NLO in BKP: Odderon and Integrability

Diffraction 2012, Lanzarote, September 11 -15, 2012

Jochen Bartels, Hamburg University

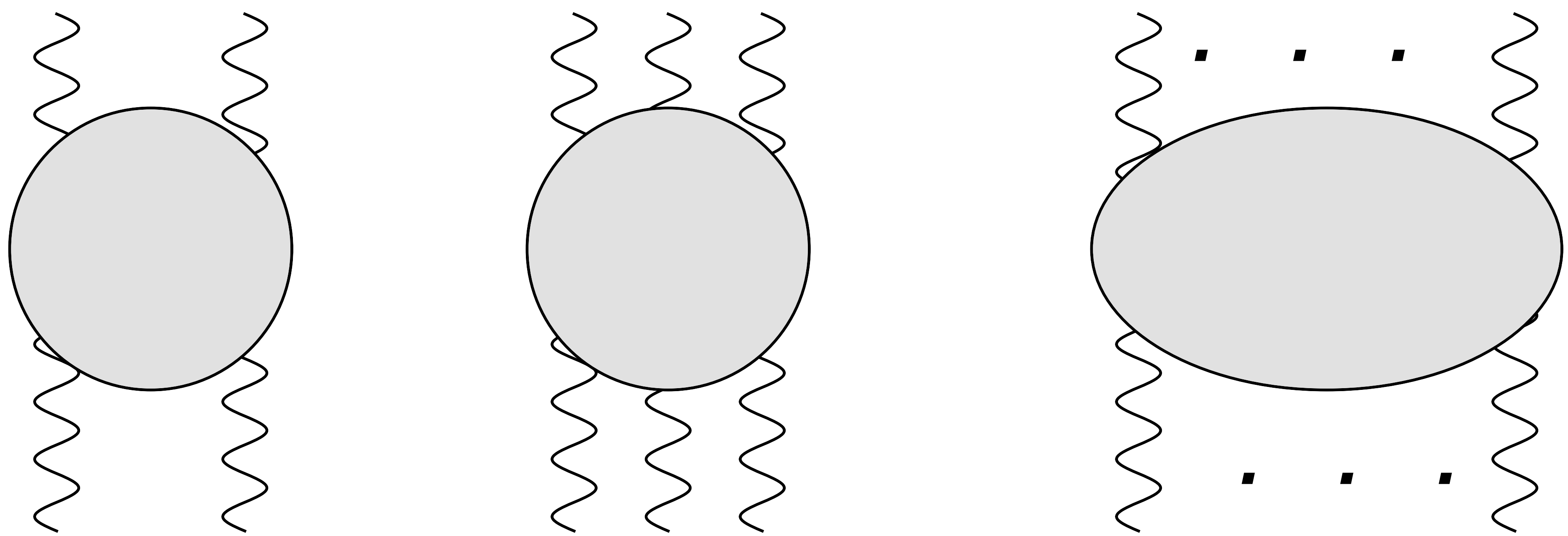
Based upon common work with: arXiv:1205.2530 [hep-th], to appear

V.S Fadin, L.N.Lipatov, G.-P.Vacca

- Introduction: motivation
- BFKL revisited
- Ward identities for reggeized gluons
- BKP kernel

Introduction

Motivations for going beyond BFKL=2 gluon state:



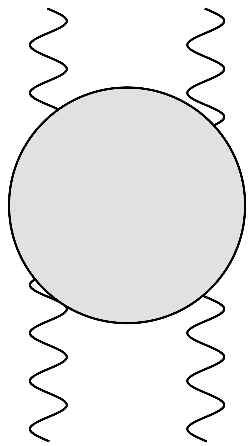
BFKL Pomeron *Odderon* *n gluon bound state*

a) Odderon in pQCD

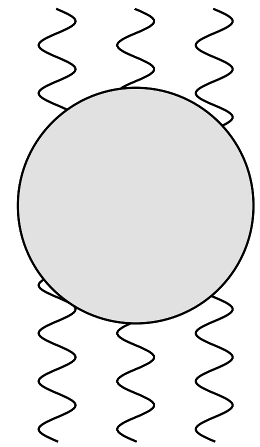
b) Integrability, AdS/CFT correspondence

Odderon:

SU(3) (QCD) has 2 Casimir operators:



BFKL Pomeron



Odderon

$$\omega_{BFKL} = \frac{4N_c \ln 2\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \qquad \omega_{Odderon} = 0 + ?$$

SU(n) has n Casimir operators:

$$\omega_2 = \frac{4N_c ln 2\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \qquad \omega_3 = 0 \qquad \omega_4 = ?$$

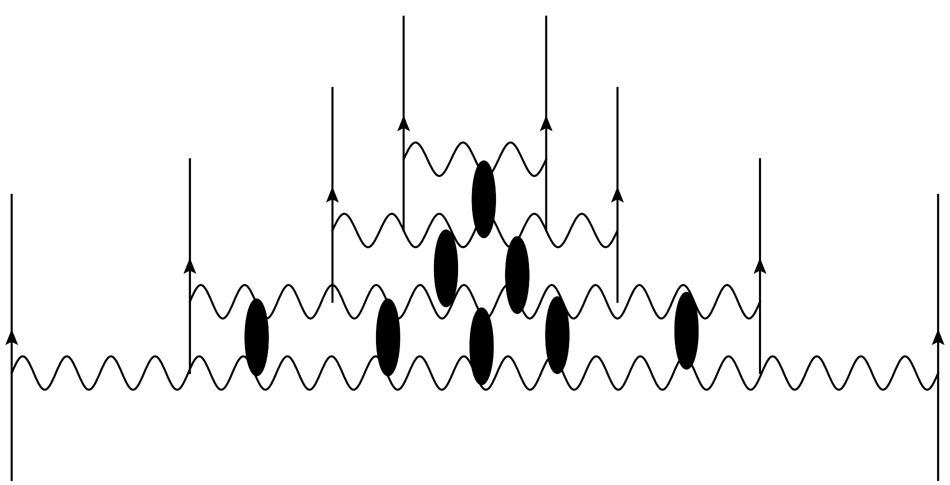
There is something to be understood

N=4 SYM may be soluble,
Integrability (anomalous dimensions)

AdS/CFT correspondence:
bridge from small to large coupling
Scattering amplitudes in multi-Regge kinematics

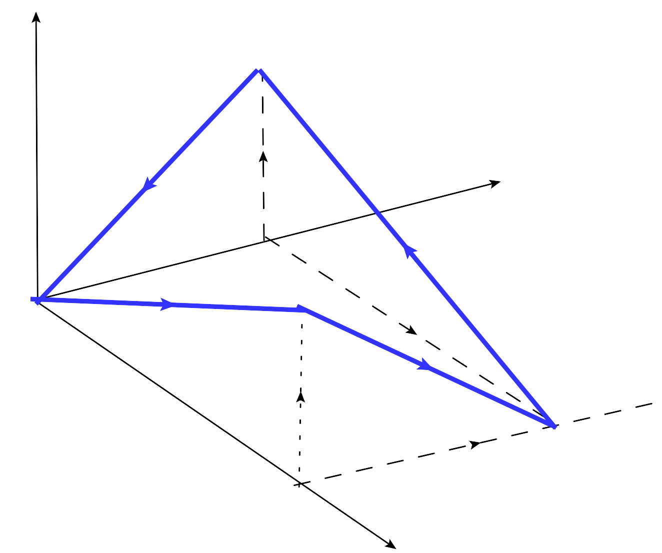
$\lambda \to 0$

leading log calculations,
integrability of BKP Hamiltonian:
needs NLO



$\lambda \to \infty$

Minimal surfaces of polygons

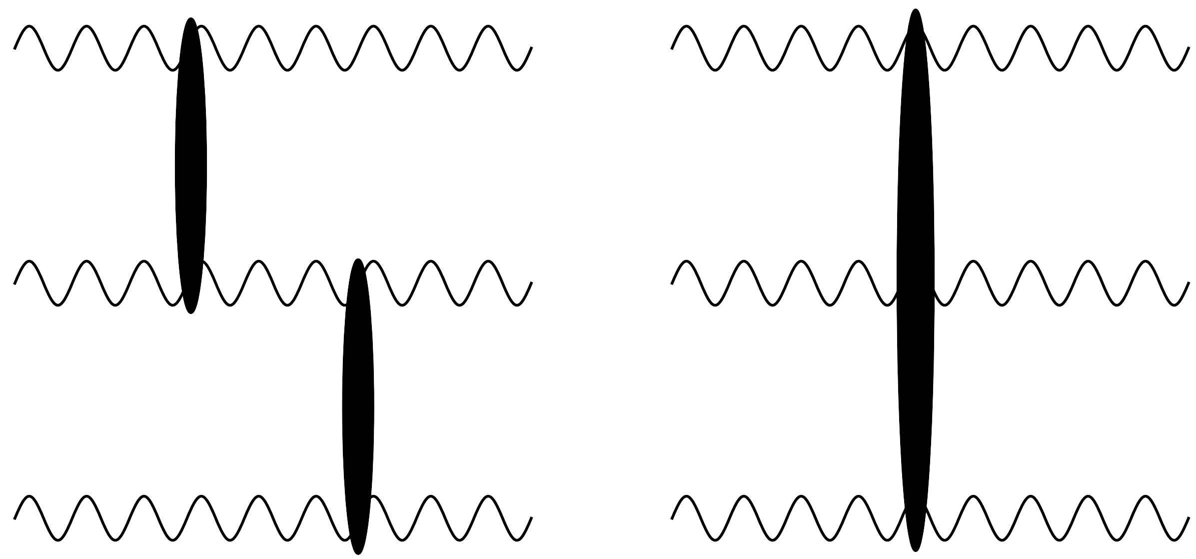


Y-equations in multiregge kinematics

JB,Kormilitzin,Kotanski,
Schomerus,Sprenger

Connect the integrable structures

NLO for BKP states need:



2 to 2 *3 to 3*

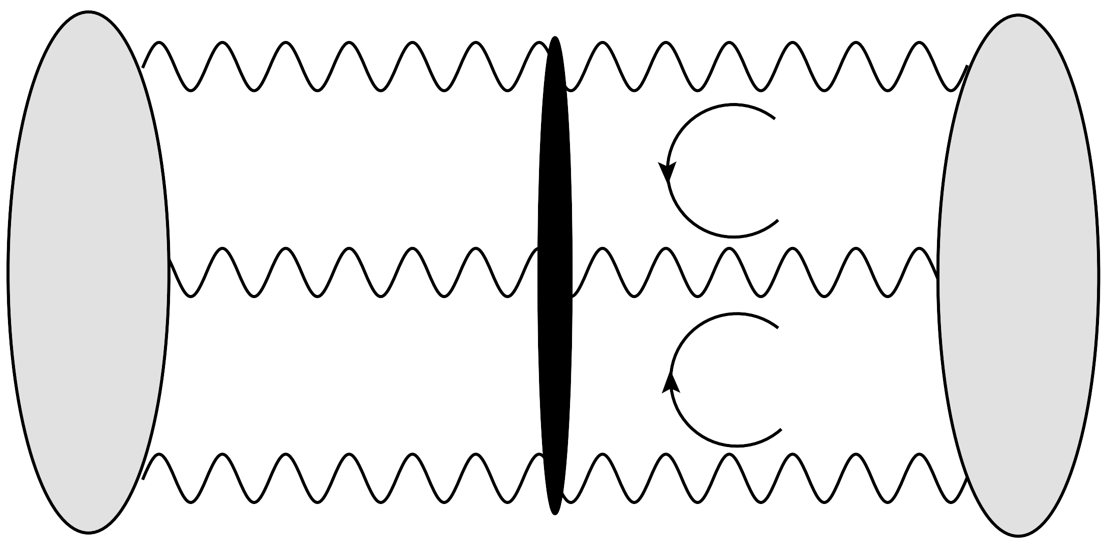
- 2 to 2 BFKL kernel in color octet state (large N_c)

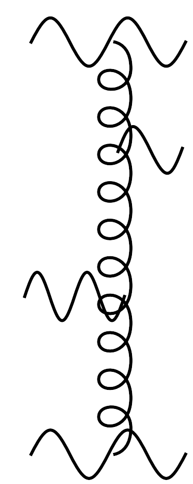
$\rightarrow$ Fadin's talk

- 3 to 3 kernel

Result simple, but not so easy to get.

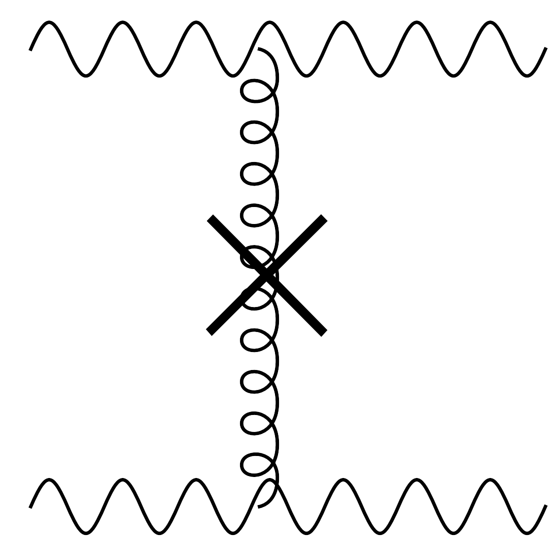
Problem is with the longitudinal integrations:





$+ \ \dots$

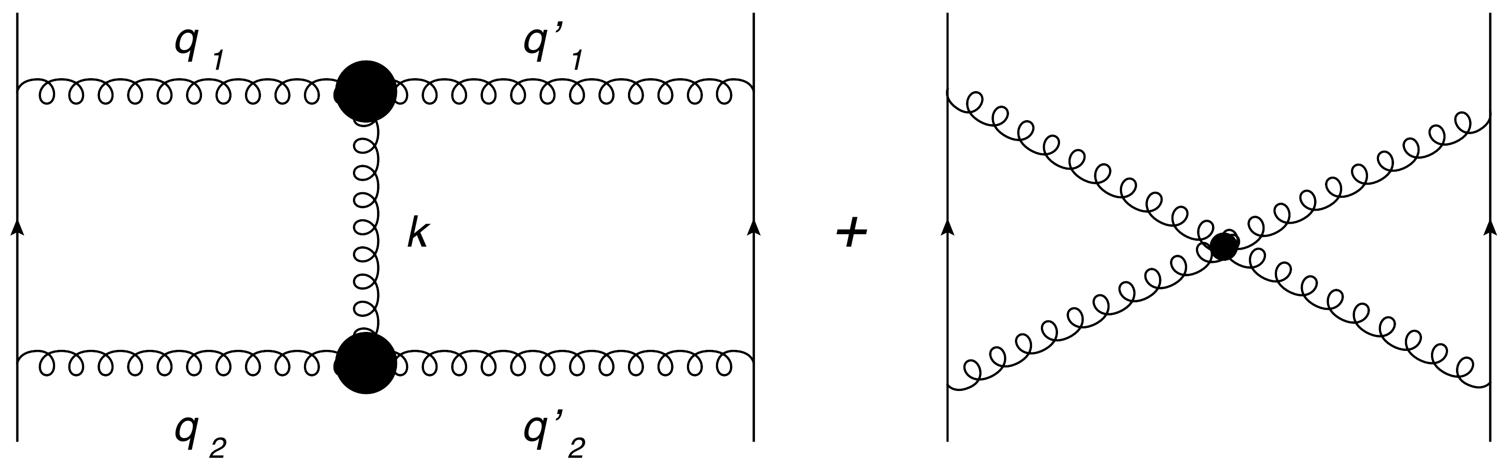
New kernel in QCD evolution equations:
cannot use unitarity, needs longitudinal integration and Bose symmetry.



New tool: Ward identities for amplitudes with reggeized gluons

BFKL, revisited

BFKL derivation without unitarity: k-line is off shell, needs longitudinal integration



+ crossed graphs

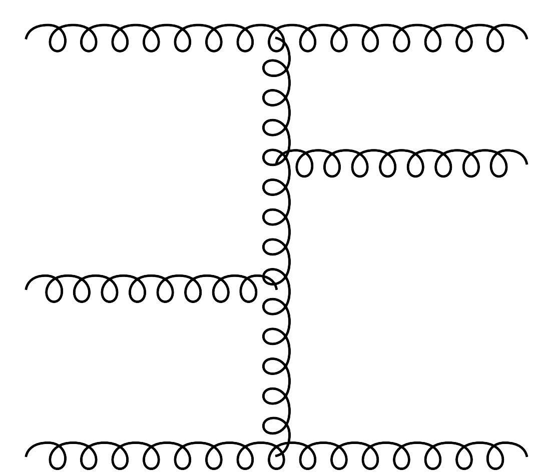
$$\frac{q^2}{k^+k^-} + \frac{-k^2q^2 + q_1^2{q'_2}^2 + {q'_1}^2q_2^2}{(k^+k^- + k^2)k^+k^-}$$

↑

needs crossed graph
to cancel the divergence

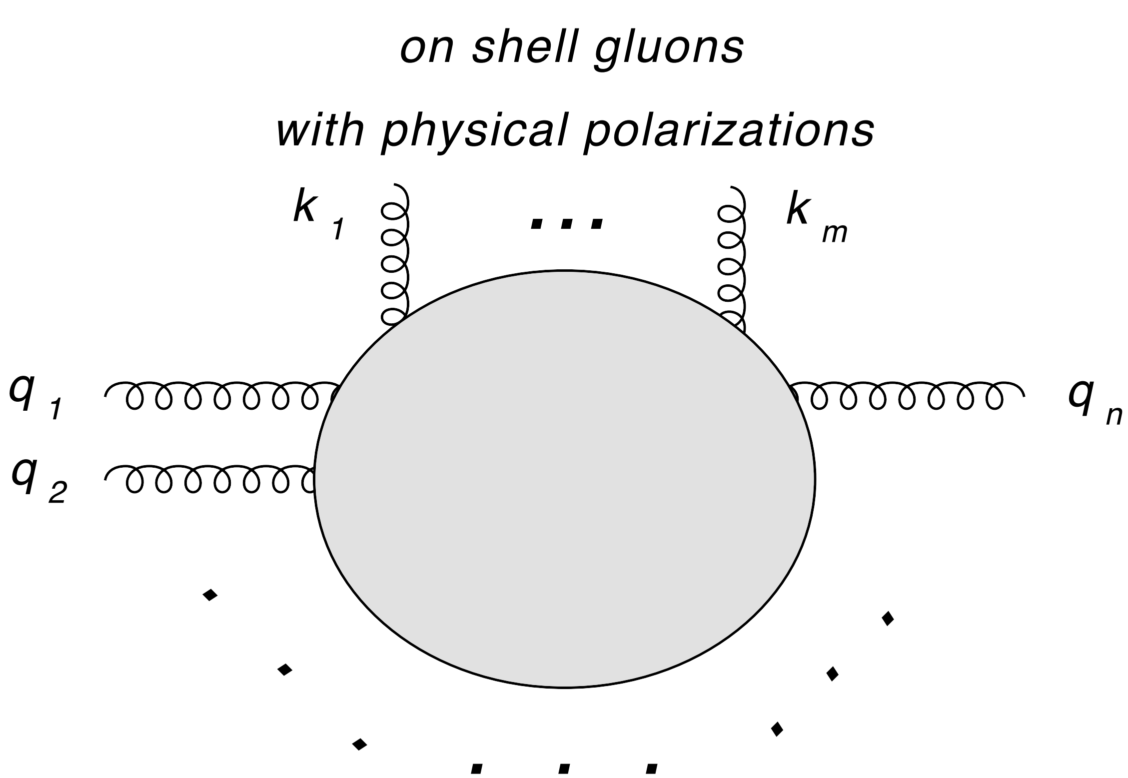
How to avoid summing over all permutations
before the integration?

Idea: Ward identities for reggeized gluons,
makes life much simpler



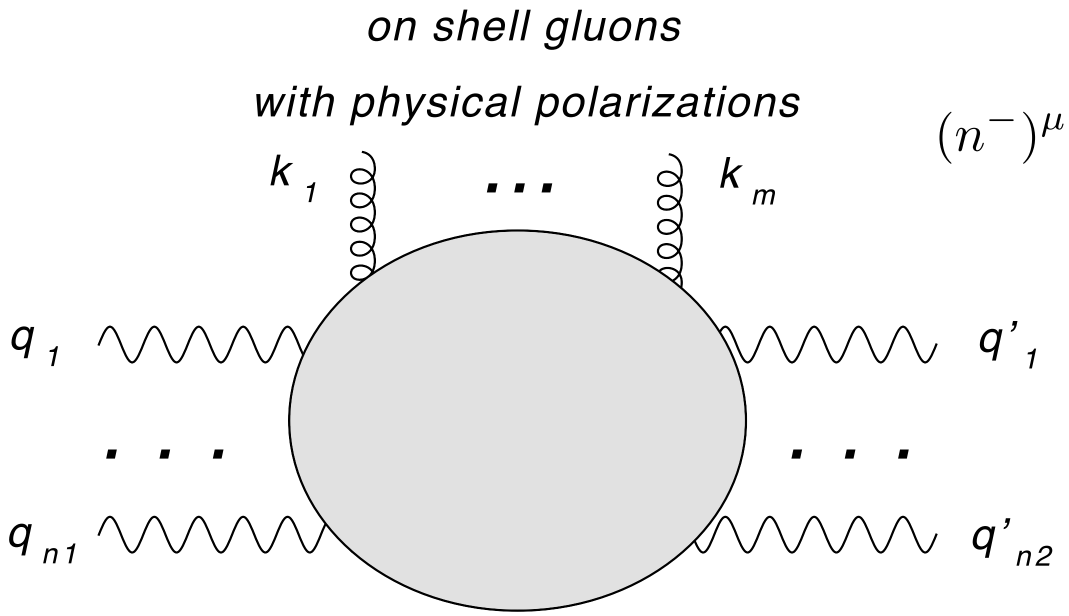
Ward identities for reggeized gluons

(Elementary) gluons:

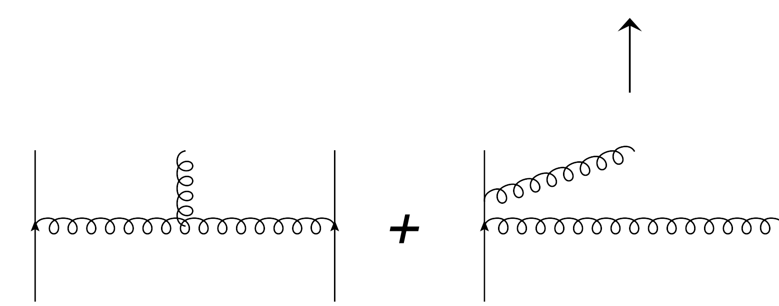


$$q_1^{\mu_1} \dots q_n^{\mu_n} M_{\mu_1 \dots \mu_n}(q_1, \dots, q_n; k_1, \dots, k_m) = 0$$

Reggeized gluons:
Lipatov's effective gauge invariant action



$$(n^-)^{\mu_1}\dots(n^-)^{\mu_{n_1}} M_{\mu_1\dots\mu_{n_1}\nu_1\dots\nu_{n_2}} (n^+)^{\nu_1}\dots(n^+)^{\nu_{n_2}} + \dots$$ induced terms



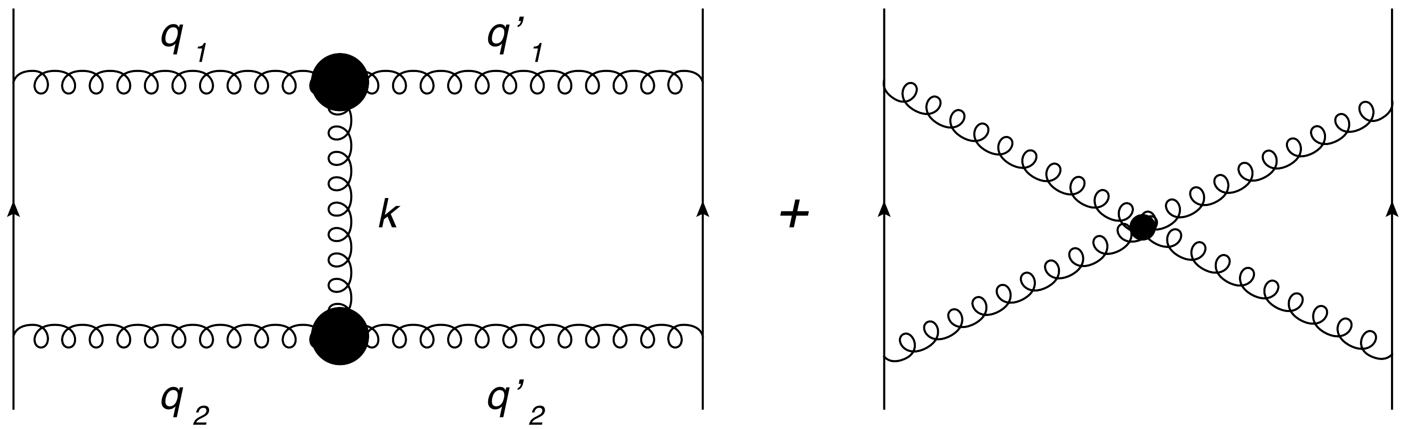
$$q_1{}^{\mu_1}\dots q_{n_1}{}^{\mu_{n_1}} M_{\mu_1\dots\mu_{n_1}\nu_1\dots\nu_{n_2}} q_1'^{\,\nu_1}\dots q_{n_2}'^{\,\nu_{n_2}} = 0$$

$$q_i^\mu = \frac{(n^-)^\mu}{2} q_i^+ + q_{i\perp}^\mu, \;\; q_i'^\mu = \frac{(n^+)^\mu}{2} q_i'^- + q_{i\perp}'^{\,\mu}$$

$$(n^-)^\mu \to -\frac{2}{q^+} q_\perp^\mu, \;\; (n^+)^\mu \to -\frac{2}{q^-} q_\perp^\mu$$

$$\frac{(q_{1\perp})^{\mu_1}}{q_1^+}\dots\frac{(q_{n_1\perp})^{\mu_{n_1}}}{q_{n_1}^+} M_{\mu_1\dots\mu_{n_1}\nu_1\dots\nu_{n_2}} \frac{(q'_{1\perp})^{\nu_1}}{q_1'^{\,-}}\dots\frac{(q'_{n_2\perp})^{\nu_{n_2}}}{q_{n_2}'^{\,-}} + \dots$$ induced terms much fewer

Apply to BFKL:
Use of Ward identities eliminates all induced terms
each diagram is convergent and has the required zero-properties.



$$\cancel{\frac{q^2}{k^+k^-}} + \frac{-k^2q^2 + q_1^2 {q'_2}^2 + {q'_1}^2 q_2^2}{(k^+k^- + k^2)k^+k^-}$$

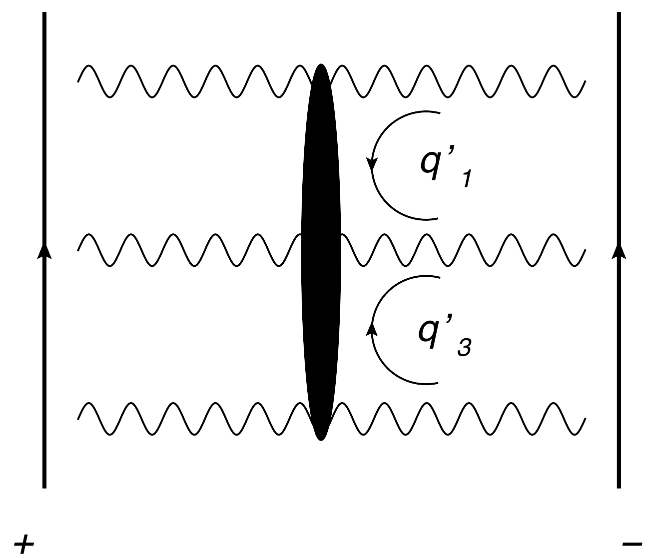
$$= \frac{8}{k^+k^-}\frac{q_1 q_2^* {q'_1}^* q'_2 + cc.}{k^+k^- + k^2}$$

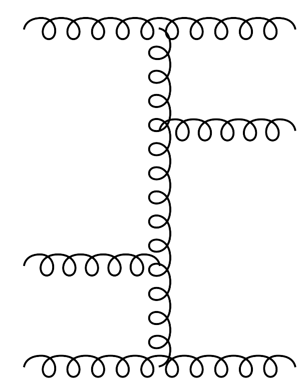
+ crossed graphs

Conclusion to be drawn: after the use of Ward identities
'physical polarization' $\sim (k_\perp)^\mu$ sometimes more convenient than
"unphysical polarization' $\sim (n^\pm)^\mu$

Apply to NLO BKP (3 to 3) kernel:

The 3 to 3 part:





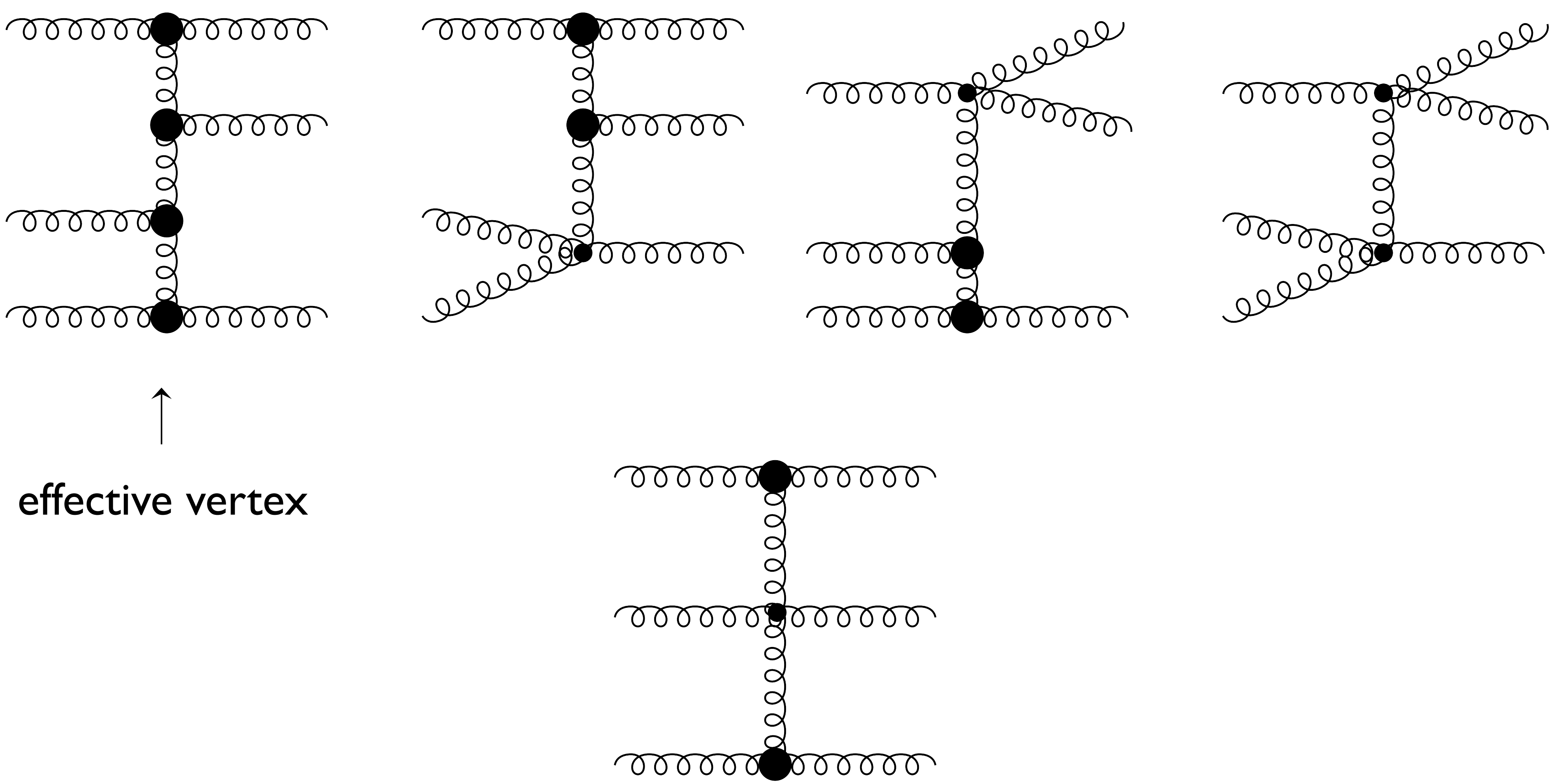
+ ...

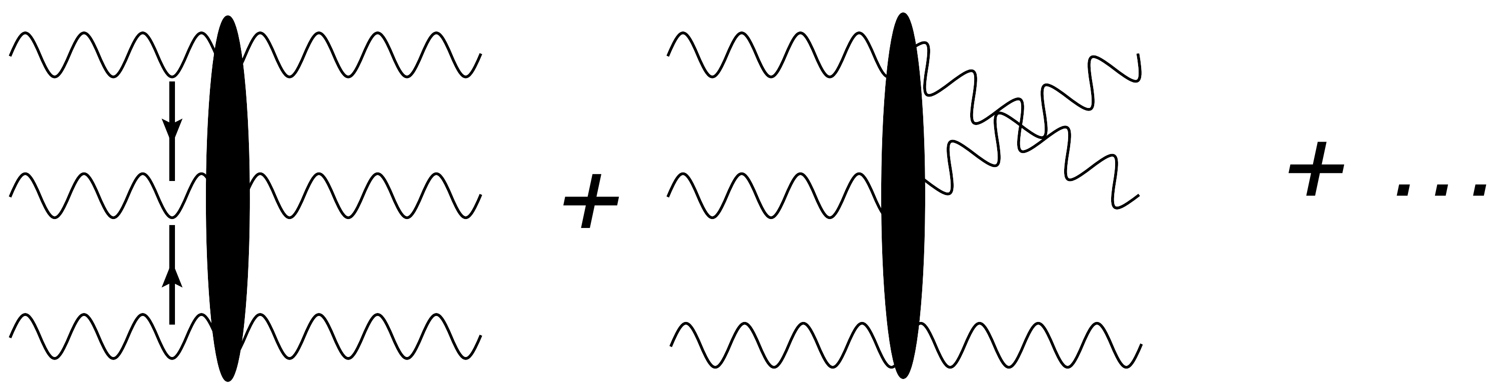
Tasks:
all s-channel lines are off-shell: longitudinal integrations
sum over permutations

Use of Ward identities for all reggeons:

$$(n^-)^\mu \to -\frac{2}{q^+} q_\perp^\mu, \ (n^+)^\mu \to -\frac{2}{q^-} q_\perp^\mu$$

Diagrams:

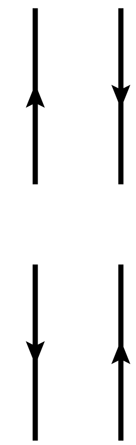


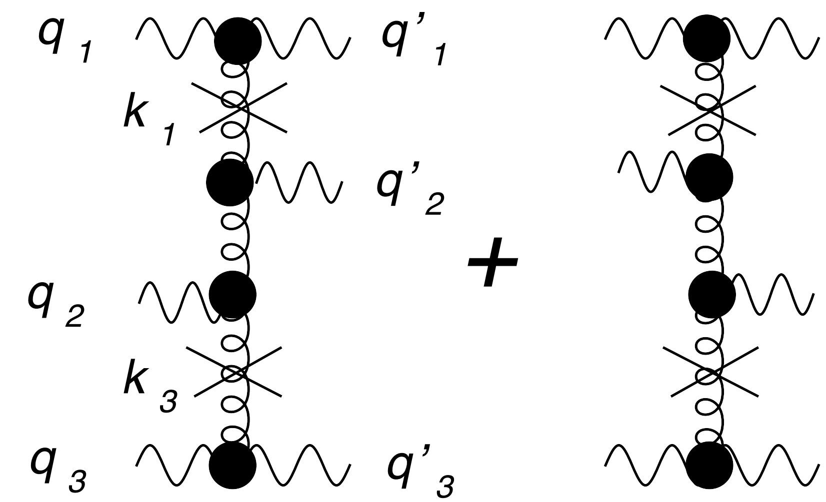


Results:

- induced terms (almost) disappeared, back to QCD diagrams:
  with this polarization, reggeized gluon $\approx$ physical gluon
- individual diagrams are convergent,
  have correct zero properties
- contribution only from 'opposite momenta'
- subtraction of LO contributions

Result:





$$
\begin{aligned}
K_{123} &= \left(\frac{q_1^\alpha}{\vec{q}_1^2} - \frac{k_1^\alpha}{\vec{k}_1^2}\right) V_{\alpha\beta} \left(\frac{q_3^\beta}{\vec{q}_3^2} - \frac{k_3^\beta}{\vec{k}_3^2}\right) \\
&= -\left(\frac{q_1^\alpha}{\vec{q}_1^2} - \frac{k_1^\alpha}{\vec{k}_1^2}\right) \Big[\ln \frac{(\vec{q}_2 + \vec{k}_1)^2}{\sqrt{\vec{k}_1^2 \vec{k}_3^2}} T_{\alpha\beta}(k_1) T_{\beta\gamma}(q_2 + k_1) + \\
&\qquad + \ln \frac{(\vec{q}_2 + \vec{k}_3)^2}{\sqrt{\vec{k}_1^2 \vec{k}_3^2}} T_{\alpha\beta}(q_2 + k_3) T_{\beta\gamma}(k_3) \Big] \left(\frac{q_3^\gamma}{\vec{q}_3^2} - \frac{k_3^\gamma}{\vec{k}_3^2}\right)
\end{aligned}
$$

where

$$
T_{\alpha\beta}(k) = g_{\alpha\beta} - 2\frac{k_\alpha k_\beta}{\vec{k}^2}.
$$

or

$$
\begin{aligned}
\tilde{K}_{123} = -\frac{1}{4}\frac{1}{q_2 q_2^*} \Bigg\{ & \log \frac{|q_2 + k_1|^2}{|k_1||k_3|} \left[\frac{q_1'^* q_3'}{q_1^* q_3} \frac{1}{k_1 k_3} \frac{(q_2 + k_1)}{(q_2 + k_1)^*} + \text{c.c.} \right] + \\
& \log \frac{|q_2 + k_3|^2}{|k_1||k_3|} \left[\frac{q_1' q_3'^*}{q_1 q_3^*} \frac{1}{k_1 k_3} \frac{(q_2 + k_3)}{(q_2 + k_3)^*} + \text{c.c.} \right] \Bigg\}.
\end{aligned}
$$

Conclusions

NLO BKP is interesting,
phenomenology + theory

Done:

- 2 to 2 kernel in NLO
- new techniques based upon Ward identities
- 3 to 3 kernel (for the Odderon)

To be done:

- extend to BKP in vacuum channel, color octet
- calculate spectrum: Odderon intercept
- prove conformal invariance, integrability