gamma_c)$	$R(\mathrm{GeV}^{-1})$	$K(y_h = 1.0)$	$K(y_h = 2.2)$	$K(y_h = 3.2)$
0.799	2.760 ± 0.130	0.190 ± 0.003	5.285 ± 0.123	4.174 ± 0.053	–	2.816 ± 0.110	2.390 ± 0.098
1.056	1.660 ± 0.137	0.186 ± 0.003	6.698 ± 0.223	4.695 ± 0.112	6.172 ± 0.379	3.783 ± 0.259	3.256 ± 0.226

Results: Predictions for LHC

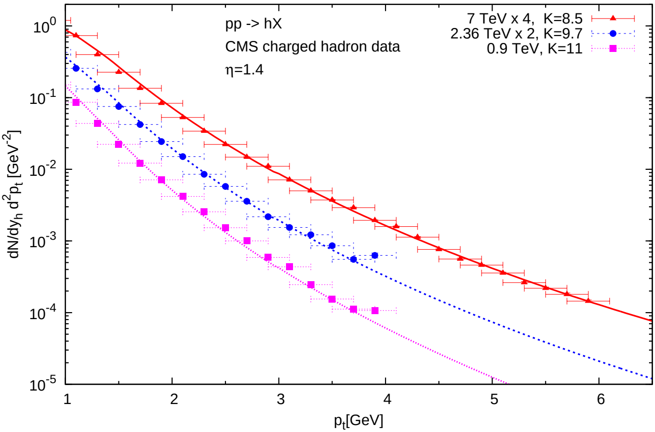
- Parameters from the fit of the forward RHIC data ($y_h \geq 2.2$)





- Suppression on the hadron yield at forward rapidities (stronger on $p + Pb$ collisions)

Results: Predictions for LHC (CMS $p + p$)



- Geometric scaling violations present at LHC energies? (AGBS,DHJ x BUW)
- Large K factors due to the inclusion of central rapidity data $\Rightarrow$ not described with AGBS + hybrid formalism

k_t-factorization

- Both colliding hadrons treated in the same way (emphasize p_t dependence)
  $\eta \sim 0 \rightarrow k_t^1 \approx k_t^2 \qquad \eta\text{forward} \rightarrow k_t^1 \ll k_t^2$
- The fit was also used to describe the LHC data within the k_t-factorization formalism [Braun '2000, Kovchegov and Tuchin '2003]

$$\frac{d\sigma^{A+B\rightarrow g}}{dyd^2p_td^2R} = K\frac{2}{C_F p_t^2}\int^{p_t}\frac{d^2k_t}{4}$$
$$\times\int d^2b\,\alpha_s(Q)\varphi\left(\frac{|p_t+k_t|}{2},x_1;b\right)\varphi\left(\frac{|p_t-k_t|}{2},x_2;R-b\right),$$

where $x_{1,2} = (p_t/\sqrt{s})e^{\pm y}$ are momentum fractions of the incoming gluons and $C_F = (N_c^2 -)/2N_c$ is the Casimir for the fundamental representation.

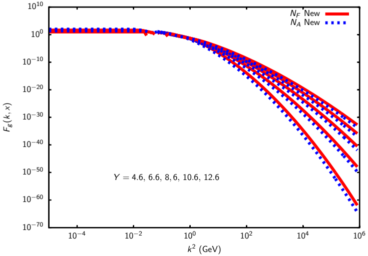
- In the large N_c limit, the unintegrated gluon distribution in either of the two colliding hadrons can be related to the dipole scatering amplitude through [Braun '2000]

$$\varphi(k,x;b) = \frac{N_c\,S_t}{2\pi^2\alpha_s(k)}k^2F_g(k,x;b).$$

k_t-factorization

$$F_g(k,x;b) = \frac{1}{2\pi}(\nabla_k^2 N_G(k,x;b)+\delta(k))$$

Spike in the low k region. Common feature of the BK amplitudes [Staśto, Xiao and Zaslavsky '2012]



- The charged hadron yield reads

$$\frac{dN_{ch}}{d\eta d^2p_t} = \frac{h(\eta)}{\sigma_{nsd}} \int d^2R \int \frac{dz}{z^2} \frac{d\sigma^{A+B\to g}}{dyd^2p_t d^2R} D_h(z = p_t/k_t)$$

  where D_h denotes de fragmentation functions and h is the Jacobian relating y to η

$$y(\eta, p_t, m) = \frac{1}{2}\ln\left[\frac{\sqrt{m^2+p_t^2\cosh^2\eta}+p_t\sinh\eta}{\sqrt{m^2+p_t^2\cosh^2\eta}-p_t\sinh\eta}\right]$$

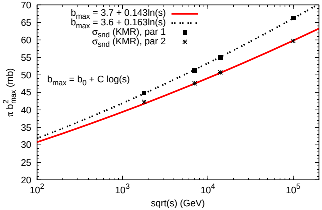
- σ_{nsd} describes the interaction area and is model dependent.

k_t-factorization

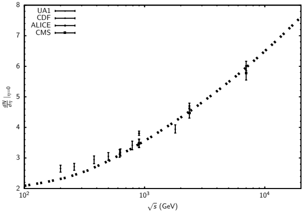
[E. Basso, MBGD and de Oliveira]

- Energy normalization model for $\sigma_{\rm nsd}$ that fits the KMR model [Khoze, Martin and Ryskin '2011]

$$\sigma_{nsd} \to \pi b_{max}^2,$$
$$b_{max} = b_0 + C\log(s).$$



- Also use central rapidity produced hadrons to fix a and b

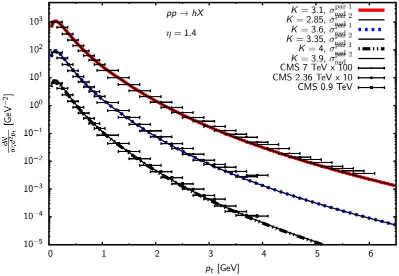


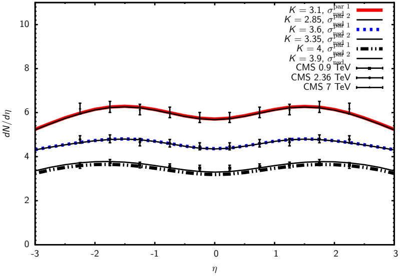
$$b_{\rm max} = 0.1 + 0.198\log(s)$$

Results $p + p$ at LHC

- Parameters from the fit of the forward RHIC data ($y_h \geq 2.2$)
- Using σ_{nsd} form KMR model
- Large-x effects ($\beta = 0.4$ and $\lambda_0 = 0 - 0.2$, $x_0 = 0.01$) [Gelis, Staśto and Venugopalan '2006]

$$\varphi(k, x; b) = \left(\frac{1-x}{1-x_0}\right)^{\beta} \left(\frac{x_0}{x}\right)^{\lambda_0} \varphi(k, x_0; b)$$

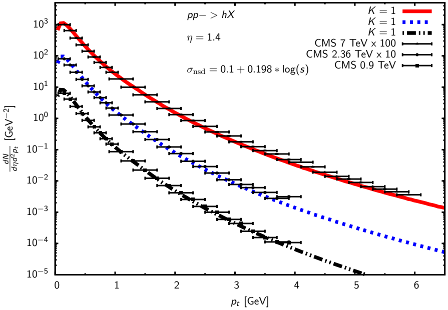


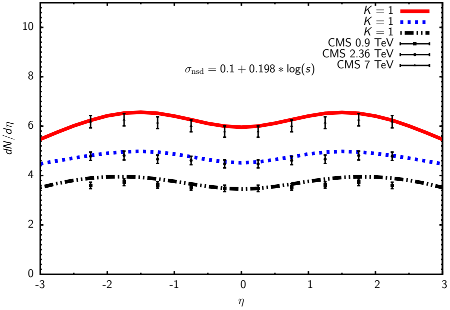


- K factors smaller compared with hybrid formalism

Results $p + p$ at LHC

- $\sigma_{\rm nsd}$ from central rapidity data

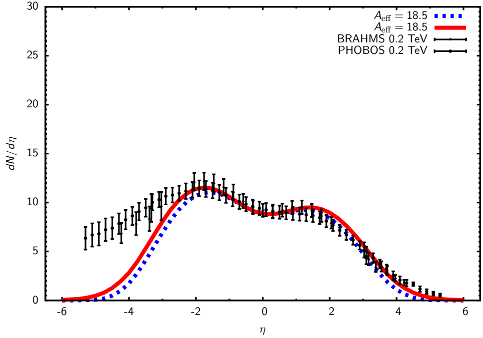




- $K \sim 1$

Results $p + A$

- $Q_s^A = A_{\rm eff}^{1/3} Q_s^p$, $A_{\rm eff} = 18.5$ for gold target



- d fragmentation region: x_2 small $\rightarrow$ small-x effects of the nuclear target included
- Au fragmentation region: x_2 large $\rightarrow$ Must include EMC, Fermi contributions

Summary

- We employed the traveling wave method to describe both DIS and inclusive hadron production using a hybrid formalism factorization (DGLAP + small-x)
  - $\Rightarrow$ Momentum space model for the scattering amplitude – AGBS
  - $\Rightarrow$ Good fit in the forward rapidity ($y_h > 2$) region of the hadron yield
- Predictions for LHC
  - $\Rightarrow$ Hadron suppression in the forward region for $p + Pb$ collisions
  - $\Rightarrow$ Good description of the LHC CMS data for $p + p$ collisions (though large K)
- k_t-factorization
  - $\Rightarrow$ Better description at $\eta \sim 0$ ($p + p$) comprade to hybrid formalism
  - $\Rightarrow$ Improved shape of nuclear targets to better describe pA collisions.
- Useful observables to constrain the model parameters
  - $\Rightarrow$ EM probes – prompt photons, dileptons

Backup Slides

simultaneous fit to HERA and RHIC

- AGBS amplitude is not Glauber like: $N \sim 1 - e^{-\Omega(x,Q_s)}$
- Large N_c limit: adjoint amplitude gotten from the fundamental one through

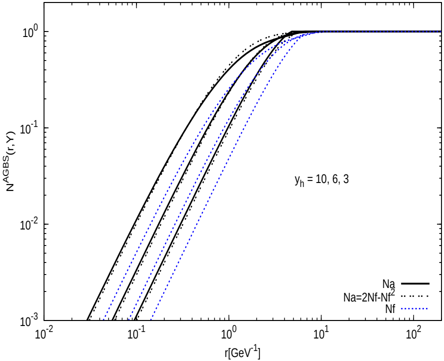
$$N_A(r,Y) = 2N_F(r,Y) - N^2{}_F(r,Y),$$

Problem?

- Analitical form of $N_F^2(r,Y)$ in momentum space
- Hankel Transform does not obey the convolution theorem

Solution

- DIS: amplitude does not includes the factor C_F/C_A, eventhough it is a N_F one (quarks)
- Simultaneous fit: reescaling of N_F through $Q_s^2 \to (C_A/C_F)Q_s^2 = (9/4)Q_s^2$ to



DHJ and BUW amplitude models

- Glauber like amplitudes

$$N_A(r_t, x_2) = 1 - \exp\left[-\frac{1}{4}(r_t^2 Q_s^2(x_2))^{\gamma(y_h, r_t)}\right] \quad (4)$$

- Saturation scales:

$$Q_s^2(x_2) = Q_0^2 A_{\mathrm{eff}}^{1/3} (x_0/x_2)^{\lambda}, \qquad \lambda = 0.3, \qquad x_0 = 3 \cdot 10^{-4} \quad (5)$$

- DHJ anomalous dimension [Dumitru, Hayashigaki and Jalilian-Marian '2006]

$$\gamma(q_t, x_2) = \gamma_s + (1 - \gamma_s)\frac{\log(q_t^2/Q_s^2(x_2))}{\lambda y + d\sqrt{y} + \log(q_t^2/Q_s^2(x_2))} \qquad y = \log 1/x(x_2)$$

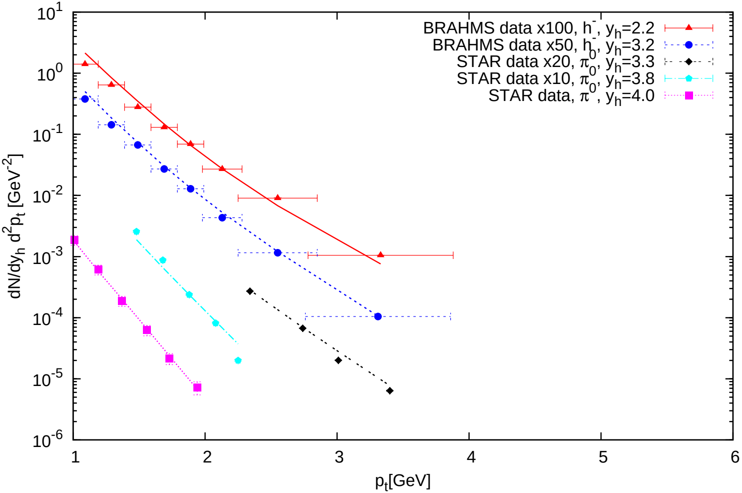
  - Violation of geometric scaling

- BUW anomalous dimension [Boer, Utermann and Wessels '2008]

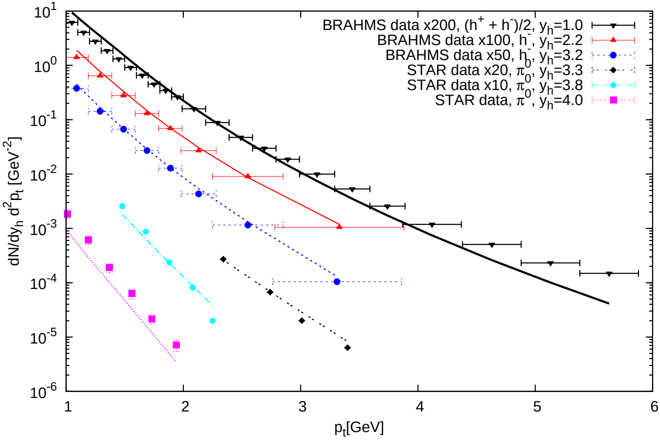
$$\gamma(w) = \gamma_1 + (1 - \gamma_1)\frac{(w^a - 1)}{(w^a - 1) + b} \qquad w = q_t^2/Q_s^2(x_2) \quad (6)$$

  - Geometric scaling preserved

Results: simultaneous fit to HERA and RHIC ($p + p$)



Results: simultaneous fit to HERA and RHIC ($p + p$)



- Model does not fit the mid-rapidity data